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Nonlinear Control of PVTOL Vehicles subjected to Drag and Lift

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Abstract— **The present study extends and encompasses a previous work on the control of thrust-propelled vehicles which focused on vehicles subjected to environmental reaction forces that are reduced to their drag component, as in the case of spherical bodies. Lift forces associated with other body shapes, like winged aerial vehicles, modifies and complicates the control problem significantly. This paper shows the existence of a generic set of drag-and-lift models for which it is possible to recast the initial control problem into the one of controlling a spherical body, thus allowing for the application of control design methods and analyses developed previously. Beside the obtention of more general nonlinear control solutions that apply to a larger class of vehicles and for which (semi) global stability results can be proved, we view this extension as a step to the automatic monitoring of flight transitions between hovering and high-velocity cruising for convertible aerial vehicles.**

I. INTRODUCTION

Feedback control of aerial vehicles in order to achieve some degree of autonomy remains an active research domain after decades of studies on the subject. The complexity of aerodynamic effects and the diversity of flying vehicles partly account for this continued interest. Lately, the emergence of small vehicles for robotic applications (helicopters, quad-rotors, etc) has also renewed the interest of the control community for these systems. This paper aims at improving and extending existing feedback control techniques by taking into account aerodynamic effects in the control design.

Most of aerial vehicles belong either to the class of fixed-wing vehicles, or to that of rotary-wing vehicles. The first class is mainly composed of airplanes. In this case, weight is compensated for by lift forces acting essentially on the wings, and propulsion is used to counteract drag forces associated with large air velocities. The second class contains several types of systems, like helicopters, ducted fans, quad-rotors, etc. In this case, lift forces are usually not preponderant and the *thrust force*, produced by one or several propellers, has also to compensate for the vehicle's weight. These vehicles are usually referred to as Vertical Take-Off and Landing vehicles (VTOLs) because they can perform stationary flight (hovering). On the other hand, energy consumption is high due to small lift-to-drag ratios. By contrast, airplanes cannot (usually) perform stationary flight, but they are much more efficient energetically than VTOLs in cruising mode. Control design techniques for airplanes and VTOLs have developed along different directions and suffer from specific limitations. Feedback control of airplanes explicitly takes into account lift forces via linearized models at low angles of attack. Based on these models, stabilization is usually achieved through linear control techniques [1]. As a consequence, the obtained stability is local and difficult to quantify. Linear techniques are used for VTOLs too, but several nonlinear feedback methods have also been proposed in the last decade to obtain (semi) global stability [2] [3] [4] [5]. These methods, however, are based on oversimplified aerodynamic models that neglect aerodynamic forces. In fact, even drag effects are but seldom taken into account [6]. Therefore, these methods are not relevant to the control of airplanes or any other aerial vehicle subjected to significant lift forces. Another drawback of the independent development of control methods for airplanes and VTOLs is the lack of tools for flying vehicles that belong to both classes. These are usually referred to as *convertible* as they can both perform stationary flight and benefit from lift properties at high airspeed via optimized aerodynamic profiles. This versatility explains the growing interest in the design and control of such systems in recent years [7] [8] [9] [10]. The control literature on this topic, however, is scarce. This can be explained by the difficulty to operate transitions between stationary flight and cruising modes, in relation to strong variations of drag and lift forces during these transitions.

In view of these observations, we believe that there is a strong potential benefit in bringing control techniques for airplanes and VTOLs closer. The present work is a step in this direction. A major difficulty for the control of winged systems is the dependence of aerodynamic forces upon the so-called *angle of attack*. Under some assumptions, we show that part of these forces can be compensated for via a change of thrust control input so that the dynamics of the transformed system does not depend on the angle of attack. The control design is then much simplified, and can be addressed with techniques recently proposed in the literature [6]. In particular, semi-global stability and robustness to unmodelled dynamics can be achieved. Application of the proposed control strategy to NACA profiles of aircraft wings [11] illustrates the pertinence of the approach. For simplicity the method and analysis are here exposed for a vehicle moving in the vertical plane (PVTOL case) solely.

The paper is organized as follows. After specifying the notation used in the paper, general dynamics equations and standard knowledge concerning the modeling of aerodynamic forces are recalled in Section II. The main result concerning the existence of simple generic drag-and-lift models which simplify the design of nonlinear controllers is presented in Section III, together with stabilizing feedback laws. Application to NACA airfoils is addressed in Section IV. Simulation results for a controlled wing-shaped body are reported in Section V. Remarks and perspectives conclude the paper.

II. Background

A. Notation

We assume that the controlled vehicle can be modeled in the first approximation by a single actuated body immersed in a fluid which exerts motion reaction forces on it.

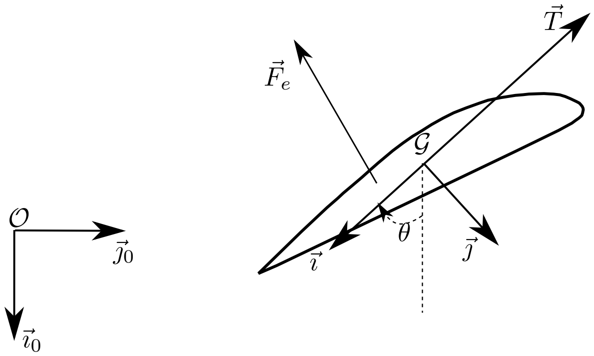

Fig. 1. Body subjected to external reaction forces

The following notation is used.

• G is the body's center of mass and *m* is the mass of the vehicle, assumed to be constant.

• $\mathcal{I} = \{0; \vec{\imath}_0, \vec{\jmath}_0\}$ is a fixed inertial frame with respect to (w.r.t.) which the vehicle's absolute pose is measured. $\mathcal{B} = \{G; \vec{\imath}, \vec{\jmath}\}$ is a frame attached to the body. The vector $\vec{\imath}$ is parallel to the thrust force axis. This leaves two possible and opposite directions for this vector. The direction here chosen ($\vec{\imath}$ pointing downward nominally) is consistent with the convention used for VTOL vehicles.

• The vector of coordinates of G in the basis of the fixed frame $\mathcal{I}$ is denoted as $x = (x_1, x_2)^T$. Therefore, $\vec{OG} = x_1\vec{\imath}_0 + x_2\vec{\jmath}_0$. We also write this relation in a more concise way as $\vec{OG} = (\vec{\imath}_0\ , \vec{\jmath}_0)x$. The vehicle's orientation is characterized by the angle θ between $\vec{\imath}_0$ and $\vec{\imath}$. The vector of coordinates associated with the linear velocity of G w.r.t. $\mathcal{I}$ is denoted as $\dot{x} = (\dot{x}_1, \dot{x}_1)^T$, and as $v = (v_1, v_2)^T$ when expressed in the basis of $\mathcal{B}$, i.e. $\vec{v} = \frac{d}{dt}\vec{OG} = (\vec{\imath}_0, \vec{\jmath}_0)\dot{x} = (\vec{\imath}, \vec{\jmath})v$.

• The wind velocity w.r.t. $\mathcal{I}$ is denoted as $\vec{v}_w = (\vec{\imath}_0, \vec{\jmath}_0)\dot{x}_w = (\vec{\imath}, \vec{\jmath})v_w$. The *airspeed* $\vec{v}_a$ of the body is the difference between the velocity of G and $\vec{v}_w$. Thus, $\vec{v}_a = (\vec{\imath}_0, \vec{\jmath}_0)\dot{x}_a = (\vec{\imath}, \vec{\jmath})v_a$, with $\dot{x}_a = \dot{x} - \dot{x}_w$ and $v_a = v - v_w$.

• The rotation matrix of an angle θ in the plane is $R(\theta)$.

• $\{e_1, e_2\}$ denotes the canonical basis in $\mathbb{R}^2$. $S = R(\pi/2)$ is a unitary skew-symmetric matrix, and $I = R(0)$ is the (2×2) identity matrix.

• Given a vector of coordinates v, its i_{th} component is denoted as v_i. Given a smooth function $f : \mathbb{R} \to \mathbb{R}$, its first and second derivative are denoted as f' and f'' respectively. Given a function f of several variables, the partial derivative of f w.r.t. one of them, say x, is denoted as $\partial_x f = \frac{\partial f}{\partial x}$.

B. System modeling

The equations of motion of the vehicle in the vertical plane are derived by considering two control inputs. The first one is a *thrust* force $\vec{T}$ along the body fixed direction $\vec{\imath}$ whose main role is to produce longitudinal motion. It is assumed that this force $\vec{T} = -T\vec{\imath}$ applies at a point lying on, or close to, the axis $\{G; \vec{\imath}\}$, so that it does not create an important torque at G. The second control input is a torque actuation, typically created via secondary propellers, rudders or flaps, control moment gyros, etc. For the sake of simplification, we assume here that any desired torque can be produced so that the vehicle's angular velocity ω can be modified at will and used as a control variable (see [6] for complementary explanations concerning this assumption). The external forces $\vec{F}_e$ acting on the body are composed of the gravity $m\vec{g}$ and the aerodynamic forces denoted by $\vec{F}_a$. Thus, $\vec{F}_e = m\vec{g} + \vec{F}_a$ and the resultant force applied to the vehicle is $\vec{F} = -T\vec{\imath} + m\vec{g} + \vec{F}_a$.

Applying the fundamental theorem of mechanics, one obtains the following equations of motion:

$$m\ddot{x} = -TR(\theta)e_1 + mge_1 + F_a(\dot{x}_a, \theta), \tag{1}$$

$$\dot{\theta} = \omega, \tag{2}$$

with g the gravity constant and F_a the aerodynamic forces expressed in the inertial frame, i.e. $\vec{F}_a = (\vec{\imath}_0\ , \vec{\jmath}_0)F_a$.

C. Aerodynamic forces

Interactions between a solid body and the surrounding fluid are governed by *Navier−Stokes equations*. These equations are complex since they consist of a set of nonlinear partial differential equations involving several scalar functions of the position x like *viscosity*, *compressibility*, and *density* of the fluid. Moreover, even when these functions can be considered constant in the first approximation, solving the equations requires spatial integration over the shape of the body which typically does not yield closed-form expressions. The complexity of the problem can be reduced when the airspeed $\dot{x}_a$ belongs to the *subsonic range*, condition for which the Mach number M, defined as the ratio of $|\dot{x}_a|$ to the *speed of sound*, is significantly smaller than one. In this case, the expression of the aerodynamic forces F_a can be approximated by a function that depends only on the constant air density ρ, the Reynolds number[1] R_e , the characteristic length of the vehicle's body Σ, the airspeed $\dot{x}_a$, and the *angle of attack* α. This latter variable is the angle between the body-fixed *zero-lift* line, along which the airspeed does not produce perpendicular forces, and the airspeed vector $\vec{v}_a$. By denoting the (constant) angle between the zero-lift line and the thrust direction $\vec{\imath}$ as μ, and the angle of the airspeed w.r.t. the fixed vertical direction $\vec{\imath}_0$ as $\xi(\dot{x}_a)$, one has (see Fig. 2):

$$\xi(\dot{x}_a) = \text{atan2}(\dot{x}_{a_2}, \dot{x}_{a_1}), \tag{3}$$

and

$$\alpha(\dot{x}_a, \theta, \mu) = \pi + \theta - \xi(\dot{x}_a) - \mu. \tag{4}$$

[1] R_e gives a measure of the ratio of inertial forces to viscous forces.

The aerodynamic force vector $\vec{F}_a$ is typically decomposed in two components: the *lift force*, perpendicular to the airspeed, and the *drag force*, parallel to the airspeed. For a single body moving within the subsonic range, and for a fixed Reynolds number, a common approximation of the function F_a is of the form [1]:

$$F_a(\dot{x}_a, \theta, \mu) = k_a|\dot{x}_a|\Big[c_L(\cdot)S - c_D(\cdot)I\Big]_{\big|\alpha(\dot{x}_a,\theta,\mu)}\dot{x}_a, \quad (5)$$

where $k_a := \frac{1}{2}\rho\Sigma$, $c_L(\alpha)$ is the *lift coefficient* and $c_D(\alpha) > 0$ is the *drag coefficient*. The functions c_L and c_D are called *aerodynamic characteristics* of the body.

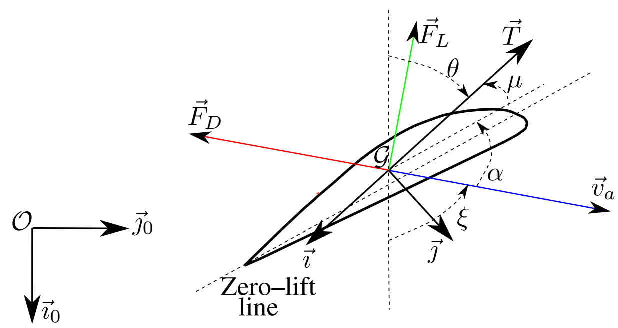

Fig. 2. Thrust-propelled body subjected to aerodynamic effects

The control design is much simplified when F_a does not depend on the vehicle's attitude θ. For example, a constant velocity flight is achieved by aligning the thrust direction $R(\theta)e_1$ in (1) with the external force $(mge_1 + F_a)$, via the angular velocity control ω, and by setting $T = |mge_1 + F_a|$ (modulo correction terms needed to compensate for tracking errors w.r.t. a given reference trajectory). Enforcing this strategy when F_a depends on θ is far from obvious because any change of the vehicle's thrust direction (i.e. the vehicle's orientation) affects the external force as well. However, we show in the next section that, for a specific class of aerodynamic characteristics, this latter case can essentially be recast into the former case.

III. MAIN RESULTS

Proposition 1 *Assume that the resultant of the aerodynamic forces is of the form* (5).

(i) If the aerodynamic characteristics are given by

$$\begin{cases} c_D(\alpha) = c_1 + 2c_2\sin^2(\alpha), \\ c_L(\alpha) = c_2\sin(2\alpha), \end{cases} \quad (6)$$

with c_1 and c_2 denoting two constant real-numbers, then the change of thrust control input

$$T \longrightarrow T_p = T + 2c_2k_a|\dot{x}_a|^2\cos(\alpha - \mu), \quad (7)$$

transforms the system's equation (1) *into:*

$$m\ddot{x} = -T_pR(\theta)e_1 + mge_1 + F_p(\dot{x}_a, \mu), \quad (8)$$

with

$$F_p(\dot{x}_a, \mu) = k_a|\dot{x}_a|\Big[\overline{c}_L(\mu)S - \overline{c}_D(\mu)I\Big]\dot{x}_a \quad (9a)$$

$$\overline{c}_D(\mu) = c_1 + 2c_2\cos^2(\mu), \quad (9b)$$

$$\overline{c}_L(\mu) = -c_2\sin(2\mu). \quad (9c)$$

(ii) If the aerodynamic characteristics c_D and c_L are respectively even and odd functions (as in the case of symmetric bodies), i.e.

$$\begin{cases} c_D(\alpha) = c_D(-\alpha), \\ c_L(\alpha) = \,-\,c_L(-\alpha), \end{cases} \quad (10)$$

then (6) *is the only family of aerodynamic characteristics for which there exists a function F_p independent of θ such that* (8) *holds true for any $\mu \in \mathbb{S}^1$.*

This proposition points out the modeling functions (6) for the aerodynamic characteristics used in the remainder of this paper and the fact that, in the case of symmetric bodies, they are the only functions which allow for the transformation of the system's equation (1) into (8) with F_p independent of the attitude θ.

Given (3), (4), (5), (6), (7) and (9), it is possible to prove the item i) of the proposition by verifying via direct calculations that $-TRe_1 + F_a(\dot{x}_a, \theta, \mu) = -T_pRe_1 + F_p(\dot{x}_a, \mu)$. As a matter of fact, Proposition 1 is a corollary of a more general result stated below. It addresses the problem of transforming System (1) into the form (8), when F_a is given by (5) without the symmetry constraints (10) being imposed upon the aerodynamic characteristics.

Theorem 1 *Assume that the resultant of the aerodynamic forces is given by* (5)*. Then,*

(i) System (1) *can be transformed into the form* (8) *with F_p independent of θ if and only if the aerodynamic characteristics $c_L(\alpha)$ and $c_D(\alpha)$ satisfy the following differential equation:*

$$(c_D'' - 2c_L')\sin(\alpha + \mu) + (c_L'' + 2c_D')\cos(\alpha + \mu) = 0. \quad (11)$$

(ii) If (11) *holds true, the vector valued function F_p depending only on $\dot{x}_a$ and μ, and the scalar function T_p such that* (8) *holds true are respectively given by:*

$$F_p(\dot{x}_a, \mu) = k_a|\dot{x}_a|\Big[\overline{c}_L(\mu)S - \overline{c}_D(\mu)I\Big]\dot{x}_a \quad (12)$$

with

$$\begin{cases} \overline{c}_D(\mu) = c_D(0) + c_L'(0)\cos 2(\mu) + c_D'(0)\sin(\mu)\cos(\mu), \\ \overline{c}_L(\mu) = c_L(0) - c_D'(0)\sin 2(\mu) - c_L'(0)\sin(\mu)\cos(\mu), \end{cases}$$

and

$$T_p = T + k_a|\dot{x}_a|\dot{x}_a^T R(\theta)\begin{bmatrix} -c_L'(\alpha) \\ c_D'(\alpha) \end{bmatrix}. \quad (13)$$

The proof of this theorem is given in the paper's appendix.

Note that condition (11) is satisfied for any value of μ only if

$$\begin{cases} c_D'' - 2c_L' = 0 \quad \forall\alpha, \\ c_L'' + 2c_D' = 0 \quad \forall\alpha. \end{cases} \quad (14)$$

The general solution to the linear differential system (14) is:

$$\begin{cases} c_D(\alpha) = b_0 + b_1 \sin(2\alpha) - b_2 \cos(2\alpha), \\ c_L(\alpha) = b_3 + b_1 \cos(2\alpha) + b_2 \sin(2\alpha), \end{cases}$$

with b_j denoting constants numbers. When the shape of the body is symmetric, the above functions must also satisfy the conditions (10). This implies that b_1 and b_3 are equal to zero. Using the fact that $\cos(2\alpha) = 1 - 2\sin^2(\alpha)$, one obtains (6) with $c_1 = b_0 - b_2$ and $b_2 = c_2$. As for relation (7), it results from (13) and (6), using the fact that $\dot{x}_a^T R(\theta) = v_a^T = |\dot{x}_a|(-\cos(\alpha+\mu), \sin(\alpha+\mu))$.

Once System (1), with F_a given by (5) and (6), is transformed into the form (8), with the "apparent external force" $mge_1 + F_p$ no longer depending on the attitude θ, the control design can be addressed by adapting the method developed for the class of systems subjected to drag forces only (see below for a possibility based on [6]). As for the model (6) of the aerodynamic characteristics, we show in the next section that it can provide a good approximation of the "low-frequency part" of the physical coefficients measured for several symmetric profiles. To illustrate the application of Proposition 1 to the control of aerial vehicles, a stabilizing feedback law for the tracking of a pre-determined reference trajectory $x_r(t)$ for System (8) is proposed next. This control solution is based on [6, Sec. III.D] and is here recalled for the sake of completeness. It is also of interest to pinpoint a few complementary issues attached to the extension here considered.

In view of (8) the *tracking error* dynamics are governed by:

$$\dot{\widetilde{x}} = R(\theta)\widetilde{v}, \tag{15a}$$

$$\dot{\widetilde{v}} = -\omega S \widetilde{v} - u e_1 + R(\theta)^T \gamma_v, \tag{15b}$$

$$\dot{\theta} = \omega, \tag{15c}$$

where $u := T_p/m$, $\gamma_p := F_p/m$, $\gamma_v := ge_1 + \gamma_p(\dot{x}_a, \mu) - \ddot{x}_r$, $\widetilde{x} := x - x_r$ is the position tracking error, and $\widetilde{v} := R^T(\theta)(\dot{x} - \dot{x}_r)$ is the velocity error expressed in the body-fixed frame.

To compensate for almost constant unmodeled additive perturbations, integral correction terms are introduced in the control law. A nonlinear "bounded integral" of the position error $\widetilde{x}$ is, for instance, the output z of the following second-order nonlinear system:

$$\ddot{z} = -2k_z\dot{z} - k_z^2[z - \text{sat}_\Delta(z)] + k_z h_z(|\widetilde{x}|^2)\widetilde{x}, \qquad k_z > 0,\ z(0) = 0,\ \dot{z}(0) = 0, \tag{16}$$

where h_z is a smooth bounded strictly positive function defined on $[0, +\infty)$ satisfying the following properties ([6, Sec. III.C]) for some positive constant numbers η_z, β_z,

$$\forall s \in \mathbb{R}, \quad |h_z(s^2)s| < \eta_z \text{ and } 0 < \frac{\partial}{\partial s}(h_z(s^2)s) < \beta_z.$$

sat_Δ is the classical *saturation function*, i.e. $\text{sat}_\Delta(z) = z\min(1, \Delta/|z|)$. Let $\widehat{F_p}$ denote an estimation of F_p and define $\widehat{\gamma}_p := \widehat{F_p}/m$. Let

$$y := \widetilde{x} + z, \tag{17}$$

$$\overline{v} := \widetilde{v} + R^T(\theta)\dot{z}, \tag{18}$$

$$\gamma := ge_1 + \widehat{\gamma}_p - \ddot{x}_r(t) + h(|y|^2)y + \ddot{z}, \tag{19}$$

with h a smooth bounded positive function satisfying the same above properties as h_z for some positive constant numbers η, β. Nonlinear controllers endowed with provable stabilizing properties in a large domain of operation are recalled next.

Proposition 2 *[6] Assume that the following regularity conditions are satisfied:*

(i) F_p is continuously differentiable and its partial derivatives are bounded uniformly w.r.t. $\dot{x}_a$ in compact sets,

(ii) the vectors $\dot{x}_w$, $\ddot{x}_w$, $\dddot{x}_w$, $\dot{x}_r$, $\ddot{x}_r$ and $\dddot{x}_r$ are bounded in norm on $\mathbb{R}_+$.

Let k_1, k_2 and k_3 denote strictly positive constants. Let $\sigma : \mathbb{R} \to \mathbb{R}$ denote a strictly increasing smooth function such that $\sigma(0) = 0$ and $\sigma(s) > -1/k_1,\ \forall s \in \mathbb{R}$. Apply the following control law

$$u = |\gamma| + k_1|\gamma|\sigma(\overline{v}_1) \quad (>0), \tag{20a}$$

$$\omega = k_2|\gamma|\left(\overline{v}_2 - \frac{\overline{v}_1\overline{\gamma}_2}{|\gamma| + \overline{\gamma}_1}\right) + \frac{k_3|\gamma|\overline{\gamma}_2}{(|\gamma| + \overline{\gamma}_1)^2} - \frac{\gamma^T S \dot{\gamma}}{|\gamma|^2}, \tag{20b}$$

to System (15) *with $\overline{\gamma} := R^T(\theta)\gamma$ and y, $\overline{v}$, γ defined by Eqs.* (17)-(19)*. Suppose that:*

(i) there exists a constant $\delta > 0$ such that $|\gamma| > \delta\ \forall t \in \mathbb{R}^+$,

(ii) the modeling error $c := \gamma_p - \widehat{\gamma}_p$ is constant,

(iii) $\lim_{s \to +\infty} h(s^2)s > |c|$.

(iv) $\Delta > |z^*|$, where z^* denotes the unique solution to the equation $h(|z^*|^2)z^* = c$ and Δ is the positive constant intervening in the function sat_Δ.

Then, for the tracking error system (15) *complemented with* (16)*, the equilibrium point $(z, \dot{z}, \widetilde{x}, \widetilde{v}, \widetilde{\theta}) = (z^*, 0, 0, 0, 0)$ of the controlled system is asymptotically stable, with the domain of attraction equal to $\mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \times (-\pi, \pi)$.*

Remarks: 1) The feedforward term $\dot{\gamma}$ in (20b) depends on $\ddot{x}_a$ via the time-derivative of $\widehat{\gamma}_p$, and the estimation of this term in practice is not simple. For this reason it is tempting to set it equal to zero in the control calculation. Simulations indicate that doing so does not significantly degrade the tracking performance in a large range of flight conditions.
2) Although the control law (20a) ensures the positivity of the variable u, the positivity of the thrust T, which is calculated from $T_p = mu$ via (7), is not guaranteed. In particular, the thrust control value may become negative when $|\dot{x}_a|$ is large or when the control has to comply with important reference decelerations. This issue thus deserves to be more thoroughly studied.

IV. Application to NACA 00XX profiles

A class of bodies well referenced in the literature is given by the NACA airfoils. These bodies are airfoil shapes for aircraft wings described by a series of digits following the word "NACA". We focus here on symmetric bodies identified by the string "NACA 00XX". In order to obtain a good adequacy between the aerodynamic characteristics of the model (6) and those measured experimentally, the following cost function is introduced:

$$E(c_1, c_2) = \sum_{i=1}^{N} [c_{L_R}(\alpha_i) - c_L(\alpha_i)]^2 + [c_{D_R}(\alpha_i) - c_D(\alpha_i)]^2,$$

where $\alpha_1, \ldots, \alpha_N$ denote values of the angle of attack for which measurements $c_{L_R}(\alpha_i)$ and $c_{D_R}(\alpha_i)$ are available. Assuming that the drag coefficient at zero-angle of attack $c_D(0)$ is known, so that $c_1 = c_D(0)$, the minimizing value of c_2 is given by:

$$c_2 = \frac{\sum_{i=1}^{N} \left[c_{L_R}(\alpha_i)\sin(2\alpha_i) + 2(c_{D_R}(\alpha_i) - c_1)\sin^2(\alpha_i)\right]}{4\sum_{i=1}^{N} \sin^2(\alpha_i)}.$$

Figure 3 shows a typical approximation result for the lift and drag characteristics when considering angles of attack over $[0, 2\pi)$. The measurements, in blue, were obtained from the experimental data presented in [11] and elaborated by [12] for the profile NACA0021. The approximation result, in red, is good almost everywhere, except for small angles of attack (modulo π), before the stall zone around $\pm 20°$. Clearly, results would be significantly different if only small angles of attack were considered. For instance, the coefficient c_2 characterizing the increase of lift with the angle of attack would be much larger. This indicates that the non-linearity of the stall phenomenon cannot be approximated by a model as simple as (6) and also suggests that a switching policy between several set of coefficients depending on the flight conditions could be of interest. This possibility could be explored in future studies. As for spherical bodies, for which no lift force is produced, the aerodynamic characteristics are well modeled everywhere by setting $c_2 = 0$. Then $T_p = T$ and $F_p(\dot{x}_a) = F_a(\dot{x}_a) = -k_a c_1 |\dot{x}_a|\dot{x}_a$.

V. Simulation: from hovering to cruising flight

We illustrate through a simulation of a classical PVTOL maneuver the performance and robustness of the proposed approach for the NACA 0021 airfoil model. The system's equations of motion are defined by Eqs. (1-2), with F_a given by (5) and the numerical data of c_L and c_D obtained by interpolation of the experimental data reported in [12] (see Figure 3). The physical parameters are: $\rho = 1.292\ [Kg/m^3]$, $m = 300\ [Kg]$, $\Sigma = 5\ [m]$. Other values are used for the calculation of the control in Proposition 2 in order to test the control robustness w.r.t. parametric errors. They are chosen as follows : $\widehat{\rho} = 1.4\ [Kg/m^3]$, $\widehat{m} = 270\ [Kg]$, $\widehat{\Sigma} = 4.9\ [m]$. The angle μ is set equal

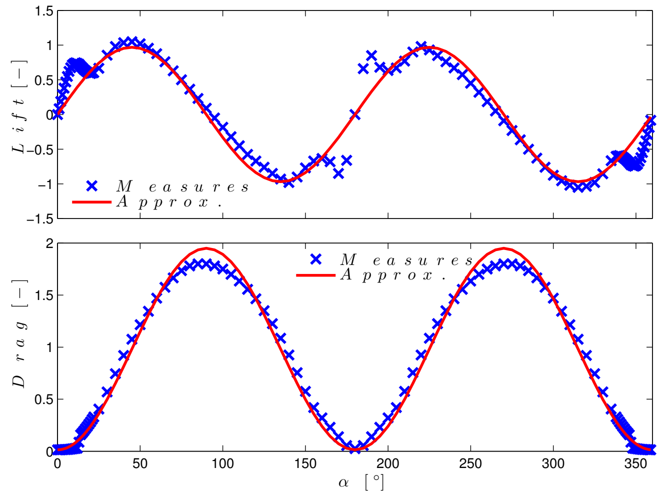

Fig. 3. Aerodynamic characteristics of the NACA0021 airfoil. Top: lift coefficients. Bottom: drag coefficients.

to zero (thrust direction aligned with zero-lift line) so that $\widehat{\gamma}_p = -\frac{(c_1+2c_2)\widehat{k}_a}{\widehat{m}}|\dot{x}_a|\dot{x}_a$, with $\widehat{k}_a = \frac{1}{2}\widehat{\rho}\widehat{\Sigma}$. The coefficients $c_1 = 0.0139$ and $c_2 = 0.9430$ are determined by applying the estimation procedure described in Section IV. The term $\ddot{x}_a$ in the expression of the feedforward term $\dot{\gamma}$ is kept equal to zero, thus providing another element to test the robustness of the controller. The thrust control input T applied to System (1-2) is calculated from $u = T_p/m$ according to (7) with m and k_a replaced by their estimated values. Also, the second term in the right hand side of equation (7) is set equal to zero when the airspeed $|\dot{x}_a|$ is smaller than some threshold here chosen equal to 2 $[m/s]$. Doing so avoids the ill-conditioned problem of evaluation of α for small airspeeds. The values for the other control parameters are:

- $k_1 = 0.1529$, $k_2 = 0.0234$, $k_3 = 6$;
- $h(s) = \beta/\sqrt{1 + \frac{\beta^2}{\eta^2}s}$ with $\beta = 0.5$ and $\eta = 10$;
- $k_z = 0.5$, $h_z(s) = \beta_z/\sqrt{1 + \frac{\beta_z^2}{\eta_z^2}s}$ with $\beta_z = 0.5$ and $\eta_z = 0.5$;
- $\text{sat}_\Delta(z) = z \min\left(1, \frac{\Delta}{|z|}\right)$ with $\Delta = 100$,
- $\sigma(s) = (0.4/k_1)\tanh(k_1 s/0.4)$.

The gains k_1, k_2, k_3, k_z, $h(0)$ and $h_z(0)$ are determined via a pole placement procedure performed on the linear approximation of the system (15-16) in hovering flight (see [13] for details).

The reference trajectory $x_r(t)$ used to simulate the aforementioned PVTOL maneuver is defined by:

$$\dot{x}_r(t) = \begin{cases} (0,0)^T & 0 \leq t < 60, \\ (0, 2(t-60))^T & 60 \leq t < 80, \\ (0,40)^T & t \geq 80. \end{cases} \quad (21)$$

with $x_r(0) = (-50, 50)^T$. Therefore, it consists of: i) a stationary point on the time interval $[0, 60)$ $[s]$; ii) an horizontal velocity ramp on the time interval $[60, 80)$ $[s]$; iii) cruising with constant horizontal velocity of 40 $[m/s]$ for $t \geq 80$ $[s]$. The applied thrust force is saturated as follows:

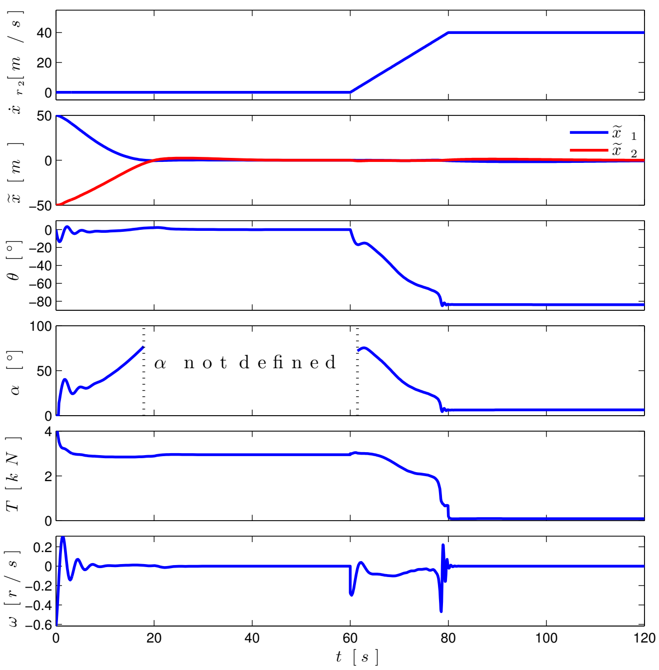

Fig. 4. From top to bottom: reference velocity $\dot{x}_{r_2}$, position errors $\widetilde{x}$, orientation θ, angle of attack α, thrust T and desired angular velocity ω.

$0 < T < 2mg$. The initial position, velocity, and attitude are $x(0) = [0, 0]$, $\dot{x}(0) = [0, 0]$, and $\theta(0) = 0$, respectively.

From top to bottom, Figure 4 depicts the evolution of the desired horizontal velocity, the position errors, the vehicle's attitude, the angle of attack, the thrust force and the desired angular velocity. No wind is blowing. On the interval $[18, 61]$ $[s]$, the angle of attack is essentially undefined due to an airspeed smaller than 2 $[m/s]$, the vehicle's attitude converges towards zero (vertical configuration), and the thrust tends to oppose the body's weight. When $t > 60$ $[s]$, the horizontal velocity of the vehicle increases and the angle of attack starts decreasing. On the time interval $[76, 80]$ $[s]$ the angle of attack crosses the stall zone and the norm of the tracking error $\widetilde{x}$ augments up to 2 $[m]$ before it decreases again to zero. The thrust force rapidly decreases when α converges to the final cruising angle of attack equal to $6.26°$.

VI. Conclusion and Future Work

This paper extends a previous work on the 2D-control of underactuated thrust-propelled vehicles subjected to environmental forces that are independent of the vehicle's attitude, as in the case of spherical bodies subjected to drag forces. In general, lift forces that depend on the vehicle's attitude also arise. They may even be essential to the vehicle's motion and control, as in the case of airplanes. The paper's main contribution is to point out a set of simple drag-and-lift models, commonly used in the literature, which simplify and unify the control design because they allow for the transformation –via a change of thrust intensity– of the original dynamical equations of the system into these of a vehicle subjected to drag forces only. At this stage, this unification paradigm is still mostly conceptual. Eventhough preliminary validation by simulation is encouraging, much has to be done to complement the study and work out a general nonlinear control design methodology for thrust-propelled vehicles. Among the many issues to be addressed or further explored, let us mention the monitoring of the transitions between hovering and cruising for convertible airplanes subjected to highly nonlinear stall phenomena. Taking into account actuators' limitations, such as saturations and the impossibility of generating a negative thrust for a number of vehicles, has to be studied. On the theoretical side, a particularly important issue concerns the possibility of extending the approach to the 3D-case. As for the applicability of the approach in practice, control implementation aspects related to available sensory data and vehicle's state-estimation, including the estimation of the critically sensitive angle of attack, need also to be addressed.

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Appendix

A. Proof of Theorem 1

Most functions' arguments are purposefully omitted to lighten the notation. Throughout the proof, it is assumed that F_a is given by (5). Note that, using the relations

$$\begin{cases} v_{a_1} = -\,|v_a|\cos(\alpha+\mu) \\ v_{a_2} = +\,|v_a|\sin(\alpha+\mu) \end{cases} \tag{22}$$

and $\dot{x}_a = Rv_a$, F_a can also be written as:

$$F_a = -k_a|\dot{x}_a|^2 RA_\mu(\alpha) \tag{23}$$

with

$$A_\mu(\alpha) := \begin{pmatrix} c_L(\alpha)\sin(\alpha+\mu) - c_D(\alpha)\cos(\alpha+\mu) \\ c_L(\alpha)\cos(\alpha+\mu) + c_D(\alpha)\sin(\alpha+\mu) \end{pmatrix} \quad (24)$$

Proof of (i)**:** First, remark that (1) can be written as (8) if and only if

$$-TRe_1 + F_a \equiv -T_pRe_1 + F_p \quad (25)$$

Assuming that this relation holds with F_p independent of θ, let us show that (11) is satisfied. It follows from (25) that

$$e_2^T R^T (F_a - F_p) \equiv 0, \quad (26)$$

This equality can also be written as

$$F_{p_2}\cos(\theta) - F_{p_1}\sin(\theta) = F_{a_2}\cos(\theta) - F_{a_1}\sin(\theta). \quad (27)$$

Differentiating w.r.t. θ and using the assumption according to which F_p does not depend upon θ yields

$$\begin{aligned} F_{p_2}\sin(\theta) + F_{p_1}\cos(\theta) = &- \partial_\theta F_{a_2}\cos(\theta) + \partial_\theta F_{a_1}\sin(\theta) \\ &+ F_{a_2}\sin(\theta) + F_{a_1}\cos(\theta). \quad (28) \end{aligned}$$

Relations (27) and (28) can be regrouped and written as:

$$R^T F_p = R^T F_a + \begin{bmatrix} \sin(\theta)\partial_\theta F_{a_1} - \cos(\theta)\partial_\theta F_{a_2} \\ 0 \end{bmatrix}.$$

Multiplying both members of this equality by R yields

$$F_p \equiv F_a + \Lambda \partial_\theta F_a, \quad (29)$$

with

$$\Lambda(\theta) := R(\theta)\begin{pmatrix} \sin(\theta) & -\cos(\theta) \\ 0 & 0 \end{pmatrix}, \quad (30)$$

Using (23), it follows that

$$F_p = -k_a|\dot{x}_a|^2 R \begin{pmatrix} -A'_{\mu,2}(\alpha) \\ A_{\mu,2}(\alpha) \end{pmatrix} \quad (31)$$

with $A_{\mu,2}$ denoting the second row of A_μ. Now, the non-dependence of F_p upon θ implies that $\partial_\theta F_p = 0$. In view of the expression (31) of F_p, this yields

$$SR\begin{pmatrix} -A'_{\mu,2} \\ A_{\mu,2} \end{pmatrix} + R\begin{pmatrix} -A''_{\mu,2} \\ A'_{\mu,2} \end{pmatrix} = 0. \quad (32)$$

Pre-multiplying the left-hand side of this equality by R^T, one obtains $A''_{\mu,2} + A_{\mu,2} = 0$. This relation, combined with (24), yields (11).

Conversely, assuming that (11) is satisfied, let us show the existence of F_p, independent of θ, and of T_p for which (25) is true. Consider the candidate function given by (31). Using (11) one verifies that $\partial_\theta F_p = 0$. Therefore, this function is independent of θ. Using (23) and (31) it is also straightforward to verify that $e_2^T R^T (F_a - F_p) = 0$. This readily implies the existence of T_p such that (25) is satisfied, i.e.

$$T_p = T + e_1^T R^T (F_p - F_a). \quad (33)$$

This concludes the proof of (i).

Proof of (ii)**:** Let us assume that (11) is satisfied so that, as shown above, (8) holds true with F_p, independent of θ, given by (31).

Let us now establish (12). Using (24), (22), and $R^T\dot{x}_a = v_a$ in (31) yields

$$F_p = -k_a|\dot{x}_a| R \left.\begin{pmatrix} c'_L + c_D & c_L - c'_D \\ -c_L & c_D \end{pmatrix}\right|_\alpha R^T \dot{x}_a.$$

Since F_p does not depend on θ, one can use any value of θ in this expression. Take, for instance, $\theta = \xi(\dot{x}_a) - \pi + \mu$. Then, in view of (4), $\alpha = 0$ and the above expression of F_p becomes

$$F_p = -k_a|\dot{x}_a| R(\xi+\mu) C R^T(\xi+\mu)\dot{x}_a, \quad (34)$$

with

$$\begin{aligned} C &= \begin{pmatrix} c'_L(0) + c_D(0) & c_L(0) - c'_D(0) \\ -c_L(0) & c_D(0) \end{pmatrix} \\ &= c_D(0)I - c_L(0)S + \begin{pmatrix} c'_L(0) & -c'_D(0) \\ 0 & 0 \end{pmatrix}. \quad (35) \end{aligned}$$

Using (35) in (34) with $R^T(\xi)\dot{x}_a = |\dot{x}_a|e_1$ and $|\dot{x}_a|R(\xi) = (\dot{x}_a, S\dot{x}_a)$ yields (12).

Finally, since F_p satisfies (29) and $\partial_\theta F_a = \partial_\alpha F_a$, (33) becomes

$$T_p = T + e_1^T R^T \Lambda \partial_\alpha F_a. \quad (36)$$

From the expression (5) of F_a, a direct calculation of the right-hand term of (36) gives (13).