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Distributed Multi-Target Tracking via Mobile Robotic Networks: a Localized Non-iterative SDP Approach

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Abstract— **We consider a robotic network composed of mobile robots capable of communicating with each other. We study the problem of collectively tracking a number of moving targets while maintaining a certain level of connectivity among the robots, by moving them into appropriate positions. The distances of the robots to each other and to the targets are used to define a communication and target tracking graph, respectively. We formulate the combined global objective as a Semi-Definite Program (SDP) and propose a non-iterative distributed solution consisting of localized SDP's which use information only from nearby neighboring robots. Numerical simulations illustrate the performance of the algorithm with respect to the centralized solution.**

I. Introduction

Groups of mobile robots capable of communicating with one another to collaboratively achieve a common goal, often referred to as Robotic Networks, offer great promise in applications ranging from underwater and space exploration [1], to search, rescue, and disaster relief [2], monitoring and surveillance [3]. Collaborative multi-target tracking is considered as a key enabling capability in the above scenarios, which require maintaining a certain level of connectivity among the robots while simultaneously ensuring that independent moving "targets" stay in the visual/detection range of the robots themselves.

The works of [4]–[10] provide a comprehensive overview of the multi-target tracking problem. Typical approaches consider a cost function based on the Fisher Information Matrix in order to determine robot movements that lead to an increase in the targets' visibility. However, even for a single target, the resulting optimization problem is nonlinear and NP-hard [10]. As a result, several alternative formulations relying on potential fields, gradient-descent, Monte Carlo methods, and linear approximations have been proposed, by sacrificing robot connectivity / target visibility guarantees, generality of the framework, or real-time applicability. Recently, an approximate formulation of the problem has been suggested using Semi-Definite Programming [11], [12], which is based on the tools of [13], [14]. Contrary to the aforementioned literature, this framework allows both the connectivity of the robotic network and the visibility of the targets to be considered *simultaneously*, in the same optimization problem. For this purpose two distance-dependent graph Laplacians are defined (communication and visibility) in order to describe the interconnection between robots and targets. The approach assumes that each robot is capable of communicating with any other team-mate and it guarantees performance for an arbitrary number of robots and targets.

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In this paper, we first describe a more general version of the approach in [11], [12] that accounts for uncertainty in the targets' positions, and we formulate the problem as a joint maximization of connectivity and visibility. Then, as our main contribution, we propose a distributed version of the solution that addresses the realistic scenario where robots are capable of communicating only with a few nearby team-mates. Our distributed approach relies on localized Semi-Definite Programs (SDPs) that are solved by each robot using information only from nearby neighbors. In contrast to iterative schemes such as super-gradient algorithms, e.g., [15], our approach does not require extensive communication and iterations among the robots to converge to the final solution, making it more suitable for real-time applications. In addition, our proposed solution is characterized by the following: *(i)* the local problems are derived via a suitable decomposition of the centralized one, *(ii)* both connectivity and tracking are guaranteed, *(iii)* the local solutions are feasible with respect to the constraints of the original centralized problem, and *(iv)* the local cost function exhibits the same improvement property as the global cost in the linearized approximation.

The paper is organized as follows. Section II formulates the approximate centralized problem. The proposed distributed approach is described in Section III. Numerical simulations are shown in Section IV to illustrate the behavior of the distributed solution and compare it with the centralized case. Conclusions and open issues are discussed in Section V.

II. Problem formulation

We consider a group of $i = 1, \ldots, N$ mobile robots (agents) and $q = 1, \ldots, M$ moving targets. We denote as $a_j(k)$ the value of variable a for agent/target j at time k, while we use the following notation for a change in its value: $\delta a_j(k) = a_j(k) - a_j(k-1)$. I_N represents the identity matrix of dimension $N \times N$ and $||\cdot||$ is the Euclidean norm.

Let $x_i(k) \in \mathbb{R}^2$ be the position of the i-th agent. For simplicity of exposition, as in [11], [13], we assume discrete-time agent dynamics of the following form:

$$x_i(k) = x_i(k-1) + v_i(k-1)\Delta t, \qquad i = 1, \ldots, N \tag{1}$$

where $v_i(k)$ is the velocity control input and Δt the sampling time. We assume $||v_i(k)|| \leq v_{\max,i}$. Let $x(k) \in \mathbb{R}^{2N}$ be the collection of the agents' positions, i.e., $x(k) = (x_1^\top(k), \ldots, x_N^\top(k))^\top$. Let $z_q(k) \in \mathbb{R}^2$ be the position of the q-th target at time k, and let $z(k) = (z_1^\top(k), \ldots, z_M^\top(k))^\top$ be the collection of the targets' positions. We assume that the agents know their own position and the position of the targets they can detect, and that they have computa-

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tion/communication capability onboard. We assume the targets can be represented as discrete-time dynamical systems

$$z_q(k) = z_q(k-1) + w_q(k-1)\Delta t, \qquad q = 1, \ldots, M \tag{2}$$

where $w_q(k) \in \mathbb{R}^2$ is a bounded noise term, i.e. $||w_q(k)|| \leq w_{\max,q}$. The set of reachable positions $\mathcal{Z}_q(k)$ at time k, is the disc centered at the previous known position $z_q(k-1)$ with radius $w_{\max,q}\Delta t$. In order to ensure that the tracking problem is solvable, we make the following assumption:

Assumption 1: (Slow targets) The maximum noise input is less than the agents' maximum velocity, i.e., $w_{\max,q} < v_{\max,i}$ for all pairs (q, i).

Let the agent clocks be synchronized, and assume perfect communication (no delays or packet losses). We model the communication network among agents as graphs. The set $\mathcal{S}$ contains the indices of the mobile agents (nodes), with cardinality $|\mathcal{S}| = N$. We use $\mathcal{E}$ to indicate the set of communication links. The communication graph $\mathcal{G}_C$ is then expressed as $\mathcal{G}_C = (\mathcal{S}, \mathcal{E})$. The graph $\mathcal{G}_C$ is assumed to be undirected. All the agents with which agent i is connected to are called neighbors and are contained in the set $\mathcal{N}_i$. Note that node i is not included in the set $\mathcal{N}_i$. We define $\mathcal{N}_i^+ = \mathcal{N}_i \cup \{i\}$ and $N_i = |\mathcal{N}_i^+|$. The collection of agents that are within the detection range of target q is defined as $\mathcal{N}_q$. These are considered as the neighboring agents of target q. We introduce the following assumptions:

Assumption 2: (Initial feasibility) At the initial time, the communication graph $\mathcal{G}_C$ is connected and $\forall q$, $|\mathcal{N}_q| > 0$.

Assumption 3: (Well-posedness) At any time $k > 0$, there exists an $x(k)$, independent of $x(k-1)$, which guarantees that the communication graph $\mathcal{G}_C$ remains connected and for each target q, $|\mathcal{N}_q| > 0$.

This last assumption ensures that the problem is well-posed, but it does not guarantee feasibility at each time step. In fact, $x(k)$ depends on $x(k-1)$ via the dynamical equation (1), therefore the $x(k)$ provided by the assumption could be unreachable, given the current position $x(k-1)$. In practice, Assumption 3 requires that the targets do not move arbitrarily far away from each other.

Let L_C be the Laplacian matrix associated with the communication graph $\mathcal{G}_C$. Its entries, ℓ_{ijC}, depend on weights, $0 \leq c_{ij} \leq 1$, which represent the "connection strength" between agents i and j. Therefore, $\ell_{ijC} = -c_{ij}$ if $(i,j) \in \mathcal{E}, i \neq j$, $\ell_{ijC} = \sum_{l \neq i} c_{il}$ if $i = j$, and $\ell_{ijC} = 0$ if $(i,j) \notin \mathcal{E}$.

In a similar way, we use the weights $0 \leq v_{qi} \leq 1$ to model the link between target q and agent i, if they fall within the detection range. The weights c_{ij} and v_{qi} are assumed to depend on the physical distance between the nodes according to

$$c_{ij} = f_C(||x_i(k) - x_j(k)||^2), \quad v_{qi} = f_V(||z_q(k) - x_i(k)||^2), \tag{3}$$

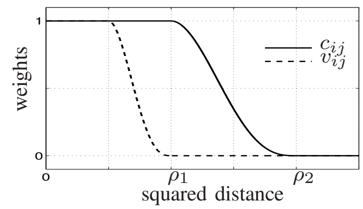
where $f_C : \mathbb{R}^2 \times \mathbb{R}^2 \to [0,1]$ and $f_V : \mathbb{R}^2 \times \mathbb{R}^2 \to [0,1]$ are smooth nonlinear functions with compact support. For the proposed distributed solution in Section III, we assume:

Assumption 4: $2\,\mathrm{supp}(f_V) < \mathrm{supp}(f_C)$, where $\mathrm{supp}(\cdot)$ stands for support. In other words, each agent that can detect a target is assumed to be able to communicate with all other agents that detect the same target.

The weights model the interconnection strength between two nodes. The closer two nodes are the higher the weight is, representing an increase in the communication / detection "quality". For simulation purposes we use the functions qualitatively represented in Tab. I, a more detailed discussion on weight selection can be found in [11].

TABLE I

THE WEIGHTING FUNCTIONS.



To characterize how a target is connected to agents we introduce the sum of the detection weights v_{qi} of a target q as $v_q = \sum_{i \in \mathcal{N}_q} v_{qi}$; we note that if $v_q > n$ then $|\mathcal{N}_q| > n$.

We are interested in maximizing visibility of the targets *and* maximizing communication connectivity among the agents. This can be posed as the joint maximization of the algebraic connectivity of the communication and detection graphs, respectively, by moving the agents into appropriate positions. This goal can be formulated in each discrete time step k as the following optimization problem, [11], [16]:

$$\mathbf{P}: \quad \max_{x(k), \gamma(k), \nu_{[1,\ldots,M]}(k)} \quad \alpha\gamma(k) + \sum_{q=1}^{M} \beta_q \nu_q(k) \tag{4a}$$

subject to:

$$\gamma(k) > 0, \nu_q(k) > 0 \qquad q = 1, \ldots, M \tag{4b}$$

$$L_C(x(k)) + \mathbf{1}\mathbf{1}^T \succ \gamma(k) I_N \tag{4c}$$

$$v_q(z_q(k), x(k)) > \nu_q(k), \quad \forall z_q(k) \in \mathcal{Z}_q(k), \quad q = 1, \ldots, M \tag{4d}$$

$$||x_i(k) - x_i(k-1)|| \leq v_{\max,i}\Delta t \quad i = 1, \ldots, N \tag{4e}$$

where the decision variables are the agents' locations and the values of $\gamma(k)$, $\nu_q(k)$'s. Here the constants $\alpha \geq 0$ and $\beta_q \geq 0$, $q = 1, \ldots, M$ model the relative weights on the maximization goals. When one selects $\alpha = 0$, as in [11], the problem becomes the maximization of detection connectivity with the targets while guaranteeing that the communication graph remains connected.

This problem (4) is non-convex given that we are optimizing over the positions x and the entries of the Laplacians are nonlinear functions of x. As a standard convex approximation to this problem formulation, [11], [13], [17], the following first-order Taylor expansions are used:

$$c_{ij}(k) = c_{ij}(x(k-1)) + \underbrace{\left(\frac{\partial f_C}{\partial x_i}\right)^{\top}_{x(k-1)}}_{a_{ij}^{\top}} (\delta x_i(k) - \delta x_j(k)) \tag{5}$$

$$v_{qi}(k) = v_{qi}(z_q(k-1), x_i(k-1)) + \underbrace{\left(\frac{\partial f_V}{\partial x_i}\right)^{\top}_{(x(k-1), z_q(k-1))}}_{h_i^{\top}} (\delta x_i(k) - \delta z_q(k)) \tag{6}$$

We employ the symbol Δ to denote such linearized entities: $\Delta L_C(x(k))$ has entries as expressed in (5), while $\Delta v_q(z_q(k), x(k))$ is composed of the weights expressed in (6). This allows us to formulate the following convex approximation of the problem (4):

$$\Delta\mathbf{P}(x(k-1), z(k-1), v_{\max,i}):$$
$$\max_{x(k),\gamma(k),\nu_{[1,\dots,M]}(k)} \alpha\gamma(k) + \sum_{q=1}^{M} \beta_q \nu_q(k) \tag{7a}$$
subject to:
$$\gamma(k) > 0, \nu_q(k) > 0 \qquad q = 1,\dots,M \tag{7b}$$
$$\Delta L_C(x(k)) + \mathbf{1}\mathbf{1}^T \succ \gamma(k) I_N \tag{7c}$$
$$\Delta v_q(z_q^*(k), x(k)) > \nu_q(k) \qquad q = 1,\dots,M \tag{7d}$$
$$||x_i(k) - x_i(k-1)|| \le v_{\max,i}\Delta t \qquad i = 1,\dots,N \tag{7e}$$

where $z_q^*(k)$ is the *worst-case* $z_q(k)$, which due to the linearity of the scalar inequality (7d) can be computed analytically (see Appendix).

Define $\nu_q^-(k) := \nu_q(k-1) - \sum_{j\in\mathcal{N}_q} h_j^\top \delta z_q^*(k)$ which represents the decrease of the detection quality due to the targets' motion. We note that $\nu_q^-(k) \ge 0$ by the definition of the weighting functions, therefore $\nu_q(k-1) \ge \sum_{j\in\mathcal{N}_q} h_j^\top \delta z_q^*(k)$. This condition is useful when checking the validity of the employed Taylor approximations.

The cost function of problem (7) at each time step k satisfies:

$$\alpha\gamma(k) + \sum_{q=1}^{M} \beta_q \nu_q(k) \ge \alpha\gamma(k-1) + \sum_{q=1}^{M} \beta_q \nu_q^-(k) \tag{8}$$

which indicates that the agents move in a way that improves the cost function if we consider only the current target locations. This inequality also implies that when the targets are stationary, the cost function is monotonically increasing.

The optimization problem that has been described in this section attempts to solve the joint connectivity and detection maximization problem in a centralized manner using linearization. In realistic application scenarios, computing the desired positions and the corresponding motion commands for the robots cannot be performed in a single centralized location due to computational and communication constraints. In the next section, we describe a solution approach that allows the problem to be solved in a distributed fashion, using local computation and limited communication resources, which increases the flexibility of the robotic network and is thus appealing in practice.

III. The Proposed Distributed Solution

In this section we present a non-iterative distributed solution to solve (7). We note that this is not a trivial task, since commonly use decomposition methods (if applicable, e.g. in [15]) typically require iterative solutions which may not be amenable to fast real-time implementations. Before presenting the main contribution of this paper, we first introduce some notation and definitions. We then proceed to describe our non-iterative distributed solution method and its properties.

In order to formulate the local problems each agent will be solving, we define subgraphs that correspond to the agents and their neighborhood. Let $\mathcal{J}_i$ denote the *enlarged* neighborhood for each agent i defined as

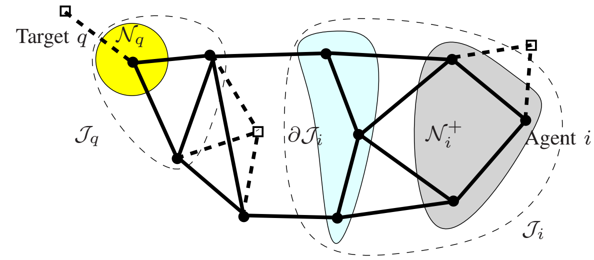


Fig. 1. Notation for the distributed solution. The targets are represented via squares, while the agents are circles. Communication links are shown via solid lines, whereas detection links via dashed lines.

$$\mathcal{J}_i = \bigcup_{l\in\mathcal{N}_i^+} \mathcal{N}_l^+, \qquad i = 1,\dots,N \tag{9}$$

whose cardinality will be J_i. We denote the vector containing all the positions of the agents in this set with $x_{\mathcal{J}_i}$. We define

$$\partial\mathcal{J}_i = \{l | l \in \mathcal{J}_i, l \notin \mathcal{N}_i^+\}, \qquad i = 1,\dots,N \tag{10}$$

as the bordering set of $\mathcal{J}_i$, while we call the set of agents belonging to $\partial\mathcal{J}_i$, the bordering agents of $\mathcal{J}_i$. We denote the communication graph Laplacian associated with subgraph $\mathcal{J}_i$ as L_{C_i}. We will assume that agent i is aware of the targets it can detect directly and also the ones his neighbors can detect. We will denote with $\mathcal{T}_i$ the set of all the targets that agent i is aware of. Correspondingly, we denote the vector containing all the positions of the targets in the set $\mathcal{T}_i$ with $z_{\mathcal{T}_i}$. Similarly to the enlarged neighborhood set for the agents we introduce the enlarged neighborhood set for the targets, indicating which agents are aware of a specific target q:

$$\mathcal{J}_q = \bigcup_{l\in\mathcal{N}_q} \mathcal{N}_l, \qquad q = 1,\dots,M \tag{11}$$

whose cardinality is J_q. We note that these neighborhood sets contain only agents, and thus the maximum allowed cardinality is N. Figure 1 provides a graphical illustration of this notation. We also introduce a scaled maximum velocity $\tilde{v}_{\max,i}$ defined as

$$\tilde{v}_{\max,i} = \frac{N}{J_i} v_{\max,i}, \quad i = 1,\dots,N \tag{12}$$

whose value varies from agent to agent. This quantity will be used to change the local constraints in such a way that the global solution constructed from the local ones satisfies the original constraint (7e).

Our algorithm consists of two phases. First, each agent solves problem $\Delta\mathbf{P}_i$ defined as

$$\Delta\mathbf{P}_i : \Delta\mathbf{P}(x_{\mathcal{J}_i}(k-1), z_{\mathcal{T}_i}(k-1), \tilde{v}_{\max,i}) \tag{13a}$$
$$\text{s.t.} \quad x_j(k) = x_j(k-1), \qquad \text{for } j \in \partial\mathcal{J}_i \tag{13b}$$

computing the solution $\hat{x}_{\mathcal{J}_i}(k)$, which is composed of $\hat{x}_{ij}(k)$ for each $j \in \mathcal{J}_i$. Thus, we will call $\hat{x}_{ij}(k)$ the position of agent j as computed by agent i. Note that the extra constraint (13b) is an important requirement for feasibility of the local problems as will be explained later in this section.

As a second phase, the solutions $\hat{x}_{\mathcal{J}_i}(k)$ are shared within the enlarged neighborhood $\mathcal{J}_i$ and averaged according to

$$x_i(k) = x_i(k-1) + \sum_{j\in\mathcal{J}_i} \frac{1}{N} \delta\hat{x}_{ji}(k), \qquad i = 1,\dots,N \tag{14}$$

Algorithm 1 summarizes the method. Note that steps 3-5 are implemented only once between subsequent robot movements, which makes the algorithm non-iterative.

Algorithm 1 Distributed algorithm.

1: Input: $x_{\mathcal{J}_i}(k-1), z_{\mathcal{T}_i}(k-1)$
2: Determine: $z^*_{\mathcal{T}_i}(k)$
3: Solve: $\Delta\mathbf{P}_i$ in (13) computing $\hat{x}_{ij}(k)$, $j \in \mathcal{J}_i$
4: Communicate: $\hat{x}_{ij}(k)$ among members of $\mathcal{J}_i$
5: Average: $x_i(k) = x_i(k-1) + \sum_{j\in\mathcal{J}_i} \frac{1}{N}\delta\hat{x}_{ji}(k)$
6: Output: $x_i(k)$

We claim that *if we consider the global position vector* $x(k) = (x_1^\top(k), \ldots, x_N^\top(k))^\top$ *resulting from* (14), then

(C1) The algebraic connectivity of the corresponding global linearized Laplacian $\Delta L_C(x(k))$ and $\Delta v_q(z^*_q(k), x(k))$ are strictly positive;
(C2) all the constraints of the global problem are met.

Furthermore we claim that, as in the centralized approach:

(C3) The improvement property (8) remains to be valid for the cost function of $\Delta\mathbf{P}$, which is monotonically increasing when the targets are stationary.

We will prove these claims in three steps: Theorems 1, 2, and 3 establish (C1), by linking the average value (14) and the algebraic connectivity through the linear dependence of the linearized Laplacians on x. The constraint (13b) plays a crucial role here to ensure the feasibility of the local solutions. Theorem 4 guarantees (C2), by showing how the scaled velocity (12) of the local problems ensure that the global solution, obtained via the average (14), satisfies the global constraints. Theorem 5 establishes (C3) by linking the variations of the local cost functions with the one of the global problem.

Consider the local problem $\Delta\mathbf{P}_i$ and its solution comprised of $\hat{x}_{ij}(k)$ for all $j \in \mathcal{J}_i$. Construct the vector

$$\hat{x}^{(i)}(k) = (x_1^\top(k-1), \ldots, \hat{x}_{ij}^\top(k), \ldots, x_N^\top(k-1))^\top \tag{15}$$

where we keep those agent positions that have not been optimized fixed, and we update the rest from the solution of the local problem.

Theorem 1: (C1.0) The new positions $\hat{x}^{(i)}(k)$ keep the global linearized Laplacian matrix connected: $\Delta L_C(\hat{x}^{(i)}(k)) + \mathbf{1}\mathbf{1}^\top \succ 0$.

Proof. At time step $k-1$, the graph $\mathcal{G}_C(x(k-1))$ is connected. We can divide this graph in two overlapping parts, $\mathcal{G}_C(x_{\mathcal{J}_i}(k-1))$, which is connected by definition, and $\mathcal{G}_C(x_{\neg\mathcal{J}_i}(k-1)) \cup \mathcal{G}_C(x_{\partial\mathcal{J}_i}(k-1))$, where with $\neg\mathcal{J}_i$ we indicate the collection of agents not present in $\mathcal{J}_i$. At time step k we know that:

$$\mathcal{G}_C(x(k)) = \mathcal{G}_C(x_{\mathcal{J}_i}(k)) \cup \left(\mathcal{G}_C(x_{\neg\mathcal{J}_i}(k-1)) \cup \mathcal{G}_C(x_{\partial\mathcal{J}_i}(k-1))\right)$$

$$\mathcal{G}_C(x_{\mathcal{J}_i}(k)) \cap \left(\mathcal{G}_C(x_{\neg\mathcal{J}_i}(k-1)) \cup \mathcal{G}_C(x_{\partial\mathcal{J}_i}(k-1))\right) = \mathcal{G}_C(x_{\partial\mathcal{J}_i}(k-1)) \neq \{\emptyset\}$$

where we use the definition of $x^{(i)}(k)$ (15) and the constraint (13b) on the bordering agents. Noticing that $\mathcal{G}_C(x_{\mathcal{J}_i}(k))$ is also connected as imposed by the local optimization problem, the claim follows. □

Lemma 1: The following equality holds:

$$\Delta L_C(\delta x(k)) = \sum_{i=1}^{N} \frac{1}{N} \Delta L_C(\delta\hat{x}^{(i)}(k)) \tag{16}$$

Proof. Let us consider the entry (i,j) of the Laplacian L on both sides of the expression. For the right side, ℓ_{ij} is

$$\ell_{ij} = a_{ij}^\top \sum_{p\in\mathcal{J}_i\cap\mathcal{J}_j} \frac{\delta\hat{x}_{pi}(k) - \delta\hat{x}_{pj}(k)}{N}$$

For the left side,

$$\ell_{ij} = a_{ij}^\top\left(\delta x_i(k) - \delta x_j(k)\right) = a_{ij}^\top\left(\sum_{p\in\mathcal{J}_i}\frac{\delta\hat{x}_{pi}(k)}{N} - \sum_{p\in\mathcal{J}_j}\frac{\delta\hat{x}_{pj}(k)}{N}\right)$$

the last expression can be divided in three parts: $p \in \mathcal{J}_i \cap \mathcal{J}_j$, $p \in \mathcal{J}_i \wedge p \notin \mathcal{J}_j$, and $p \in \mathcal{J}_j \wedge p \notin \mathcal{J}_i$. Since $a_{ij}^\top \neq 0$ only if (i,j) are first order neighbors, we can make the key observation that: $\{p | p \in \mathcal{J}_i \wedge p \notin \mathcal{J}_j\} \subseteq \partial\mathcal{J}_i$ and $\{p | p \in \mathcal{J}_j \wedge p \notin \mathcal{J}_i\} \subseteq \partial\mathcal{J}_j$ which leads to:

$$\ell_{ij} = a_{ij}^\top \sum_{p\in\mathcal{J}_i\cap\mathcal{J}_j} \frac{\delta\hat{x}_{pi}(k) - \delta\hat{x}_{pj}(k)}{N} + \underbrace{a_{ij}^\top \sum_{p\in\mathcal{J}_i\wedge p\notin\mathcal{J}_j}\frac{\delta\hat{x}_{pi}(k)}{N}}_{=0} - \underbrace{a_{ij}^\top \sum_{p\in\mathcal{J}_j\wedge p\notin\mathcal{J}_i}\frac{\delta\hat{x}_{pj}(k)}{N}}_{=0}$$

where the last two terms are 0 due to constraint (13b). □

Theorem 2: (C1.1) The algebraic connectivity of the global linearized Laplacian $\Delta L_C(x(k))$ is strictly positive, $\Delta L_C(x(k)) + \mathbf{1}\mathbf{1}^\top \succ 0$ where $x(k)$ is computed by the average (14).

Proof. Theorem 1 implies $(\Delta L_C(\hat{x}^{(i)}(k)) + \mathbf{1}\mathbf{1}^\top)/N \succ 0$ for all i. Thus summing over all agents leads to

$$\sum_{i=1}^{N} \frac{1}{N}(\Delta L_C(\hat{x}^{(i)}(k)) + \mathbf{1}\mathbf{1}^\top) \succ 0$$

or

$$(\Delta L_C(x(k-1)) + \mathbf{1}\mathbf{1}^\top) + \sum_{i=1}^{N}\frac{1}{N}\Delta L_C(\delta\hat{x}^{(i)}(k)) \succ 0$$

Considering the weighted sum $x_i(k)$ in (14), and the associated global vector $x(k)$, by Lemma 1 follows the desired consistency property $\Delta L_C(x(k)) + \mathbf{1}\mathbf{1}^\top \succ 0$. □

Theorem 3: (C1.2) The local constraint $\hat{\nu}_q(k) > 0$ is a sufficient condition for all the targets to be connected at least to one agent, i.e., $\nu_q(k) > 0, \forall q = 1, \ldots, M$.

Proof. Consider target q, which appears in the local constraints of subproblem $\Delta\mathbf{P}_p$, $p \in \mathcal{J}_q$ as

$$\sum_{j\in\mathcal{N}_q} \Delta v_{qj}(z^*_q(k), \hat{x}_{pj}(k)) > \hat{\nu}_{qp}(k) > \hat{\nu}_q(k) > 0$$

for a suitable $\hat{\nu}_q(k)$. This constraint can be written in the equivalent form:

$$\sum_{j\in\mathcal{N}_q} h_j^\top \delta\hat{x}_{pj} + b_j > \hat{\nu}_q(k) > 0 \tag{17}$$

where:

$$0 \leq \sum_{j\in\mathcal{N}_q} b_j = \sum_{j\in\mathcal{N}_q} v_{qj}(z_q(k-1), x_{pj}(k-1)) - \sum_{j\in\mathcal{N}_q} h_j^\top \delta z^*_q(k) = \nu_q^-(k)$$

and we note that due to Assumption 4, $\forall p \in \mathcal{J}_q$ we have $\mathcal{N}_q \subseteq \mathcal{J}_p$, therefore constraint (17) can indeed be computed locally. Starting from Equation (17), summing over the p's and dividing by N:

$$\sum_{p\in\mathcal{J}_q}\sum_{j\in\mathcal{N}_q} h_j^\top \frac{\delta\hat{x}_{pj}(k)}{N} + \frac{b_j}{N} > \sum_{p\in\mathcal{J}_q}\frac{\hat{\nu}_q(k)}{N}$$

or

$$\sum_{p\in\mathcal{J}_q}\sum_{j\in\mathcal{N}_q} h_j^\top \frac{\delta\hat{x}_{pj}(k)}{N} > \frac{J_q}{N}\hat{\nu}_q(k) - \frac{J_q}{N}\sum_{j\in\mathcal{N}_q} b_j \tag{18}$$

Globally one would like to have

$$\sum_{j\in\mathcal{N}_q} h_j^\top \delta x_j(k) + b_j > \nu_q(k) > 0$$

substituting the average (14):

$$\sum_{j\in\mathcal{N}_q}\sum_{p\in\mathcal{J}_j} h_j^\top \frac{\delta\hat{x}_{pj}(k)}{N} + b_j > \nu_q(k) \tag{19}$$

since

$$\bigcup_{j\in\mathcal{N}_q}\mathcal{J}_j = \mathcal{J}_q \cup \left(\bigcup_{j\in\mathcal{N}_q}\partial\mathcal{J}_j\right)$$

and due to constraint (13b), then the expression (19) becomes:

$$\sum_{j\in\mathcal{N}_q}\sum_{p\in\mathcal{J}_q} h_j^\top \frac{\delta\hat{x}_{pj}(k)}{N} + b_j > \nu_q(k)$$

where we can switch the sum operators and substituting expression (18), we obtain

$$\frac{J_q}{N}\hat{\nu}_i(k) + \sum_{j\in\mathcal{J}_q} b_j - \frac{J_q}{N}b_j = \nu_q(k) > 0$$

for some $\nu_q(k)$. This leads to

$$\hat{\nu}_q(k) + \frac{N - J_q}{J_q}\nu_q^-(k) = \frac{N}{J_q}\nu_q(k). \tag{20}$$

Since we want $\nu_q(k) > 0$, knowing that $\nu_q^-(k) \geq 0$, this implies $\hat{\nu}_q(k) > 0$. □

Theorem 4: (C2) The global constraints (7b)-(7e) are met with the average solution (14).

Proof. The constrains (7b)-(7d) are ensured via Theorems 2-3. Consider now the constraint (7e), for each subproblem we have

$$||\delta\hat{x}_{ii}(k)|| < \tilde{v}_{\max,i} = \frac{N}{J_i}v_{\max,i}$$

and for the global problem:

$$||\delta x_i(k)|| < \sum_{j\in\mathcal{J}_i}\frac{1}{N}||\delta\hat{x}_{ji}(k)|| < v_{\max,i}$$

Thus $x(k)$ satisfies also (7e) and (C2) is established. □

Theorem 5: (C3) Similarly to the centralized case, the global cost function of $\Delta\mathbf{P}$ satisfies the following improvement property:

$$\alpha\gamma(k) + \sum_{q=1}^{M}\beta_q\nu_q(k) \geq \alpha\gamma(k-1) + \sum_{q=1}^{M}\beta_q\nu_q^-(k) \tag{21}$$

where the solution at time k is computed from the local problems with the average (14).

Proof. We start rewriting (21) in the equivalent semi-definite form:

$$\alpha\left(\Delta L_C(x(k)) + \mathbf{1}\mathbf{1}^\top\right) + I_N\sum_{q=1}^{M}\beta_q\nu_q(k) \succeq \alpha\left(\Delta L_C(x(k-1)) + \mathbf{1}\mathbf{1}^\top\right) + I_N\sum_{q=1}^{M}\beta_q\nu_q^-(k)$$

For optimality of the local problems, in each $\Delta\mathbf{P}_i$:

$$\alpha\left(\Delta L_{C_i}(\hat{x}_{\mathcal{J}_i}(k)) + \mathbf{1}\mathbf{1}^\top\right) + I_{|J_i|}\sum_{q\in\mathcal{T}_i}\beta_q\hat{\nu}_q(k) \succeq \alpha\left(\Delta L_{C_i}(\hat{x}_{\mathcal{J}_i}(k-1)) + \mathbf{1}\mathbf{1}^\top\right) + I_{|J_i|}\sum_{q\in\mathcal{T}_i}\beta_q\nu_q^-(k)$$

or

$$\alpha\Delta L_{C_i}(\delta\hat{x}_{\mathcal{J}_i}(k)) + I_{|J_i|}\sum_{q\in\mathcal{T}_i}\beta_q(\hat{\nu}_q(k) - \nu_q^-(k)) \succeq 0$$

For constraint (13b) and Assumption 4, this can be written as:

$$\alpha\Delta L_{C_i}(\delta\hat{x}^{(i)}(k)) + I_N\sum_{q\in\mathcal{T}_i}\beta_q(\hat{\nu}_q(k) - \nu_q^-(k)) \succeq 0$$

Summing over the agents and dividing by N:

$$\alpha\sum_{i=1}^{N}\frac{1}{N}\Delta L_{C_i}(\delta\hat{x}^{(i)}(k)) + I_N\sum_{i=1}^{N}\frac{1}{N}\sum_{q\in\mathcal{T}_i}\beta_q(\hat{\nu}_q(k) - \nu_q^-(k)) \succeq 0$$

the second term can be written as:

$$\sum_{i=1}^{N}\frac{1}{N}\sum_{q\in\mathcal{T}_i}\beta_q(\hat{\nu}_q(k) - \nu_q^-(k)) = \sum_{q=1}^{M}\frac{J_q}{N}\beta_q(\hat{\nu}_q(k) - \nu_q^-(k))$$

by Theorem 2 and Eq. (20):

$$\alpha\Delta L_C(\delta\hat{x}(k)) + I_N\sum_{q=1}^{M}\beta_q(\nu_q(k) - \nu_q^-(k)) \succeq 0$$

□

Corollary 1: (Stationary targets) When the targets are stationary the global cost function of $\Delta\mathbf{P}$ with the average (14) is monotonically increasing.

The proof is straightforward by noticing that in this case $\nu_q^-(k) = \nu_q(k)$.

IV. Simulation Results

In this section we present simulation results comparing the centralized approximation with our distributed approach. We consider $N = 10$ agents and $M = 3$ targets. The agents lie initially close to the x-axis, while the targets start at $(0, 0)$. We consider $v_{\max,i} = 0.25$, $w_{\max,q} = v_{\max,i}/15$, and $\Delta t = 1$. For the weighting function of Table I, we take $\rho_1 = 0.75$, $\rho_2 = 3$ for c_{ij} and $\rho_1 = 0.75$, $\rho_2 = 1.25$ for v_{qi}.

We select $\alpha = 1$ and $\beta_q = 10^4$ for all three targets. We drive the targets in opposite direction with a non zero-mean bounded noise process. Figure 2 shows the trajectories of the agents/targets for both the centralized and the distributed solutions. Although the trajectories are different in the two approximations, in both cases the agents maintain a certain level of connectivity, while keeping track of the moving targets. The differences are due to the linearizations, which are trajectory dependent.

V. Conclusions and future work

We proposed a distributed and non-iterative solution for the problem of collectively tracking mobile targets while maintaining a certain level of connectivity, using a robotic network. More complex agent dynamics and full-state dependent Laplacians, where the connectivity depends also on the relative velocities, are topics of current research. Collision avoidance as well as minimal distance separation

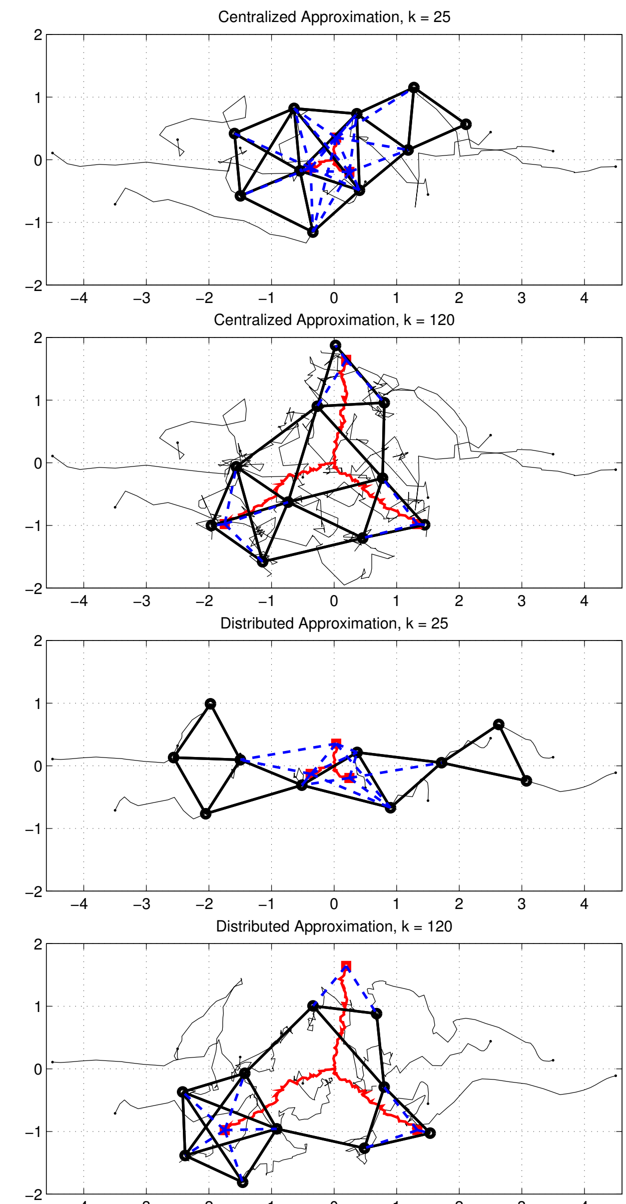


Fig. 2. Centralized and distributed approximations at two different discrete time instances $k = 25$ and $k = 120$. The thin black lines represent the agents' trajectories, which start from the points marked with black tiny dots. The targets' trajectories are in red and start from $(0, 0)$. Black circles represent the agents, squares the targets, solid lines are the communication links and dashed lines the detection links.

are considered to be of paramount importance for real applications and they will be addressed in the near future. The sub-optimality characterization for the distributed approach along with extensive numerical simulations and experimental validations are currently being investigated.

APPENDIX

Consider the linearized version of constraint (4d):

$$\sum_{j\in\mathcal{N}_q} h_j^\top \delta x_j(k) - h_j^\top \delta z_q(k) > d(k), \qquad \forall z_q(k) \in \mathcal{Z}_q(k)$$

for some constant $d(k)$. The worst case, $z_q^*(k)$ is the one that minimizes $-\sum_{j\in\mathcal{N}_q} h_j^\top \delta z_q(k)$. Therefore:

$$z_q^*(k) = \arg \min_{z_q(k)\in\mathcal{Z}_q(k)} - \sum_{j\in\mathcal{N}_q} h_j^\top \delta z_q(k)$$

This problem can be solved analytically, resulting in

$$z_q^*(k) = z_q(k-1) - [\bar{h}_{(2)} - \bar{h}_{(1)}]^\top w_{\max,q}\Delta t$$

where $\bar{h}_{(1)}$ and $\bar{h}_{(2)}$ are the normalized components of $\sum_{j\in\mathcal{N}_q} h_j$.

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