2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Distributed Coordination-by-Constraint Strategies for Multi-agent Networked Systems

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***Abstract*— In this paper we present a novel distributed supervision strategy for networks of linear locally regulated and possibly dynamically coupled systems connected via data links and subject to coordination constraints on the evolutions of relevant variables of them. Such a *coordination-by-constraint* paradigm is accomplished by resorting to a Distributed Command Governor approach where each agent of the strategy is in charge to locally modify, whenever necessary and on the basis of a reduced amount of data exchanged with the other agents, the prescribed set-points to the regulated subsystems so as that the global pointwise-in-time coordination constraints are satisfied along the overall network evolutions. A sequential strategy, where only one agent per time is allowed to manipulate its own reference signal, is fully described and analyzed. The constrained coordination of a network of dynamically coupled eight water tanks is presented as an example in order to show the effectiveness of the proposed method.**

I. Introduction

The problem of interest here is the design of distributed supervision strategies based on Command Governor (CG) ideas for multi-agent systems in situations where the use of a centralized coordination unit is impracticable because requiring unrealistic or unavailable communication infrastructures. A centralized solution to this problem has been recently proposed in [1] in the quite general context depicted in Fig. 1. There, the master station is in charge of supervising and

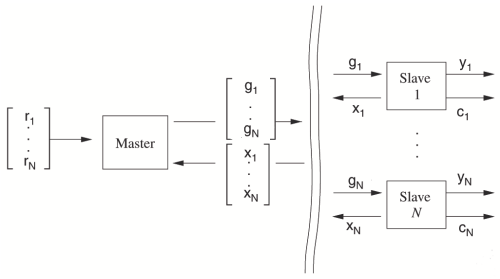

Fig. 1. Multi-agent master/slave architectures

coordinating the slave systems via a data network. In particular, r_i, g_i, x_i, y_i and c_i represent respectively: the nominal references, the feasible references, the states, performance-related and coordination-related outputs of the slave systems. In such a context, the supervision task can be expressed as the requirement of satisfying some tracking performance, viz. $y_i \approx r_i$, whereas the coordination task consists of enforcing some pointwise-in-time constraints $c_i \in \mathcal{C}_i$ and/or $f(c_1, c_2, ..., c_N) \in \mathcal{C}$ on each slave system and/or on the overall network evolutions. To this end, the supervisor is in charge of modifying the nominal references into the feasible ones, when the tracking of the nominal references would produce constraint violations and hence loss of coordination.

In this paper we move towards distributed strategies for solving the above task in large scale applications based on novel CG ideas recently proposed in [2]. The novel distributed context is depicted in Figure 2, where the supervisory task is now distributed amongst many agents which are assumed to be able to communicate amongst them and with the regulated plants as well. See also [3]-[6] for recent results on distributed MPC schemes of relevance here.

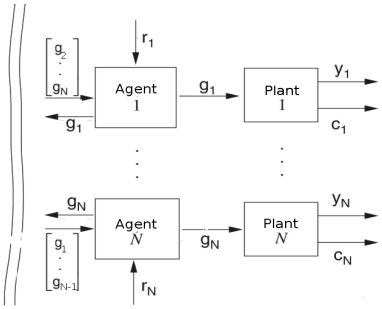

Fig. 2. Multi-agent architectures

The CG approach (see [1], [7]) is a well known and established methodology that provides a simple and effective way to enforce pointwise-in-time constraints along the trajectories of a closed-loop system. The CG is a nonlinear device which is added to a pre-compensated control system. Whenever necessary, the CG modifies the reference to the closed-loop system so as to avoid constraint violations.

In the above "traditional" contexts, the CG action is determined on the basis of the knowledge of the actual measured state. In this paper, on the contrary, we will make use of a recently proposed *Feed-Forward* CG solution [2] to the CG design problem, that, at the price of some additional conservativeness, is able to accomplish the CG task in the absence of an explicit measure or estimate of the state.

This peculiarity of the FF-CG scheme makes it an attractive solution for distributed frameworks based on model predictive control ideas because it alleviates the need to make the entire aggregate state, or substantial parts of it, known to all agents at each time instant, the latter being unrealistic or requiring unrealistic communication infrastructures in some large scale applications. It is important to remark that the use of such a novel FF-CG scheme introduces many technical

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[1]This paper was supported by the Belgian Network DYSCO (Dynamical Systems, Control and Optimization) funded by the Interuniversity Attraction Poles Program, initiated by the Belgian State, Science Policy Office.

challenges for the development of distributed schemes which have to be carefully managed. In this respect, this paper makes clear several theoretical aspects of this novel sequential distributed scheme, not discussed in the preliminary version [8]. Aspects related to the *liveliness* of the method, that is the analysis of conditions avoiding the occurrence of deadlock situations, are fully analyzed in the companion paper [9].

The main advantages of such a scheme are in its simplicity and in the low communication rates required for its implementation, remarkably lower than other distributed approaches -e.g those based on consensus mechanisms. The basic idea is that only one agent per time is allowed to modify its own reference signal. This approach, although behaving increasingly slower for an increasing number of agents, is anyway of interest in all situations whereby the coordination problem consists of few and slow set-point adjustments, e.g. in all small/medium-scale situations where the set-points change infrequently or slower than the system dynamics. Its derivation is also instrumental to built up faster "parallel" version of the scheme in which, whenever possible, all agents are allowed to modify their own reference signals simultaneously.

The feasibility and stability properties of the presented approach are analyzed and the coordination of an eight-tank cascaded water system is considered as an example.

II. System description and Problem Formulation

Consider a set of N subsystems $\mathcal{A} = \{1, \dots, N\}$, each one being a LTI closed-loop dynamical system regulated by a local controller which ensures stability and good closed-loop properties when the constraints are not active (small-signal regimes when the coordination is effective). Let the i-th closed-loop subsystem be described by the following discrete-time model

$$\left\{ \begin{array}{rcl} x_i(t+1) & = & \Phi_{ii}x_i(t)+G_ig_i(t)+\sum\limits_{j\in\mathcal{A}-\{i\}}\Phi_{ij}x_j(t) \\ y_i(t) & = & H_i^y x_i(t) \\ c_i(t) & = & H_i^c x(t) + L_i g(t) \end{array} \right. \quad (1)$$

where: $t \in \mathbb{Z}_+$, $x_i \in \mathbb{R}^{n_i}$ is the state vector (which includes the controller states under dynamic regulation), $g_i \in \mathbb{R}^{m_i}$ the manipulable reference vector which, if no constraints (and no CG) were present, would coincide with the desired reference $r_i \in \mathbb{R}^m$ and $y_i \in \mathbb{R}^{m_i}$ is the output vector which is required to track r_i. Finally, $c_i \in \mathbb{R}^{n_i^c}$ represents the local constrained vector which has to fulfill the set-membership constraint

$$c_i(t) \in \mathcal{C}_i, \ \forall t \in \mathbb{Z}_+, \quad (2)$$

$\mathcal{C}_i$ being a convex and compact polytopic set. It is worth pointing out that, in order to possibly characterize global (coupling) constraints amongst states of different subsystems, the vector c_i in (1) is allowed to depend on the aggregate state and manipulable reference vectors $x = [x_1^T, \dots, x_N^T]^T \in \mathbb{R}^n$, with $n = \sum_{i=1}^N n_i$, and $g = [g_1^T, \dots, g_N^T]^T \in \mathbb{R}^m$, with $m = \sum_{i=1}^N m_i$. Moreover, we denote by $r = [r_1^T, \dots, r_N^T]^T \in \mathbb{R}^m$, $y = [y_1^T, \dots, y_N^T]^T \in \mathbb{R}^m$ and $c = [c_1^T, \dots, c_N^T]^T \in \mathbb{R}^{n^c}$, with $n^c = \sum_{i=1}^N n_i^c$, the other relevant aggregate vectors. The overall system arising by the composition of the above N subsystems can be described as

$$\left\{ \begin{array}{rcl} x(t+1) & = & \Phi x(t) + Gg(t) \\ y(t) & = & H^y x(t) \\ c(t) & = & H^c x(t) + Lg(t) \end{array} \right. \quad (3)$$

where

$$\Phi = \begin{pmatrix} \Phi_{11} & \dots & \Phi_{1N} \\ \vdots & \ddots & \vdots \\ \Phi_{N1} & \dots & \Phi_{NN} \end{pmatrix}, G = \begin{pmatrix} G_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & G_N \end{pmatrix}$$

$$H^y = \begin{pmatrix} H_1^y & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & H_N^y \end{pmatrix}, H^c = \begin{pmatrix} H_1^c \\ \dots \\ H_N^c \end{pmatrix}, L = \begin{pmatrix} L_1 \\ \dots \\ L_N \end{pmatrix}.$$

It is further assumed that

A1. The overall system (3) is asymptotically stable.

A2. System (3) is off-set free i.e. $H^y(I_n - \Phi)^{-1}G = I_m$.

Roughly speaking, the CG design problem we want to solve is that of locally determine, at each time step t and for each agent $i \in \mathcal{A}$, a suitable reference signal $g_i(t)$ which is the best approximation of $r_i(t)$ such that its application never produces constraints violation, i.e. $c_i(t) \in \mathcal{C}_i, \forall t \in \mathbb{Z}_+, i \in \mathcal{A}$.

Classical centralized solutions to the above stated CG design problem (see [1]) have been achieved by finding, at each time t, a CG action $g(t)$ as a function of the current reference $r(t)$ and measured state $x(t)$

$$g(t) := \underline{g}(r(t), x(t)) \quad (4)$$

such that $g(t)$ is the best approximation of $r(t)$ under the condition $c(t) \in \mathcal{C}$, where $\mathcal{C} \subseteq \{\mathcal{C}_1 \times ... \times \mathcal{C}_N\}$ is the global admissible region. In [2], the *Feed-Forward* CG (FF-CG) approach has been proposed, where a CG action having the following structure

$$g(t) \ = \ \underline{g}(r(t), g(t-\tau)) \quad (5)$$

was proved to have similar properties of the standard CG state-based approach when computed every τ steps and kept constant between two subsequent updating, without hinging upon on the explicit knowledge of the state vector.

III. The Feed-Forward CG approach

In this section we recall the basic ideas and notation of the FF-CG approach proposed in [2] which will be relevant for the the forthcoming discussion. To this end, consider, for a given $\delta > 0$, the sets:

$$\begin{array}{rcl} \mathcal{C}_\delta & := & \mathcal{C} \sim \mathcal{B}_\delta \\ \mathcal{W}_\delta & := & \{g \in \mathbb{R}^m : \ c_g \in \mathcal{C}^\delta\} \end{array} \quad (6)$$

where $\mathcal{B}_\delta$ is the ball of radius δ centered at the origin and $\mathcal{A} \sim \mathcal{E}$ is the Pontryagin set difference defined as $\{a : a + e \in \mathcal{A}, \forall e \in \mathcal{E}\}$. In particular, $\mathcal{W}^\delta$, which we assume non-empty, is the convex and closed set of all constant commands g whose corresponding equilibrium points $c_g := H^c(I_n - \Phi)^{-1}Gg + Lg$ satisfy the constraints with margin δ. Let introduce also the *virtual evolutions* of the c-variable

$$\hat{c}(k, x(t), g(t)) := H^c\left(\Phi^k x(t) + \sum_{i=0}^{k-1}\Phi^{k-i-1}Gg(t)\right) + Lg(t) \quad (7)$$

along the *virtual time* k, from the initial condition $x(t)$ at time $k = 0$ under the application of a constant command $g(t)$, $\forall k$. The virtual c-variable evolution (7) can be rewritten as the sum of two amounts: a steady-state component represented by $c_{g(t)}$ and the transient evolution $H^c\Phi^k(x(t) - x_{g(t)})$:

$$\hat{c}(k, x(t), g(t)) = c_{g(t)} + H^c\Phi^k(x(t) - x_{g(t)}). \quad (8)$$

Because $g(t) \in \mathcal{W}_\delta$ and, in turn, $c_{g(t)} \in \mathcal{C}_\delta$ at each time t, then, a sufficient condition to ensure that the constraints are satisfied, although in a quite arbitrary and conservative way, is to ensure that the transient component is confined into a ball of radius $\rho_{g(t))}$

$$\|H^c\Phi^k(\hat{x}(t)-x_{g(t)})\| \leq \rho_{g(t)}, \forall k \geq 0 \quad (9)$$

where $\rho_{g(t)}$ represents the minimum distance between $c_{g(t)}$ and the border of $\mathcal{C}$

$$\begin{array}{rcl} \rho_g & := & \arg\max_\rho \rho \\ \text{subject to} & & \mathcal{B}^\rho(c_g) \subseteq \mathcal{C}. \end{array} \quad (10)$$

where $\mathcal{B}^\rho(c_g)$ represents the ball of radius ρ centered in c_g. Details on the computation of $\rho_{g(t)}$ can be found in [2].

Then, the FF-CG design problem translates into the problem of defining an algorithm that is able to select, at each time t, a reference value $g(t)$ such that (9) holds true for all $k \geq 0$. This has been achieved n [2] by selecting a suitable integer τ, referred to as a *Generalized Settling Time*, and a sequence of positive scalars $\rho(t)$ such that the following more strict condition than (9) is satisfied at each time t.

$$\|H^c\Phi^k(x(t)-x_{g(t)})\| \leq \rho(t) \leq \rho_{g(t)}, \forall k \geq 0 \quad (11)$$

Then, if condition (11) were holding true at time $t - \tau$ and a certain command $g(t - \tau)$ were constantly applied to the system, then the transient contribution from t onwards could be bounded as follows

$$\| H^c\Phi^k(\hat{x}(t) - x_{g(t-\tau)}) \| \leq \gamma\rho(t-\tau) \leq \gamma\rho_{g(t-\tau)}, \quad \forall k \geq 0 \quad (12)$$

with $\gamma < 1$ because of asymptotical stability. Then, if the FF-CG action were computed every τ sampling steps and kept constant between two successive updating, at time t our goal would be that to select a new command $g(t)$ such that

$$\| H^c\Phi^k(\hat{x}(t) - x_{g(t)}) \| \leq \rho(t) \leq \rho_{g(t)}, \forall k \geq 0. \quad (13)$$

is satisfied for some $\rho(t) > 0$. By introducing the τ-step incremental reference $\Delta g(t) = g(t) - g(t - \tau)$, and by observing that $x_{\Delta g(t)} = x_{g(t)} - x_{g(t-\tau)}$, a sufficient condition for (13) to hold is

$$\| H^c\Phi^k x_{\Delta g(t)} \| \leq \rho_{g(t-\tau)+\Delta g(t)} - \gamma\rho(t-\tau)\}, \forall k \geq 0. \quad (14)$$

Please note that the latter inequalities always hold true for Δg in a sufficiently small ball centered in $\Delta g = 0$. Finally, by taking the definition of $x_{\Delta g}$ into account, we can formulate the Feed-Forward CG selection algorithm as follows

The FF-CG Algorithm

REPEAT AT EACH TIME t

1.1 IF $(t == \kappa\tau, \kappa = 1, 2 \ldots)$

1.1.1 SOLVE

$$g(t) = \arg\min_g \| g - r(t) \|^2_\Psi \quad (15)$$

$$\text{SUBJECT TO}: \begin{cases} g \in \mathcal{W}_\delta \\ (g - g(t-\tau)) \in \Delta\mathcal{G}(g(t-\tau), \rho(t-\tau)) \end{cases} \quad (16)$$

1.2 ELSE $g(t) = g(t - 1)$

2.1 APPLY $g(t)$

3.1 UPDATE $\rho(t) = \gamma\rho(t - \tau) + max_{k\geq 0} \| H_c\Phi^k(I - \Phi)^{-1}G\Delta g(t) \|$.

where $\Psi > 0$ is a weighting matrix and $\Delta\mathcal{G}(g, \rho)$ is the set of all possible τ-step incremental commands Δg which ensure (14) to hold true

$$\Delta\mathcal{G}(g, \rho) := \{\Delta g : \|H^c\Phi^k(I-\Phi)^{-1}G\Delta g\| \leq \rho_{g+\Delta g} - \gamma\rho, \forall k \geq 0\}. \quad (17)$$

It is worth to note that the sets $\mathcal{W}_\delta$, $\Delta\mathcal{G}(g, \rho)$ and the generalized settling time τ can be computed from the outset. Finally the following properties can be proved [2]

Proposition *1:* - Let assumptions **A1-A2** be fulfilled. Consider system (3) along with the **FF-CG** selection rule and let an admissible command signal $g(0) \in \mathcal{W}_\delta$ be applied at $t = 0$ such that (9) holds true. Then:

1) the minimizer in (15), computed every τ steps, uniquely exists and can be obtained by solving a convex constrained optimization problem;
2) constraints are fulfilled for all $t \in \mathbb{Z}_+$;
3) the overall system is asymptotically stable and whenever $r(t) \equiv r$, the sequence of $g(t)$ converges in finite time either to r or to its best steady-state admissible approximation: $g(t) \to \hat{r} := \arg\min_{g\in\mathcal{W}_\delta} \|g - r\|^2_\Psi$.□

For simplicity, in the forthcoming distributed analysis a simplified variant of the above FF-CG method will be considered by setting $\rho(t) = \rho_{g(t)}, \forall t > 0$. In this case, point 3.1 of the FF-CG Algorithm can be skipped and the set $\Delta\mathcal{G}(g, \rho)$ depends only on the current command g

$$\Delta\mathcal{G}(g) := \{\Delta g : \|H^c\Phi^k(I-\Phi)^{-1}G\Delta g\| \leq \rho_{g+\Delta g} - \gamma\rho_g, \forall k \geq 0\}. \quad (18)$$

The above simplification, although conservative, lead to a simpler analysis and has the merit that the agents don't need to communicate their instances of the scalar $\rho(t)$ amongst them.

IV. Distributed Sequential FFCG (S-FFCG)

Here we introduce a distributed CG scheme based on the above FF-CG approach, inspired by the serial method presented in [4], by assuming that the agents are connected via a communication network. Such a network may be modeled by a *communication graph*: an undirected graph $\mathcal{G} = (\mathcal{A}, \mathcal{B})$, where $\mathcal{A}$ denotes the set of the N subsystems and $\mathcal{B} \subset \mathcal{A} \times \mathcal{A}$ the set of edges representing the communication links amongst agents. More precisely, the edge (i, j) belongs to $\mathcal{B}$ if and only if the agents governing the i-th and the j-th subsystems are able to directly share information within τ sampling times. The communication graph is assumed to be connected, i.e. for each couple of agents $i \in \mathcal{A}, j \in \mathcal{A}$ there exists at least one sequence of edges connecting i and j, with the minimum number of edges connecting the two agents denoted by $d_{i,j}$. The set of all agents with a direct connection with the i-th agent will be referred to as *Neighborhood of the i-th agent* $\mathcal{N}_i = \{j \in \mathcal{A} : d_{i,j} = 1\}$.

Let $\mathcal{G}$ be a Hamiltonian graph and assume, without loss of generality, that the sequence $\mathcal{H} = \{1, 2, ..., N - 1, N\}$ is a Hamiltonian cycle. The idea behind the approach is that

only one agent at decision time is allowed to manipulate its local command signal $g_i(t)$ while all others are instructed to hold their previous values. After each decision, the agent in charge will update the global command received from the previous updating agent and will forward this new value to the next updating agent in the cycle. Such a polling policy implies that, eventually after a preliminary initialization cycle, at each time instant the "agent in charge" always knows the whole aggregate vector $g(t-\tau)$. By exploiting this observation we can define the following distributed FF-CG algorithm:

Sequential-FFCG Algorithm (S-FFCG) *- Agent i*

REPEAT AT EACH TIME t

1.1 IF$(t==\kappa\tau, \kappa=0,1,\ldots)\&\&(\kappa \mod N) == i$

1.1.1 RECEIVE $g(t-\tau)$ FROM THE PREVIOUS AGENT IN THE CYCLE $\mathcal{H}$

1.1.2 SOLVE

$$g_i(t) = \arg\min_{g_i} \| g_i - r_i(t) \|^2_{\Psi_i}$$
$$\text{SUBJECT TO :} \begin{cases} g(t)=[g_1^T(t-\tau),..,g_i^T,...,g_N^T(t-\tau)]^T \in \mathcal{W}_\delta \\ (g_i - g_i(t-\tau)) \in \Delta\mathcal{G}_i^0(g(t-\tau)) \end{cases} \quad (19)$$

1.1.3 APPLY $g_i(t)$

1.1.4 UPDATE $g(t)=[g_1^T(t-\tau), ..., g_i^T(t), ..., g_N^T(t-\tau)]^T$

1.1.5 TRANSMIT $g(t)$ TO THE NEXT AGENT IN $\mathcal{H}$

1.2 ELSE

1.2.1 APPLY $g_i(t) = g_i(t-1)$

where $\Psi_i > 0$ is a weighting matrix, $\kappa \mod N$ is the remainder of the integer division κ/N and

$$\Delta\mathcal{G}_i^0(g) := \left\{\Delta g_i : [0_{m_1}^T, 0_{m_2}^T, \ldots, \Delta g_i^T, \ldots, 0_{m_N}^T]^T \in \Delta\mathcal{G}(g)\right\} \quad (20)$$

is the set of all possible command variations for g_i in the case that the commands of all other agents are frozen.

In order to present properties of the above algorithm let us introduce some important notions and assumptions.

Definition (*Admissible direction*) - Let a point $g \in \mathcal{W}_\delta$. The vector $v \in \mathbb{R}^m$ represents an admissible direction for $g \in \mathcal{W}_\delta$ if there exists a real $\bar{\lambda} > 0$ such that $(g + \lambda v) \in \mathcal{W}_\delta, \lambda \in [0, \bar{\lambda}]$. □

Definition (*Decision Set of agent i*) - The Decision Set $\mathcal{V}_i(g)$ of the agent i at a point $g \in \mathcal{W}_\delta$ represents the set of all admissible directions belonging to $\mathbb{R}_i^m$ that such an agent could move along in updating its action when all other agents held their commands unvaried, viz. $\mathcal{V}_i(g) := \{d \in \mathbb{R}^{m_i} : [0_1^T, \ldots, 0_{i-1}^T, d^T, 0_{i+1}^T, \ldots, 0_N^T]^T$ *is an admissible direction for* $g \in \mathcal{W}_\delta\}$. □

Definition (*Viability property*) - A point $g \in \mathcal{W}_\delta$ is said to be "*viable*" if, for any admissible direction $v = [v_1^T, ..., v_N^T]^T \in \mathbb{R}^m$, $v_i \in \mathbb{R}^{m_i}$ with $\sum_{i=1}^N m_i = m$, at least one subvector $v_i \neq 0$ there exists such that $v_i \in \mathcal{V}_i(g)$. □

Definition *(Pareto Optimal Solution)* - Let vectors r_i, $i = 1, 2, ..., N$ be given. Consider the following multi-objective problem:

$$\min_g [\| g_1 - r_1 \|^2_{\Psi_1}, \ldots, \| g_i - r_i \|^2_{\Psi_i}, \ldots, \| g_N - r_N \|^2_{\Psi_N}]$$
$$\text{subject to } g=[g_1^T,..,g_i^T,...,g_N^T]^T \in \mathcal{W}_\delta \quad (21)$$

A solution $g^* \in \mathcal{W}_\delta$ is a Pareto Optimal solution of the optimization problem (21) if there not exist $g \in \mathcal{W}_\delta$, such that: $\| g_i - r_i \|^2_{\Psi_i} \leq \| g_i^* - r_i \|^2_{\Psi_i}$ $\forall i \in \{1, \ldots, N\}$ and $\| g_j - r_j \|^2_{\Psi_j} < \| g_i^* - r_i \|^2_{\Psi_i}, j \in \mathcal{A}$. □

The above definitions are instrumental to characterize deadlock situations that, unlike the centralized solution, may exist in this decentralized scheme when the same constraint set $\mathcal{W}_\delta$ of the centralized scheme is used. The rationale is that by acting one agent per time, certain viable paths existing in the centralized scheme are precluded and the agents could get stuck indefinitely. In order to avoid this deadlock situations we have to introduce the following assumption for the points belonging to the border of $\mathcal{W}_\delta$

A3. Each point belonging to $\partial(\mathcal{W}_\delta)$ is viable, $\partial(\mathcal{W}_\delta)$ denoting the border of $\mathcal{W}_\delta$.

For space limitations no other details are given here on the fulfilment of **A3**. The characterization of viable points, a computable way of checking if the viability property **A3** is satisfied by the polyhedral set $\mathcal{W}_\delta$ at hands and a geometrical method allowing one to compute suitable inner approximations of $\mathcal{W}_\delta$ satisfying **A3** are presented in the companion paper [9]. For other details, please see also [10].

Finally, we present some properties enjoyed by the above described S-FFCG scheme

Theorem *1:* Let assumptions **A1-A2-A3** be fulfilled. Consider system (3) as the composition of N subsystems in form (1) along with the distributed **S-FFCG** selection rule (19) and let an admissible aggregate command signal $g(0) = [g_1^T(0), \ldots, g_N^T(0)]^T \in \mathcal{W}_\delta$ be applied at $t = 0$ such that (9) holds true. Then

1) for each agent $i \in \mathcal{A}$, at each decision time $t = k\tau, k \in \mathbb{Z}_+$, the minimizer in (19) uniquely exists and can be obtained by locally solving a convex constrained optimization problem;
2) the overall system acted by the agents implementing the S-FFCG policy never violates the constraints, i.e. $c(t) \in \mathcal{C}$ for all $t \in \mathbb{Z}_+$;
3) whenever $r(t) \equiv [\, r_1^T, \ldots, r_N^T \,]^T, \forall t$, with r_i a constant set-point, the sequence of solutions $g(t) = [g_1^T(t), \ldots, g_N^T(t)]^T$ asymptotically converges to a Pareto-Optimal stationary (constant) solution of (21), which is given by r whenever $r \in \mathcal{W}_\delta$, or by any other Pareto-Optimal solution $\hat{r} \in \mathcal{W}_\delta$ otherwise. □

V. AN EIGHT-TANK WATER DISTRIBUTION SYSTEM APPLICATION

Consider the water tank network depicted in Figure 3. The system consists of the interconnection of four cascaded two-tank models. Each cascaded subsystem is described by the following non-linear equations

$$\begin{cases} \rho S_i^1 \dot{h}_i^1 = -\rho A_i^1 \sqrt{2gh_i^1} + u_i \\ \rho S_i^2 \dot{h}_i^2 = -\rho A_i^2 \sqrt{2gh_i^2} + \rho A_i^1 \sqrt{2gh_i^1} + \sum_{j \in \mathcal{S}^i} \rho A_j^1 \sqrt{2gh_i^1} \end{cases}$$

where u_i is the water flow supplied by the pump whose command is the voltage V_i, $i \in \mathcal{A} := \{1, .., 4\}$. Moreover, for each $q = 1, 2$, S_i^q are the tank sections, h_i^q, the water level in the tanks, A_q^i the section of pipes connecting the tanks, and g and ρ the gravity constant and the water density respectively. Their values are specified in Tables 1-2.

Subsystem 1			
Tank 1	Value	Tank 2	Value
S_1^1	2500 cm^2	S_1^2	2500 cm^2
A_1^1	4 cm^2	A_1^2	8 cm^2
$\overline{h}_i^1$	80 cm	$\overline{h}_i^2$	70 cm
$\underline{h}_i^1$	1 cm	$\underline{h}_i^2$	1 cm
Subsystem $i = \{2, 3, 4\}$			
Tank 1	Value	Tank 2	Value
S_i^1	2500 cm^2	S_i^2	2500 cm^2
A_i^1	8 cm^2	S_i^2	8 cm^2
$\overline{h}_i^1$	80 cm	$\overline{h}_i^2$	70 cm
$\underline{h}_i^1$	1 cm	$\underline{h}_i^2$	1 cm

TABLE I

TANKS AND CONSTRAINTS VALUES

Parameters	Value
g	980 $cm/(sec^2)$
ρ	$10^{(-3)}$ $Kg/(cm^3)$
V_{max}	4
T_c	$0.8sec$

TABLE II

PARAMETER VALUES

With $\mathcal{S}^i$ we denote the set of subsystems which provide water to the downstream tank of the i-th subsystem; in our case $\mathcal{S}^1 := \{2\}$, $\mathcal{S}^2 = \{3\}$, $\mathcal{S}^3 = \{4\}$ and $\mathcal{S}^4 = \emptyset$. Each cascaded two-tank subsystem has a related *decision maker* or *agent* in charge of regulating the levels $h_i^2(t)$, $i \in \mathcal{A}$ by modifying properly their set-points and by exchanging relevant data with the other agents. Local decentralized tracking LQ output feedback controllers ([11]) are implemented, which act properly on the incoming water flows $u_i(t)$, in such a way that the offset property **A2** is satisfied. A simple static equation is used to model the relationship between the input voltage $V_i(t)$ and the incoming mass of water

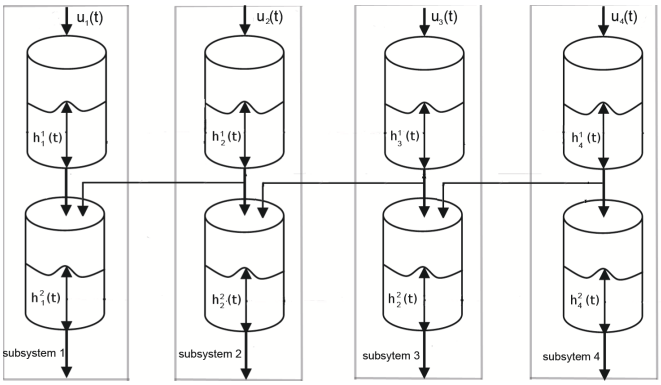

Fig. 3. A four cascaded two-tank water system

$$u_i(t) = \begin{cases} V_i(t) & if \quad V_i(t) \geq 0 \\ 0 & if \quad V(t) < 0 \end{cases}$$

The following local and global constraints are to be enforced at each time instant

$$\underline{h}_i^1 \leq h_i^1 \leq \overline{h}_i^1, \ \forall \ i \in \mathcal{A},$$
$$\underline{h}_i^2 \leq h_i^2 \leq \overline{h}_i^2, \ \forall \ i \in \mathcal{A},$$
$$0 \leq V_i \leq V_{max}, \ \forall \ i \in \mathcal{A},$$
$$|h_1^1 - h_2^1| <= 5cm, \ |h_2^1 - h_3^1| <= 5cm, \ |h_3^1 - h_4^1| <= 5cm$$

The system is linearized around the equilibrium $\bar{V}_i = \bar{u}_i^{eq} = 2$, $i \in \mathcal{A}$ $\bar{h}_i^j = 32cm$ and discretized with sampling time $T_c = 0.8$ sec.

The reported simulations investigate the behavior of the overall system when the desired set-points to the water levels of the downstream tanks have the profiles depicted in Figure 5 (red dashed line). At the beginning, the desired references $r_i = 32cm$, $i \in \mathcal{A}$ correspond to an equilibrium. At time $t = 30$ sec, the reference r_1 related to the downstream tank of subsystem 1 is changed from 32 cm to 42 cm. At the same time, also the reference r_2 is modified from 32 cm to 34 cm. These values are kept constant until time instant $t = 400$ sec when they are changed back to their initial values. Simultaneously, the desired references r_3 and r_4 change their values at time $t = 300$ sec from 32 cm to 27.85 cm and, respectively, 28.5 cm. After that, these new values are kept constant up to time $t = 800$ sec, when are brought back to the previous values.

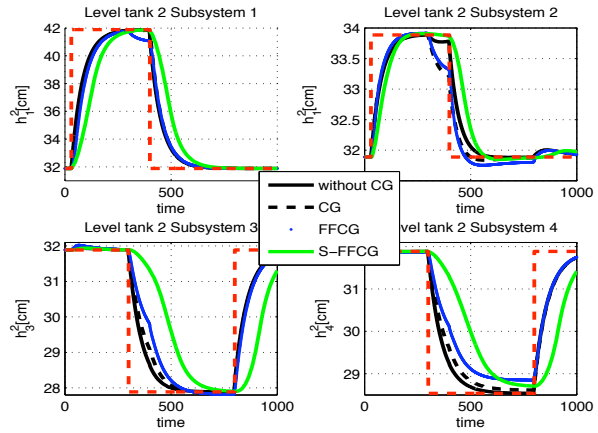

Fig. 4. Water levels in the downstream tanks

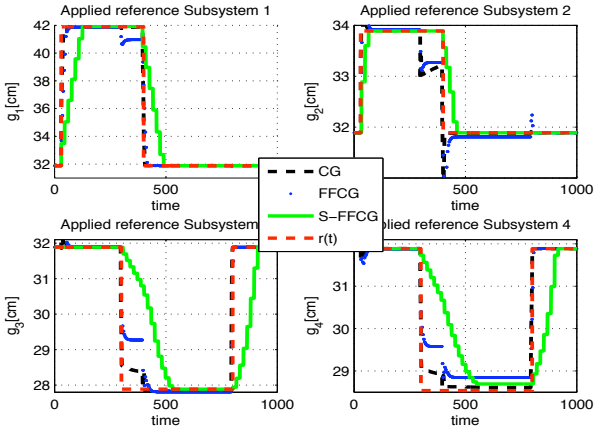

Fig. 5. CG actions

In Figures 6-7, the constrained vector responses can be observed. It is important to note how such a vector violates the constraints at several time instants when no CG unit is used. On the contrary, this never happens when a CG unit is

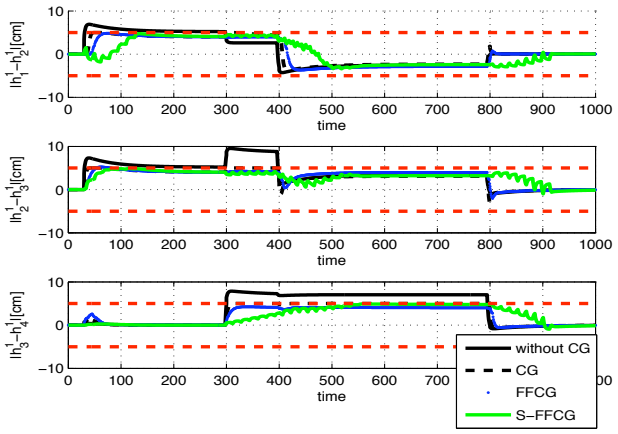

Fig. 6. Coordination constraints

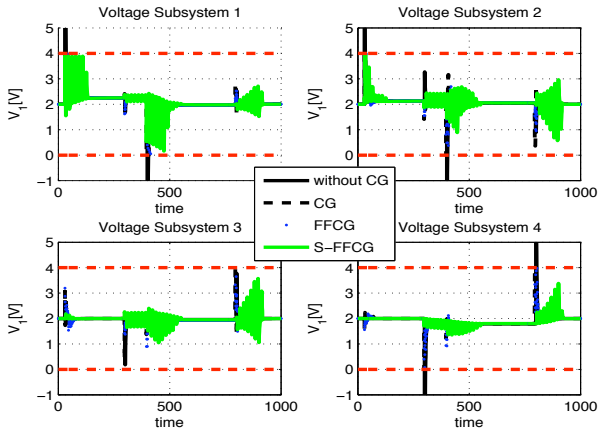

Fig. 7. Applied Voltages

used. In particular, the responses of the classical CG, the FF-CG and the S-FFCG units are all reported for comparisons.

The evolutions of the downstream water levels are depicted in Figure 4 while in Figure 5 the various CG actions are reported. The standard CG and the FF-CG centralized schemes have similar coordination performance. On the contrary, the distributed S-FFCG exhibits, as expected, a slower response to changed conditions. Nevertheless, the related performance, especially during the equilibrium phases, are quite good even if compared to the centralized algorithms.

Although the performance of a centralized solution, especially those based on the direct measure of the state, are expected to remarkably outperform any decentralized solution here, on the contrary, the difference is modest as it can be observed in Figures 6-7.

However, the main advantage of S-FFCG is reported in Table 3 where the required CPU execution time and the rate of data exchanged is shown for a single supervisory agent and for the centralized solution. It results that a single agent in the S-FFCG scheme has a computational burden which is an order of magnitude lower than a centralized CG. Also the rate of exchanged data is lower than centralized strategies,

	CG	FFCG	S-FFCG
CPU Time [ms]	0.61	0.43	0.058
RX/TX Data (bit/agent)	512	256	192

TABLE III

CPU TIME (SECONDS PER STEP): IT IS RELATED, IN THE DISTRIBUTED CASE TO SINGLE AGENT AND IN THE CENTRALIZED CASE TO THE UNIQUE SUPERVISOR IN CHARGE. EXCHANGED INFORMATION: IN THE DISTRIBUTED CASE IT IS THE AMOUNT OF INFORMATION EXCHANGED BY AN AGENT WITH THE REST OF THE NETWORK, IN THE CENTRALIZED CASE IT IS THE INFORMATION RECEIVED AND TRANSMITTED BY THE UNIQUE CG OR FFCG DEVICE

and it is very low if compared with other distributed MPC techniques.

VI. CONCLUSIONS

In this paper, a distributed FFCG schemes has been proposed for the supervision of dynamically coupled interconnected linear systems subject to local and global constraints and used for solving constrained coordination problems in networked control system.

A sequential distributed strategy has been proposed and its feasibility and stability properties analyzed in full details. Its effectiveness has been demonstrated in the final example where also comparisons with centralized solutions have been reported for the same problem.

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