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An Approach to Data-Driven Design of Feedback Control Systems with Embedded Residual Generation

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Abstract— **Motivated by the increasing needs in the process industry for designing fault tolerant feedback control systems based on process data, data-driven design of feedback control systems with embedded residual generation is addressed. For this purpose, an extended internal model control (EIMC) structure aiming at accessing the residuals embedded in control loop is first proposed. Based on the identification of the so-called parity subspace and a well-established mapping between the parity vector and the solution of the Luenberger equations, a direct design scheme of EIMC from process data is developed. The achieved results are illustrated by an academic example.**

I. INTRODUCTION

Due to the system complexity, the first principle modelling of industrial processes is a critical issue. In addition, both the modelling and design of advanced (model-based) control and monitoring schemes may demand for considerable engineering efforts. In practice, data-driven techniques often offer alternative solutions, which make use of the available process data to simplify the modeling and design procedure aiming at (considerably) reducing the engineering efforts. Among numerous schemes, the well-established subspace identification method (SIM) [10], [17], [18] is a powerful technique, based on which, for instance, the subspace predictive control approaches [8], [9], [12] are developed for constructing the predictive controller without explicitly identifying a system model. Recently, Ding et al. [5] proposed an SIM aided data-driven design of observer-based fault detection and isolation (FDI) systems, based on a direct identification of parity vectors from the process data proposed in [19], [20]. In this way, the FDI system design becomes more convenient for application engineers without special knowledge of e.g. observer-based residual generators or solving Luenberger equations. Moreover, its recursive form reduces the on-line computation and the demand for working memory, and thus such an FDI system can be easily integrated into the process advanced control systems.

Associated with the increasing demands for high system performance and more system safety and reliability, the model-based FDI technique has received, both in application and research domains, more and more attention since the early 70's of the last century. For the design of the so-call observer-based FDI systems, a well-established theoretical framework and some design tools are available [1], [2], [4], [11], [13]. A trend of integrating FDI into the feedback control scheme as a possible solution with low needs for resource can be currently observed [14],[15],[16].

[3] and [21] have studied the extraction of residual signals from the access points in a feedback control loop instead of a separate design and construction of an observer-based residual generator. Zhou and Ren [24] proposed a fault tolerant control (FTC) architecture, the so-called generalized internal model control (GIMC) structure, whose core is the reconstruction of the standard control loop. Alternatively, Ding et al. [6] proposed a so-called EIMC structure that is established from the viewpoint of the relationship between feedback control loop and embedded residual generation (for the purpose of FDI). It has been proven that all stabilization controllers (the so-called Youla parametrization) can be equivalently realized in an observer-based residual generator form.

Strongly motivated by the above-mentioned results, our study in this paper focuses on the data-driven design of *control systems with embedded residual generation*. Following the idea presented in [5] and [6], we shall first propose an EIMC structure and, based on it, a data-driven approach to the design of the EIMC controller. The basic idea consists in constructing the controller using its residual generation form and *identifying the residual generator directly from the process data without a special design step*. In this context, our approach is called data-driven. It is of practical interest and allows an engineer to constrcut a controller with embedded residual signals without a special design procedure.

The paper is organized as follows. After the needed preliminaries are reviewed, the problems addressed in this paper will be briefly formulated in Section II. In Section III, the data-driven design approach will be presented and some associated issues discussed. To illustrate the results, an academic example is given in Section IV.

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II. PRELIMINARIES AND PROBLEM FORMULATION

A. Plant model, system factorization and Youla parameterization

In this paper, we consider a standard feedback control loop sketched in Fig.1. It is supposed that the plant model

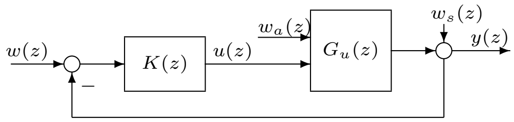

Fig. 1. Feedback control loop

is described by

$$y(z) = G_u(z)u(z) + G_a(z)w_a(z) + w_s(z) \tag{1}$$

with the minimal state space realization given by

$$x(k+1) = Ax(k) + Bu(k) + w_a(k) \tag{2}$$
$$y(k) = Cx(k) + Du(k) + w_s(k) \tag{3}$$

where $u(k) \in \mathbf{R}^l, y(k) \in \mathbf{R}^m$, and $x(k) \in \mathbf{R}^n$ represent process input, output and state variable vectors respectively. $w_a(k) \in \mathbf{R}^n$ and $w_s(k) \in \mathbf{R}^m$ denote noise sequences that are statistically independent of $u(k)$ and $x(0)$, and

$$G_u(z) = C(zI - A)^{-1}B + D, G_a(z) = C(zI - A)^{-1}$$

It is further assumed that the controller is described by

$$u(z) = K(z)\left(w(z) - y(z)\right)$$

with $w \in \mathbf{R}^m$ denoting the vector of the reference signals and $K(z)$ the control law. Let

$$G_u(z) = \hat{M}^{-1}(z)\hat{N}(z) \tag{4}$$
$$\hat{M}(z) = I - C(zI - A_L)^{-1}L \tag{5}$$
$$\hat{N}(z) = D + C(zI - A_L)^{-1}B_L \tag{6}$$

be a left coprime factorization of transfer matrix $G_u(z)$ with $A_L = A - LC, B_L = B - LD$ and L ensuring a stable A_L (i.e. all of its eigenvalues are located in the unit circle). The well-known Youla parameterization [4], [23] of all stabilizing controllers can be described by

$$K(z) = \left(\hat{X}(z) - Q(z)\hat{N}(z)\right)^{-1}\left(\hat{Y}(z) - Q(z)\hat{M}(z)\right)$$

with $Q(z) \in \mathcal{RH}_\infty$ as a parameter matrix and

$$\hat{X}(z) = I - F(zI - A_L)^{-1}B_L \tag{7}$$
$$\hat{Y}(z) = F(zI - A_L)^{-1}L \tag{8}$$

where F is a matrix of appropriate dimensions and ensures that $A + BF$ is stable.

B. Data structure, parity vectors and identification of parity subspace

In practice, the system matrices, A, B, C and D, as well as the process order n in plant model (2)-(3) are unknown *a prior*. We assume that a data set including process input and output records is available. The block Hankel matrices for the output are defined as

$$Y_f = \begin{bmatrix} y(k) & y(k+1) & \cdots & y(k+N-1) \\ y(k+1) & y(k+2) & \cdots & y(k+N) \\ \vdots & \vdots & \ddots & \vdots \\ y(k+s) & & & y(k+s+N-1) \end{bmatrix}$$

$$Y_p = \begin{bmatrix} y(k-s) & y(k-s+1) & \cdots & y(k-s+N-1) \\ y(k-s+1) & y(k-s+2) & \cdots & y(k-s+N) \\ \vdots & \vdots & \ddots & \vdots \\ y(k) & y(k+1) & \cdots & y(k+N-1) \end{bmatrix}$$

where $Y_f \in \mathbf{R}^{(s+1)m \times N}, Y_p \in \mathbf{R}^{(s+1)m \times N}, s$ and N are user defined parameters. Similar to Y_f and Y_p, Hankel structures for $U_f, U_p, W_{a,f}$ and $W_{s,f}$ are defined. Using the above data structure, an extended state space model for the system in (2)-(3) can be written as

$$Y_f = \Gamma_s X_k + H_{u,s}U_f + H_{a,s}W_{a,f} + W_{s,f} \tag{9}$$

where

$$\Gamma_s = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^s \end{bmatrix}, H_{u,s} = \begin{bmatrix} D & O & \cdots & O \\ CB & D & & \vdots \\ \vdots & & \ddots & \\ CA^{s-1}B & \cdots & & D \end{bmatrix}$$

Similar is $H_{a,s}$ and the state sequence

$$X_k = [x(k)\ x(k+1)\ \cdots\ x(k+N-1)] \in \mathbf{R}^{n \times N}$$

A parity vector v_s solves the following equation

$$v_s\Gamma_s = 0, v_s \in \mathbf{R}^{(s+1)m} \tag{10}$$

and thus belongs to $\Gamma_s^\perp$, the so-called parity subspace, i.e.

$$v_s \in \Gamma_s^\perp, \Gamma_s^\perp\Gamma_s = 0$$

Using parity vector v_s, we are able to generate a residual sequence, for instance in the form

$$r = v_sY_f - v_sH_{u,s}U_f \tag{11}$$

In general, the problem of identifying the parity space, aiming at residual generation, can be formulated as finding $\Gamma_s^\perp$ and $\Gamma_s^\perp H_{u,s}$. Define $Z_f = \left[Y_f^T\ U_f^T\right]^T$ and $Z_p = \left[Y_p^T\ U_p^T\right]^T$. For our identification purpose, the following algorithm is proposed [19].

Algorithm D2PS (from data to parity space)

- *Step 1: Generate datasets Z_f and Z_p and construct $Z_fZ_p^T$*
- *Step 2: Do SVD on $\frac{1}{N}Z_fZ_p^T$*

$$\frac{1}{N}Z_fZ_p^T = \mathcal{U}_z \begin{bmatrix} \Sigma_{z,1} & O \\ O & \Sigma_{z,2} \end{bmatrix} \mathcal{V}_z^T$$

with $\mathcal{U}_z = \begin{bmatrix} \mathcal{U}_{z,11} & \mathcal{U}_{z,12} \\ \mathcal{U}_{z,21} & \mathcal{U}_{z,22} \end{bmatrix}, \mathcal{U}_{z,12} \in R^{s_fm \times \eta}, \mathcal{U}_{z,11} \in R^{s_fm \times (s_fl+n)}$, $\mathcal{U}_{z,22} \in R^{s_fl \times \eta}$, *and* $\Sigma_{z,2} = 0 \in R^{\eta \times \eta}$, $s_f = s+1, \eta = s_fm - n$

- *Step 3: Set* $\Gamma_s^\perp = \mathcal{U}_{z,12}^T$, $\Gamma_s^\perp H_{u,s} = -\mathcal{U}_{z,22}^T$.

Remark 1: In [19] and [20], the existence conditions both for the open- and closed-loop systems have been extensively studied. In our study, we assume that the input excitation condition as given [5] is satisfied so that $\Gamma_s^\perp$, $\Gamma_s^\perp H_{u,s}$ given in *D2PS* Algorithm can be well identified.

C. Data-driven design of DO-based FDI systems

Observer-based technique is well-established in the framework of model-based FDI [1], [2], [4], [11], [13]. Suppose that A, B, C, D in (2)-(3) are known. The design of an observer-based residual generator, also called diagnostic observer (DO), is achieved by solving the Luenberger equations,

$$\begin{aligned} TA - A_z T = L_z C, c_z T = gC, \\ B_z = TB - L_z D, d_z = gD \end{aligned} \tag{12}$$

where $A_z \in \mathbf{R}^{s\times s}, B_z \in \mathbf{R}^{s\times l}, c_z \in \mathbf{R}^{1\times s}, d_z \in \mathbf{R}^{1\times l}, g \in \mathbf{R}^{1\times m}, L_z \in \mathbf{R}^{s\times m}$ together with the transformation matrix $T \in \mathbf{R}^{s\times n}$. It follows then the construction of the DO

$$z(k+1) = A_z z(k) + B_z u(k) + L_z y(k) \in \mathbf{R}^s \tag{13}$$

$$r(k) = gy(k) - c_z z(k) - d_z u(k) \in \mathbf{R} \tag{14}$$

In the above equations, $r(k)$ is called residual signal and s the order of the observer.

In [4], [22], a relationship between a parity vector and the above DO has been established. For a given parity vector $\alpha_s = [\alpha_{s,0}\ \alpha_{s,1}\ \cdots\ \alpha_{s,s}] \in \mathbf{R}^{(s+1)m}$, $\alpha_{s,i} \in \mathbf{R}^m, i = 0, 1, \cdots, s$, matrices

$$A_z = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}, L_z = -\begin{bmatrix} \alpha_{s,0} \\ \alpha_{s,1} \\ \vdots \\ \alpha_{s,s-1} \end{bmatrix} \tag{15}$$

$$c_z = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix} \in \mathbf{R}^s, g = \alpha_{s,s} \in \mathbf{R}^m \tag{16}$$

$$T = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_s \end{bmatrix} = \begin{bmatrix} \alpha_{s,1} & \alpha_{s,2} & \cdots & \alpha_{s,s} \\ \alpha_{s,2} & \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_{s,s} & 0 & \cdots & 0 \end{bmatrix} \Gamma_{s-1} \tag{17}$$

solve Luenberger equations (12). Moreover, B_z, d_z can be expressed in terms of $\alpha_s H_{s,u}$:

$$B_z = \begin{bmatrix} B_{z,1} \\ B_{z,2} \\ \vdots \\ B_{z,s} \end{bmatrix} = \begin{bmatrix} t_1 B - \alpha_{s,0} D \\ t_2 B - \alpha_{s,1} D \\ \vdots \\ t_s B - \alpha_{s,s-1} D \end{bmatrix} = \begin{bmatrix} \alpha_s H_{s,0} \\ \alpha_s H_{s,1} \\ \vdots \\ \alpha_s H_{s,s-1} \end{bmatrix}$$

$$d_z = \alpha_s H_{s,s} \text{ with } H_{u,s} = \begin{bmatrix} H_{s,0} & \cdots & H_{s,s} \end{bmatrix} \tag{18}$$

By a combined use of (15)-(18) and *D2PS Algorithm*, [5] have proposed a data-driven scheme for the design of a DO (13)-(14) (based on an identified parity vector), which delivers a scalar residual signal. We summarize it into the following algorithm.

Algorithm PS2DO (from parity vector to diagnostic observer)

- *Step 1: Select $\alpha_s \in \Gamma_s^{\perp}$ and the corresponding $\beta_s \in \Gamma_s^{\perp} H_{u,s}$, and form them as*

$$\alpha_s = \begin{bmatrix} \alpha_{s,0} & \alpha_{s,1} & \cdots & \alpha_{s,s} \end{bmatrix}, \alpha_{s,i} \in \mathbf{R}^m$$
$$\beta_s = \begin{bmatrix} \beta_{s,0} & \beta_{s,1} & \cdots & \beta_{s,s} \end{bmatrix}, \beta_{s,i} \in \mathbf{R}^l$$

- *Step 2: Set A_z, c_z, L_z, g according to (15)-(16) and*

$$B_z = \begin{bmatrix} \beta_{s,0}^T & \cdots & \beta_{s,s-1}^T \end{bmatrix}^T, d_z = \beta_{s,s}$$

- *Step 3: Construct the DO according to (13)-(14)*

D. Problem formulation

It is well-known that for a stable plant the Youla parameterization control loop can be equivalently realized in the form of the IMC structure [23], [24]. Recently, Ding et al. [6] have demonstrated the residual generator based realization of the Youla parameterized controllers. Inspired by this work, a relationship between the Youla parameterization and the generalized form of residual generators will be studied and an EIMC structure with an integrated access to residual signals will be proposed. In this way, issues of designing a feedback controller can be addressed by constructing a residual generator.

A further motivation of our study comes from the successful application of the data-driven technique in FDI [5]. Using *D2PS Algorithm*, parity subspace can be directly identified from process data. Based on *PS2DO Algorithm*, the observer-based residual generator can then be established.

The major task of our study is to integrate the above results and, based on which, to develop an approach to the data-driven design of feedback controllers with embedded residual generation.

III. DATA-DRIVEN DESIGN OF THE EIMC

A. Extended IMC structure

Assume that $G_u(z)$ is stable. Setting $F = 0$ leads to a Youla parameterization of the form

$$K(z) = \left(I - Q(z)\hat{N}(z)\right)^{-1} Q(z)\hat{M}(z)$$

The feedback control loop with Youla parameterization can then be sketched in Fig.2. Recall that

$$r(z) = \hat{M}(z)y(z) - \hat{N}(z)u(z) \tag{19}$$

delivers a residual vector [4] which can be, e.g. used for the FDI purpose. Motivated by this structure and noticing that

$$u(z) = \left(I - Q(z)\hat{N}(z)\right)^{-1} Q(z)\hat{M}(z)\left(w(z) - y(z)\right) \Longrightarrow$$
$$u(z) = Q(z)\left(\hat{M}(z)w(z) - \left(\hat{M}(z)y(z) - \hat{N}(z)u(z)\right)\right)$$

we re-structure the control loop as shown by Fig.3, and denote it by EIMC. Fig.3 reveals that the residual signals $r(z)$ is embedded in the control loop which can be directly extracted and used, e.g. for the FDI purpose. From the viewpoint of controller design, it is interesting to notice that designing a controller can be now equivalently dealt by

- constructing the residual generator (19) and
- selecting the parameter matrix $Q(z) \in \mathcal{RH}_\infty$

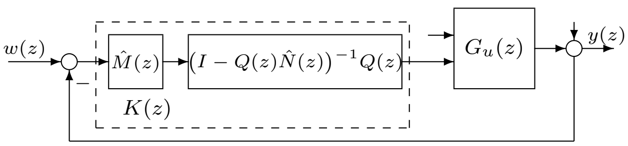

Fig. 2. An equivalent realization of the Youla parameterization

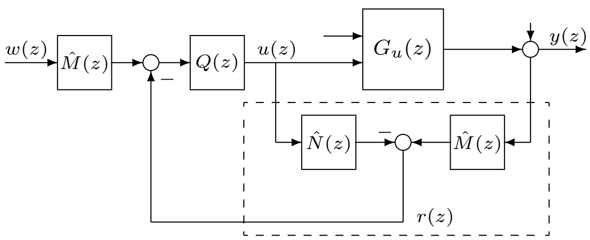

Fig. 3. EIMC structure

B. Data-driven design of the residual generator to be embedded in the EIMC

In this subsection, we are going to address the data-driven design of residual generator (19), which builds the core of the EIMC and thus of our approach.

It is well-known that the residual generator (19) delivers a residual vector

$$r(k) = y(k) - \hat{y}(k) \in \mathbf{R}^m$$

where $\hat{y}(k)$ denotes an estimate for the plant output vector delivered by a full-order observer [4]. Thus, for our purpose, we are going to extend *Algorithm PS2DO* to the vector-valued case.

Suppose that $\Gamma_s^{\perp}$ and $\Gamma_s^{\perp} H_{u,s}$ are identified using *Algorithm D2PS*. Select m (linearly independent) parity vectors $\alpha_{s_i} \in \Gamma_s^{\perp}, i = 1, \cdots, m,$

$$\alpha_{s_i} = \begin{bmatrix} \alpha_{s_i,0} & \alpha_{s_i,1} & \cdots & \alpha_{s_i,s_i} \end{bmatrix} \in \mathbf{R}^{(s+1)m}$$

satisfying

$$rank \begin{bmatrix} \alpha_{s_1,s_1}^T & \cdots & \alpha_{s_m,s_m}^T \end{bmatrix}^T = m \tag{20}$$

and the associated vectors

$$\beta_{s_i} = \alpha_{s_i} H_{u,s} \in \Gamma_s^{\perp} H_{u,s}, i = 1, \cdots, m$$

Using $\alpha_{s_i}, \beta_{s_i}, i = 1, \cdots, m$, we now construct the following residual generator

$$z(k+1) = A_z z(k) + B_z u(k) + L_z y(k) \in \mathbf{R}^s \tag{21}$$
$$r(k) = y(k) - G^{-1}\left(C_z z(k) + D_z u(k)\right) \in \mathbf{R}^m \tag{22}$$
$$A_z = diag(A_{z_1}, \cdots, A_{z_m}), C_z = diag(c_{z_1}, \cdots, c_{z_m}) \tag{23}$$
$$G = \left[\alpha_{s_1,s_1}^T, \cdots, \alpha_{s_m,s_m}^T\right]^T, B_z = \left[B_{z_1}^T, \cdots, B_{z_m}^T\right]^T \tag{24}$$
$$L_z = \left[L_{z_1}^T, \cdots, L_{z_m}^T\right]^T, D_z = \left[d_{z_1}^T, \cdots, d_{z_m}^T\right]^T \tag{25}$$

where $s = \sum_{i=1}^m s_i$ is the order of the above system, $A_{z_i}, B_{z_i}, c_{z_i}, d_{z_i}, L_{z_i}, i = 1, \cdots, m$, are those matrices as defined by (15), (16) and (18). Let

$$T = \begin{bmatrix} T_1^T & \cdots & T_m^T \end{bmatrix}^T \in \mathbf{R}^{s \times n}$$

with $T_i, i = 1, \cdots, m$, as defined in (17). Notice that, following (12), the system matrices in (21)-(22) solve the Luenberger equations

$$TA - A_z T = L_z C \in \mathbf{R}^{s \times n}, C_z T = GC \in \mathbf{R}^{m \times n} \tag{26}$$
$$B_z = TB - L_z D \in \mathbf{R}^{s \times l}, D_z = GD \in \mathbf{R}^{m \times l} \tag{27}$$

and the system (21) delivers an estimate for $Tx(k)$,

$$\hat{y}(k) = G^{-1}\left(C_z z(k) + D_z u(k)\right)$$

is an estimate for $y(k)$. For our purpose, we now study residual generator (21)-(22).

Recall that under the observability assumption and considering the features of those (independent) parity vectors [4] we have $s \geq n$. Moreover, it has been proven in [7] that

$$rank\,(T) = n \tag{28}$$

if (20) holds. Below, we consider two cases, $s = n$ and $s > n$, separately.

For $s = n$, system (21) is a full-order observer and the residual generator (21)-(22) can be equivalently written into, according to Luenberger equations (26)-(27),

$$\begin{aligned} r(z) &= \hat{M}(z)y(z) - \hat{N}(z)u(z) \\ \hat{M}(z) &= I - G^{-1}C_z(zI - A_z)^{-1}L_z \\ &= I - C\left(zI - A_L\right)^{-1} L, L = T^{-1}L_z \\ \hat{N}(z) &= G^{-1}\left(D_z + C_z(zI - A_z)^{-1}B_z\right) \\ &= D + C\left(zI - A_L\right)^{-1} B_L \end{aligned} \tag{29}$$

As a result, residual generator (21)-(22) and $\hat{M}(z)$ satisfying (29) can be directly embedded into the EIMC shown in Fig.3. Note that for $m = 1$, the order-reduction algorithm introduced in [7] delivers a parity vector with $s = n$. We would like to mention that there do exist m parity vectors in the identified parity subspace such that $s = n$. To find those m parity vectors, an algorithm is needed.

For $s > n$, we first introduce the following result.

Theorem: Given residual generator (21)-(22) with matrices $A_z, B_z, C_z, D_z, L_z, G$ and the associated matrix T solving Luenberger equations (26)-(27) and $s > n$, then

$$r(z) = \hat{M}_d(z)y(z) - \hat{N}_d(z)u(z) \tag{30}$$
$$= R(z)\left(\hat{M}(z)y(z) - \hat{N}(z)u(z)\right) \tag{31}$$
$$\hat{M}_d(z) = I - G^{-1}C_z(zI - A_z)^{-1}L_z \tag{32}$$
$$\hat{N}_d(z) = G^{-1}\left(D_z + C_z(zI - A_z)^{-1}B_z\right) \tag{33}$$

where

$$\hat{M}(z) = I - C\left(zI - A_L\right)^{-1} L, L = T^{-}L_z \tag{34}$$
$$\hat{N}(z) = D + C\left(zI - A_L\right)^{-1} B_L \tag{35}$$
$$R(z) = I - G^{-1}C_z T_o\left(zI - A_r\right)^{-1} T_o^{-} L_z \tag{36}$$
$$\begin{bmatrix} T^- \\ T_o^- \end{bmatrix} \begin{bmatrix} T & T_o \end{bmatrix} = \begin{bmatrix} I_{n \times n} & 0 \\ 0 & I_{(s-n)\times(s-n)} \end{bmatrix} \tag{37}$$
$$T_o A_r - A_z T_o = 0, A_r \in \mathbf{R}^{(s-n)\times(s-n)} \tag{38}$$

The proof of this theorem can be found in [4] (see Theorem 5.15) and thus omitted here.

It follows from the above theorem that for $s > n$ the residual generator (21)-(22) can be written into two parts: a full-order observer-based residual generator described by

$$\hat{M}(z)y(z) - \hat{N}(z)u(z)$$

and a dynamic system $R(z)$. In the FDI study [4], $R(z)$ in (31) is called post-filter. It can be seen from (38) that the poles of $R(z)$ are some of the eigenvalues of A_z, which ensures $R(z) \in \mathcal{RH}_\infty$. Note also that $\hat{M}_d(z) = R(z)\hat{M}(z)$. With the residual generator (21)-(22) and $\hat{M}_d(z)$ given in (32), the control loop can now be constructed as shown in Fig.4 with $\bar{Q}(z) \in \mathcal{RH}_\infty$ being the parameter matrix. Note that $Q(z)$ given in Fig.3 equals to

$$Q(z) = \bar{Q}(z)R(z) \in \mathcal{RH}_\infty$$

In summary, we conclude that once the parity subspace is identified, we are able to construct residual generator (21)-(22), and associated with it, $\hat{M}_d(z) = R(z)\hat{M}(z)$ using the following algorithm.

Algorithm D2EIMC (from data to extended IMC)

- *Step 1: Identify* $\Gamma_s^\perp, \Gamma_s^\perp H_{u,s}$ *using Algorithm D2PS*
- *Step 2: Compute* $A_z, B_z, C_z, D_z, L_z, G$ *as defined in (23)-(25) based on Algorithm D2DO*
- *Step 3*: *Construct residual generator (21)-(22)*
- *Step 4: Compute* $\hat{M}_d(z)(= \hat{M}(z)$ for $s = n)$ *given in (32)*

Note that the control structure given in Fig.4 can be equivalently replaced by Fig.5 with $R(z) = I$ for $s = n$. The control structure given in Fig.5 is useful for the system analysis.

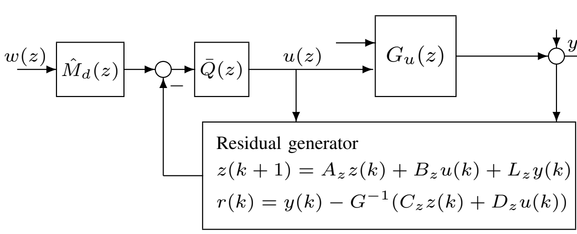

Fig. 4. A data-driven realization of the EIMC structure

To complete the controller design, the remaining task is to analyze the system dynamics and, based on it, to determine the parameter matrix $\bar{Q}(z) \in \mathcal{RH}_\infty$ (or $Q(z) \in \mathcal{RH}_\infty$ for $s = n$).

C. System analysis and determination of $\bar{Q}(z)(Q(z))$

It is straightforward that the dynamics of the control loop shown in Fig.5 is governed by

$$\hat{M}_d(z)y(z) =$$

$$\left[\begin{array}{cc} \hat{N}_d(z)\bar{Q}(z)\hat{M}_d(z) & \left(I - \hat{N}_d(z)\bar{Q}(z)\right) G_{\bar{w}}(z) \end{array} \right] \left[\begin{array}{c} w(z) \\ \bar{w}(z) \end{array} \right]$$

$$G_{\bar{v}}(z)\bar{w}(z) = \left[\begin{array}{cc} I & 0 \end{array} \right] + C\left(zI - A\right)^{-1} \left[\begin{array}{cc} 0 & I \end{array} \right] \left[\begin{array}{c} w_s(z) \\ w_a(z) \end{array} \right]$$

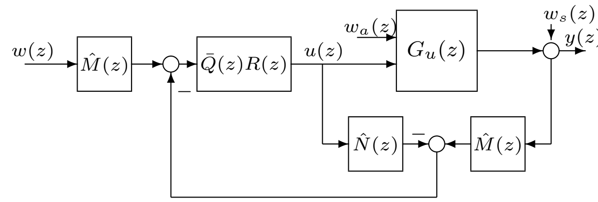

Fig. 5. An equivalent form of the data-driven realization of the EIMC structure

In order to minimize the control error, $e(z) = w(z) - y(z)$, we can theoretically , for instance, solve the following norm optimization problem

$$J = \min_{\bar{Q}(z) \in \mathcal{RH}_\infty} \|I - \hat{N}_d(z)\bar{Q}(z)\|$$

for determining $\bar{Q}(z)$, where $\|\cdot\|$ stands for some system norm, e.g. the $\mathcal{H}_\infty$- or the $\mathcal{H}_2$-norm of a transfer function matrix. For a practical solution, it is often required that

- $\bar{Q}(z)$ should be as much simple as possible, and
- only the steady state behavior (corresponding to constant w) has to be taken into account.

In this case, $\bar{Q}(z)$ can be set to be a constant matrix

$$\bar{Q} = \left(\left(D_z + C_z(I - A_z)^{-1}B_z\right)\right)^{-1} G \tag{39}$$

if $D_z + C_z(I - A_z)^{-1}B_z$ is invertible, otherwise

$$\bar{Q} = \left(\left(D_z + C_z(I - A_z)^{-1}B_z\right)\right)^{-} G \tag{40}$$

where $(\cdot)^-$ denotes the pseudo inverse of a matrix.

Remark 2: The context of data-driven design, the optimization of the controller and FDI subsystem e.g. as mentioned above, should be done based on a further data processing. This builds the major future work in this domain.

IV. AN ACADEMIC EXAMPLE

In this section, an academic example is considered to illustrate the application of the data-driven EIMC controller scheme presented in the last section. The system matrices in the plant model (2)-(3) are assumed to be as follows:

$$A = \left[\begin{array}{ccccc} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right], \ B = \left[\begin{array}{c} 0.00098 \\ 0.01299 \\ 0.01859 \\ 0.0033 \\ -0.00002 \end{array} \right]$$

$$C = \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \end{array} \right], \ D = 0$$

$$w_a(k) \sim \mathcal{N}(0, 0.01^2), \ w_s(k) \sim \mathcal{N}(0, 0.01^2)$$

$\mathcal{N}(\mu, \sigma^2)$ denotes normal distribution with mean μ and variance σ^2. We would like to emphasize that this plant model is only used for producing the necessary input and output data. For the controller design, the system matrices A, B, C, D and noises $w_a(k), w_s(k)$ are unknown a *prior*. The design procedure of the EIMC controller is as follow.

Phase I: Collect the plant data with the reference excitation consisting of 1000 samples by simulating (2)-(3). Identify a parity vector with the minimum order $s = 5$. Set A_z, C_z from (15)-(16) and perform *Algorithm D2DO,*

$$B_z = \left[\begin{array}{ccccc} 0.0043 & 0.0155 & 0.0225 & 0.0109 & 0.0008 \end{array}\right]^T,$$
$$L_z = \left[\begin{array}{ccccc} 0.0010 & -0.0061 & 0.0297 & -0.1417 & -0.5910 \end{array}\right]^T,$$
$$D_z = 0, G = 0.7919.$$

Phase II: Construct residual generator (21)-(22), $\hat{M}_d(z)$ by (32) and the matrix $\bar{Q} = 14.6754$ by (39). Note that $m = 1$, thus, $s = n$, $R(z) = 1$, $\hat{M}(z) = \hat{M}_d(z), Q = \bar{Q}$.

Phase III: The following simulation is done:

$$w = \begin{cases} 1, 0 \leq k < 100 \\ 2, k \geq 100 \end{cases}, \; y = y^* + f, f = \begin{cases} 0, 0 \leq k < 500 \\ 0.1, k \geq 500 \end{cases}$$

where y^* is the fault-free output and f represents an additive sensor fault. Fig.6 shows the output signal obtained from experiment and fault detection can be successfully achieved by the embeded residual signal shown in Fig.7.

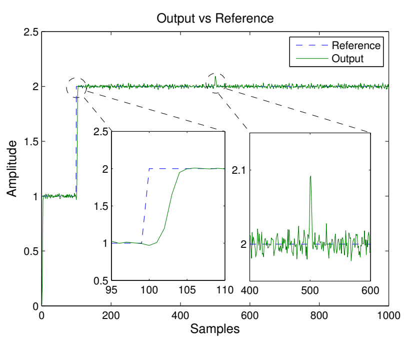

Fig. 6. The performance of EIMC controller

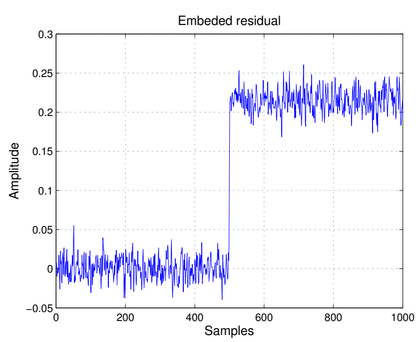

Fig. 7. Embedded residual generation

V. CONCLUSIONS

In this paper, we have proposed an EIMC structure and its data-driven design scheme. The main results are summarized in the form of some algorithms. The core of the proposed approach is a data-driven design and construction of an observer-based residual generator which builds the basis of the EIMC. In this way, a class of feedback controllers with an embedded residual generation can be directly designed and constructed in a data-driven manner.

The results achieved in this study build the basis for the future work on the data-driven development of advanced control schemes and for the realization of process monitoring, fault diagnosis and for FTC in the plug-and-play manner. Our recent work is also dedicated to the tests on a real industrial benchmark.

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