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# Bias Reduction in DAE Estimators by Model Augmentation: Observability Analysis and Experimental Evaluation

Erik Höckerdal, Erik Frisk, and Lars Eriksson

***Abstract*— A method for bias compensation in model based estimation utilizing model augmentation is developed. Based on a default model, that suffers from stationary errors, and measurements from the system a low order augmentation is estimated. The method handles models described by differential algebraic equations and the main contributions are necessary and sufficient conditions for the preservation of the observability properties of the default model during the augmentation.**

**A characterization of possible augmentations found through the estimation, showing the benefits of adding extra sensors during the design, is included. This enables reduction of estimation errors also in states not used for feedback, which is not possible with for example PI-observers. Beside the estimated augmentation the method handles user provided augmentations, found through e.g. physical knowledge of the system.**

**The method is evaluated on a nonlinear engine model where its ability to incorporate information from additional sensors during the augmentation estimation is clearly illustrated. By applying the method the mean relative estimation error for the exhaust manifold pressure is reduced by 55 %.**

## I. INTRODUCTION

Accurate information of the internal state of systems is important for fulfilling the increasing demands on control accuracy and fault detection. At the same time the overall cost has to be kept as low as possible, which often implies that it is insufficient to rely upon only physical sensors. As a consequence, model based estimation has attracted attention.

A common situation is that models based on first principles exist and it is desirable to be able to use them for observer design. However, these models often have undesired properties that prevent them from being directly applicable for estimation in embedded systems, such as engine control units (ECU). One such deficiency is that, even if the system dynamics is well described, the models can suffer from stationary errors, or biases [1]. Figure 1 illustrates this with experimental data from an engine where the model captures dynamics well while there is a bias in estimate.

The number of models described by differential algebraic equations (DAE) are increasing, partly due to modern modeling tools such as DYMOLA and SIMSCAPE that often deliver DAE models and since DAE:s are a way of describing systems with both fast and slow dynamics. The latter arise when approximating fast dynamics with algebraic constraints, i.e. instantaneous dynamics. DAE applications range from electrochemical and reactive distillation processes [2] to combustion engines [3], [4].

The objective is to develop a method that enables usage of biased default models for estimation with reduced estimation bias where the reduction is achieved using model augmentation. Central in the method is preservation of the observability properties of the, biased, default model. The method is as an extension of the method developed for ordinary differential equations (ODE) in [5] to systems described by DAE:s.

E. Höckerdal, E. Frisk and L. Eriksson are with the Department of Electrical Engineering, Linköping University, 581 83 Linköping.
E-mail: {hockerdal,frisk,larer}@isy.liu.se

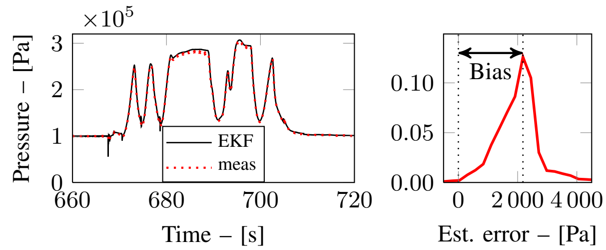


Fig. 1. Biased estimates for intercooler pressure in a heavy duty diesel engine. **Left:** Measured and estimated intercooler pressure. **Right:** Normalized histogram of the corresponding EKF estimation error which clearly is biased.

## II. PROBLEM FORMULATION AND SOLUTION OUTLINE

Designing an observer based on a model that predicts the system dynamics well but suffers from stationary errors will result in biased estimates [1]. Common ways to reduce bias in observers are; i) to use so called PI-observers [6], [7] that introduce integrators for the feedback signals, ii) by physical knowledge introduce extra states to compensate for known model deficiencies [8], or iii) to estimate a minimal augmentation that reduces the bias [5].

The objective is to reduce bias in estimates for observers based on DAE:s in a systematic manner without involving extensive modeling efforts in a similar way as for ODE:s in [5]. The starting point is a default, semi-explicit, DAE model

$$\dot{x} = f(\bar{x}, u) \tag{1a}$$
$$0 = g(\bar{x}, u) \tag{1b}$$
$$y = h(\bar{x}, u), \tag{1c}$$

where $\bar{x} = \binom{x}{z} \in \mathbb{R}^{n_{\bar{x}}}$, $x$ and $z$ denote differential and algebraic variables respectively, and measurements, $(u, y)$, $y \in \mathbb{R}^{n_y}$, $u \in \mathbb{R}^{n_u}$. The method generally handles DAE:s of differential index 1, i.e. systems where $\frac{\partial \binom{g}{h}}{\partial z}$ have full column rank. For index 1 DAE:s, without loss of generality, it is henceforth assumed that $\frac{\partial g}{\partial z}$ has full column rank. It is assumed that the model described by (1) captures the system dynamics well but suffers from stationary errors, i.e. the model is biased. A possible solution is obtained by considering the bias as an offset error during stationary operation of the system.

A way to compensate for stationary errors is to adjust the default states of the system according to $(\bar{x} - A_q q)$, where $q$ is a bias state and $A_q$ is its effect on the default states. By introducing $q$ as new states with constant derivatives, $\dot{q} = 0$, and driving noise, it is possible to describe a bias that varies with operating point. It is desirable to have as few bias states $q$ as possible and the method describes a way to estimate a low order augmentation $A_q$ from measurement data.

The objective is then to design an observer based on the augmented model,

$$\dot{x} = f(\bar{x} - A_q q, u) \tag{2a}$$
$$0 = g(\bar{x} - A_q q, u) \tag{2b}$$
$$\dot{q} = 0 \tag{2c}$$
$$y = h(\bar{x}, u), \tag{2d}$$

where $q \in \mathbb{R}^{n_q}$, denoted Aug., which will have better stationary estimation accuracy than an observer based on the default model. The observer design used is the discrete time extended Kalman filter (EKF) [9] and specifically the for DAE:s modified version presented in [2].

## III. EKF FOR DAE SYSTEMS

The EKF algorithm used originates from [2]. It is similar to the standard EKF; it consists of two steps, prediction and measurement update. The differential part of the DAE fits nicely into the standard EKF. To enable use of the standard EKF for both differential and algebraic states the linearized algebraic subsystem is differentiated once

$$\begin{aligned} \dot{x} &= A_t x + B_t z \\ 0 &= C_t x + D_t z \end{aligned} \quad \Rightarrow \quad \begin{aligned} \dot{x} &= A_t x + B_t z \\ \dot{z} &= -D_t^{-1} C_t \dot{x}, \end{aligned} \tag{3}$$

as in [2]. In [2], the partial derivatives

$$\begin{pmatrix} A_t & B_t \\ C_t & D_t \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial z} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial z} \end{pmatrix},$$

evaluated in ($x_t$, $z_t$, $u_t$), are assumed constant.

The algorithm used is summarized in Algorithm 1, where $\bar{G}_{t|t}$ is a result of the differentiation in (3) and implies that system noise is present in the differential variables of the DAE only and (4) is the linear approximation of the corresponding transition matrix $\phi = e^{\bar{A}_{t|t} T_s}$.

## IV. OBSERVABILITY OF THE AUGMENTED MODEL

In estimation the concept of observability, or detectability, is used to analyze the estimators' ability to provide consistent estimates that asymptotically converge to the true states. This section addresses the observability of the augmented model (2) given the observability properties of the default model (1), i.e., which $A_q$:s are allowed, given that the observability of the default model must not be compromised. Since the observability of a linearization in a stationary operating point is a sufficient condition for local observability of the nonlinear system [10, Theorem 6.4], the observability analysis is conducted on model linearizations.

**Algorithm 1** Extended Kalman Filter for DAE:s

1. Initialization:
$$\begin{pmatrix} \hat{x}_{0|0} \\ \hat{z}_{0|0} \end{pmatrix} = \begin{pmatrix} x_0 \\ z_0 \end{pmatrix} \quad \text{and} \quad P_{0|0} = \Pi_0,$$
where $\left(\begin{smallmatrix} x_0 \\ z_0 \end{smallmatrix}\right)$ is the initial state estimate and $\Pi_0 = \operatorname{cov}\left(\begin{smallmatrix} x_0 \\ z_0 \end{smallmatrix}\right)$. Let $t = 0$.
2. Prediction:
$$\begin{aligned} \hat{x}_{t+1|t} &= \tilde{f}(\hat{x}_{t|t}, \hat{z}_{t|t}, u_t) \\ 0 &= g(\hat{x}_{t+1|t}, \hat{z}_{t+1|t}) \end{aligned} \quad \Rightarrow \quad \begin{pmatrix} \hat{x}_{t+1|t} \\ \hat{z}_{t+1|t} \end{pmatrix}$$
$$\bar{P}_{t+1|t} = \bar{A}_{t|t} \bar{P}_{t|t} \bar{A}_{t|t}^T + \bar{G}_{t|t} Q \bar{G}_{t|t}^T,,$$
where the implication indicates solving $\hat{x}_{t+1|t}$ and $\hat{z}_{t+1|t}$ from the system of equations,
$$\bar{A}_{t|t} = I + T_s \begin{pmatrix} A_{t|t} & B_{t|t} \\ -D_{t|t}^{-1} C_{t|t} A_{t|t} & -D_{t|t}^{-1} C_{t|t} B_{t|t} \end{pmatrix}, \tag{4}$$
$T_s$ the sampling time, and
$$\bar{G}_{t|t} = \begin{pmatrix} I \\ -D_{t|t}^{-1} C_{t|t} \end{pmatrix},$$
from (3).
3. Measurement update:
$$\begin{aligned} S_{t+1} &= \bar{H}_{t+1|t} \bar{P}_{t+1|t} \bar{H}_{t+1|t}^T + R \\ \bar{K}_{t+1} &= \bar{P}_{t+1|t} \bar{H}_{t+1|t}^T S_{t+1}^{-1} \\ \hat{x}_{t+1|t+1} &= \hat{x}_{t+1|t} + \bar{K}_{t+1} \left( y_{t+1} - h(\hat{x}_{t+1|t}, \hat{z}_{t+1|t}, u_{t+1}) \right) \\ 0 &= g(\hat{x}_{t+1|t+1}, \hat{z}_{t+1|t+1}) \quad \Rightarrow \quad \hat{z}_{t+1|t+1} \\ \bar{P}_{t+1|t+1} &= \bar{P}_{t+1|t} - \bar{P}_{t+1|t} \bar{H}_{t+1|t}^T S_{t+1}^{-1} \bar{H}_{t+1|t} \bar{P}_{t+1|t}, \end{aligned}$$
where
$$\bar{H}_{t+1|t} = \begin{pmatrix} \frac{\partial h}{\partial x} & \frac{\partial h}{\partial z} \end{pmatrix} \Big|_{\left(\begin{smallmatrix} x \\ z \end{smallmatrix}\right) = \left(\begin{smallmatrix} \hat{x} \\ \hat{z} \end{smallmatrix}\right)_{t+1|t}}.$$
4. Let $t = t + 1$ and repeat from 2.

### A. DAE Observability

Before presenting the main results an overview of definitions and theorems used to assess observability for systems described by differential algebraic equations is given.

For DAE:s there exist several concepts of observability, i.e. complete observability, observability within the reachable set, and impulse observability [11], [12], [13].

*Definition 4.1 ($\mathcal{C}$-observable):* The system

$$\begin{aligned} F(t, x, \dot{x}, u) &= 0 \\ y - H(t, x) &= 0, \end{aligned} \tag{5}$$

is completely observable if the zero output of the descriptor system with $u = 0$ implies that this system has the trivial solution $x = 0$ only.

Generally, descriptor systems are not $\mathcal{C}$-observable since they contain algebraic constraints that force the solution and output, onto a specific manifold. For this reason the observability within the reachable set, i.e. $\mathcal{R}$-observability, is introduced. This concept needs an appropriate projection of the dynamical part of the system, sometimes referred to as the slow sub-system, onto a manifold defined by the algebraic equations, or fast sub-system. For linear time-invariant systems

$$\begin{aligned} E\dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned} \tag{6}$$

that are regular, i.e. that $\det(\alpha E - \beta A) \neq 0$ for some $(\alpha, \beta) \in \mathbb{C}^2$, this can be expressed in terms of the system matrices $E$, $A$, and $C$.

*Definition 4.2 ($\mathcal{R}$-observable):* The system (6) is called observable within the reachable set if the zero output of the descriptor system with $u = 0$ implies that all solutions of this system satisfy $P_r x = 0$, where $P_r$ denotes the projection onto the right deflating sub-space corresponding to the finite eigenvalues of $\lambda E - A$.

A last observability definition concerns the problem that arises when descriptor systems that are not strangeness-free [14], i.e. have differentiation index larger than 1, are driven by inputs that are only piecewise continuous [11]. Then, since the solution may depend on the derivative of the input, any classical solution may not exist.

*Definition 4.3 ($\mathcal{I}$-observable):* The system (6) is called impulse observable if the output is continuous when a step is used as input.

In the linear time-invariant case, (6), these concepts can be characterized algebraically in terms of $E$, $A$, and $C$, see for example [12] for theorems and proofs.

*Theorem 4.1:* The system (6) is

1) $\mathcal{C}$-observable if and only if

$$\begin{pmatrix} \alpha E - \beta A \\ C \end{pmatrix}$$

has full column rank for all $(\alpha, \beta) \in \mathbb{C}^2 \backslash \{(0, 0)\}$.

2) $\mathcal{R}$-observable if and only if

$$\begin{pmatrix} \lambda E - A \\ C \end{pmatrix}$$

has full column rank for all $\lambda \in \mathbb{C}$.

3) $\mathcal{I}$-observable if and only if

$$\begin{pmatrix} E \\ K_{E^T}^T A \\ C \end{pmatrix}$$

has full column rank, where the rows of $K_{E^T}$ span $\operatorname{Ker} E^T$.

Note that the system (6) is $\mathcal{C}$-observable if and only if it is $\mathcal{R}$-observable and $\begin{pmatrix} E \\ C \end{pmatrix}$ has full column rank. Furthermore, a descriptor system (6) with a regular pencil $\lambda E - A$ with differentiation index less than two is $\mathcal{I}$-observable [11], [12].

### *B. Possible Augmentations*

Based on the theory presented in Section IV-A, necessary and sufficient conditions for preserving the observability properties of the default model throughout the augmentation, are now given and proven.

Using Theorem 4.1 it is possible to characterize the allowable model augmentations for a descriptor system,

$$\begin{aligned} E\dot{x} &= A\left(x - A_q q\right) + Bu \\ \dot{q} &= 0 \\ y &= Cx, \end{aligned} \tag{7}$$

i.e. augmentations $A_q$ that preserve the observability of the default model (6).

*Theorem 4.2:* The observability of (6) is preserved during model augmentation according to (7) if and only if

$$A \begin{pmatrix} A_q & N_C \end{pmatrix},$$

has full column rank, where the columns of $N_C$ span $\operatorname{Ker} C$.

Worth noting is that even though there are three observability concepts, there is only one requirement. This due to the structure of the augmented system with $\bar{E} = \left(\begin{smallmatrix} E & 0 \\ 0 & I \end{smallmatrix}\right)$ which gives full column rank of the augmented subsystem.

*Proof:* The different observability properties of (7) are preserved if and only if $x = 0$, $q = 0$ is the only solution to the corresponding algebraic conditions in Theorem 4.1.

Rewriting the augmented system according to

$$\begin{aligned} \overbrace{\begin{pmatrix} E & 0 \\ 0 & I \end{pmatrix}}^{\bar{E}} \overbrace{\begin{pmatrix} \dot{x} \\ \dot{q} \end{pmatrix}}^{\dot{\bar{x}}} &= \overbrace{\begin{pmatrix} A & -AA_q \\ 0 & 0 \end{pmatrix}}^{\bar{A}} \overbrace{\begin{pmatrix} x \\ q \end{pmatrix}}^{\bar{x}} + \begin{pmatrix} B \\ 0 \end{pmatrix} \\ y &= \underbrace{\begin{pmatrix} C & 0 \end{pmatrix}}_{\bar{C}} \begin{pmatrix} x \\ q \end{pmatrix} \end{aligned} \tag{8}$$

and applying Theorem 4.1 to the augmented system (8):

**$\mathcal{R}$-observability** is preserved if and only if $x = 0$, $q = 0$ is the only solution to

$$(\lambda E - A)\, x + AA_q q = 0 \tag{9a}$$

$$\lambda I q = 0 \tag{9b}$$

$$y = Cx \tag{9c}$$

for all $\lambda \in \mathbb{C}$. For $\lambda \neq 0$ it is immediate from (9b) that $q = 0$. Then the assumption that (6) is $\mathcal{R}$-observable together with (9a), (9c) and Theorem 4.1 gives that $x = 0$. Thus only $\lambda = 0$ needs further investigation. For $\lambda = 0$ in (9) the augmented model is $\mathcal{R}$-observable if and only if $x = 0$, $q = 0$ is the only solution to

$$-Ax + AA_q q = 0, \tag{10a}$$

$$Cx = 0. \tag{10b}$$

Let the columns of $N_C$ be a basis for $\operatorname{Ker} C$, then, from (10b), $x = N_C \xi$ for some arbitrary $\xi$ and $\mathcal{R}$-observability is equivalent to $q = 0$, $\xi = 0$ being the only solution to

$$-A\left(N_C \xi - A_q q\right) = 0,$$

which is equivalent to the matrix

$$A \begin{pmatrix} N_C & A_q \end{pmatrix}$$

having full column rank.

$\mathcal{C}$**-observability** is equivalent to $\mathcal{R}$-observability with the additional requirement that

$$\begin{pmatrix} \bar{E} \\ \bar{C} \end{pmatrix} = \begin{pmatrix} E & 0 \\ 0 & I \\ C & 0 \end{pmatrix} \tag{11}$$

has full column rank [12]. The fact that the rank of a block diagonal matrix is equal to the sum of the ranks of the blocks [15] and the assumption that

$$\begin{pmatrix} E \\ C \end{pmatrix}$$

has full column rank now gives that also $\mathcal{C}$-observability is preserved if and only if

$$A \begin{pmatrix} N_C & A_q \end{pmatrix}$$

has full column rank.

$\mathcal{I}$**-observability** is preserved if and only if

$$\begin{pmatrix} \bar{E} \\ K_{\bar{E}^T}^T \bar{A} \\ \bar{C} \end{pmatrix} = \begin{pmatrix} E & 0 \\ 0 & I \\ K_{E^T}^T A & -K_{E^T}^T A A_q \\ C & 0 \end{pmatrix}$$

has full column rank. The Kernel of $\bar{E}$ is equal to the Kernel of $E$, padded with zeros to the dimension of $\bar{E}$. Again using that the rank of a block diagonal matrix is the sum of the ranks of the blocks and the assumption that

$$\begin{pmatrix} E \\ K_{E^T}^T A \\ C \end{pmatrix}$$

has full column rank the preservation of the $\mathcal{I}$-observability follows.

■

## V. Augmentation Estimation

After establishing a theoretical basis for all possible augmentations that preserve the default model observability properties, the method of estimating a low order model augmentation is presented.

The augmentation estimation procedure is divided into two steps; i) estimate the bias, and ii) compute a basis for the bias space. The bias estimation is performed by augmenting the default model fully, that is introduce as many bias states as possible without compromising the observability criteria in Theorem 4.2. That is, full column rank of

$$A \begin{pmatrix} N_C & A_q \end{pmatrix},$$

with dimension $n_{\bar{x}} \times (n_{\bar{x}} - n_y + n_q)$, i.e. $n_{\bar{x}} - n_y + n_q \leq n_{\bar{x}}$. This means that the augmentation $A_q$ can have at most as many columns as there are measurements, $n_q \leq n_y$. Also, from Theorem 4.2, these columns have to be linearly independent of the columns of $N_C$ and can not lie in $\operatorname{Ker} A$.

A simple way to construct such an augmentation is to use $C^\dagger$, where $\dagger$ denotes the Moore-Penrose inverse [15, Exercise 5.1.7 and Proposition 12.8.2], and exclude the columns that become zero when multiplied by $A$ from the left. Based on the fully augmented model, an observer that estimates both $\hat{x}$ and $\hat{q}$, enabling the computation of bias estimates,

$$\beta_t = C^\dagger \hat{q}_t,$$

can be constructed.

Central in the bias estimation is that the entire operating region of the system is spanned, otherwise the estimated bias samples might not represent the actual bias for all operating points.

Given bias estimates, a basis for the bias is computed using a singular value decomposition (SVD) [15] of the bias estimates. To increase the computational efficiency and allow easier weighting of the biases from different stationary operating points, the average of the bias samples from each operating point is computed. These averaged bias samples from $N$ operating points are collected

$$\bar{\beta}^{n_{\bar{x}} \times N} = \begin{pmatrix} w^1 \beta^1 & \cdots & w^N \beta^N \end{pmatrix}, \qquad \sum_1^N w^i = 1,$$

for which the corresponding SVD is computed

$$\bar{\beta} = U \Sigma V^*, \tag{12}$$

where $\beta^i$ indicates the averaged bias in operating point $i$, and $w^i$ is the corresponding importance weight. In (12) the columns of $U$ contains orthonormal vectors spanning the bias space and $\Sigma$ the corresponding singular values. The augmentation dimension can be found by analyzing the singular values and pick out the most significant ones. Then $\hat{A}_q$ is constructed by assembling the corresponding columns of $U$.

### A. Augmentation Properties

In Section IV-B the set of possible augmentations is analyzed, and it is apparent that the measurement equation plays a central role in which augmentations that are possible to find. This is also given by the bias estimation in Section V, i.e. that $\beta = C^\dagger \hat{q}$.

From an engineering perspective this is interesting since it means that it is possible to, temporarily augment the measurement equation in for example a development environment, to increase the set of possible augmentations. Increasing the set of possible augmentations like this gives the possibility to reduce bias also in states not used for feedback in the final application.

Also note that, even though the main idea with the method is to estimate an augmentation using system measurements, it is possible to provide an augmentation found through physical knowledge or engineering intuition, as long as it fulfills Theorem 4.2.

## VI. Experimental Evaluation

The method is evaluated on a heavy duty Scania diesel engine with exhaust gas recirculation (EGR), variable geometry turbocharger (VGT), and intake manifold throttle. The evaluation is based on experimental data collected in an

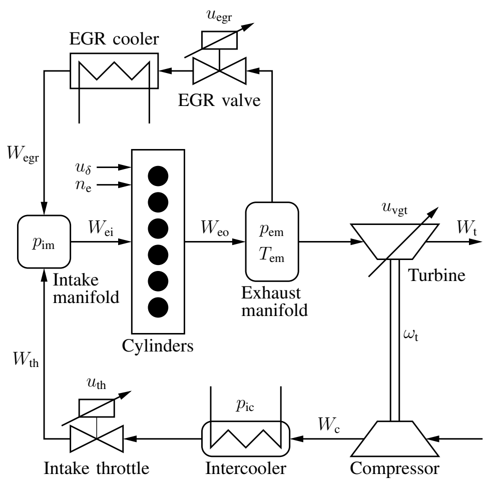


Fig. 2. Schematics of the diesel engine used in the evaluation, showing the differential states ($p_{\mathrm{im}}$, $p_{\mathrm{em}}$, $T_{\mathrm{em}}$, and $\omega_{\mathrm{t}}$), algebraic state ($p_{\mathrm{ic}}$), inputs ($u_{\mathrm{egr}}$, $u_{\mathrm{vgt}}$, $u_\delta$, $u_{\mathrm{th}}$, and $n_{\mathrm{e}}$), and flows between components ($W_{\mathrm{c}}$, $W_{\mathrm{th}}$, $W_{\mathrm{egr}}$, $W_{\mathrm{ei}}$, $W_{\mathrm{eo}}$, and $W_{\mathrm{t}}$).

engine test cell. The model used, presented with states, mass-flows, inputs, and outputs in Figure 2, originally developed in [16]. The modifications are twofold; i) removal of actuator dynamics, and ii) elimination of the intercooler pressure state with fast dynamics, both with the aim to improve the computational efficiency of the resulting EKF. By the latter modification the original model, described by ODE:s, is transformed into a system of DAE:s.

Designing a standard EKF on the default model directly gives biased estimates which is obvious from Figure 4, where the normalized estimation error histograms for all states are biased, i.e., the solid histograms are not centered at zero. The objective is to use the proposed method from Section V to reduce the estimation bias of all system states except the exhaust manifold temperature, using only measurements of intake manifold pressure, intercooler pressure, and turbine speed. The reason for not including $T_{\mathrm{em}}$ in the evaluation is that there is no reference measurement for that state available.

### A. Augmentation Estimation

The model augmentation, $A_q$, is computed using a measurement sequence containing a large number of different stationary operating points, spanning the whole operating region of the engine. In the experimental environment of the engine test cell, the sensor setup is larger than in the intended customer application which is used to improve the augmentation. That is, extra sensors during the augmentation estimation allows bias compensation of also non-measured states, recall the discussion in Section V-A. In the studied example the application includes measurements of the states $\omega_{\mathrm{t}}$, $p_{\mathrm{im}}$, and $p_{\mathrm{ic}}$ while the experimental setup also allows measurement of $p_{\mathrm{em}}$, which is utilized in the augmentation estimation.

Bias estimates, estimated according to Section V, from 42 stationary operating points is used to find a low order augmentation $A_q$. In this case each operating point is weighted

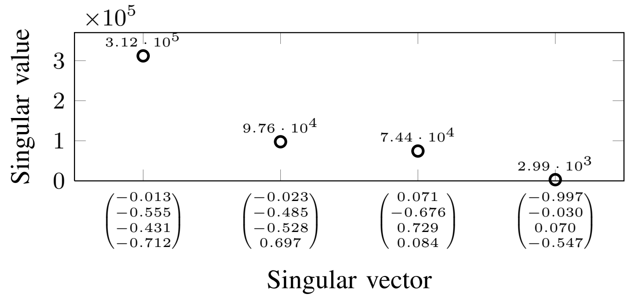


Fig. 3. SVD singular values presented with the corresponding singular vector used for determining the augmentation, $A_q$. The vector components, from top to bottom, correspond to $\omega_{\mathrm{t}}$, $p_{\mathrm{em}}$, $p_{\mathrm{im}}$, and $p_{\mathrm{ic}}$.

equally, i.e. $w^i = \frac{1}{N}$ since no information of a suitable distribution is available. The resulting singular values and vectors used to determine the augmentation are presented in Figure 3. The construction of $A_{q,i}$, where $i$ indicates augmentation dimension, is done by assembling the $i$ singular vectors that have the largest singular values, e.g.,

$$A_{q,3} = \begin{pmatrix} -0.013 & -0.023 & 0.071 \\ -0.555 & -0.485 & -0.676 \\ -0.431 & -0.528 & 0.729 \\ -0.712 & 0.697 & 0.084 \end{pmatrix}.$$

Three augmented observers, $A_{q,1}$, $A_{q,2}$, and $A_{q,3}$, are designed, where the maximal dimension, i.e. $\dim A_{q,3} = 3$, is limited by the three dimensional measurement equation in the EKF.

### B. Estimation Performance

The estimation performance is evaluated by comparing the observers with different augmentation dimension and an observer based on the default model using data from a WHTC [17]. As performance measures the mean relative error, and the normalized estimation error histogram, are used. Together these measures capture both estimation bias and variance. The results are presented in Table I and Figure 4.

Figure 4 shows that with a one dimensional augmentation, $A_{q,1}$, a significant reduction of the estimation error is achieved for the states where feedback is available, i.e. $\omega_{\mathrm{t}}$, $p_{\mathrm{im}}$, and $p_{\mathrm{ic}}$. A further indication of the performance of the augmented observer is given by the estimation error of the observer output that is not explicitly used for feedback, i.e. $p_{\mathrm{em}}$. Table I clearly shows that in this case a mean estimation error reduction of approximately 55 % is achieved for $p_{\mathrm{em}}$ using a three dimensional augmentation, $A_{q,3}$. This shows a clear advantage of the proposed method compared to for example normal PI-observers [6] that have integrators affecting the feedback variables only, i.e. the PI-observers have no ability to incorporate information from extra sensors during the design.

## VII. Conclusions

A method for estimating a low dimension model augmentation for bias compensation given a default model and system measurements that is applicable to models described by DAE:s, is developed. A theorem that characterizes all possible augmentations that preserve the observability properties of

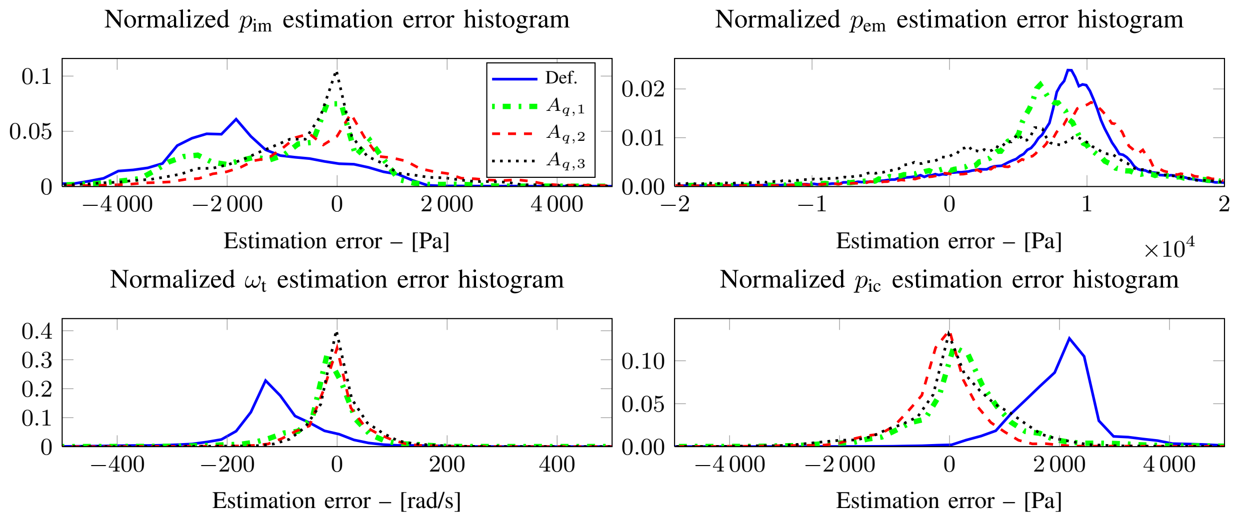


Fig. 4. Normalized histogram for the estimation errors during a WHTC. It is obvious that all outputs that are compensated through feedback, i.e. $p_{\mathrm{im}}$, $p_{\mathrm{ic}}$, and $\omega_{\mathrm{t}}$, benefit from any choice of augmentation dimension. For $p_{\mathrm{em}}$ however, a three dimensional augmentation is required to achieve a significant bias reduction.

TABLE I

MEAN REL. ERROR FOR THE DEFAULT AND AUGMENTED OBSERVERS. ALL WITH FEEDBACK FROM $p_{\mathrm{IM}}$, $\omega_{\mathrm{T}}$, AND $p_{\mathrm{IC}}$ ONLY.

| Outputs | Mean relative error – [%] | | | |
|---|---|---|---|---|
| | Def. | $A_{q,1}$ | $A_{q,2}$ | $A_{q,3}$ |
| $\omega_{\mathrm{t}}$ | -2.1 | 0.1 | 0.3 | 0.5 |
| $p_{\mathrm{em}}$ | 6.8 | 5.6 | 8.5 | 3.0 |
| $p_{\mathrm{im}}$ | -1.6 | -1.5 | -0.1 | -0.2 |
| $p_{\mathrm{ic}}$ | 0.1 | 0.06 | -0.09 | 0.07 |

the default model is given, and a characterization of the augmentations that are possible to estimate is presented. Common for both are that they are mainly limited by the available measurements. Beside the estimated augmentation the method allows user defined augmentations.

A main advantage of the proposed model augmentation method, compared to for example PI-observers, is its ability to incorporate information from additional sensors during the design to estimate an augmentation that can also reduce the estimation errors in states not used for feedback.

The method is applied to a heavy-duty diesel engine with EGR, VGT, and intake throttle, using a nonlinear default DAE and measurements from an engine test cell. The data used is collected during a WHTC. It is shown that a one dimensional augmentation significantly reduces the mean estimation error for states where feedback is available. It is also shown that a 3 dimensional augmentation reduces the mean estimation error by as much as 55 % for the non-measured exhaust manifold pressure.

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