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# Stochastic MPC for real-time market-based optimal power dispatch

Panagiotis Patrinos, Sergio Trimboli and Alberto Bemporad

*Abstract*— **We formulate the problem of dynamic, real-time optimal power dispatch for electric power systems consisting of conventional power generators, intermittent generators from renewable sources, energy storage systems and price-inelastic loads. The generation company managing the power system can place bids on the real-time energy market (the so-called regulating market) in order to balance its loads and/or to make profit. Prices, demands and intermittent power injections are considered to be stochastic processes and the goal is to compute power injections for the conventional power generators, charge and discharge levels for the storage units and exchanged power with the rest of the grid that minimize operating and trading costs. We propose a scenario-based stochastic model predictive control algorithm to solve the real-time market-based optimal power dispatch problem.**

## I. Introduction

Liberalization and deregulation of electricity markets has led to a competitive environment consisting of market participants (usually termed as generation companies or balance responsible parties) that are legally entitled to trade electricity on the various markets in order to satisfy their loads and earn profit. On the other hand, adoption levels of renewable resources are continuously increasing due to the need for a decrease of production costs and greenhouse emissions from electricity generation by conventional fossil-fueled power plants (e.g. coal, gas, etc.). Efficient integration of intermittent generation into the existing power grid is a major bottleneck due to high variability and low predictability of renewable resources, especially wind [1].

Although market structures vary with respect to each country, they share some common characteristics. Specifically, in the majority of electricity markets, participants (power suppliers and consumers) place their bids on the day-ahead market regarding commitments for each hour of the following day. At the end of the day-ahead auction, the independent system operator (ISO) selects the accepted and rejected bids according to some clearing mechanism [2]. Due to uncertainties in power demand and generation, the existence of a real-time market operated from the ISO is mandatory in order to counteract real-time energy imbalances. In real-time markets (also called *balancing* or *regulating* markets), market participants place their bids at every PTU (Program Time Unit), which is usually an interval of 5 to 15 minutes long [2]. Unlike day-ahead prices, real-time prices are characterized by large volatility, sudden spikes that are hard to predict, and counterintuitive phenomena like negative values. Negative prices result from positive imbalance, where supply is larger than demand, especially during the night [3].

On the other hand, intermittent resources are uncontrollable, thus cannot be immediately dispatched when needed, i.e., at periods when demand is high. A partial remedy to this problem is the use of energy storage systems [4] (e.g. pumped hydro storage, thermal energy storage, compressed air energy storage, fuel cells). Energy storage systems can use electricity during off-peak hours in order to store energy which can be converted back to electricity during peak hours. The value of storage for providing balancing services for power systems with significant wind penetration is indicated in [5].

In this paper we consider a combined power system consisting of conventional generators, intermittent generators and energy storage units. The exogenous signals acting on the power system are the load, the real-time price and power outputs from intermittent resources. These are all treated as stochastic processes. The goal is to compute in real-time, power outputs for the conventional power generators, charge and discharge power levels for the energy storage systems and exchanged power with the real-time market, so as to satisfy demand and physical constraints while minimizing expected multi-period production and purchase costs.

The problem studied in this paper bears some resemblance with the dynamic economic dispatch (DED) and optimal control dynamic dispatch (OCDD) problems tackled in [6], [7]. However, in their classical setup the demand is assumed to be a periodic and deterministic signal, there is no intermittent generation or storage, the objective is to minimize production costs over a finite-horizon (equal to the period of the demand), and the resulting policies are of open-loop nature. In [8] a model predictive control (MPC) approach to the dynamic dispatch problem through the OCDD framework is proposed. In [9], MPC is proposed for real-time dispatch of power systems consisting of conventional generators and intermittent resources. However, loads and intermittent generation are assumed to be accurately predicted by point forecasts (certainty-equilvalent MPC). In [10], a general modeling framework is proposed for energy storage in power systems in order to manage intermittent power feeds. Their framework is exemplified by an MPC strategy whose purpose is to balance storage conversion losses and thermal load setpoint deviations against wind curtailments in order to avoid unnecessary generator ramping and load shedding.

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P. Patrinos & A. Bemporad are with IMT Institute for Advanced Studies Lucca, Piazza San Ponziano 6, 55100 Lucca, Italy. {panagiotis.patrinos, alberto.bemporad}@imtlucca.it
S. Trimboli is with the Department of Mechanical and Structural Engineering, University of Trento, Via Mesiano 77, 38100, Italy. {trimboli@ing.unitn.it

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Demand and intermittent generation are considered to be deterministic and there is no economic incentive included in the controller's performance index. To the best of the authors' knowledge there are only few papers that consider the market-based dispatch problem [11], but only in the deterministic setting (demand and prices are deterministic) producing open-loop solutions. Therefore, our model can be seen as an extension of DED or OCDD to a setting that is more suitable to the needs of today's deregulated energy markets with high penetration of renewable resources.

Studies in a stochastic framework have been mainly conducted for optimal bidding along with the unit commitment problem for the day-ahead in an open-loop manner [12], [13]. Instead, our algorithm considers an optimal dispatch problem based on the real-time market, can take as input the results of the unit commitment, i.e., which power units are on/off during which periods, taking advantage of more accurate predictions for the exogenous inputs to produce cost-efficient power management solutions.

The contributions of the paper are a modeling framework for real-time market-based optimal power dispatch for power systems with energy storage systems and the application of scenario-based *stochastic model predictive control* (SMPC) for its solution without any *a-priori* assumption on the distribution of the underlying stochastic process. SMPC [14] has been successfully applied to problems such as automotive power management and adaptive cruise control, [15], networked control systems, [16], [17], option hedging [18], [19] and portfolio optimization [20].

## II. Notation

Let $\mathbb{R}$, $\mathbb{Z}_+$, $\mathbb{R}^n$, $\mathbb{R}^{m\times n}$ denote the field of real numbers, the set of non-negative integers, the set of column real vectors of length $n$ and the set of $m$ by $n$ real matrices, respectively. The transpose of a matrix $A \in \mathbb{R}^{m\times n}$ is denoted by $A'$, while $\mathbf{1}_n$ denotes a column vector of $n$ elements all being equal to 1. For any $k_1, k_2 \in \mathbb{Z}_+$ with $k_1 \le k_2$, the finite set of integers $\{k_1, \dots, k_2\}$ is denoted by $\mathbb{N}_{[k_1,k_2]}$. For any $x_1, x_2 \in \mathbb{R}$ with $x_1 \le x_2$, $[x_1, x_2]$ denotes the closed interval $\{x \in \mathbb{R} | x_1 \le x \le x_2\}$. The Cartesian product of $X_i \subset \mathbb{R}^{n_i}$, $i \in \mathbb{N}_{[k_1,k_2]}$, is denoted by $\Pi_{i=k_1}^{k_2} X_i$. If $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{p\times q}$, $A \otimes B$ denotes the Kronecker product of $A$ and $B$. The direct sum of matrices $A \in \mathbb{R}^{n_i \times m_i}$, $i \in \mathbb{N}_{[k_1,k_2]}$ is denoted by $\bigoplus_{i=k_1}^{k_2} A_i$. If $(\Omega, \mathfrak{F}, \mathbb{P})$ is a probability space and $\xi : \Omega \to \mathbb{R}^n$ is a random vector on $(\Omega, \mathfrak{F}, \mathbb{P})$, then $\mathbb{E}[\xi]$ denotes its expected value.

## III. Model Formulation

Consider a power system consisting of $n_p$ conventional power generators, $n_r$ intermittent power generators and $n_s$ energy storage systems. The length of the PTU is denoted by $T$ (in hours). At each time instant (PTU) $k \in \mathbb{Z}_+$, let $p_i(k) \in \mathbb{R}$ (MW), $i \in \mathbb{N}_{[1,n_p]}$ denote the power output of the $i$-th power generator, $x_i(k) \in \mathbb{R}$ (MWh), $u_i^{\rm c}(k) \in \mathbb{R}$, $u_i^{\rm d}(k) \in \mathbb{R}$ (MW), $i \in \mathbb{N}_{[1,n_s]}$ denote the amount of energy stored (state-of-charge, SOC), the amount of electricity converted to some other form of stored energy and the amount of energy converted to electricity respectively for the $i$-th storage unit, $r_i(k) \in \mathbb{R}$ (MW), $i \in \mathbb{N}_{[1,n_r]}$ the power output of the $i$-th intermittent generator, $p^{\rm ex}(k) \in \mathbb{R}$ (MW) the amount of electricity exchanged with the market, $d(k)$ (MW) the local load that needs to be served by the power system, and $\lambda(k)$ the real-time energy price.

### A. Conventional Power Generators

At each time instant $k \in \mathbb{Z}_+$, the power output of each conventional power generator, $p_i(k)$, must lie in the closed interval $[p_i^{\min}, p_i^{\max}]$ with $0 \le p_i^{\min} \le p_i^{\max}$, $i \in \mathbb{N}_{[1,n_p]}$, i.e.:

$$p_i^{\min} \le p_i(k) \le p_i^{\max},\ i \in \mathbb{N}_{[1,n_]} \quad (1)$$

and satisfy the following ramp-rate constraints:

$$\Delta p_i^{\min} \le p_i(k) - p_i(k-1) \le \Delta p_i^{\max},\ i \in \mathbb{N}_{[1,n_p]} \quad (2)$$

Letting $p(k) \triangleq [p_1(k) \cdots p_{n_p}(k)]'$, $P \triangleq \Pi_{i=1}^{n_p}[p_i^{\min}, p_i^{\max}]$ and $\Delta P \triangleq \Pi_{i=1}^{n_p}[\Delta p_i^{\min}, \Delta p_i^{\max}]$, constraints (1), (2) can be expressed as

$$p(k) \in P \quad (3a)$$

$$p(k) - p(k-1) \in \Delta P \quad (3b)$$

### B. Energy Storage Systems

The existence of energy storage systems is the distinct characteristic that makes the power system under investigation a dynamical system. Specifically, the dynamics of the $i$-th energy storage system is described by the following state equation:

$$x_i(k+1) = \alpha_i x_i(k) + T(\alpha_i^{\rm c} u_i^{\rm c}(k) - (\alpha_i^{\rm d})^{-1} u_i^{\rm d}(k)) \quad (4)$$

Parameter $\alpha_i \in (0, 1]$ accounts for self-discharge, i.e. internal energy losses associated with energy storage (e.g. heat storages lose energy due to a difference between internal storage and ambient temperature). Parameters $\alpha_i^{\rm c} \in (0, 1]$, $\alpha_i^{\rm d} \in (0, 1]$ represent efficiency of the charge (conversion from electricity to energy) and discharge (conversion from energy to electricity) processes, respectively.

At each PTU $k \in \mathbb{Z}_+$, the amount of energy stored in each energy storage system, $x_i(k)$, must lie in the closed interval $[x_i^{\min}, x_i^{\max}]$ with $0 \le x_i^{\min} \le x_i^{\max}$, $i \in \mathbb{N}_{[1,n_s]}$, i.e.:

$$x_i^{\min} \le x_i(k) \le x_i^{\max},\ i \in \mathbb{N}_{[1,n_s]} \quad (5)$$

and satisfy the following ramp-rate constraints:

$$\Delta x_i^{\min} \le x_i(k) - x_i(k-1) \le \Delta x_i^{\max},\ i \in \mathbb{N}_{[1,n_s]} \quad (6)$$

Furthermore, for any $k \in \mathbb{Z}_+$ one must have

$$u_i^{\rm c,min} \le u_i^{\rm c}(k) \le u_i^{\rm c,max},\ i \in \mathbb{N}_{[1,n_s]} \quad (7a)$$

$$u_i^{\rm d,min} \le u_i^{\rm d}(k) \le u_i^{\rm d,max},\ i \in \mathbb{N}_{[1,n_s]} \quad (7b)$$

with $0 \le u_i^{\rm c,min} \le u_i^{\rm c,max}$, $0 \le u_i^{\rm d,min} \le u_i^{\rm d,max}$, $i \in \mathbb{N}_{[1,n_s]}$. Letting $x(k) \triangleq [x_1(k) \cdots x_{n_s}(k)]'$, $u(k) = [u_1^{\rm c}(k)\ u_1^{\rm d}(k) \cdots u_{n_s}^{\rm c}(k)\ u_{n_s}^{\rm d}(k)]'$, and $b_i \triangleq T[\alpha_i^{\rm c}\ -(\alpha_i^{\rm d})^{-1}]$, $i \in \mathbb{N}_{[1,n_s]}$ the dynamics of the energy storage systems (eq. (4)) can be expressed by the following linear time-invariant system

$$x(k+1) = Ax(k) + Bu(k) \quad (8)$$

where $A \triangleq \bigoplus_{i=1}^{n_s} \alpha_i$ and $B \triangleq \bigoplus_{i=1}^{n_s} b_i$. Furthermore, letting $X \triangleq \Pi_{i=1}^{n_s}[x_i^{\min}, x_i^{\max}]$, $\Delta X \triangleq \Pi_{i=1}^{n_s}[\Delta x_i^{\min}, \Delta x_i^{\max}]$, $u_i^{\min} \triangleq [u_i^{\mathrm{c},min} \;\; u_i^{\mathrm{d},min}]'$ and $U \triangleq \Pi_{i=1}^{n_s}[u_i^{\min}, u_i^{\max}]$, constraints (5), (6), (7) can be expressed as

$$x(k) \in X \tag{9a}$$
$$x(k) - x(k-1) \in \Delta X \tag{9b}$$
$$u(k) \in U \tag{9c}$$

### C. Power balance

At each time instant $k \in \mathbb{Z}_+$ the following power balance must be satisfied:

$$\sum_{i=1}^{n_p} p_i(k) + \sum_{i=1}^{n_s}(u_i^{\mathrm{d}}(k) - u_i^{\mathrm{c}}(k)) + \sum_{i=1}^{n_r} r_i(k) - p^{\mathrm{ex}}(k) = d(k) \tag{10}$$

Notice that according to (10), we make the convention that $p^{\mathrm{ex}}(k)$ takes positive values if electricity outflows from the power system to the rest of the grid.

By letting $E \triangleq I_{n_s} \otimes [-1 \;\, 1]$, (10) can be expressed as

$$\mathbf{1}_{n_p}' p(k) + Eu(k) + \mathbf{1}_{n_r}' r(k) - p^{\mathrm{ex}}(k) = d(k) \tag{11}$$

where $r(k) \triangleq [r_1(k) \cdots r_{n_r}(k)]'$.

### D. Stage Cost

With each conventional power generator $i \in \mathbb{N}_{[1,n_p]}$, we associate a convex quadratic fuel cost function $\ell_i : \mathbb{R} \to \mathbb{R}_+$:

$$\ell_i(p_i) \triangleq Q_i p_i^2 + q_i p_i + c_i \tag{12}$$

The total production cost for the power system at time $k \in \mathbb{Z}_+$ can be expressed as:

$$\ell_p(p(k)) \triangleq \sum_{i=1}^{n_p} \ell_i(p_i(k)) = p(k)'Qp(k) + q'p(k) + c \tag{13}$$

where $Q \triangleq \bigoplus_i^{n_p} Q_i$, $q \triangleq [q_1 \cdots \; q_{n_p}]'$ and $c \triangleq \sum_{i=1}^{n_p} c_i$. At time instant $k \in \mathbb{Z}_+$, the profit of trading $p^{\mathrm{ex}}(k)$ MW of power to the real-time market is given by $T\lambda(k)p^{\mathrm{ex}}(k)$ (price $\lambda(k)$ is in $\$/\mathrm{MWh}$). The total cost that incurs for the power system at time $k \in \mathbb{Z}_+$ is equal to the total production cost minus the trading profit:

$$\ell(p(k), p^{\mathrm{ex}}(k), \lambda(k)) = \ell_p(p(k)) - T\lambda(k)p^{\mathrm{ex}}(k) \tag{14}$$

### E. Exogenous Inputs

It is assumed that load $\{d(k)\}_{k\in\mathbb{Z}_+}$, real-time price $\{\lambda(k)\}_{k\in\mathbb{Z}_+}$ and intermittent production $\{r_i(k)\}_{k\in\mathbb{Z}_+}$, $i \in \mathbb{N}_{[1,n_r]}$ are real-valued stochastic processes defined on a probability space $(\Omega, \mathfrak{F}, \mathbb{P})$. Let $\xi(k) \triangleq [d(k) \;\, \lambda(k) \;\, r(k)']' \in \mathbb{R}^{n_r+2}$. Then $\{\xi(k)\}_{k\in\mathbb{Z}_+}$ is a vector-valued stochastic process on $(\Omega, \mathfrak{F}, \mathbb{P})$. Notice that we do not assume any particular *a-priori* assumption about its properties (e.g. Markovianity, etc.).

## IV. Scenario-Based SMPC for Real-time Optimal Market-Based Power Dispatch

### A. Stochastic Model Predictive Control

For $k \in \mathbb{Z}_+$, let $z(k) \triangleq [p(k) \;\, p^{\mathrm{ex}}(k) \;\, u(k)']'$, $n_z \triangleq n_p + 2n_s + 1$. Given $z(k-1)$, $x(k-1)$ and $\xi(k)$, let $\mathcal{Z}(z(k-1), x(k-1), \xi(k))$ denote the set of $(z(k), x(k)) \in \mathbb{R}^{n_z} \times \mathbb{R}^{n_s}$ that satisfy (3), (9) and (11). Extending the classical receding horizon philosophy to a stochastic setting, a finite-horizon stochastic optimal control problem is solved on-line at every time $k \in \mathbb{Z}_+$, given $x(k-1)$, $p(k-1)$ and after measurements for $x(k)$, $\xi(k)$ are received:

$$\inf_{z(l)} \mathbb{E}\left[\sum_{l=k}^{k+N} \ell(p(l), p^{\mathrm{ex}}(l), \lambda(l)) \middle| \xi(k)\right] \tag{15a}$$

$$\text{s.t. } z(l) \text{ is } \mathfrak{F}_k^l - \text{measurable}, l \in \mathbb{N}_{[k,k+N]} \tag{15b}$$
$$x(l+1) = Ax(l) + Bu(l), l \in \mathbb{N}_{[k,k+N]} \tag{15c}$$
$$(z(l), x(l)) \in \mathcal{Z}(z(l-1), x(l-1), \xi(l)), l \in \mathbb{N}_{[k,k+N]} \tag{15d}$$
$$x(k+N+1) \in X \tag{15e}$$
$$x(k+N+1) - x(k+N) \in \Delta X \tag{15f}$$

According to (15a), the goal is to minimize the expected value of the sum of future costs up to a prediction horizon $N \in \mathbb{Z}_+$, conditioned on the present information $\xi(k)$. In (15b), $\mathfrak{F}_k^l \triangleq \sigma(\xi_{[k,k+l]})$, i.e., $\mathfrak{F}_k^l$ is the natural filtration generated by $\xi_{[k,k+l]} = \{\xi(k), \ldots, \xi(k+l)\}$ and $\mathfrak{F}_k^k = \{\emptyset, \Omega\}$, since $\xi(k)$ is known at time $k$. The $\mathfrak{F}_k^l$-measurability condition (cf. (15b)) means that $z(k) : \Omega \to \mathbb{R}^{n_z}$ depends only on information up to time $k$. It is also known as nonanticipativity in the stochastic programming community [21] or causality in the control literature. Implicit in this assumption is that at any time $k \in \mathbb{Z}_+$, we know the current load $d(k)$, real-time price $\lambda(k)$ and intermittent production $r(k)$. However, we only have a probabilistic information about their future evolution. The satisfaction of constraints (15d) is to be understood in the $\mathbb{P}$-almost sure sense. After solving (15), only the first member of the optimal finite-horizon policy is kept and applied to the power system, i.e., the SMPC control law is $\kappa_{\mathrm{SMPC}}(x(k), p(k-1), x(k-1), \xi(k)) \triangleq z(k)$. Notice that optimization takes place over closed-loop policies due to the nonanticipativity constraint (15b).

### B. Scenario-based Stochastic Model Predictive Control

There are some inherent difficulties regarding the SMPC formulation (15). First, explicit knowledge of the probability distribution of the underlying stochastic process $\{\xi(k)\}_{k\in\mathbb{Z}_+}$ is required. However, such a probability distribution is usually very hard to estimate, if not impossible, especially for quantities such as wind speed and real-time prices. In case such distribution is available but has infinite support, then still the SMPC problem (15) is an infinite-dimensional optimization problem, hence intractable, since one has to optimize with respect to mappings $z(k) : \Omega \to \mathbb{R}^{n_z}$.

A practical solution to these problems is to approximate the stochastic process $\{\xi(l)\}_{l\in\mathbb{N}_{[k,k+N]}}$ by a process having finitely many sample paths (called scenarios) exhibiting a tree structure and emanating from the current value $\xi(k)$. A *scenario tree* has a finite number of nodes for each stage $l \in \mathbb{N}_{[k,k+N]}$. At stage $k$ there is only one node called the *root node*, which is labeled by 0 and whose value is equal to $\xi(k)$. At a general stage $l > k$, each node $j \in \mathcal{N}$ ($\mathcal{N}$ denotes the set of nodes of the tree) is connected to a unique node at stage $l-1$, called the *ancestor node* and denoted by $\mathcal{A}(j) \in \mathcal{N}$, and is also connected to (possibly more than one) nodes at stage $l+1$, called the *children nodes*. The set of nodes corresponding to the final stage $k+N$ is called the set of *leaf nodes* and is denoted by $\mathcal{L} \subset \mathcal{N}$. By construction, there is a one-to-one correspondence between scenarios and the leaf nodes. Algorithms that perform such kind of approximations are called scenario tree generation methods, see the survey [22]. Various scenario tree generation methods have been proposed in the literature, e.g., [23], [24], [25] to mention a few. In the case of the stochastic process $\{\xi(l)\}_{l\in\mathbb{N}_{[k,k+N]}}$ being modeled by a scenario tree, the SMPC problem (15) becomes a (large-scale) convex QP which can be solved efficiently either by off-the-self algorithms or by specialized decomposition methods.

In the present work we applied the method proposed by [25] to generate scenario trees in order to approximate $\{\xi(l)\}_{l\in\mathbb{N}_{[k,k+N]}}$. More precisely, the underlying idea of the method is to independently generate a finite number of scenarios that are obtained either by resampling from historical data or by sampling from a time-series model calibrated using historical data. The result of this step is named a *scenario fan*. However a scenario fan does not exhibit an appropriate tree structure capable of modeling the stagewise decision process. Specifically, the nonanticipativity condition is violated since the decision maker is allowed to see into the future. Furthermore, in order to obtain a good approximation of the true probability distribution the number of scenarios may be very large. Therefore, the scenario fan is further processed in order to arrive to a scenario tree with a reduced number of nodes and the appropriate information structure.

Specifically, *forward tree construction* ([25], alg. 4.5) is based on successive clustering of scenarios, starting for the root node. The algorithm takes as input the scenario fan and the relative tolerance $\epsilon_{\rm rel} \triangleq \epsilon/\epsilon_{\max}$ ($\epsilon_{\max}$ is the best possible distance between the probability distribution of the initial scenario fan and the distribution of one of its scenarios endowed with unit mass) and at each stage, each cluster of the previous stage is further subdivided into sub-clusters using scenario reduction according to a probability metric. The outcome of forward tree construction is a scenario tree, consisting of a reduced set of nodes, $\mathcal{N}$, the value of the process $\xi^j \triangleq (d^j, \ell^j, r^j)$ and the probability $\pi^j$ for each node $j \in \mathcal{N}$, with $\xi^0 = \xi(k)$ and $\pi^0 = 1$ for the root node.

The resulting scenario-based SMPC (SSMPC) problem solved at each time $k \in \mathbb{Z}_+$, given $x(k-1)$, $p(k-1)$, $x(k)$ and $\xi(k)$ becomes:

$$
\begin{aligned}
&\min_{\{z^j\}_{j\in\mathcal{N}}} \sum_{j\in\mathcal{N}} \pi^j \ell(p^j, p^{\mathrm{ex},j}, \lambda^j) && \text{(16a)}\\
&\text{s.t. } (z^0, x(k)) \in \mathcal{Z}(z(k-1), x(k-1), \xi^0) && \text{(16b)}\\
&\quad x^j = Ax^{\mathcal{A}(j)} + Bu^{\mathcal{A}(j)}, \quad j \in \mathcal{N}\setminus\{0\} && \text{(16c)}\\
&\quad (z^j, x^j) \in \mathcal{Z}(z^{\mathcal{A}(j)}, x^{\mathcal{A}(j)}, \xi^j), j \in \mathcal{N}\setminus\{0\} && \text{(16d)}\\
&\quad Ax^j + Bu^j \in X, \quad j \in \mathcal{L} && \text{(16e)}\\
&\quad Ax^j + Bu^j - x^j \in \Delta X, \quad j \in \mathcal{L} && \text{(16f)}
\end{aligned}
$$

Since constraints (3), (9) and (11) are linear, the stage cost (cf. (14)) is convex quadratic in $(p^j, p^{\mathrm{ex},j})$ and $\pi^j$ are positive, it follows that (16) is a convex quadratic program. To sum up, according to the proposed SSMPC algorithm for real-time optimal market-based power dispatch, at each time $k \in \mathbb{Z}_+$, after we receive measurements about the current load $d(k)$, real-time price $\lambda(k)$, intermittent production $r(k)$ and storage levels $x(k)$, we generate scenarios of length $N$ emanating from $\xi(k)$, then we apply forward tree construction to generate a scenario tree and finally we solve the convex QP (16), to obtain the SSMPC control action $\kappa_{\rm SSMPC}(x(k-1), p(k-1), x(k), \xi(k)) \triangleq z^0$ which we apply to the power system (Fig. 1).

Unlike standard MPC where the main goal is to achieve stability of some equilibrium point or tracking of a reference signal, here the main concern is to satisfy load in the power system while minimizing operational costs. Notice that (16) is always feasible since there is an option of buying electricity from the grid to satisfy load.

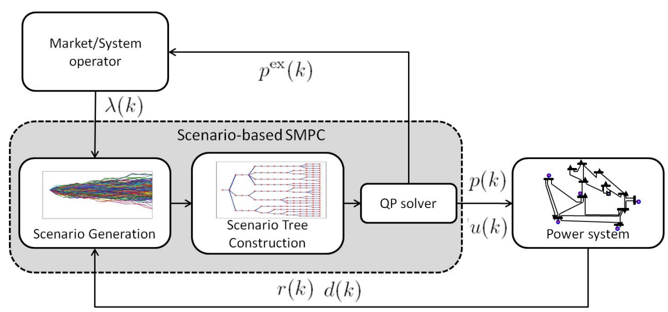



Fig. 1: Scenario-based SMPC for optimal power dispatch

## V. Case Study

In order to exemplify the merits of the proposed SSMPC approach, we simulate the 12-bus power system (modified from [9]) shown in Figure 2. The system consists of three conventional power generators, i.e., two coal power generators ($P_1$, $P_2$) and one natural gas power generator ($P_3$), two intermittent generators, i.e., a wind farm ($R_1$) and a photovoltaic (PV) generator ($R_2$), and one hydro storage unit ($S_1$). The characteristics of the conventional power generators are given in Tables I and II, while the parameters of the storage unit are summarized in Table III.

The PTU is assumed to be equal to 10 minutes ($T = 1/6$). Real historical data were used in the simulations. Load and real-market price data are obtained from the

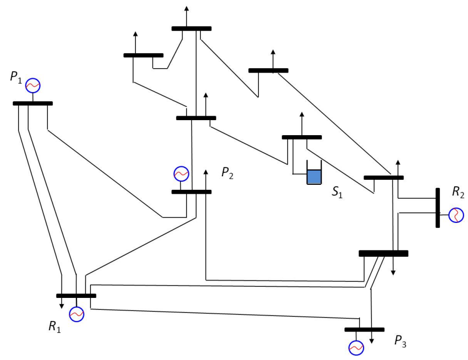



Fig. 2: 12–bus power system

TABLE I: Generator Cost Data

| Unit | $Q_i$ | $q_i$ | $c_i$ |
|---|---|---|---|
| P1 | 0.0015 | 5.063 | 66.338 |
| P2 | 0.0038 | 12.225 | 48.713 |
| P3 | 0.0081 | 10.248 | 81.659 |

New York ISO (`http://www.nyiso.com/public/market_data/`), while meteorological data regarding wind speed and solar radiation are obtained by the National Data Buoy Center (`http://www.ndbc.noaa.gov/`). Specifically, data for the first 22 days of January 2011 were used for creating scenarios at every time instant $k \in \mathbb{Z}_+$, while the power system in closed-loop with the SSMPC controller is simulated for the 23rd of January. Figure 3 depicts the load, total intermittent generation (sum of wind farm and PV outputs) and the real-time price for that day.

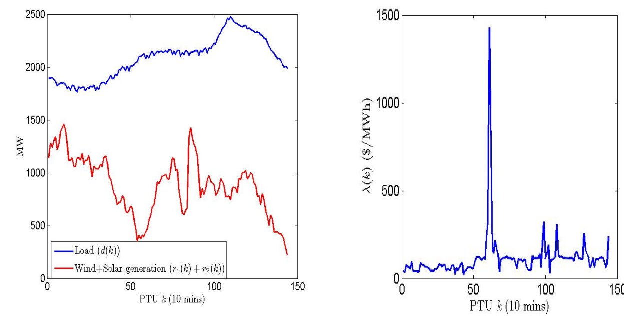
SSMPC was compared against *prescient optimal control* (Prescient-OC) where complete knowledge of the realization of the stochastic exogenous inputs is assumed, and *certainty-equivalent MPC* (CE-MPC), where uncertain parameters are substituted by their time-varying average values based on the historical data. The prediction horizon $N$ was set equal to 16 for both SSMPC and CE-MPC. SSMPC was tested for various values of the relative error parameter of forward tree construction. Table IV summarizes the results of the simulations. As expected, the average number of nodes is decreasing while the simulation cost is increasing as $\epsilon_{\rm rel}$

TABLE II: Generator Data

| Unit | $p_i^{\min}$ | $p_i^{\max}$ | $\Delta p_i^{\min}$ | $\Delta p_i^{\max}$ |
|---|---|---|---|---|
| P1 | 450 | 1100 | -250 | 250 |
| P2 | 50 | 500 | -200 | 200 |
| P3 | 50 | 100 | -75 | 75 |

TABLE III: Storage Data

| Unit | $x_i^{\min}$ | $x_i^{\max}$ | $\Delta x_i^{\min}$ | $\Delta x_i^{\min}$ | $\alpha_i$ | $\alpha_i^{\rm c}$ | $\alpha_i^{\rm d}$ |
|---|---|---|---|---|---|---|---|
| S1 | 15 | 300 | -120 | 120 | 1 | 0.85 | 0.90 |
| | $u_i^{\rm c,min} = u_i^{\rm d,min}$ | | | $u_i^{\rm c,max} = u_i^{\rm d,max}$ | | | |
| | 0 | | | 300 | | | |




(a) (b)

Fig. 3: (a) Load, intermittent generation, (b) real-time price, for January 23rd, 2011.

TABLE IV: Comparison of scenario-based SMPC with prescient optimal control and certainty-equivalent MPC

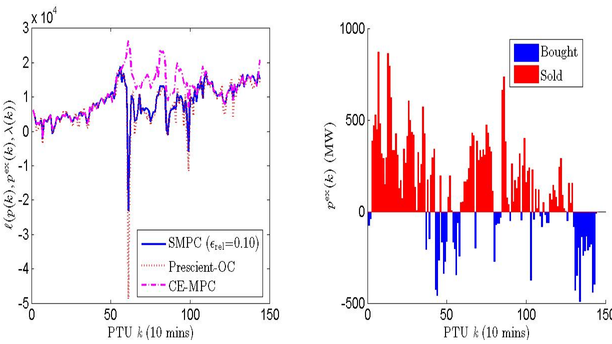
| Algorithm | Storage Cost | No Storage Cost | Avg # of nodes |
|---|---|---|---|
| Prescient-OC | 1071329 | 1146623 | |
| CE-MPC | 1629791 | 1636586 | |
| SSMPC ($\epsilon_{\rm rel} = 0.1$) | 1189097 | 1207660 | 350 |
| SSMPC ($\epsilon_{\rm rel} = 0.2$) | 1190668 | 1208233 | 335 |
| SSMPC ($\epsilon_{\rm rel} = 0.3$) | 1191415 | 1208367 | 172 |
| SSMPC ($\epsilon_{\rm rel} = 0.4$) | 1196641 | 1210750 | 87 |
| SSMPC ($\epsilon_{\rm rel} = 0.5$) | 1204152 | 1211249 | 50 |
| SSMPC ($\epsilon_{\rm rel} = 0.6$) | 1206664 | 1212901 | 38 |
| SSMPC ($\epsilon_{\rm rel} = 0.7$) | 1209915 | 1216337 | 31 |
| SSMPC ($\epsilon_{\rm rel} = 0.8$) | 1209207 | 1218682 | 26 |
| SSMPC ($\epsilon_{\rm rel} = 0.9$) | 1210070 | 1219773 | 22 |
| SSMPC ($\epsilon_{\rm rel} = 1.0$) | 1210070 | 1219773 | 20 |

increases. It is clear from Table IV that SSMPC outperforms CE-MPC. In order to examine the value of employing energy storage systems for power systems with intermittent generation, we also compare against the case where there is no energy storage unit in the system.

Figure 4(a) illustrates the operational costs of the three approaches during the simulation. Unlike CE-MPC, it is clear that SSMPC can take advantage of the high profit opportunities appearing when upward real-time price spikes occur. Figure 4(b) displays the exchanged power with the real-time market for the power system in closed-loop with the SSMPC controller. Figure 5 depicts power outputs for the three conventional generators (Fig. 5(a)) and state of charge of the storage unit (Fig. 5(b)) for the power system in closed-loop with the SSMPC controller ($\epsilon_{\rm rel} = 0.1$).

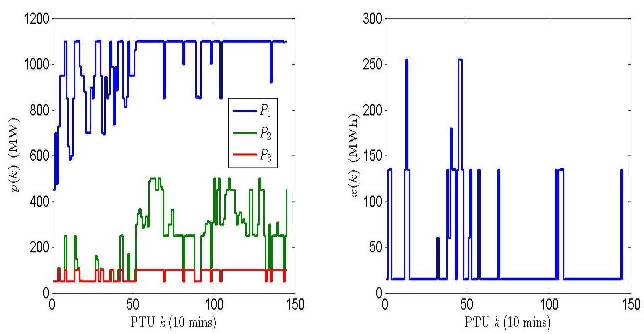
## VI. Conclusions and Future Work

In this paper we formulated a real-time market-based optimal power dispatch problem for power systems that can be seen as balance responsible parties participating in the deregulated electricity market. Specifically, the power system must balance its own loads while respecting operational constraints, minimizing production costs, and making as large profit as possible by trading power on the real-time market. The power system can contain intermittent




(a) (b)

Fig. 4: (a) Operational cost comparison, (b) exchanged electricity with the real-time market.




(a) (b)

Fig. 5: (a) Conventional power generation, (b) state of charge of the energy storage unit.

generation and storage energy systems, characteristics that will become prevalent in future power systems.

We proposed a novel scenario-based SMPC algorithm for the solution of the real-time market-based optimal power dispatch problem. The algorithm uses a scenario tree generation algorithm in order to construct a tree suitable for multistage stochastic optimization from a scenario fan and solves a convex QP at every sampling time. The algorithm is very flexible in the sense that the process of creating scenarios is separated from the solution procedure. Specifically, the user can provide scenarios based on historical data or coming from a time-series model of the underlying stochastic process. The value of incorporating stochastic information was examined on a non-trivial 12-bus system using real historical data for simulation, showing clear advantages over certainty-equivalent MPC and achieving large cost savings which stronlgy encourage the use of SMPC for managing electric power systems in today's (and tomorrow's) markets.

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