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Control of Discrete Linear Repetitive Processes using Non-local Previous Pass Information

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Abstract

A repetitive process makes a series of sweeps or passes through dynamics defined on a finite duration termed the pass length. The process output is termed the pass profile and when each pass is completed, resetting to the starting location ready for the start of the next one. On any pass the previous pass profile acts as a forcing function on, and hence contributes to, the dynamics of the next one. There has been a considerable volume of profitable work on the development of a control theory for these processes with more recent focus on the design of control laws. The novel contribution of this paper is a new design algorithm which makes more use of available previous pass profile information and reduces the conservativeness present in existing alternatives.

1. INTRODUCTION

Repetitive processes are characterized by a series of sweeps, termed passes, through dynamics defined over a finite duration known as the finite pass length. Once a pass is completed the process resets to the initial position and the next one commences. Each pass profile acts as a disturbance function on, and hence contributes to, the dynamics of the next one [1]. This interaction between successive pass profiles leads to the unique control problem where the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.

To introduce a formal definition in the case of discrete dynamics let $y_k(p)$, $0 \leq p \leq \alpha - 1$, $k \geq 0$, denote the, scalar or vector valued, pass profile of finite duration α on pass k. Then in a repetitive process $y_k(p)$ acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile $y_{k+1}(p)$, $0 \leq p \leq \alpha - 1$, $k \geq 0$.

Repetitive processes have their origins in the coal mining industry [1] where in the long-wall mode of operation coal is extracted by a series of passes of the finite length coal face by a coal cutting machine. During this operation, the coal cutting machine rests on the pass profile produced during the previous pass, that is, the height of the coal/stone interface above some datum line. The result can be undulations in the pass profile that increase in amplitude from pass-to-pass and require a suspension of productive work to enable their removal.

Applications exist where adopting a repetitive process setting for analysis can be used to productive effect. Examples include iterative algorithms for solving nonlinear dynamic optimal control problems based on the maximum principle [2]. In this case, use of the repetitive process setting provides the basis for the development of highly reliable and efficient solution algorithms. More recent work on the use of this setting for the analysis of optimal control/optimization problems includes a gas pipeline application [3]. Also iterative learning control algorithms can be designed in the repetitive process setting with very good agreement between predicted and measured results [4].

For some discrete linear repetitive processes it is possible to check stability by using necessary and sufficient tests developed for 2D discrete linear systems described by Roesser [5] and Fornasini Marchesini [6] state-space models. Such tests do not, however, provide a basis for control law design except in relatively low-order examples. A generally applicable alternative is to use a Lyapunov function interpretation of stability, leading to Linear Matrix Inequality (LMI) based conditions that also produce formulas for the design of im-

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plementable control laws. These LMI based algorithms are based on sufficient but not necessary stability tests and hence they can be conservative, a critical issue in terms of applications that is addressed in this paper.

Consider a discrete linear repetitive process on pass k and instance p, denoted by (k,p). Then in terms of control the most obvious route is feedback activated by the state or pass profile vector at (k,p), but in general the use of information from both the current and previous passes is required, such as current pass state action coupled with a term actuated by the previous pass profile. Most of the currently available control law design algorithms are locally actuated where, as one example, at (k,p) the control law is actuated by the state or pass profile vector at (k,p) and the pass profile vector at $(k-1,p)$. However, at (k,p) the complete previous pass profile has already been generated and is therefore available for use in control law design and hence the non-locally actuated control law is possible. This paper develops algorithms for designing control laws that at any instance on the current pass are partially actuated using the pass profile vector at other instances along the previous pass. The outcome is again LMI based designs and an example is given where the new approach is less conservative than existing LMI based control law design algorithms..

Throughout this paper, the null matrix and the identity matrix with compatible dimensions are denoted by 0 and I, respectively. Also $\bigoplus$ (and $\oplus$) denotes direct sum of matrices and $\otimes$ denotes the Kronecker product of matrices, $\mathbf{diag}(W_1,W_2)$ denotes a block diagonal matrix with diagonal blocks W_1 and W_2, $\mathbf{tr}(.)$ denotes the matrix trace, $M>0$ (<0) denotes a real symmetric positive (negative) definite matrix, $X \leq Y$ is used to represent the case when $X-Y$ is a negative semi-definite matrix, and $\star$ denotes a block entry in a symmetric matrix. Finally, $||\cdot||$ denotes the norm in the considered Banach space.

2. PRELIMINARIES

Discrete linear repetitive processes evolve over the subset of the positive quadrant in the 2D plane defined by $\{(p,k): 0 \leq p \leq \alpha-1, k \geq 0\}$, and the most basic state-space model for their dynamics has the following form [1]

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p)+Bu_{k+1}(p)+B_0y_k(p), \\ y_{k+1}(p) &= Cx_{k+1}(p)+Du_{k+1}(p)+D_0y_k(p). \end{aligned} \tag{1}$$

On pass k in this model $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the pass profile vector, and $u_k(p) \in \mathbb{R}^r$ is the vector of control inputs. Note that when $p=\alpha-1$, $x_{k+1}(p+1)$ is equal to $x_{k+1}(\alpha)$ and hence α is termed the pass length.

To complete the process description it is necessary to specify the boundary conditions, that is, the pass state initial vector sequence and the initial pass profile. The simplest form of these is $x_{k+1}(0)=d_{k+1}$, $k \geq 0$, where the $n\times 1$ vector d_{k+1} has known constant entries, and $y_0(p)=f(p)$, where $f(p)$ is an $m\times 1$ vector whose entries are known functions of p.

The state-space model (1) has strong structural similarities with the well known Roesser [5] and Fornasini Marchesini [6] state-space models for 2D discrete linear systems. These similarities have led to the use of results from the extensive literature for these models to solve systems theoretic questions for processes described by (1). There are, however, important systems theoretic questions for these processes which cannot be answered in this way. For example, pass profile controllability requires that for given boundary conditions there exists a computable control input sequence such that an example described by (1) produces a pre-defined pass profile vector either on some pass or with the pass number also pre-defined. This property is well defined in terms of applications and has no 2D Roesser or Fornasini-Marchesini state-space model interpretation, nor can conditions for its existence be established using systems theory for these models.

As discussed in the previous section long-wall coal cutting can be modeled as a repetitive process. In this application, the pass profile is the height of the stone/coal interface above some datum line. Also the cutting machine rests on the previous pass profile during the production of the current one. It is therefore unrealistic to assume that at instance p on the current pass the only previous pass profile contribution comes from the same instance. An improved model is

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p)+\hat{B}u_{k+1}(p)+\sum_{i=-\varepsilon}^{\gamma} B_i y_k(p+i), \\ y_{k+1}(p) &= Cx_{k+1}(p)+\hat{D}u_{k+1}(p)+\sum_{i=-\varepsilon}^{\gamma} D_i y_k(p+i), \end{aligned} \tag{2}$$

where ε and γ are positive integers and the boundary conditions applied in this paper are those for (1), but with the additional assumption that

$$\begin{aligned} y_0(i) &= 0, \quad -\varepsilon \leq i \leq -1, \\ y_0(i) &= 0, \quad \alpha \leq i \leq \alpha+\gamma-1. \end{aligned} \tag{3}$$

The dynamics are again defined over $\{(p,k): 0 \leq p \leq$

$\alpha - 1, k \geq 0\}$ and hence

$$\begin{aligned} y_k(i) &= 0, \quad -\varepsilon \leq i \leq -1, \\ y_k(i) &= 0, \quad \alpha \leq i \leq \alpha + \gamma - 1. \end{aligned} \tag{4}$$

Setting $\varepsilon = 0$ and $\gamma = 0$ recovers the previous model.

In this alternative model at instance (k, p) the previous pass profile contribution is modeled as a linear sum of those at $0 \leq p - \varepsilon \leq p \leq p + \gamma \leq \alpha - 1$. The resulting model structure has no 2D Roesser or Fornasini-Marchesini state-space model interpretation.

3. STABILITY ANALYSIS

The stability theory [1] for linear repetitive processes is based on an abstract model in a Banach space setting which includes a wide range of examples as special cases. In terms of their dynamics it is the pass-to-pass coupling, noting again their unique feature, which is critical. This is of the form $y_{k+1} = L_\alpha y_k$, where $y_k \in E_\alpha$, E_α a Banach space and L_α is a bounded linear operator mapping E_α into itself. It is routine to show that the model of (1) can be written in the abstract model form, with $E_\alpha = \ell_2^m[0, \alpha]$. A similar construction holds for processes described by (2).

In terms of the abstract model, the stability theory for linear repetitive processes [1] is defined in bounded-input bounded-output (BIBO) terms and characterized in terms of properties of L_α. In particular, a bounded initial pass profile is required to produce a bounded sequence of pass profiles where boundedness is defined in terms of the norm on E_α. The stability theory has two forms termed asymptotic and along the pass, respectively, where the former demands this property over the finite and fixed pass length α for a given example and the latter for all possible values of the pass length.

Asymptotic stability is equivalent to the existence of finite real scalars $M_\alpha > 0$ and $\lambda_\alpha \in (0, 1)$ such that $||L_\alpha^k|| \leq M_\alpha \lambda_\alpha^k$, $k \geq 0$, where $||\cdot||$ also denotes the induced operator norm. It can be shown [1] that asymptotic stability holds if and only if $r(L_\alpha) < 1$, where $r(\cdot)$ denotes the spectral radius. Also if this property holds then the strong limit $y_\infty := \lim_{k\to\infty} y_k$ is termed the limit profile and is the unique solution of $y_\infty = L_\alpha y_\infty + b_\infty$.

In the case of processes described by (1), it is known [1] that asymptotic stability holds if and only if $r(D_0) < 1$. Moreover, if asymptotic stability holds and the control input sequence applied is strongly convergent in the pass-to-pass direction, the limit profile is described by a standard, also termed 1D in the repetitive process literature, discrete linear systems state-space model with state matrix $A + B_0(I - D_0)^{-1}C$. Hence the dynamics can converge to and unstable limit profile as the simple case when $A = -0.5, B_0 = 0.5 + \beta$, $C = 1$, $D = 0$, and $D_0 = 0$, where β is a real scalar with $|\beta| \geq 1$ demonstrates. The reason why this case arises is due to the finite pass length, over which duration even an unstable 1D linear system can produce a bounded output. This analysis is easily extended to examples described by (2).

To prevent examples such as the one given above from arising, stability along the pass demands the BIBO property for all possible values of the pass length. This requires the existence of finite real scalars $M_\infty > 0$ and $\lambda_\infty \in (0, 1)$, which are independent of α, such that $||L_\alpha^k|| \leq M_\infty \lambda_\infty^k$, $k \geq 0$. In the case of processes described by (1), the following result gives necessary and sufficient conditions that be tested by direct application of 1D discrete linear systems stability tests.

Lemma 1 *[1] A discrete linear repetitive process described by (1) is stable along the pass if and only if*

i) $r(D_0) < 1$,

ii) $r(A) < 1$, *and*

iii) all eigenvalues of the transfer-function matrix

$$G(z) = C(zI - A)^{-1}B_0 + D_0,$$

have modulus strictly less than unity for all $|z| = 1$.

In the case of processes described by (2) introduce the Lyapunov function

$$V(k, p) = V_1(k, p) + V_2(k, p), \tag{5}$$

where

$$V_1(k, p) = \sum_{i=-\varepsilon}^{\gamma} y_k^T(p+i) Q_i y_k(p+i), \tag{6}$$

and

$$V_2(k, p) = x_k^T(p) P x_k(p), \tag{7}$$

with $P > 0$ and $Q_i > 0$, $i = -\varepsilon, 1, \ldots, \gamma$. The term $V_1(k, p)$ captures the pass-to-pass energy change and $V_2(k, p)$ the change in energy along a pass. Moreover, the associated increment is

$$\begin{aligned} \Delta V(k, p) = {} & y_{k+1}^T(p) \Big(\sum_{i=-\varepsilon}^{\gamma} Q_i \Big) y_{k+1}(p) \\ & + x_{k+1}^T(p+1) P x_{k+1}(p+1) \\ & - \sum_{i=-\varepsilon}^{\gamma} y_k^T(p+i) Q_i y_k(p+i) - x_{k+1}^T(p) P x_{k+1}(p). \end{aligned} \tag{8}$$

Summing over $p = 0$ to $p = \alpha - 1$ gives the global Lyapunov function

$$V(k) = \sum_{p=0}^{\alpha-1} V(k, p), \tag{9}$$

with associated increment

$$\Delta V(k) = \sum_{p=0}^{\alpha-1} \Delta V(k,p). \tag{10}$$

The proof of the next result follows by routine extensions to that for a process described by (1) given in [1] (setting $\varepsilon = 0$ and $\gamma = 0$ recovers this previous case) and hence the details are omitted.

Theorem 1 *A discrete linear repetitive process described by (2) is stable along the pass if*

$$\Delta V(k) < 0, \tag{11}$$

for all possible values of the pass length.

4. CONTROL DESIGN

An extensively analyzed [1] static control law for processes described by (1) has the following form over $0 \leq p \leq \alpha - 1, k \geq 0$

$$u_{k+1}(p) = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x_{k+1}(p) \\ y_k(p) \end{bmatrix}, \tag{12}$$

where K_1 and K_2 are compatibly dimensioned matrices to be designed. This control law is composed of the weighted sum of current pass state feedback and feedforward of the previous pass profile, where achieving stability along the pass by current pass state or pass profile vector feedback alone is not possible in most cases.

The design of (12) could be attempted using Lemma 1, but this is not very tractable and extension to robustness analysis is also problematic. An alternative that extends naturally to robustness analysis is to use an LMI setting for analysis and design.

Theorem 2 *[1] Suppose that a control law of the form* (12) *is applied to a discrete linear repetitive process described by* (1). *Then the resulting controlled process is stable along the pass if there exist matrices* $W = \textbf{diag}(W_1, W_2)$, $W_1 > 0, W_2 > 0$, *G, and*

$$N = \begin{bmatrix} \overline{N}_1 & \overline{N}_2 \\ \overline{N}_1 & \overline{N}_2 \end{bmatrix}, \tag{13}$$

such that

$$\begin{bmatrix} -G - G^T + W & (\Phi G + \Psi N)^T \\ \Phi G + \Psi N & -W \end{bmatrix} < 0, \tag{14}$$

where

$$\Phi = \begin{bmatrix} A & B_0 \\ C & D_0 \end{bmatrix}, \Psi = \begin{bmatrix} B & 0 \\ 0 & D \end{bmatrix}. \tag{15}$$

If the LMI of (14) holds, stabilizing K_1 and K_2 in the control law (12) *are given by*

$$K = NG^{-1}, \tag{16}$$

where

$$K = \begin{bmatrix} K_1 & K_2 \\ K_1 & K_2 \end{bmatrix}. \tag{17}$$

In implementation terms, the control law (12) requires that all elements of the current pass state vector are available for measurement. If this is not true then an observer will be required to reconstruct the current pass state vector or else the state vector is replaced by the current pass profile vector. The pass profile vector is the process output and in this work it is assumed that any unwanted effects, such as noise, are negligible.

The conditions of Theorem 2 are sufficient but not necessary and hence there is an associated level of conservativeness, which means that if Theorem 2 does not hold a stabilizing control law may exist. Hence there is a need to investigate ways of reducing this level of conservativeness and in this paper the route is augment the control law (12) with extra previous pass profile contributions. The resulting control law over $k \geq 0$ and $0 \leq p \leq \alpha - 1$, is given by

$$u_{k+1}(p) = K_x x_{k+1}(p) + \sum_{i=-\varepsilon}^{\gamma} K_i y_k(p+i). \tag{18}$$

The controlled process state-space model obtained by applying the control law (18) to (1) is

$$\begin{aligned} x_{k+1}(p+1) &= \mathbb{A} x_{k+1}(p) + \sum_{i \in \mathscr{I}} \mathbb{B}_i y_k(p+i) + \mathbb{B}_0 y_k(p), \\ y_{k+1}(p) &= \mathbb{C} x_{k+1}(p) + \sum_{i \in \mathscr{I}} \mathbb{D}_i y_k(p+i) + \mathbb{D}_0 y_k(p), \end{aligned} \tag{19}$$

where $\mathscr{I}\{-\varepsilon, \ldots, -1, 1, \ldots, \gamma\}$ and

$$\begin{aligned} \mathbb{A} &= A + BK_x, \quad \mathbb{B}_i = BK_i, \quad \mathbb{B}_0 = B_0 + BK_0, \\ \mathbb{C} &= C + DK_x, \quad \mathbb{D}_i = DK_i, \quad \mathbb{D}_0 = D_0 + DK_0, \end{aligned} \tag{20}$$

which is of the form (2).

To apply the result of Theorem 1, introduce the following notation

$$\eta = \varepsilon + \gamma + 2, \tag{21}$$

and

$$\widetilde{B}=\begin{bmatrix}\mathbb{A} & \mathbb{B}_{-\varepsilon} & \cdots & \mathbb{B}_{-1} & \mathbb{B}_0 & \mathbb{B}_1 & \cdots & \mathbb{B}_\gamma\\ 0 & & & & & & & 0\\ \vdots & & & & \ddots & & & \vdots\\ 0 & & & & & & & 0\end{bmatrix},$$

$$\widetilde{D}=\begin{bmatrix}\mathbb{C} & \mathbb{D}_{-\varepsilon} & \cdots & \mathbb{D}_{-1} & \mathbb{D}_0 & \mathbb{D}_1 & \cdots & \mathbb{D}_\gamma\\ \vdots & & & & \ddots & & & \vdots\\ \mathbb{C} & \mathbb{D}_{-\varepsilon} & \cdots & \mathbb{D}_{-1} & \mathbb{D}_0 & \mathbb{D}_1 & \cdots & \mathbb{D}_\gamma\end{bmatrix},$$

$$\widehat{Q}=\bigoplus_{i=-\varepsilon}^{\gamma} Q_i, \quad \widehat{Z}=P\oplus\widehat{Q}, \quad \widehat{P}=I_\eta\otimes P, \tag{22}$$

where the block entry matrices $\widetilde{B}$ and $\widetilde{D}$ are of dimensions $\eta\times\eta$ and $(\eta-1)\times\eta$, respectively. Using this notation, the condition of Theorem 1 can be written as

$$\widetilde{D}^T\widehat{Q}\widetilde{D}+\widetilde{B}^T\widehat{P}\widetilde{B}-\widehat{Z}<0. \tag{23}$$

This condition is not in LMI form, and hence no effective methods are available to compute the control law matrices. The following result removes this difficulty.

Theorem 3 *Suppose that a control law of the form* (18) *is applied to a discrete linear repetitive process described by* (1). *Then the resulting controlled process is stable along the pass if exist matrices $\check{P}>0, N_x$, $\check{Q}_i>0$, and N_i, $i=-\varepsilon,\dots,\gamma$, such that the following LMI holds*

$$\begin{bmatrix}-\bar{Z} & \star & \star\\ \widehat{A}\bar{Q}+\widehat{B}\widehat{N} & -\bar{P} & \star\\ \widehat{C}\bar{Q}+\widehat{D}\widehat{N} & 0 & -\bar{Q}\end{bmatrix}<0, \tag{24}$$

where

$$\bar{Q}=\bigoplus_{i=-\varepsilon}^{\gamma}\check{Q}_i, \quad \bar{Z}=\check{P}\oplus\bar{Q},$$

$$\bar{P}=I_\eta\otimes\check{P}, \quad \check{P}=P^{-1},$$

$$\widehat{A}=\begin{bmatrix}A & 0 & \cdots & 0 & B_0 & 0 & \cdots & 0\\ 0 & & & & & & & 0\\ \vdots & & & & \ddots & & & \vdots\\ 0 & & & & & & & 0\end{bmatrix},$$

$$\widehat{C}=\begin{bmatrix}C & 0 & \cdots & 0 & D_0 & 0 & \cdots & 0\\ \vdots & & & & \ddots & & & \vdots\\ C & 0 & \cdots & 0 & D_0 & 0 & \cdots & 0\end{bmatrix}, \tag{25}$$

$$\widehat{B}=\begin{bmatrix}B & \cdots & B\\ 0 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 0\end{bmatrix}, \widehat{D}=\begin{bmatrix}D & \cdots & D\\ \vdots & \ddots & \vdots\\ D & \cdots & D\end{bmatrix},$$

$$\widehat{N}=N_x\bigoplus\left(\bigoplus_{i=-\varepsilon}^{\gamma}N_i\right).$$

If the LMI of (24) holds, stabilizing matrices in the control law (18) *are given by*

$$K_x=N_x\check{P}^{-1},\, K_i=N_i\check{Q}_i^{-1},\, i=-\varepsilon,\dots,\gamma. \tag{26}$$

Proof 1 *Follows immediately on application of appropriate congruence transforms to* (23)*, use of the Schur's complement formula, and substitution of (26) into the result of the previous two steps.*

Direct application of Theorem 3 will, in the case when the pass profile is a scalar for simplicity of presentation, frequently produce solutions characterized by very small values of the control law matrices $K_i, i=-\varepsilon,\dots,\gamma$, which, in turn, can result in very large control signals. If the LMI (24) is feasible, there are infinitely many other solutions and one method of selecting an appropriate set of $K_i, i=-\varepsilon,\dots,\gamma$, is to use an additional optimization procedure.

Define the matrix function $\mathbf{SumElm}(M)$ on, say, an $n\times m$ matrix M as

$$\mathbf{SumElm}(M)=\sum_{r=1}^{n}\sum_{c=1}^{m}M(r,c), \tag{27}$$

where $M(r,c)$ denotes the element in row r and column c of M. Then the following optimization procedure can be employed to obtain an appropriate set of control law matrices $K_i, i=-\varepsilon,\dots,\gamma$.

$$\text{maximize}\left(\sum_{i=-\varepsilon}^{\gamma}\mathbf{tr}\left(\check{Q}_i\right)+\sum_{i=-\varepsilon}^{\gamma}\mathbf{SumElm}(N_i)\right),$$

subject to

$$\begin{bmatrix}-\bar{Z} & \star & \star\\ \widehat{A}\bar{Q}+\widehat{B}\widehat{N} & -\bar{P} & \star\\ \widehat{C}\bar{Q}+\widehat{D}\widehat{N} & 0 & -\bar{Q}\end{bmatrix}<0, \tag{28}$$

with the notation and requirements of Theorem 3, which can be undertaken using LMI solvers.

5. NUMERICAL EXAMPLE

Consider the case of (1) with $\alpha=26$ and

$$\begin{aligned}A &= \begin{bmatrix}0.1 & -0.78 & -0.77\\ 0.89 & -0.1 & -0.81\\ -0.56 & 0.79 & -0.71\end{bmatrix}, B=\begin{bmatrix}0.03\\ -0.47\\ -0.28\end{bmatrix},\\ C &= \begin{bmatrix}0.95 & -0.86 & -0.24\end{bmatrix}, D=-0.67,\\ B_0^T &= \begin{bmatrix}-0.18 & -0.82 & -0.65\end{bmatrix}, D_0=-0.31\end{aligned}$$

and boundary conditions $x_{k+1}(0)=0$, $k\geq 0$, $y_0(p)=1$, $0\leq p\leq\alpha-1$. This system is asymptotically stable

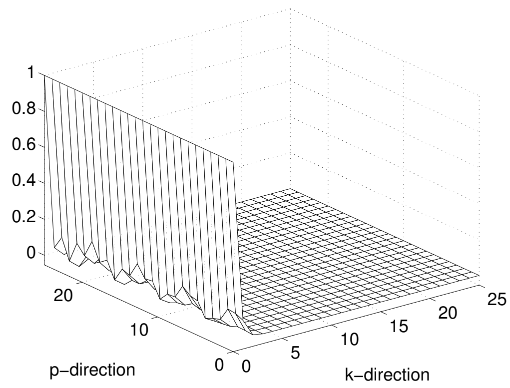

Figure 1. Pass profile dynamics for the controlled process.

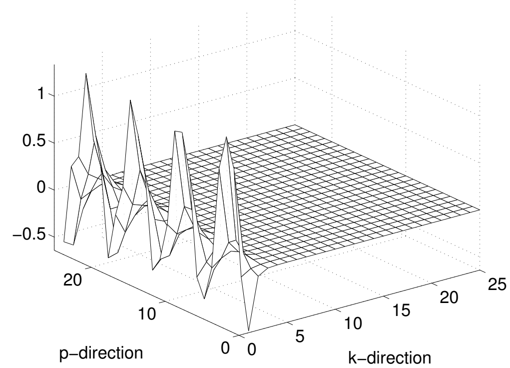

Figure 2. Control input signal for the controlled process.

since $r(D_0) = 0.31$ but unstable along the pass since one eigenvalue of A is -1.0356.

For this example, Theorem 2 does not produce a stabilizing control law of the form (12). Hence we try to find a stabilizing control law of the form (18) with $\varepsilon = 1$ and $\gamma = 1$. Application of Theorem 3 gives

$$\begin{aligned} K_x &= \begin{bmatrix} 1.3862 & -1.1204 & -0.4803 \end{bmatrix} \\ K_{-1} &= K_1 = -5.5765 \times 10^{-3}, K_0 = -0.6163, \end{aligned}$$

where the computations were undertaken using YALMIP [7] and the SeDuMi solver [8].

Figures 1 and 2, respectively, show the pass profile and control input sequences generated by the controlled process and confirm that stability along the pass is achieved but some redesign may be required to obtain lower control signal values in the early passes.

6. CONCLUSIONS AND FURTHER WORK

The paper has considered how to reduce the conservativeness present in LMI based design of a stabilizing control law for discrete linear repetitive processes. The method is to add extra previous pass profile actuated terms to the control law and an example has been given where this new law can be computed but not the original. Ongoing work includes replacing the current pass state vector component in the control law by a current pass profile term, where the pass profile is a directly measured output. The structure of the previous pass profile contribution to the current pass state and pass profile dynamics can take many possible forms and further research is needed to find the best possible form to use in a given application. Another area of possible future research is to further extend the control law (12) to make use of the complete previous pass profile.

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