

A High Order Sliding Mode Control Scheme Based on Adaptive Radial Basis Function Neural Network

W. Q. Tang, Y. L. Cai

Abstract—A high order sliding mode control algorithm for uncertain nonlinear systems is presented. This problem can be considered as finite time stabilization of higher order input-output dynamic systems with bounded uncertainties. The algorithm developed is based on the concept of integral sliding mode and includes two steps. One is the controller for nominal system using geometric homogeneity. The other is one compensating for uncertainties utilizing sliding mode control. In addition, to overcome the difficulty in determining the boundaries of uncertainties, the adaptive radial basis function neural network is designed to estimate bounded uncertainties. The proposed procedure ensures establishment of high order sliding mode and provides easy implementation. An illustrative example of a car control shows feasibility of the approach.

I. INTRODUCTION

SLIDING mode control (SMC) is a main tool to deal with systems running under uncertainty conditions [1], [2]. The corresponding approach contains two aspects. First, a sliding manifold is designed according to the desired performance index. Next, the design of a discontinuous control law is conducted such that the system trajectories reach and stay, in finite time, on the manifold by means of high-frequency control switching. Although insensitive to internal and external disturbance, the resulting controller has a disadvantage, known as chattering phenomenon. In addition, the conventional sliding mode demands that the relative degree of system is 1 with respect to sliding variable, i.e., the control has to appear explicitly in its first total derivative of time [3].

Holding the primary advantages of the conventional SMC, a technique, called high order sliding mode control (HOSMC), has been proposed to reduce and (or) remove the chattering effect [4]–[6]. At the same time, the technique can achieve better accuracy than the standard SMC [7]. On second order sliding mode, many scholars devote their attention to it and give various kinds of algorithm [8]–[10]. Arbitrary order sliding mode controllers have been proposed by Levant [11]–[13], Laghrouche [14], [15], Defoort [16]. In [11], the homogeneity properties of the known controller indicate that it can simplify the proof and develop new families of high

order sliding mode controllers (HOSMCs) based on homogeneity approach. The method called quasi-continuous HOSMC was presented in [12], which allowed the control practically continuous function of time with the relative degree $r > 1$. In [13], integral sliding mode approach was used to design HOSMC, and the advantage of the approach enabled to prescribe transient dynamics. The algorithms proposed by Levant allowed the tracking of reference signals by adjusting only one sufficiently large gain parameter with a known permanent relative degree. However, these algorithms didn't provide constructive conditions on gain tuning. Laghrouche [14] combined standard SMC with linear quadratic optimal control, which is over a finite time interval with a settled final value, to design a practical HOSMC and offer constructive conditions on gain's adjustment. The controllers proposed in [15], [16] implied that the higher order sliding mode control was equivalent to finite time stabilization of higher order integrator chain system with bounded nonlinear uncertainties. The design was based on integral sliding mode control, that is to say, the controller contained two parts, the first one was the finite time convergence controller which guaranteed the finite time stabilization of nominal system at the origin, the second one was the discontinuous controller which enabled to reject the uncertainties and ensured that system trajectories stay on the sliding manifold. The resulting controller achieved robustness throughout the entire response. The difference between two methods was that the former used the approach of finite time convergence optimal control to design the continuous controller for single input single output (SISO) system which can choose convergence time dependent of initial conditions of the system in advance, the latter utilized the technique of geometric homogeneity to design the continuous controller for multi-input multi-output (MIMO) system which eliminated the requirement of the initial condition. However, the convergence time was not available in advance.

High order sliding mode control method is a robust control technique, suitable to the control of uncertain systems. The algorithms mentioned above, in the application, are under the assumption that the boundaries of uncertainty exist and are available. However, in many cases, it is very difficult or even impossible to determine the uncertain boundaries in many practical applications. Neural networks are capable of learning and reconstructing complex nonlinear mappings and have been widely studied in control community in the identifica-

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tion analysis and design of control systems [17]. The RBF network is developed to estimate the uncertainties in on-line way and the weight adaptation law is derived by utilizing Lyapunov synthesis approach.

The rest of this paper is organized as follows. Section 2 states the problem and corresponding hypotheses. In section 3, the proposed high order sliding mode control is derived. To verify effectiveness of the developed algorithm, a simulation of a car control is carried out in section 4.

I. PROBLEM FORMULATION

Consider a dynamic system of the form

$$\begin{aligned} \dot{x} &= f(x, t) + g(x, t)u \\ y &= s(x, t) \end{aligned} \quad (1)$$

where $x \in R^n$, $u \in R$ are the state variable and the control input, respectively. $f(x, t)$, $g(x, t)$ are uncertain smooth functions, and y is a smooth measurable output. The control objective is to make the output y vanish in finite time and to keep $y \equiv 0$ thereafter.

Assumption 1. The relative degree r of system (1) with respect to $s(x, t)$ is constant and known, and the associated zero dynamics are stable.

Definition 1 ([18]). Consider a smooth dynamic system with a smooth output function $s(x, t)$, and let the system be closed by some possibly dynamical discontinuous feedback. Then, provided that the successive total time derivatives $s(x, t)$, $\dot{s}(x, t), \dots, s^{(r-1)}(x, t)$ are continuous functions of the closed system state space variables, and the set $\Theta = \{x | s(x, t) = 0, \dot{s}(x, t) = 0, \dots, s^{(r-1)}(x, t) = 0\}$ is non-empty and consists locally of Filippov trajectories [19], the motion on the set Θ is said to exist in r -sliding mode (r th-order sliding mode). The r th derivative $s^{(r-1)}(x, t)$ is mostly supposed to be discontinuous or nonexistent.

The r -order SMC allows the finite time stabilization to zero of the sliding variable $s(x, t)$ and its $(r-1)$ first time derivatives by defining a suitable discontinuous control function. Calculating the r th total time derivative of $s(x, t)$ along the trajectories of the system (1) gets the under equation

$$s^{(r)}(x, t) = \varphi(x, t) + \gamma(x, t)u \quad (2)$$

with $\varphi(x, t) = L_{f(x,t)}^r s(x, t)$, $\gamma(x, t) = L_{g(x,t)} L_{f(x,t)}^{r-1} s(x, t)$ being uncertain functions.

Assumption 2. The solutions are understood in the Filippov sense [19], and system trajectories are supposed to be infinite extendible in time for any bounded Lebesgue measurable input. In practice it means that the system is weakly minimum phase.

Assumption 3. There exist $K_m \in R^+$, $K_M \in R^+$, $C \in R^+$ such that the following inequalities hold at least locally.

$$K_m \leq \gamma(x, t) \leq K_M, |\varphi(x, t)| \leq C \quad (3)$$

The problem of r -order SMC of system (1) with respect to the sliding variable $s(x, t)$ is equivalent to the finite time stabilization of [15], [16]

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_{r-1} &= z_r \\ \dot{z}_r &= \varphi(x, t) + \gamma(x, t)u \end{aligned} \quad (4)$$

in which z is the vector $[z_1, z_2, \dots, z_r]^T = [s(x, t), \dot{s}(x, t), \dots, s^{(r-1)}(x, t)]^T$.

II. HIGH ORDER SLIDING MODE CONTROL DESIGN

In practice, the system (4) can be divided into the nominal part, known a priori, and uncertain part, so it can be rewritten as

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_{r-1} &= z_r \\ \dot{z}_r &= \varphi_0(x, t) + \gamma_0(x, t)u + F(x, t) \end{aligned} \quad (5)$$

where $\varphi_0(x, t)$, $\gamma_0(x, t)$ are determinate terms, and $F(x, t) = \Delta\varphi(x, t) + \Delta\gamma(x, t)u$ represents the whole perturbation. Furthermore, design the following control law

$$u = \frac{1}{\gamma_0(x, t)} (-\varphi_0(x, t) + w) \quad (6)$$

with w being a auxiliary control input. Thus, the system (5) can be expressed as follows

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_{r-1} &= z_r \\ \dot{z}_r &= w + F(x, t) \end{aligned} \quad (7)$$

For system (7), the control law is composed of two parts [20], like

$$w = w_0 + w_1 \quad (8)$$

where w_0 is the ideal control stabilizing the nominal system (7) in finite time at the origin and w_1 is design to be discontinuous for rejecting the perturbation term $F(x, t)$.

A. Ideal Control Design

A constructive feedback control law which renders the closed-loop system asymptotically stable and homogeneous of negative degree with respect to a suitable dilatation for

finite time stabilization of arbitrary-order integrator chain system without uncertainty has been proposed in [21]. One can see it for further details.

Theorem 1 ([16]). Let the positive constants k_1, k_2, \dots, k_r be such that polynomial $p^r + k_r p^{r-1} + \dots + k_2 p + k_1$ is Hurwitz. There is $\varepsilon \in (0, 1)$ such that for every $v \in (1 - \varepsilon, 1)$ the nominal system is stabilized at the origin in finite time under the feedback

$$w_0 = -k_1 \text{sign}(z_1) |z_1|^{v_1} - k_2 \text{sign}(z_2) \cdot |z_2|^{v_2} - \dots - k_r \text{sign}(z_r) |z_r|^{v_r} \quad (9)$$

in which the notation $\text{sign}(\cdot)$ denotes the signum function and v_1, v_2, \dots, v_r satisfy

$$v_{j-1} = \frac{v_j v_{j+1}}{2v_{j+1} - v_j} \quad (10)$$

with $v_{j+1} = 1, j = (2, 3, \dots, r)$.

B. Discontinuous Control Design

The discontinuous control law w_1 is designed to make sure that the sliding motion on the set $\Omega = \{x | s(x, t) = 0\}$ is reached in the presence of uncertainties. In the case of the boundaries of uncertainties available, the controller can be designed easily to some extent (see [15] for more details). In fact, these boundaries are difficult to be obtained exactly, sometimes even impossible, so this results in increasing the complexity in designing. One way to solve this problem is to identify the uncertainties. Here, the RBF neural network with an adaptive rule adjusting the weights by using the reaching condition of SMC is used to model the uncertainties of the system. The Gaussian function is employed as the activation function of each neuron in the hidden layer. The excitation values of these Gaussian functions are distances between the input values of z , and the central positions of Gaussian functions, described as

$$d_j = \|z - c_j\| \quad (11)$$

where c_j denotes the central position vector of neuron j , $\|\cdot\|$ indicates Euclidean norm. The weightings, w_j , between input layer neurons and hidden layer neurons are specified as constant 1. The weightings, w_k , between hidden layer neurons and output layer neurons are adjusted based on an adaptive rule. Then the output of the network is [22–24]

$$\hat{F}(x, t) = \sum_{k=1}^n \omega_k \phi_k(\|z - c_k\|) = \omega^T \phi(z) \quad (12)$$

in which $\phi_k(z) = \exp\left(-\frac{\|z - c_k\|^2}{\sigma_k^2}\right)$ is Gaussian function

and k is the k th neuron of the hidden layer. σ_k, c_k are the spread factor and central position of the Gaussian function, respectively. n is the number of neurons and z is the input value [25].

Assumption 4 ([26]). For a given positive number ζ , there exist an optimal weight ω^* and an integer n such that $\hat{F}(x, t)$ can arbitrarily approximate $F(x, t)$, i.e.

$$|\tilde{F}| = |F(x, t) - \omega^{*T} \phi(z)| < \zeta \quad (13)$$

with \tilde{F} implying approximation error.

Design the sliding variable $\sigma(z) \in R$, associated with w_1 and adaptive law as follows

$$\begin{aligned} \sigma(z) &= z_r + \xi \\ \dot{\xi} &= -w_0 \\ w_1 &= -\hat{F} - h \cdot \text{sign}(\sigma), h > \zeta \\ \dot{\hat{\omega}} &= \gamma \sigma \phi(z) \end{aligned} \quad (14)$$

where ξ is an auxiliary variable and induces the integral term, and $\dot{\cdot}$ represents the first derivative.

Theorem 2. Consider the nonlinear system (1) with a relative degree r with respect to the sliding variable $s(x, t)$ and suppose that hypotheses 1 – 4 are fulfilled. Then the control law

$$u = \frac{1}{\gamma_0(x, t)} (-\varphi_0(x, t) + w_0 + w_1) \quad (15)$$

in which w_0, w_1 are given by Eqs. (9) and (14), respectively.

The control law allows the establishment of an r -order sliding mode with respect to the sliding variable $s(x, t)$ in finite time.

Proof: Choose the Lyapunov function as

$$V = \frac{1}{2} \sigma^2 + \frac{1}{2\gamma} \tilde{\omega}^T \tilde{\omega}, \gamma > 0$$

with $\tilde{\omega} = \omega^* - \hat{\omega}, \dot{\tilde{\omega}} = -\dot{\hat{\omega}}$. The time derivative of V along the system trajectories is expressed as

$$\dot{V} = \sigma \dot{\sigma} - \frac{1}{\gamma} \tilde{\omega}^T \dot{\tilde{\omega}}$$

Substituting Eqs. (12) and (14) into the above equation achieves

$$\begin{aligned} \dot{V} &= \sigma(-\hat{F} - h \cdot \text{sign}(\sigma) + F) - \frac{1}{\gamma} \tilde{\omega}^T \dot{\tilde{\omega}} \\ &= -h|\sigma| + \sigma(F - \hat{F}) - \frac{1}{\gamma} \tilde{\omega}^T \dot{\tilde{\omega}} \\ &= -h|\sigma| + \sigma(\omega^{*T} \phi(z) + \tilde{F} - \hat{\omega}^T \phi(z)) \end{aligned}$$

$$\begin{aligned}
& -\frac{1}{\gamma}(\omega^{*\top} - \hat{\omega}^\top)\dot{\hat{\omega}} \\
& = -h|\sigma| + \sigma\tilde{F} + \sigma(\omega^{*\top} - \hat{\omega}^\top)\phi(z) \\
& -\frac{1}{\gamma}(\omega^{*\top} - \hat{\omega}^\top)\dot{\hat{\omega}} \\
& \leq -h|\sigma| + |\sigma|\|\tilde{F}\| + (\omega^{*\top} - \hat{\omega}^\top)[\sigma\phi(z) - \frac{1}{\gamma}\dot{\hat{\omega}}] \\
& < -h|\sigma| + |\sigma|\zeta + (\omega^{*\top} - \hat{\omega}^\top)[\sigma\phi(z) - \frac{1}{\gamma}\dot{\hat{\omega}}] \\
& = -|\sigma|(h - \zeta) + (\omega^{*\top} - \hat{\omega}^\top)[\sigma\phi(z) - \frac{1}{\gamma}\dot{\hat{\omega}}] \\
& = -\eta|\sigma| \leq 0, \eta = h - \zeta
\end{aligned}$$

Remark 1. As mentioned above, the implementation of controller requires real-time robust estimation of the higher order total output derivatives. The problem is solved by arbitrary order robust exact finite time convergent differentiators. The r -order sliding controller combined with the $(r-1)$ -order differentiator produces an output feedback universal controller for SISO processes [3], [11].

III. SIMULATION

This part displays the control of a car (see Fig. 1). It has been used to verify the control strategy of HOSMC in [11], [15]. Here, the effectiveness of the proposed controller is proved by using the car control. The mathematical model of car is formulated as

$$\begin{aligned}
\dot{x}_1 &= v \cos(x_3) \\
\dot{x}_2 &= v \sin(x_3) \\
\dot{x}_3 &= v/L \cdot \tan(x_4) \\
\dot{x}_4 &= u
\end{aligned} \tag{16}$$

with x_1 and x_2 being the Cartesian coordinates of the rear-axle middle point, x_3 the orientation angle, x_4 the steering angle, v the longitudinal velocity, L the length between the two axles ($L = 5$ m), u the control input. The control objective is to steer the car from a given initial position to the trajectory $x_{2d} = 10 \sin(0.05x_1) + 5$. The variables x_3 and x_4 are constrained to take their values in

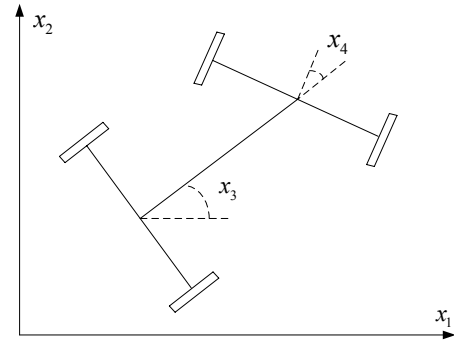


Fig. 1. Kinematic car model

$X = \{x \mid |x_4| \leq \pi/4, |x_3| \leq \pi/4\}$. Design the following sliding variable $s = x_2 - x_{2d}$, so the relative degree of the system is 3 and a 3rd order controller with 2nd differentiator is needed. The state is initialized at $x(0) = [0, 0, 0, 0]^\top$, which indicates $z(0) = [-5, -5, 0]^\top$. The 3rd time derivative of $s(x, t)$ read as

$s^{(3)}(x, t) = \varphi(x, t) + \gamma(x, t)u = a(x, t)v^3 + b(x, t)v^2 \cdot u$ with

$$\begin{aligned}
a(x, t) &= \left[\frac{1}{800} \cos\left(\frac{x_1}{20}\right) (\cos(x_3))^2 - \frac{1}{40L} \sin\left(\frac{x_1}{20}\right) \cdot \right. \\
& \quad \left. \sin(x_3) \tan(x_4) \right] \cos(x_3) + \left[-\frac{1}{20} \sin\left(\frac{x_1}{20}\right) \cdot \right. \\
& \quad \left. \cos(x_3) \sin(x_3) + \frac{\tan(x_4)}{L} \left(\frac{1}{2} \cos\left(\frac{x_1}{20}\right) \cos(x_3) \right. \right. \\
& \quad \left. \left. - \sin(x_3) \right) \right] \frac{\tan(x_4)}{L}
\end{aligned}$$

$$b(x, t) = \frac{1}{L} \left[\frac{1}{2} \cos\left(\frac{x_1}{20}\right) \sin(x_3) + \cos(x_3) \right] [1 + \tan^2(x_4)]$$

Suppose that the velocity v is an uncertain variable with a nominal value v_0 of 10 m/s, that is $v = v_0 + \Delta v$ and there is no knowledge of Δv . Therefore, $\varphi_0(x, t)$, $\gamma_0(x, t)$ and $F(x, t)$ are given by

$$\varphi_0(x, t) = a(x, t)v_0^3,$$

$$\gamma_0(x, t) = b(x, t)v_0^2 \cdot u$$

$$\begin{aligned}
F(x, t) &= a(x, t)(3v_0^2\Delta v + 3v_0\Delta v^2 + \Delta v^3) + b(x, t) \cdot \\
& \quad (2v_0\Delta v + \Delta v^2) \cdot u
\end{aligned}$$

The following 2-order differentiator originates from [12], which provides the time derivative calculation of $s(x, t)$.

$$\dot{z}_0 = v_0, v_0 = -14.7361|z_0 - s|^{2/3} \text{sign}(z_0 - s) + z_1$$

$$\dot{z}_1 = v_1, v_1 = -30|z_1 - v_0|^{1/2} \text{sign}(z_1 - v_0) + z_2$$

$$\dot{z}_2 = -440 \text{sign}(z_2 - v_1)$$

The weight $\hat{\omega}$ is initialized as $\hat{\omega}^T = 0.5 + \text{rand}(1,5)$ ($\text{rand}(\cdot)$ denotes uniform distribution) at $t = 0$, which implies 5 neurons in hidden layer. Let $k_1 = 40, k_2 = 40, k_3 = 15, c_i = [0, 0, 0]^T (i = 1, 2, 3), h = 10, \gamma = 5, \tau = 10^{-4}$ (τ is the sampling interval). A simulation is conducted and the following results have been achieved.

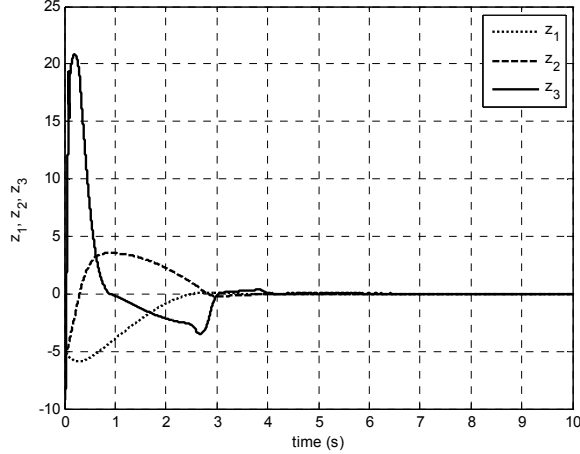


Fig. 2. $s(x,t), \dot{s}(x,t), \ddot{s}(x,t)$ versus time

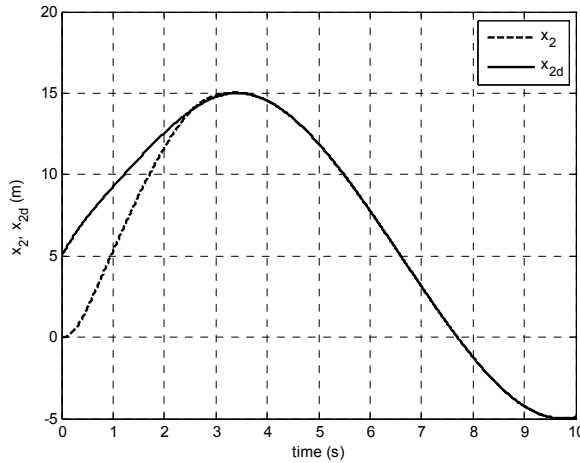


Fig. 3. x_2, x_{2d} versus time

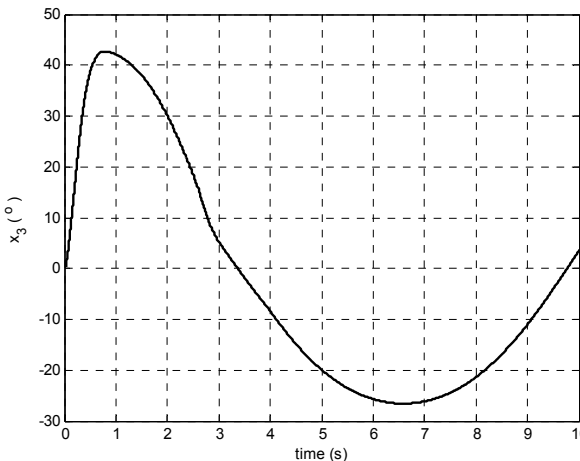


Fig. 4. x_3 versus time

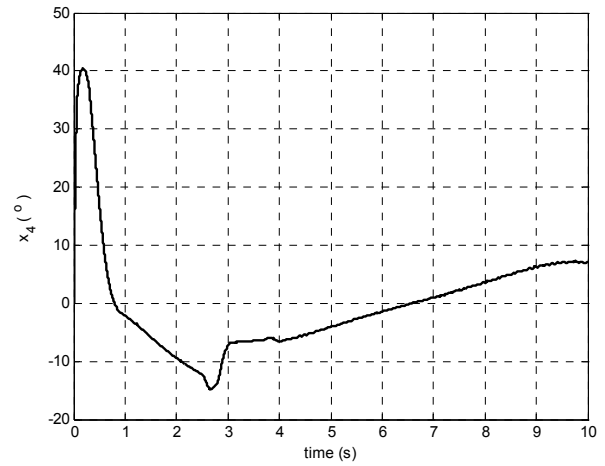


Fig. 5. x_4 versus time

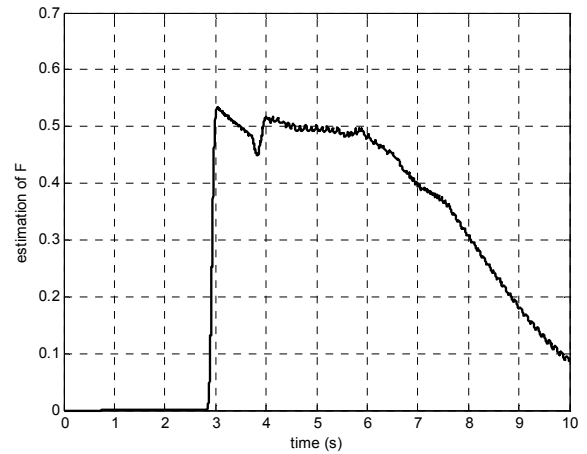


Fig. 6. The estimation of $F(x,t)$ versus time

In order to highlight the advantages of the proposed controller, a comparative study is made of two controllers. The following simulation results Fig. 7 and Fig. 8 are achieved using the controller given in [12].

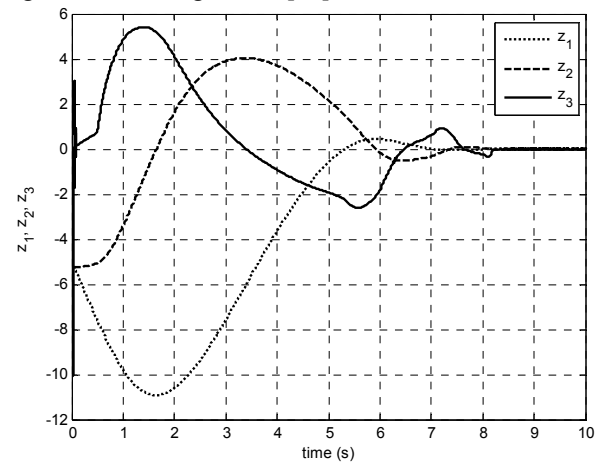


Fig. 7. $s(x,t), \dot{s}(x,t), \ddot{s}(x,t)$ versus time

Fig. 2 and Fig. 7 show the convergence of $s(x,t), \dot{s}(x,t), \ddot{s}(x,t)$ and 3-order sliding mode can be established in finite time. A conclusion is easily obtained that the proposed controller has faster transient process than the con-

troller given in [12]. The tracking performance of the desired trajectory by x_2 under unknown perturbation is displayed in Fig. 3 and Fig. 8. The conclusion is made that the output of the system can track the reference signal, but the former has great advantage over the latter. Fig. 4 and Fig. 5 display the evolvement of x_3 and x_4 with respect to time, obviously, they are in the range of the constraint. Fig. 6 describes the identification result of the perturbation term $F(x, t)$ with $\Delta v = 0.5$. It should be paid attention to the fact that the estimated value may not be in accordance with its real one. As the persistent excitation condition should be satisfied for the estimated value to converge to its real value [27].

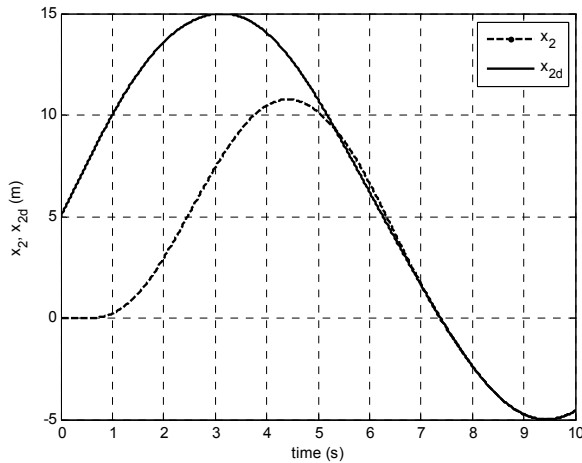


Fig. 8. x_2, x_{2d} versus time

IV. CONCLUSION

Many systems operate in uncertain environment. The available control model constantly includes uncertainties, for example, parameters variation and external disturbance. During the design of robust control, it is supposed that the boundaries of uncertainties are available in advance, but it may be an intractable problem in reality. So, the identification plays a very important role in design. In this paper, adaptive RBF neural network is used to estimate the uncertain term, aiding the design of the control. In next investigation, other approximation approaches can be developed to identify the unknown terms.

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