

Optimal PI Tuning Rules for Flow Loop, Based on Modified Relay Feedback Test

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Abstract—Normally PID controller tuning rules are derived using linear models of the plant or process, with nonlinearities neglected. This factor often results in the deterioration of loop performance which is tuned using these rules. In the present paper, optimal tuning rules are derived based on a more accurate nonlinear model of the flow process, in which the nonlinearity of the pneumatic actuator (most widely used in the process industries) is considered. This nonlinearity is dynamic and exists even if the valve static characteristic is linear. The proposed tuning rules are coupled with the Modified Relay Feedback Test (MRFT) that was recently proposed in the literature. It is shown that the use of the presented tuning rules along with MRFT provides an advantageous result for control performance of flow loops. Simulations are provided.

I. INTRODUCTION

Despite the successful application of different types of modern control methods and techniques, the PID control still remains the main type of control widely used in process industries. It is because of its simple implementation, relatively good performance and availability of tuning rules that it received popularity and found extensive applications. It is practically the only type of control for flow control loops, which are the subject of research in the present paper. The flow loops are commonly used in the process industries by itself and as basic blocks in combination with other types of process control loops. In overall the flow loop is one of the widest spread controls in the process industry. Analysis of the power plant distributed control system, undertaken by the authors, showed that the share of the flow control loops was around 27% of all control loops. The flow process is also the simplest process from the point of view of the models used for identification of its dynamics and controller tuning. Normally, if the valve characteristic is linear, only linear models of the flow process are used for identification and tuning. Given the two circumstances mentioned above one can figure out how important the subject PID controller tuning for a flow loop would be. Tuning of a PID controller is an important problem from both theoretical and practical points of view. Several tuning methods have been proposed so far but the problem is still of interest to the researchers. Tuning is based on such tests as the step test, Ziegler-Nichols's closed loop test [14], Astorm-Hagglund's relay feedback test (RFT) [1] and recently proposed algorithm such as Modified relay feedback test (MRFT) [4], [5] among others, which generally provide a satisfactory performance despite inherent low accuracy (in the non-parametric setup) of those methods. The source of the inherent low accuracy is well known, which is the use of only two measurements (three in the case of the step test and method used in [14]).

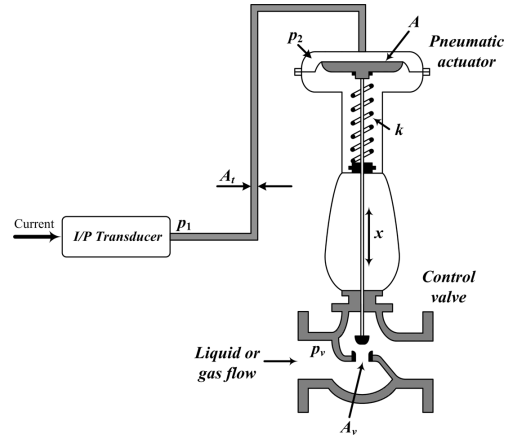


Fig. 1: Control valve connected to an I/P transducer

However, the use of precise models when generating tuning rules for a specific process, as shown in the present paper, allows one to significantly enhance performance of tuning.

This paper is organized as follows: In the second section, a brief overview of the flow model is presented. In the third section, the motivation for obtaining optimal tuning formulas for PID controller parameters for a flow loop is briefly discussed, and analysis of performance of a few tuning rules coupled with the nonlinear flow loop model is provided. The contribution of this work is thus justified as aimed at the performance enhancement. In the fourth section a short review of PID controller tuning through Modified Relay Feedback Test is presented. The optimization problem and optimized parameters and constraints are defined, and a quick review of the gradient decent as a method of optimization is given in the fifth section. In the sixth section, PI optimal tuning rules optimized through ISE criterion for the flow loop are proposed and the simulation results are presented. The last section is devoted to the conclusion.

II. MODEL OF FLOW PROCESS

The model of a simple flow loop control consists of two main blocks: firstly, a process block which includes a current-to-pressure (I/P) transducer, pneumatic actuator, control valve, gas or liquid flow through the valve (Fig. 1), and secondly, a PID controller and a flow sensor connected to the process block. The movement of the stem is a function of the pressure on the diaphragm, spring position, the fluid forces on the valve plug, and friction. The valve motion is described by the following equation.

$$m\ddot{x} + b\dot{x} + kx = (p_2 - p_a)A - \Delta p_v A_v + F_f \quad (1)$$

In formula (1), m is the mass of the valve stem, diaphragm and of other moving parts, b is the coefficient of viscous friction, k is the spring rate, p_2 is the pressure (absolute) above the diaphragm, p_a is the atmospheric pressure, and $\Delta p_v A_v$ is the force exerted on the plug due to the pressure drop across the valve, F_f is the Coulomb friction. For simplicity, the fluid force on the valve plug ($P_v A_v$) is neglected and the Coulomb friction is accounted for as some equivalent viscous friction. We will, therefore, assume that $F_f = 0$ and b account for the Coulomb friction too.

The volumetric flow through the valve is assumed proportional to the valve opening (linear valve characteristic):

$$q = C_v \sqrt{\frac{\Delta p_v}{\rho}} (x/x_{max}) \quad (2)$$

where C_v is the CV of the valve, ρ is the specific gravity, and x_{max} is the maximum travel. We will assume that Δp_v is constant and does not depend on the position of the valve. This represents a relatively rare situation. However, very often such pressure source as the centrifugal pump driving the liquid through the valve with equal percentage characteristic can be reduced to the considered case.

By assuming that the reaction of the I/P transducer to the electric current change is fast and considering the fact that the I/P transducer produces the output pressure p_1 proportional to the supplied current, we model the dynamics of the I/P transducer by a small time delay or even neglect it, as this time delay can be accounted for in the processing delay of the control system (which usually varies from 200ms to 1s for flow loops). The dependence of the transducer output pressure on the input current is linear, with 0-100% ranges as follows: 4-20mA current range corresponds to 3-15psi (gage) pressure range (this is the most common industry standard). However, because of the transmission line (tube) between the two chambers the actuator pressure p_2 is not equal to the transducer pressure p_1 (in transients). Using the St. Venant and Wantzel formula [3], we can model the mass air flow from chamber 1 to chamber 2 as follows:

$$G_{12} = A_t c_d p_1 \sqrt{\frac{g\gamma}{RT_1} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \Psi\left(\frac{p_2}{p_1}\right)} \quad (3)$$

where A_t is the smallest cross-sectional area of the tube between the I/P transducer and the actuator, c_d is the discharge coefficient (so that $A_t c_d$ is the effective cross-sectional area of the tube), g is the gravity constant, R is the universal gas constant, T_1 is the air temperature in chamber 1, γ is the isentropic coefficient ($\gamma = 1.4$ for air), $\Psi(p_2/p_1)$ is the flow function given by

$$\Psi\left(\frac{p_2}{p_1}\right) = \begin{cases} 1 & \text{if } \frac{p_2}{p_1} < \beta_c \\ K \sqrt{\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}}} & \text{if } \frac{p_2}{p_1} > \beta_c \end{cases} \quad (4)$$

where $K = \sqrt{\frac{2}{\gamma-1} \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{\gamma-1}}}$ and $\beta_c = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma+1}}$ is the critical pressure ratio that is $\beta_c = 0.528$ for air. This formula is detailed in [6].

Considering the fact that air could flow in both directions from the transducer to the actuator and vice versa, the air flow from the actuator to the transducer (where it is released to the atmosphere) G_{21} can be described by the same equations (3) and (4) with pressures p_1 and p_2 swapped in the formulas.

Considering the introduced functions and the equation for pressure change in chamber 2 based on the ideal gas equation $p_2 V = \frac{m}{\mu}$, where $V = V_0 + Ax$ is the volume of chamber 2, with V_0 being the volume at $x = 0$, and subject to $T_2 = T_1 = T$, we write the state equations of the plant (process) as follows.

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= \frac{1}{m} (-bv - kx + (p_2 - p_a)A) \\ \dot{p}_2 &= \frac{1}{V_0 + Ax} \left(\frac{G_{12}}{\mu} RT - p_2 Av \right) \end{aligned} \quad (5)$$

Equations (5) are a set of three equations for the stem (valve) position, stem (valve) velocity, and pressure in chamber 2. Pressure in chamber 1 is considered a control input.

III. MOTIVATING EXAMPLE ILLUSTRATING THE PROBLEM

Over the years, many different design and tuning methods with different rules and coefficients for PI and PID controller, have been presented. A fairly complete survey up to 1993 has been done by Astrom, et al [2]. Many of these methods are based on the closed-loop test proposed by Ziegler and Nichols in the 1940s [14], with subsequent modifications aimed at the improvement of tuning performance [1], [9]. Some effort has also been aimed at finding an analytical approach for tuning and obtaining optimal PI and PID parameters but this has been limited to linear approximations and low-order plant models [10], [7]. It is a known fact that most of the actual processes in industries are nonlinear and involve high-order dynamics. As a result many loops in practice display different performance from the one that is predicted by respective linear models. Application of tuning rules developed according to [14] and [1] requires the knowledge of some specific information about the system such as ultimate gain and ultimate period. These characteristics of the system are obtained through the closed-loop ZN test [14] or relay feedback test [1]. The same characteristics defined in a different way (which is advantageous for the PI/PID controller tuning) can be obtained via the MRFT [4], [5]. The MRFT is used in the present paper as a foundation for designing the tuning rules. MRFT is a method that focuses on obtaining critical information of a system, namely K_u and T_u through the introduction of a discontinuous control algorithm in a closed-loop system. MRFT has such advantages over the other two methods mentioned above as providing the desired value of gain margin exactly in a linear system with a PI/PID controller, thus, better suiting the purpose of ensuring the required stability in comparison with original RFT introduced in [1].

A summary of six classic and recently proposed tuning rules for PI and PID controllers are given in Table I. These six rules are introduced and assessed below based on the

TABLE I: The six tuning rule coefficients

Rule Name	Tuning Parameters		
	C_1	C_2	C_3
Classic Z-N(PID) [14]	0.6	0.5	0.125
Classic Z-N(PI) [14]	0.45	0.83	0
IAE (Pessen) [11]	0.7	0.4	0.15
SOR [12]	0.33	0.5	0.33
NOR [12]	0.2	0.5	0.33
Non-parametric tuning [4]	0.327	0.8	0

following transfer function of a PID controller.

$$W_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right) \quad (6)$$

where K_p , T_i and T_d are the PID parameters. The PI and PID controller parameters are simply calculated via following formulas:

$$\begin{cases} K_p = C_1 \cdot K_u \\ T_i = C_2 \cdot T_u \\ T_d = C_3 \cdot T_u \end{cases} \quad (7)$$

where the coefficients C_1 , C_2 , and C_3 for different tuning rules are given in Table I. (please also note the difference of the definition of K_u , T_u between MRFT and other methods).

The response of the presented flow loop model with the controller tuned through each of the tuning rules from Table I is shown in Fig. 2 to illustrate the problem we are dealing with. It is worth noting that mostly overshoot and settling time are considered as criteria for the purpose of performance comparison. Fig. 2 shows the step response of the closed-loop flow process system (introduced in Section II) with a PI/PID controller at 10% increment from the steady state condition corresponding to 50% of the valve opening. The set point increment of 10% of the steady state value was applied at the 30th second after the system reached the steady state conditions. Each line shows the performance of the flow loop system tuned through a particular tuning rule.

One can see that the loop response can show oscillatory transients in the half of the cases. From the practical point of view, even if these oscillatory responses would not lead to system instability, they could be a source of disturbance for other loops in the system. Also it is noticeable that even when we have a smooth response by using NOR or Non-parametric tuning rules, the response is slow. In fact, the mentioned tuning rules work quite well when the system is linear, and the process response is monotonic, sluggish, and dominated by a single-pole exponential "lag". However, linear models of the process do not always adequately describe it. In fact, if the process is controlled by a control valve with pneumatic, hydraulic or electric actuation, the presence of the valve makes the dynamics nonlinear. Therefore, characteristics of the tuned nonlinear loop differ from those of a linear loop. This is the problem we are going to solve by directly using a nonlinear model of flow loop instead of using linearization or approximation methods for the development of optimal tuning rules. Another advantage of the developed tuning rules comes from the use of MRFT instead of the original relay feedback test.

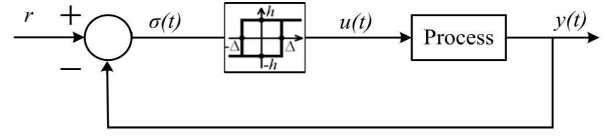


Fig. 3: Modified relay feedback test for increments of process variable and control

IV. PID CONTROLLER TUNING THROUGH MODIFIED RELAY FEEDBACK TEST

MRFT is given by the following algorithm: [5]

$$u(t) = \begin{cases} h & \text{if } \sigma(t) \geq \Delta_1 \\ & \text{or } (\sigma(t) \geq -\Delta_2 \text{ and } u(t-) = h) \\ -h & \text{if } \sigma(t) \leq \Delta_2 \\ & \text{or } (\sigma(t) \leq -\Delta_1 \text{ and } u(t-) = -h) \end{cases} \quad (8)$$

where $\Delta_1 = \beta \sigma_{max}$, $\Delta_2 = -\beta \sigma_{min}$ and σ_{max} and σ_{min} are last singular points of the error signal $\sigma(t) = r - y(t)$ corresponding to a last maximum or minimum value of $\sigma(t)$ after crossing the zero level, β is a positive constant parameter, r is the reference signal (set point), $y(t)$ is the output of the system (process variable). MRFT over the process can be illustrated by the diagram Fig. 3, in which Δ is a variable.

The describing function of the algorithm (8) is obtained in [5] as follows:

$$N(a) = \frac{4h}{\pi a} \left(\sqrt{1 - \beta^2} - j\beta \right) \quad (9)$$

Parameters of the oscillations generated by MRFT can be found from the harmonic balance equation:

$$W_p(j\Omega_0) = -\frac{1}{N(a_0)} \quad (10)$$

where a_0 is the amplitude of the periodic motions and Ω_0 is its frequency. The negative reciprocal of the DF is given as follows:

$$-\frac{1}{N(a)} = -\frac{\pi a}{4h} \left(\sqrt{1 - \beta^2} + j\beta \right) \quad (11)$$

Finding a periodic solution in the system Fig. 3 with MRFT has a simple graphic interpretation (Fig. 4). If the process is linear (or linearized about an operating point), the point of intersection of the Nyquist plot of the process and of the negative reciprocal of the DF, which is a straight line that begins in the origin and makes a counterclockwise angle with the negative part of the real axis, gives the point of the periodic solution, with respective values of the amplitude and the frequency. The results presented in [6] show that in the flow control loop the change of an operating point does not result in a significant change of the tuning coefficients. Tuning coefficients have stronger dependence on the amplitude of the relay control in MRFT, which for the flow loop ranges from 5-20% of valve motion. It is also shown in [6] that the use of smaller amplitudes in MRFT may

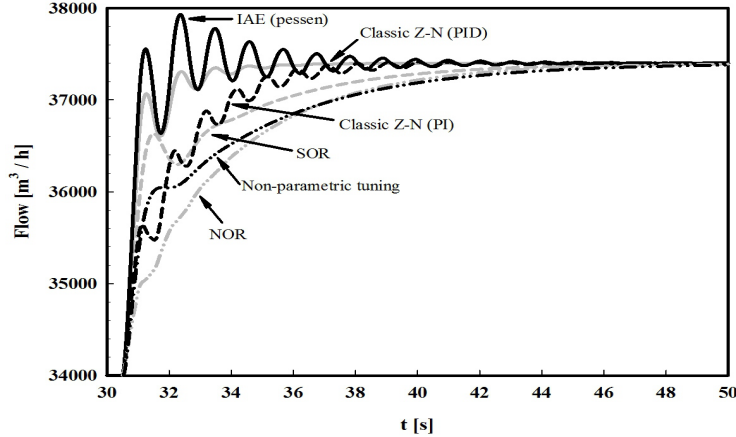


Fig. 2: Performance of the Flow loop system tuned by the six the tuning rules for PI and PID controllers

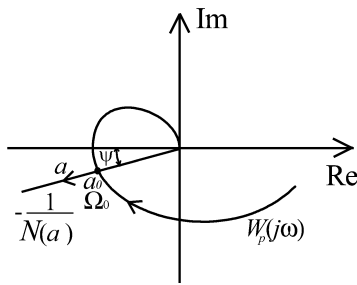


Fig. 4: Finding periodic solution

result in the deterioration of stability (oscillatory response) when the error (difference between the set point and the actual flow) becomes high. However, the use of the MRFT amplitudes from the range of 10 to 20% provides tuning rules consistent with the modes of operation of the flow loop. Therefore, with a reasonable accuracy we can use the MRFT amplitude of 10% and methodology normally applicable to linear systems, in the problem being solved that involves a nonlinear model.

Frequency Ω_0 and amplitude a_0 are unknown variables in this equation. They are found from the complex equation (10) for the problem of analysis of periodic motions. In the problems of identification and tuning, Ω_0 and a_0 are obtained from the MRFT, and on the basis of the measurements obtained either parameters of the underlying model are calculated (for parametric tuning) or tuning parameters are calculated immediately from Ω_0 and a_0 (for non-parametric tuning).

For the PID controller given by the transfer function described by equation (6) and the tuning rules format described above (7) where C_1 , C_2 and C_3 are constant parameters that define the tuning rules, the following equality constraint must be satisfied for the closed-loop system with PID controller to be stable with gain margin γ_m : [5]

$$\gamma_m C_1 \sqrt{1 + \left(2\pi C_3 - \frac{1}{2\pi C_2}\right)^2} = 1 \quad (12)$$

MRFT must be executed with the following value of parameter β to provide the specified gain margin:

$$\beta = -\text{sinarctan}\left(2\pi C_3 - \frac{1}{2\pi C_2}\right) \quad (13)$$

Using the described MRFT, we can now extend the approach to finding the optimal values of the tuning coefficients for the nonlinear model of the flow process.

V. OPTIMAL TUNING COEFFICIENT FOR PID CONTROLLER

A. Performance criteria and optimization problem definition

In optimization problems, when the plant model is known, the parameters of the PID controller and tuning rules may be optimized by minimizing one of the performance criteria listed above. It is worth mentioning that integral performance criteria are time-domain criteria and represent certain characteristics of the step response of the closed-loop system. The authors chose Integral of the Squared Error function (ISE) as a performance criterion to derive the tuning rules, because, as the undertaken simulation shows, this criterion (the step response pattern) better suits the goal of tuning of the flow loop than other criteria. In particular, because of the square term, both overshoot and undershoot contribute a lot to the value of the cost function, which eventually leads to the tuning rules without significant overshoot/undershoot. Therefore, the ISE criteria is suitable for conditions where small overshoot and short settling time are required, which makes it suitable for other types of process loops too: pressure, temperature, and level control loops. At the same time, the robustness of the designed PID controller is guaranteed by the selected gain margin, which is ensured by the tuning rules coupled with MRFT. It is also worth noting that ISE can easily be computed from a step response.

Given above description about performance criteria candidates, the optimization problem and associated cost function can be defined as the solution for a constrained nonlinear

TABLE II: Optimal tuning coefficients for PI controller optimized by ISE criteria

Tube Size	Coefficient	GAIN MARGIN			
		2	3	4	5
0.5A _t	C ₁	0.43	0.26	0.17	0.14
	C ₂	0.26	0.20	0.15	0.15
A _t	C ₁	0.44	0.27	0.17	0.14
	C ₂	0.30	0.22	0.15	0.15
2A _t	C ₁	0.44	0.28	0.17	0.16
	C ₂	0.30	0.25	0.15	0.21

programming problem as described by the following equation:

$$\begin{cases} \min f(x_i) \\ g(x_i) = 0 \quad i = 1, 2, 3 \end{cases} \quad (14)$$

where $(x_1, x_2, x_3) = (C_1, C_2, C_3)$, and the constraint $g(x_i)$ is the group of all constrain. In the case of the considered optimization problem, the constraint is given by (12), with relation between the coefficient of the tuning rules and controller parameters K_p , T_i and T_d are given by (7). It represents the PID (or PI) controller parameters such as proportional gain, integral gain and derivative gain. Function $f(x_i)$ is the cost function (optimization criterion) with the arguments being the coefficients C_1 , C_2 , C_3 of the tuning rules. Function $f(x_i)$ represents the integral performance criterion.

B. Optimization method and configuration

Gradient descent is selected as the optimization method for this work. This choice is made in favor of the simplicity, efficiency and accurate performance of the Gradient descent [13]. Given the Gradient method as the optimization method the Optimization algorithm consists of following four steps:

- 1) Take a sample for C_2 . Smart selection of initial PID settings would help the optimization process in point of view of fast convergence and result accuracy.
- 2) Calculate C_1 from the constraint (for a given gain margin) from equation (12)
- 3) Compute β per (13) and run MRFT; using the nonlinear process model; measure Ω_0 and a_0
- 4) Calculate K_p , T_i
- 5) Run step test and find the ISE value of the cost function
- 6) Compute coordinates of a new point (C_2) via the gradient descent and return to step 2

It is important to note that we optimize the tuning rules but not the controller parameters. Also, because of the consideration of the PI controller, we set $C_3 = 0$.

VI. SIMULATION AND RESULTS

All simulations and optimization were done in the MATLAB/Simulink environment. A flow model with typical set of parameters was selected for the tuning rules optimization task. It is worth noting that if the valve and actuator are matching and properly adjusted (so that the maximum force developed by the actuator, and the selected spring allow for the travel of the valve between 0 and 100%) then the

equation of the valve motion becomes invariant (at least approximately) to the valve and actuator size: the increase of A , m , b , and k by the same number of times would result in the same motion of the valve (subject to the same $p_2(t)$). On the other hand, it becomes difficult to provide the same law of change of $p_2(t)$ because of the use of the same I/P transducers for all valve sizes. Therefore, the parameter that best describes the variety of flow applications is the orifice of the pneumatic line (that includes the transducer too) A_t . We totally realize that this is a simplified view at the variety of flow applications but, as simulations show, it makes sense and allows us to produce a useful result.

The model of the flow process involves typical parameters of the control valve and actuator with instrument air pressure in the actuator varying within 3–15psig and valve stroke 42mm. Other actuator and valve parameters are as follows [8]: effective diaphragm surface area $A = 0.0028m^2$, effective tube surface area $A_t = 0.00001963m^2$, spring rate $k = 5200N/m$, mass of stem and associated moving parts $m = 1.36kg$, viscous friction $b = 2425kg/s$, maximal water flow (at 100% of valve opening) 68,000kg/h, valve characteristic is linear. Additionally, the model contains a 0.5s dead time, which corresponds to the typical execution period of the DCS (we consider that this time also includes some small dead time in the transducer model).

For the considered type of process (flow loop), only PI controller tuning rules were generated, as this corresponds to the industrial practice, where PID controllers are not used for flow control due to the noisy flow signal and even theoretically provide just marginal advantage over PI control in terms of performance. The proposed optimal tuning coefficients are given in Table II for PI controller controller. The optimal tuning coefficients are given for three different tube sizes and for gain margin varying between 2 and 5. These values of the gain margin are selected to cover the range of possible desired types of tuning ranging from “fast” to “slow”. The values of the ultimate gain are determined in the optimization algorithm as $\frac{4h}{a_0}$, where a_0 is the oscillations amplitude.

Results of application of the 10% step change to the set point of the optimally tuned PI controller, from the steady state corresponding to 50% of flow, at the 30th second, are given in Fig. 5.

Fig. 5 shows that the performance of the flow loop tuned through the presented approach provides a good step response pattern, with a small overshoot for gain margin 2 and no overshoot for higher values of gain margin, and sufficiently fast response (also better in comparison with all the above mentioned tuning rules). Also, the results show that the larger the tube size the slower the response, which agrees with real process observations. It can be translated into the valve size as the responses for smaller tube size corresponding to application with larger valve size (larger size of the actuator chamber requires more time for the pressure to change by the same value, which can be equivalently simulated by a smaller actuator and a smaller cross-sectional area of the pneumatic line).

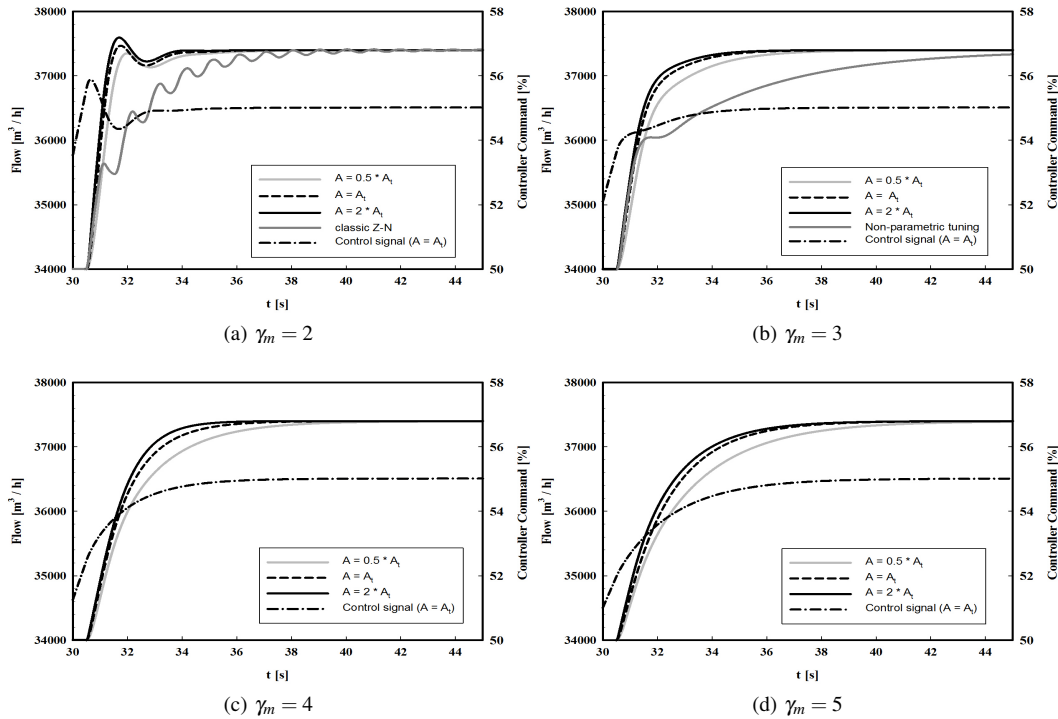


Fig. 5: Step response of flow loop with PI controller tuned and optimized by ISE criteria per different gain margins

VII. CONCLUSION

An attempt to design PI controller tuning rules specific for the flow loop has been made in the present paper. The authors believe that tuning rules designed for a specific types of processes (like flow, pressure, temperature, level) can be more efficient than generic tuning rules. The designed tuning rules are coupled with the modified relay feedback test that ensures that the selected gain margin can be provided exactly, which guarantees the necessary degree of robustness. The results are generated and presented in the table format (Table II). The results of this table can be used as follows. At first, the user determines whether the application is a small, medium-size or large valve. This classification is approximate but can provide a better tuning than having uniform tuning rules. For small valve size, the 3rd line ($2A_t$) should be used, for medium-size - the second line (A_t), and for large valve - the first line ($0.5A_t$). Then the user determines what kind of tuning he/she needs: from fast to slow and selects the respective gain margin value. And after that the MRFT is executed. It is worth noting that the above-noted selection must be made before the test because the test itself depends on the coefficients that are defined by the above selections. The authors find the proposed approach to tuning advantageous and intend to continue their research in the direction of generating process-specific tuning rules, extending the present approach to pressure, temperature, and level loops.

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