2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Multi-agent Path Planning for an Unknown Number of Targets over Dynamic Space Partitions

Jared Wood and J. Karl Hedrick

Abstract— **Searching a surveillance area for an unknown number of targets by making observations with small fields of view relative to the surveillance area is a task performed in settings such as search and rescue, area patrol, and reconnaissance. Based on observations, locations of possible target existence within the surveillance area can be estimated through time. This estimation can then be used to determine search paths. Typically this is done by optimization directly over the target estimation. Due to complicating factors such as the number of targets being unknown, this approach is not taken. Instead, time-evolving characteristic regions within the surveillance area are learned and paths are planned based on these regions. Based on the target estimation, three types of regions can be identified. These types are 1) regions of possible target existence, 2) regions of almost certain target existence, and 3) regions of almost certainly no targets. According to these regions, the surveillance area can then be partitioned into areas characteristic of required search or tracking and resource needed to perform the search or tracking. In order to accomplish this partitioning, a set of features is determined and used for classification. Once the surveillance area is partitioned, path planning is performed over the partitions. Because several partitions have been constructed, they can be allocated as tasks to a team of autonomous agents. The path planning is then separated into two levels. The first level is agent route planning over the partitions. The second level is agent path planning by optimization over partition level target estimation. Results are presented for a team of autonomous aerial vehicles searching for an unknown number of targets.**

I. INTRODUCTION

The task of searching a large area to find an unknown number of targets is challenging especially when the field of view of one observation is much smaller than the entire area. Because the observation field of view is small relative to the surveillance area, it is necessary to maintain an estimate of target locations and area coverage as the search progresses over time and use this estimate to optimize the evolving search path [1]. As knowledge of the search area changes with new observations, decisions must be made on how to adjust the search strategy. The purpose of this paper is to present an approach toward planning paths for a team of agents searching for an unknown number of targets. Fig. 1 shows an overview of this approach. In Fig.1 block (1), observations update an estimate of target density over the search area. Based on the target density estimate, regions in the surveillance area are classified into partitions, Fig. 1 block (2). After classication, the partitions are passed to a path planning algorithm, Fig. 1 in block (3). Path planning is separated into two levels as depicted in Fig. 2. First, the partitions are viewed as tasks and agents are allocated to them. Algorithms already developed for task allocation [2], [3] can then be utilized to accomplish this step. Second, agent paths are chosen by optimizing directly over partition level target density distributions. Approaches developed for path planning by direct probability distribution optimization [4], [5], [6], [7] can then be extended to accomplish this step of path planning. Buidling on these approaches, their extension to partition level target density distributions is presented in this paper.

To motivate the path planning approach taken in this paper, three types of search are now considered. The types of search are 1) there is only one target, 2) there is a known number of multiple targets, and 3) there is an unknown number of multiple targets. For each search type, some applicable existing path planning approaches will be mentioned.

For the first case, a single probability distribution can be used to estimate the target position, so implementations of standard recursive Bayesian estimation work well. Since there's a single probability distribution, paths can be opti-

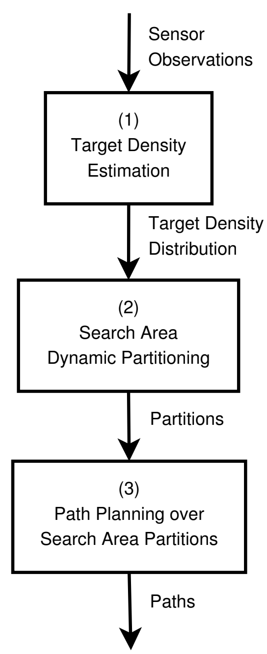

Fig. 1. Structure of search planning decomposition into dynamic partition classification and path planning over the partitions.

Jared Wood is a Ph.D. candidate in the Vehicle Dynamics Lab, Mechanical Engineering Department, UC Berkeley jwood@vehicle.berkeley.edu

J. Karl Hedrick is the James Marshal Wells Professor and head of the Vehicle Dynamics Lab, Mechanical Engineering Department, UC Berkeley khedrick@me.berkeley.edu

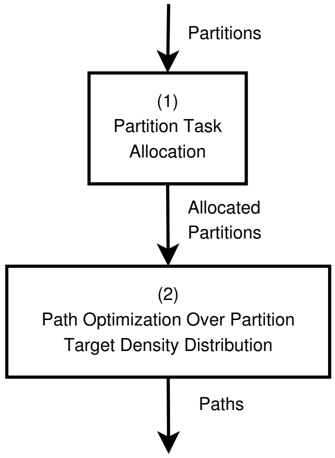

Fig. 2. Structure of path planning separated into task allocation over search partitions and path planning by optimizing directly over the target density distribution.

mized directly over it [5], [6]. And there are several information metrics that can be used for the path optimization, such as probability of detection [5], entropy gain [8], mutual information [4], [9], Kullback-Leibler divergence [10], and other generalizations of information [11], [12]. Due to these many well-defined metrics, many path planning strategies have been developed for this case. The path an agent chooses to follow for each of these methods ultimately is determined by optimization over a probability distribution. At this point either single or multiple-step finite horizon path optimization determines the path for the agent to follow. However, there are performance limitations with finite-horizon optimization. Ways to get around these limitations were proposed in [13], [7].

For the second case, variations on Bayesian estimation have been developed, such as multiple hypothesis tracking [14]. The most naive approach is to maintain an independent probability distribution for each target and assume that the target association of observations is given. In reality the target association must be addressed and methods for this case cease to be purely Bayesian because track tables are maintained [14], [15]. Yet in most of these methods a set of distributions are constructed (one for each target) and the path planning optimization strategies developed for the first case can be extended to account for optimization over multiple probability distributions [16], [17].

For the third case, the complexity of maintaining a probability distribution for each target (including unknown targets) becomes difficult. This difficulty inspired development of a generalization of the standard recursive Bayesian estimation framework from random variables to random sets that provides a target density distribution over the surveillance area [15]. For this method there's one distribution over the surveillance area that combines all targets. Path planning strategies developed for the first two cases are then no longer applicable without modification. One possible approach for path planning based on a target density distribution has been developed [18]. This approach chooses paths by maximizing the expected number of targets. However, analysis of the target density distribution can be done to classify the surveillance area into a set of time-evolving partitions characteristic of particular target search or tracking. Methods have been developed to perform this analysis [19]. Building upon this type of analysis, the search plan can then consist of two layers. The first layer can be task allocation of the partitions to the team of agents. The second layer can be partition level target density distribution path optimization.

In this paper the target density distribution will be defined and methods for analyzing target density distributions to dynamically partition the surveillance area will be presented. More details of these methods can be found in [19]. Then methods for path planning over a set of partitions will be presented. The planning consists of two layers. The first layer views the search partitions as tasks and allocates them to the team of agents. Existing vehicle routing algorithms [2], [3] can be implemented here. The second layer chooses agent paths by optimizing over partition level target density distributions. In order to accomplish this, existing methods for path optimization directly over probability distributions are generalized and extended.

II. DYNAMIC PARTITIONING FROM TARGET DENSITY DISTRIBUTION

A. Target Density Distribution

The work presented in this paper relies on target positions being estimated by a target density distribution. Consider the surveillance area S and some subarea $A \subset S$. A target density distribution $f(x)$ is defined over the surveillance area as

$$\mathrm{E}\, N_A = \int_{x \in A \subset S} f(x)\, d\mu(x) \qquad (1)$$

where N_A is the expected number of targets in the subarea A. Notice that the target density distribution provides the estimated number of targets within regions of the surveillance area. This means that the expected number of targets within a region can be evaluated simply by integrating the target density distribution over the region. The ability to quickly evaluate the number of expected targets within a region by integrating over a target density distribution is used frequently in the dynamic partitioning classification [19] that will be presented in this paper.

Target density distribution estimation is outside the scope of this paper. However, recursive estimation of target density distributions has been developed [18]. So the estimation problem can be treated similarly to the case of single target estimation, in which a probability distribution is recursively estimated.

B. Dynamic Partitions Classification

The surveillance area is dynamically partitioned into characteristic regions. The partitions are composed of one partition for exploration and a set of ordered partitions for search and tracking, following the approach presented in [19]. Fig. 3 presents the structure of dyanamic partitioning classification. Observing this figure, note that the classification is accomplished by a cascade of classifiers. The levels of the cascade classify points in the surveillance area into

1) One partition for exploration and one for searching (block (1) in Fig. 3).
2) Search and tracking partitions based on local expected number of targets (block (2) in Fig. 3).
3) Search and tracking partitions accounting for level of search and tracking required (block (4) in Fig. 3).

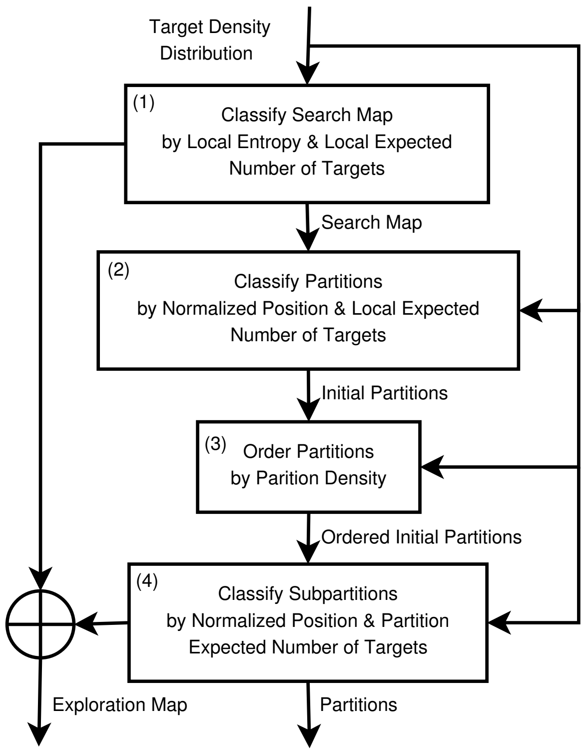

Fig. 3. Cascade of classifiers for partitioning the surveillance area into an exploration partition and an ordered set of search and partitions.

To partition the surveillance area, the shape of the target density distribution must be analyzed. To do this, features are computed over local areas [19] as defined by

$$S_r(x_0) := \{x \in S : d(x, x_0) < r\} \tag{2}$$

where $d(x, x_0)$ is some measure of distance. The features used in the classification are local entropy H_r, local expected number of targets I_r, and normalized position. These features were defined in [19].

In the first cascade level (Fig. 3 block (1)), the search map S_{search} is computed. S_{search} is a classification of points in the surveillance area into either an exploration partition or a partition for informed search and tracking. In order to determine S_{search}, 1) the set of points with high local entropy S_{HE} and 2) the set of points with no information S_{NI} must first be determined. S_{HE} is defined as

$$S_{HE} := \{x \in S :\mid \mathrm{H}_r(x) - \mathrm{H}_{max} \mid < \epsilon\} \tag{3}$$

where H_{max} is the maximum local entropy possible [19]. S_{NI} is defined as

$$S_{NI} := \{x \in S : \mathrm{I}_r(x) < U + \epsilon\} \tag{4}$$

where U is the local expected number of targets corresponding with unbiased points in the surveillance area. The selection of ϵ in Eq. 3 and Eq. 4 is tuned according to resulting simulation sensitivity. The search map is then

$$S_{\text{search}} := S \setminus (S_{HE} \cap S_{NI}) \tag{5}$$

Most of the computation involved in dynamic partitioning classification is done by block (2) of Fig. 3. In this level of the cascade, partitions are dynamically constructed and points in the surveillance area are classified. The features used for this classification are local expected number of targets and normalized position. When the search task initiates, partition features for this classifier are intialized by a K-means algorithm [20]. In subsequent time instances a Gaussian Mixture Model [20] is used for classification. The partition features at time t are then initialized by the previous partition features at time step $t-1$.

The final layer of the cascade (Fig. 3 blocks (3) and (4)) determines if some partitions need to be further subpartitioned to account for the required level of search and tracking for each partition. To accomplish this, the classifier first orders the partitions according to partition density ρ_{P_i} [19], where P_i is the ith partition. The classifier then iterates through the partitions in descending order of partition density. If a partition has an expected number of targets N_{P_i} such that

$$N_{P_i} = \text{ceil}\left(\int_{x \in P_i} f(x) d\mu(x)\right) > 1 \tag{6}$$

then the partition P_i is subpartitioned. This classifier continues until a specified maximum number of partitions is reached.

III. PATH PLANNING

Recall Fig. 1. After estimating a target density distribution (block (1) in Fig. 1), the target density distribution is analyzed and the surveillance area is partitioned into a partition for exploration and an ordered set of partitions for search and tracking (block (2) of Fig. 1). Once the partitions are computed, agent path planning is then performed (block (3) of Fig. 1). Path planning is accomplished by separating the planning into two layers as depicted in Fig. 2. The layers of the planning are

1) Partitions are considered tasks and allocated to the team of agents (block (1) of Fig. 2).
2) Partition level target density distribution path optimization (block (2) of Fig. 2).

A. Partition Task Allocation

The partitions generated by the classifier define areas over which subsets of the target density distribution can be extracted. This suggests the application of some kind of task allocation algorithm that takes each of the partitioned search areas as tasks with varying level of certainty or priority. The exact method of task allocation is beyond the scope of this paper. However, there are methods that have been developed [3], [2] and are applicable to the type of task allocation required for the planning structure presented in this paper. In particular, task allocation algorithms have been developed for projects at the Center for Collaborative Control of Unmanned Vehicles [2].

B. Partition Level Path Optimization

As agents are allocated to partitions, their paths within the partitions are determined by optimizing directly over the partition level target density distribution. In order to optimize over a target density distribution, the utitlity of points in the surveillance area must be defined. To do this, consider an agent's observation coverage $f_C(x, x_0)$ about a point x_0 in the surveillance area. An agent's observation coverage is determined by the properties of the agent's sensor. For example, if an agent can make observations perfectly within a radius r, then the observation coverage is

$$f_C(x, x_0) = \begin{cases} 1, & \text{if } \|x - x_0\| < r \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

However, in general the observation coverage is determined by the sensor's capable field of view as well as the resolution of observable points within the field of view, and the probability of missed detection [21].

To motivate derivation of utility, consider a zero horizon path. The observation coverage is used to define the utility of a point x_0 as

$$V_0(x_0) := \int_{x \in S} f(x) f_C(x, x_0) d\mu(x) \quad (8)$$

where $f(x)$ is the target density distribution. Extending this to finite horizon planning, define the H-step horizon observation coverage over the path $x_{0:H} = (x_0, ..., x_H)$ as

$$f_C(x, x_{0:H}) := 1 - \prod_{t=0:H} (1 - f_C(x, x_t)) \quad (9)$$

The utility of a point $x_0 \in S$ is then defined as

$$V_H(x_0) := \max_{\substack{x_i \in \mathrm{R}(x_{i-1}) \\ i=1,...,H}} \int_{x \in S} f(x) f_C(x, x_{0:H}) d\mu(x) \quad (10)$$

where $\mathrm{R}(x)$ is the set of all points within the reach set of x [13]. Intuitively, Eq. 10 represents the expected number of targets within the sensor coverage over a H-step path originating from the point x_0. Maximizing Eq. 10 then corresponds to choosing the point $x_0^\star$ that yields the maximum expected number of targets within the observation coverage of a path originating from $x_0^\star$.

To extend utility as defined in Eq. 10 to optimization over partition level target density distributions, let the set of partitions defined over the surveillance area be $P = \{P_1, ..., P_n\}$. The partition level target density distribution $f_{P_i}(x)$ for the partition $P_i \in P$ is defined as

$$f_{P_i}(x) := \begin{cases} f(x) & \text{if } x \in P_i \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

The partition level utility is then defined as

$$V_H^{P_i}(x_0) := \max_{\substack{x_i \in \mathrm{R}(x_{i-1}) \\ i=1,...,H}} \int_{x \in S} f_{P_i}(x) f_C(x, x_{0:H}) d\mu(x) \quad (12)$$

With Eq. 12 the paths of the agents are computed. Yet, depending on sensor dynamics, the point in the surveillance area chosen for observation may still have to be chosen. By choosing agent paths that optimize observation coverage, the sensor's point of observation can be chosen by optimizing over the area local to an agent's predicted position x_t at the next observation time t. The optimal observation point can be determined by computing the expected number of targets by observing particular points local to the agent's predicted position. To do this, first determine the sensor's no-detection observation likelihood function $L(x; z = \neg D, x_{obs}, x_t)$ [21], where $z = \neg D$ specifies that no target is detected, and x_{obs} is the point in the surveillance area to observe local to x_t. Define the observation specific sensor coverage as $f_C^{x_{obs}}(x, x_t) := 1 - L(x; z = \neg D, x_{obs}, x_t)$. The optimal observation point $x_{obs}^\star$ is then the x_{obs} that maximizes

$$V_{obs}(x_{obs}) = \int_{x \in S_r(x_t)} f_C^{x_{obs}}(x, x_t) f(x) d\mu(x) \quad (13)$$

where $S_r(x_t)$ was defined in Eq. 2.

IV. RESULTS

The performance of the approach presented in this paper to autonomously plan search paths to find an unknown number of targets was tested by construcing a simulation environment. In this environment the team of agents consisted of six autonomous aircraft equipped with visual spectrum gimballed camera sensors. The capabilities of these agents were designed to closely represent behaviors observed in flight experiments [22]. The camera characteristics were designed to represent a field of view resulting from a 0.9273 rad view angle. Effects of resolution were included by limiting the distance of observations to 250 m. The agents were designed to fly at 25 m/s and 100 m altitude with a maximum turn rate of 0.2 rad/s. There were six targets, but this was unknown to the search planning. The targets were allowed to move according to a transition model defined by

$$x_{T,t} = x_{T,t-1} + r \begin{bmatrix} cos(\theta) \\ sin(\theta) \end{bmatrix} \quad (14)$$

where r and θ were distributed as

$$\begin{aligned} r &\sim \text{Gaussian}\left(\mu, \sigma^2\right) \\ \theta &\sim Uniform([0, 2\pi)) \end{aligned} \quad (15)$$

with $\mu = 2$ m in one second and $\sigma^2 = 10$ m^2. The time interval of each simulation iteration was 4 seconds.

Several simulation samples were performed with various initial target and agent positions as well as various prior density distributions. The performance of the search planning is dependent on not only the path planning (Fig. 2) but also the dynamic partitioning classification (Fig. 1 block (2)) which is represented in Fig. 3.

The performance of the dynamic partitioning classification is affected by the quality and diversity of observations made over the surveillance area. Dynamic partitioning classification should perform well according to the observations it receives. Dynamic partitioning classiciation performance is analyzed over simulation time by measuring the

- Number of targets in search and tracking partitions.
- Number of search and tracking partitions.
- Average search and tracking partition size.
- Average size of search and tracking partitions containing targets.

Recall the types of partitions generated by the classification represented in Fig. 3. The exploration partition contains points in the surveillance area that either 1) still need to be explored or 2) belong to large regions that have been repeatedly observed without any target detections. The other partitions are an ordered set of search and tracking partitions. As an immediate check to see if the surveillance area is being partitioned to capture the targets, the sample mean number of targets within search and tracking partitions is plotted in Fig. 4. From Fig. 4, observe that all targets are eventually within search and tracking partitions.

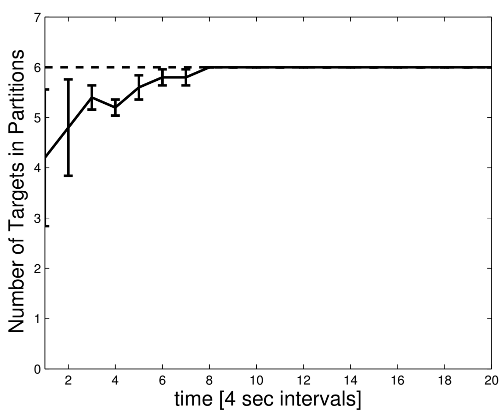

Fig. 4. Sample mean number of targets within search and tracking partitions over simulation time. Error bars represent sample standard deviation.

To further analyze the search and tracking partitions, consider their quantity and average size over simulation time. The sample mean number of search and tracking partitions are plotted in Fig. 5. Observe that the number of search and tracking partitions gradually increases but appears to level out. To make sense of this gradual increase in number of partitions, the sample mean average size of the search and tracking partitions is plotted in Fig. 6. From Fig. 6 it is observed that the average partition size tends to remain constant. The only way for the number of paritions to increase and the average size of the partitions to remain constant is for the additional partitions to be small tracking partitions and to have the average size of the search partitions increase. This is confirmed by observing the sample mean average partition size of partitions in which targets exist, plotted in Fig. 7. In these simulations, there were six agents and six targets. Consequently, once six targets are localized within tracking partitions and are being tracked by six agents, the remaining search partitions will persist unless additional agents come to assist.

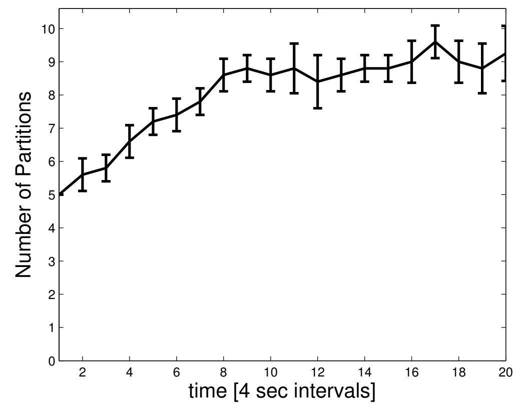

Fig. 5. Sample mean number of search and tracking partitions over simulation time. Error bars represent sample standard deviation.

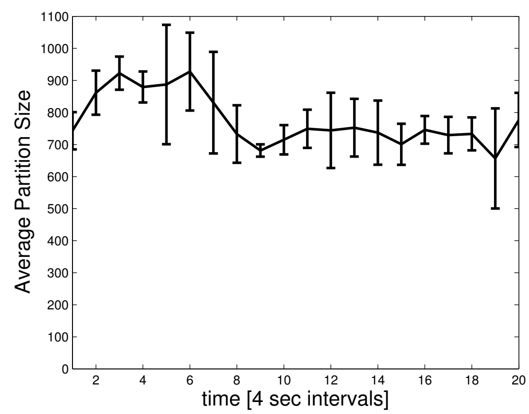

Fig. 6. Sample mean average search and tracking partition size over simulation time. Error bars represent sample standard deviation.

Path planning is affected by the performance of dynamic partitioning classification. Agent paths are determined by task allocation over the partitions and then path optimization directly over partition level target density distributions. The performance of path planning is analyzed by observing statistics of the number of localized targets over simulation time. Fig. 8 shows the sample mean number of localized targets over simulation time. As observed in Fig. 8 the general trend was to approach localization of all targets by the end of the simulation. This result suggests path planning based on dynamic partitioning classification performs well to search for an unknown number of targets.

V. CONCLUSIONS

The main objective of this paper was to present an autonomous search planning structure in which the vast amount

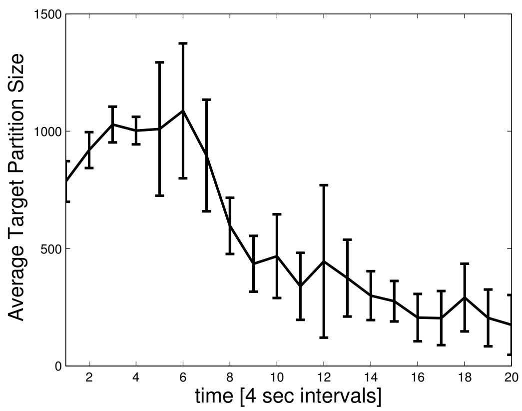

Fig. 7. Sample mean average partition size of partitions containing targets over simulation time. Error bars represent sample standard deviation.

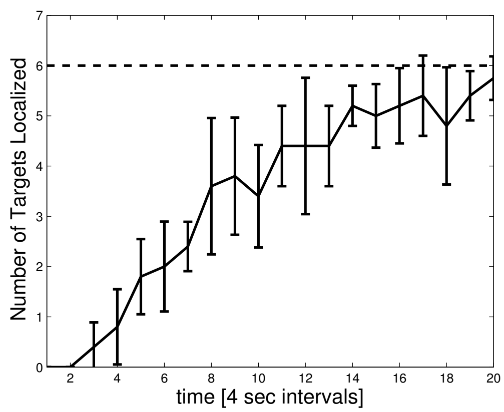

Fig. 8. Sample mean number of targets localized over simulation time. Error bars represent sample standard deviation.

of already developed path planning methods can be extended to the case of searching for an unknown number of targets. Additionally, extensions of these path planning methods to partition level target density distributions was presented. An overview of the planning structure developed to accomplish this is shown in Fig. 1. According to this planning structure, a target density distribution is used to estimate the expected number of targets over the surveillance area. The target density distribution is analyzed to dynamically partition the surveillance area into one partition for exploration and an ordered set of partitions for search or tracking, as represented by Fig. 3. Path planning is separated into two layers as depicted in Fig. 2. The first layer of path planning views the partitions as tasks and allocates them to the team of agents. This layer of path planning utilizes the work that has already been developed for task allocation among a team of autonomous agents. The second layer of the path planning determines agent search paths by optimizing directly over partition level target density distributions. This layer of path planning required an approach that generalizes and extends path planning approaches based on direct probability distribution optimization. Results suggest the planning structure of this paper performs well to dynamically partition the surveillance area, plan paths for autonomous agents, and localize an initially unknown number targets.

REFERENCES

[1] L. Stone, *Theory of Optimal Search*. New York, NY: Academic Press, 1975.

[2] M. Godwin and J. K. Hedrick, "Stochastic approximation of an online vehicle routing problem for autonomous aircraft," in *Infotech@Aerospace Conference*, 2011.

[3] M. Alighanbari and J. P. How, "A robust approach to the UAV task assignment problem," *International Journal of Robust and Nonlinear Control*, vol. 18, no. 2, pp. 118–134, 2008.

[4] B. Grocholsky, A. Makarenko, and H. Durrant-Whyte, "Information-theoretic coordinated control of multiple sensor platforms," in *Robotics and Automation, 2003. Proceedings. ICRA '03. IEEE International Conference on*, vol. 1, 2003, pp. 1521 – 1526 vol.1.

[5] F. Bourgault, T. Furukawa, and H. Durrant-Whyte, "Optimal search for a lost target in a bayesian world," in *Field and Service Robotics*, ser. Springer Tracts in Advanced Robotics, S. Yuta, H. Asama, E. Prassler, T. Tsubouchi, and S. Thrun, Eds. Springer Berlin / Heidelberg, 2006, vol. 24, pp. 209–222, 10.1007/10991459_21.

[6] C. Kreucher, A. Hero, K. Kastella, and M. Morelande, "An information-based approach to sensor management in large dynamic networks," *Proceedings of the IEEE*, vol. 95, no. 5, pp. 978 –999, May 2007.

[7] J. Tisdale, Z. Kim, and J. Hedrick, "Autonomous uav path planning and estimation," *Robotics Automation Magazine, IEEE*, vol. 16, no. 2, pp. 35 –42, 2009.

[8] D. V. Lindley, "On a Measure of the Information Provided by an Experiment," *The Annals of Mathematical Statistics*, vol. 27, no. 4, pp. 986–1005, 1956.

[9] G. Hoffmann and C. Tomlin, "Mobile sensor network control using mutual information methods and particle filters," *Automatic Control, IEEE Transactions on*, vol. 55, no. 1, pp. 32 –47, 2010.

[10] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. John Wiley & Sons, Inc., 2001.

[11] I. Csiszar, "Information-type measures of difference of probability distributions and indirect observations," in *Sudia Sci. Math. Hungar.*, vol. 2, 1967, pp. 299–318.

[12] A. Renyi, "On measures of entropy and information," in *Proc. 4th Berkeley Symp. on Mathematical Statistics and Probability*, vol. 1, 1961, pp. 547–561.

[13] J. Wood, B. Kehoe, and J. K. Hedrick, "Target estimate pdf-based optimal path planning algorithm with application to uav systems," in *Proceedings Dynamic Systems and Control Conference*. ASME, 2010.

[14] L. D. Stone, T. L. Corwin, and C. A. Barlow, *Bayesian Multiple Target Tracking*, 1st ed. Norwood, MA, USA: Artech House, Inc., 1999.

[15] R. P. S. Mahler, *Statistical Multisource-Multitarget Information Fusion*. Norwood, MA, USA: Artech House, Inc., 2007.

[16] E.-M. Wong, F. Bourgault, and T. Furukawa, "Multi-vehicle bayesian search for multiple lost targets," in *Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on*, 2005, pp. 3169 – 3174.

[17] T. Furukawa, F. Bourgault, B. Lavis, and H. Durrant-Whyte, "Recursive bayesian search-and-tracking using coordinated uavs for lost targets," in *Robotics and Automation, 2006. ICRA 2006. Proceedings 2006 IEEE International Conference on*, May 2006, pp. 2521 –2526.

[18] R. Mahler, "Unified sensor management using cphd filters," in *Information Fusion, 2007 10th International Conference on*, 2007, pp. 1 –7.

[19] J. Wood and J. K. Hedrick, "Space partitioning and classification for multi-target search and tracking by heterogeneous unmanned aerial system teams," in *Infotech@Aerospace 2011*. AIAA, 2011.

[20] R. O. Duda, P. E. Hart, and D. G. Stork, *Pattern Classification*. New York, NY, USA: John Wiley & Sons, Inc., 2001.

[21] Z. Kim and R. Sengupta, "Target detection and position likelihood using an aerial image sensor," in *Proceedings International Conference on Robotics and Automation*, 2008.

[22] J. Garvey, B. Kehoe, B. Basso, M. Godwin, J. Wood, J. Love, S.-Y. Liu, Z. Kim, S. Jackson, Y. Fallah, T. Fu, R. Sengupta, and J. K. Hedrick, "An autonomous unmanned aerial vehicle system for sensing and tracking," in *Infotech@Aerospace Conference*, 2011.