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A Distributed Fault Detection Filtering Approach for a Class of Interconnected Continuous-Time Nonlinear Systems

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Abstract— **This paper develops a filtering approach for distributed fault detection of a class of interconnected continuous-time nonlinear systems with modeling and measurement uncertainties. A distributed fault detection scheme and corresponding thresholds are designed based on filtering certain signals so that the effect of high frequency measurement uncertainty is diminished. The analysis of the proposed distributed fault detection scheme shows that the derived thresholds guarantee that there are no false alarms and characterizes quantitatively the class of detectable faults.**

I. Introduction

The smooth and reliable operation of large-scale distributed systems is a key requirement in the modern technological world. Examples of such systems include manufacturing systems and critical infrastructure systems such as telecommunication networks, electric power systems and water distribution networks. The need for early detection of developing faults before they lead to major failures is of crucial importance. The problem of Fault Detection and Isolation (FDI) is not new and there are many important survey papers [1]–[3] and books [4]–[6] relying on the model-based analytical redundancy approach.

In the last years significant research has been conducted for FDI based on adaptive approximation methods [7], [8]. Most approaches for fault detection and accommodation so far have been based on a centralized architecture, where information about the state of the system is gathered and processed centrally. Motivated by advances in wireless communications, computing devices and software, there has recently been significant interest in distributed and hierarchical fault diagnosis methods [9]–[14]. An important issue that is often overseen is the presence of high frequency measurement uncertainty. In most real world applications such measurement uncertainty may influence significantly the performance of fault detection schemes by causing false alarms. The primary task of this paper is to fill the gap for the case of uncertain state measurements in addition to modeling uncertainty by using a distributed fault detection scheme for a class of interconnected continuous-time nonlinear systems.

In order to accomplish this task, the state measurements which are corrupted by measurement uncertainty are filtered by p-th order low pass filters. The filtering is important because it dampens the effect of high-frequency noise but at the same time it imposes some new challenges such as how to properly design the estimator and derive the corresponding thresholds.

In this paper, by assuming that the filtering extinguishes the measurement uncertainty, adaptive fault detection thresholds are obtained under a rigorous analytical framework guaranteeing no-false alarms. Further on, a fault detectability condition is derived that characterizes the class of detectable faults. The distributed fault detection scheme is based on local fault filtering schemes with each one assigned to monitor one subsystem. Each local fault detection scheme receives the input and output measurements of the subsystem it monitors and also the output measurements of all the interconnected subsystems that influence the subsystem under consideration. Finally, the local fault detection scheme provides a decision regarding the health of the subsystem it monitors. The implementation of the scheme is presented in detail, along with practical issues and potential solutions.

The paper is organized as follows: in Section II, a problem formulation for distributed fault detection of a class of nonlinear dynamical systems with modeling and measurement uncertainties is presented. In Section III the design of the distributed fault detection scheme based on a filtering approach is presented in detail. In Section IV the implementation of the proposed scheme is demonstrated and a practical issue is discussed along with a proposed solution. In Section V the fault detectability condition that characterizes the class of faults detectable by the proposed methodology is derived, and finally, Section VI provides some concluding remarks.

II. Problem Formulation

Consider a large scale distributed nonlinear dynamic system which is comprised of N subsystems Σ_I, $I \in \{1, ..., N\}$ and each subsystem is described by the following differential equation:

$$\Sigma_I : \begin{cases} \dot{x}_I = f_I(x_I, u_I) + \sum_{\substack{J=1 \\ J \neq I}}^{N} h_{I,J}(x_I, x_J, u_I) \\ \qquad + \eta_I(x_I, u_I, t) + \beta_I(t - T_0)\phi_I(x_I, u_I) & (1) \\ y_I(t) = x_I(t) + \xi_I(t) & (2) \end{cases}$$

where $x_I \in \mathbb{R}^{n_I}$, $u_I \in \mathbb{R}^{m_I}$ and $y_I \in \mathbb{R}^{n_I}$ are the state, input and measured output vectors of the I-th subsystem respectively, $f_I : \mathbb{R}^{n_I} \times \mathbb{R}^{m_I} \mapsto \mathbb{R}^{n_I}$ is the nominal function dynamics of the I-th subsystem, $\eta_I : \mathbb{R}^{n_I} \times \mathbb{R}^{m_I} \times \mathbb{R}^{+} \mapsto \mathbb{R}^{n_I}$ is the modeling uncertainty, $\xi_I \in \mathbb{R}^{n_I}$ is the measurement uncertainty and $h_{I,J} : \mathbb{R}^{n_I} \times \mathbb{R}^{n_J} \times \mathbb{R}^{m_I} \mapsto \mathbb{R}^{n_I}$ represents the interconnection functions between the I-th and J-th

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subsystem, $J \in \{1, ..., N\} \backslash I$. The term $\beta_I(t-T_0)\phi_I(x_I, u_I)$ characterizes the fault function dynamics affecting the I-th subsystem including its time evolution. More specifically, the term $\phi_I : \mathbb{R}^{n_I} \times \mathbb{R}^{m_I} \mapsto \mathbb{R}^{n_I}$ is the fault function and the term $\beta_I(t-T_0) : \mathbb{R} \mapsto \mathbb{R}^+$ determines the time evolution of the fault, where T_0 is the time of the fault occurrence. The fault time profile $\beta_I(t-T_0)$ can be used to model abrupt or incipient faults using a decaying exponential type function:

$$\beta_I(t-T_0) \triangleq \begin{cases} 0 & \text{if } t < T_0 \\ 1 - e^{-b_I(t-T_0)} & \text{if } t \geq T_0 \end{cases} \tag{3}$$

where $b_I > 0$. Large values of b_I indicate abrupt faults, whereas smaller values of b_I indicate slowly developing faults (incipient faults).

The objective is to design and analyze a distributed fault detection scheme, where for each subsystem Σ_I, corresponds a local fault detection algorithm, which receives local measurements and information from interconnected systems. In this paper the interconnection functions $h_{I,J}$ are considered known. In the case of sparsely interconnected systems most of the interconnection functions will be zero. The fault functions ϕ_I are unknown and the fault occurrence time T_0 is also unknown. It is assumed that there exist feedback controllers for selecting u_I such that some desired control objectives are achieved. In this paper, we do not deal explicitly with the control problem, but instead consider the fault detection issue in the presence of faults ϕ_I, modeling uncertainties η_I and measurement uncertainties ξ_I. The following assumptions are used throughout the paper:

Assumption 1: For each subsystem Σ_I, $I \in \{1, ..., N\}$ the local state variables x_I and the local inputs u_I remain bounded before and after the occurrence of a fault.

Assumption 1 is required for well-posedness since in this work we address the fault detection problem, not the control design and fault accommodation problem.

Assumption 2: The modeling uncertainty η_I in each subsystem is unstructured and possibly unknown nonlinear function of x_I, u_I and t but bounded by a known functional $\bar{\eta}_I$, i.e.,

$$|\eta_I(x_I, u_I, t)| \leq \bar{\eta}_I(y_I, u_I),\ \forall (x_I, u_I) \in \mathcal{D}_I, \forall t \geq 0, \tag{4}$$

where $\bar{\eta}_I(y_I, u_I) \geq 0$ is a known bounding function and $\mathcal{D}_I \subset \mathbb{R}^{n_I} \times \mathbb{R}^{m_I}$ is some region of interest.

Assumption 2 characterizes the class of modeling uncertainties considered. In practice, the system is sometimes more accurately modeled in certain regions of the state space. Therefore, the fact the bound $\bar{\eta}_I$ is a function of y_I and u_I (as opposed to a constant) provides more flexibility by allowing the designer to take into consideration any prior knowledge of the system.

In order to minimize the effect of measurement uncertainty $\xi_I(t)$, each measured state variable $y_I^{(k)}$ (k-th component of y_I) is filtered by a p-th order low-pass filter with transfer function:

$$H_p(s) = \frac{\alpha^p}{(s+\alpha)^p}. \tag{5}$$

Generally, the values of α and p can be different for each subsystem and for each state variable, but in this paper without loss of generality we consider them to be the same for all the output variables in order to simplify the notation and presentation. The order p of the low-pass filter regulates the damping effect of the high frequency noise, whereas the value α of the filter determines the cutoff frequency at which the damping begins. In the rest of the paper, the notation $y_{I,j}^{(k)}$ indicates the measurement of the k-th state variable of the I-th subsystem after being filtered by j first order filters. Moreover, the operator D_i indicates the i-th time derivative operator:

$$D_i \triangleq \frac{\mathrm{d}^i}{\mathrm{d}t^i}. \tag{6}$$

For example, $D_0 y \equiv y, D_1 y \equiv \dot{y}, D_2 y \equiv \ddot{y}$, etc.

Assumption 3: There exists a p-th order filter of the form $H_p(s)$ such that the output of the filtered measurement uncertainty $\xi_I(t)$ is zero, that is $\xi_{I,p}^{(k)}(t) = 0$, or equivalently:

$$y_{I,p}^{(k)}(t) = x_{I,p}^{(k)}(t). \tag{7}$$

It is important to note that filtering the output measurements is crucial to the proposed fault detection scheme as it helps dampening the effect of noise and therefore the derived detection thresholds are less conservative. In addition, the filtering results in noise-free residuals and therefore the case of false alarms due to the noise is avoided. At this point, we must stress that although the method is particularly tailored to the case of uncertain state measurements, it can also be applied to the case of error-free measurements as it was assumed in earlier work, since Assumption 3 holds by default. In that case, the order p of the filter used can be set to $p = 1$ and the value of α can be somewhat larger in order to reduce the detection time.

III. Distributed Fault Detection

By filtering each output signal $y_I^{(k)}(t)$ with the p-th order low-pass filter given by (5) we obtain the filtered output $y_{I,p}^{(k)}(t)$, given by:

$$y_{I,p}^{(k)}(t) = H_p(s)\left[y_I^{(k)}(t)\right]. \tag{8}$$

By using standard polynomial expansion, (8) can be written as:

$$y_{I,p}^{(k)} = \frac{\alpha^p}{\sum_{i=0}^{p} c_i s^i}\left[y_I^{(k)}\right],\ \text{where } c_i = \binom{p}{i}\alpha^{p-i}. \tag{9}$$

By rearranging terms we obtain:

$$D_p y_{I,p}^{(k)} = -\sum_{i=0}^{p-1} c_i D_i y_{I,p}^{(k)} + \alpha^p y_I^{(k)}. \tag{10}$$

Identically, the same holds for the state variables $x_I^{(k)}$:

$$D_p x_{I,p}^{(k)} = -\sum_{i=0}^{p-1} c_i D_i x_{I,p}^{(k)} + \alpha^p x_I^{(k)}. \tag{11}$$

In this work, the residual $\epsilon_I^{(k)}(t)$ to be used for fault detection is defined as:

$$\epsilon_I^{(k)}(t) \triangleq D_1 y_{I,p}^{(k)}(t) - z_I^{(k)}(t), \tag{12}$$

where $z_I^{(k)}(t)$ will be specified below. This residual constitutes the basis of the fault detection scheme and it is readily measurable as it will be shown later on. The detection of a fault in the large-scale system is made when $\left|\epsilon_I^{(k)}(t)\right| > \bar{\epsilon}_I^{(k)}(t)$, for at least one component k in any subsystem Σ_I, where $\bar{\epsilon}_I^{(k)}(t)$ is the detection threshold (to be specified). The first term $D_1 y_{I,p}^{(k)}$ of the residual is simply the difference of the input-output signals of the last first order filter multiplied by the coefficient α, as it can be seen from Figure 1 or mathematically from Lemma 1. The signal $z_I^{(k)}(t)$ is given by:

$$\begin{aligned} z_I^{(k)}(t) = H_p(s)\Big[& f_I^{(k)}(y_I(t), u_I(t)) \\ & + \sum_{\substack{J=1 \\ J\neq I}}^{N} h_{I,J}^{(k)}(y_I(t), y_J(t), u_I(t))\Big]. \end{aligned} \tag{13}$$

with zero initial conditions for the filter $H_p(s)$, i.e. $D_i z_I^{(k)}(0) = 0,\ i = 0, 1, \ldots, p-1$. Similarly to (10) and (11), the signal $z_I^{(k)}$ can be rewritten in the following form:

$$\begin{aligned} D_p z_I^{(k)} = & -\sum_{i=0}^{p-1} c_i D_i z_I^{(k)} \\ & + a^p\Big(f_I^{(k)}(y_I, u_I) + \sum_{\substack{J=1 \\ J\neq I}}^{N} h_{I,J}^{(k)}(y_I, y_J, u_I)\Big). \end{aligned} \tag{14}$$

By successively differentiating $p-1$ times the residual signal given by (12) we obtain:

$$\begin{aligned} D_1\epsilon_I^{(k)} &= D_2 y_{I,p}^{(k)} - D_1 z_I^{(k)} \\ &\vdots \\ D_{p-1}\epsilon_I^{(k)} &= D_p y_{I,p}^{(k)} - D_{p-1} z_I^{(k)}. \end{aligned} \tag{15}$$

Equations (12) and (15) can be represented collectively as:

$$D_i\epsilon_I^{(k)} = D_{i+1} y_{I,p}^{(k)} - D_i z_I^{(k)},\ i = 0, 1, \ldots, p-1, \tag{16}$$

where $y_{I,0}^{(k)} \triangleq y_I^{(k)}$.
By using (7) and (11), the last equation of (15) becomes:

$$D_{p-1}\epsilon_I^{(k)} = -\sum_{i=0}^{p-1} c_i D_i x_{I,p}^{(k)} + \alpha^p x_I^{(k)} - D_{p-1} z_I^{(k)} \tag{17}$$

By differentiating (17) one more time it becomes:

$$D_p\epsilon_I^{(k)} = -\sum_{i=0}^{p-1} c_i D_{i+1} x_{I,p}^{(k)} + \alpha^p \dot{x}_I^{(k)} - D_p z_I^{(k)} \tag{18}$$

Then, by using (14), equation (18) becomes:

$$\begin{aligned} D_p\epsilon_I^{(k)} = & -\sum_{i=0}^{p-1} c_i\Big(D_{i+1} x_{I,p}^{(k)} - D_i z_I^{(k)}\Big) \\ & + \alpha^p\Big(\dot{x}_I^{(k)} - f_I^{(k)}(y_I, u_I) - \sum_{\substack{J=1 \\ J\neq I}}^{N} h_{I,J}^{(k)}(y_I, y_J, u_I)\Big). \end{aligned} \tag{19}$$

Finally, by using (7) and (16), equation (19) can be rewritten as:

$$\begin{aligned} & \sum_{i=0}^{p} c_i D_i \epsilon_I^{(k)} = \\ & \alpha^p\Big(\dot{x}_I^{(k)} - f_I^{(k)}(y_I, u_I) - \sum_{\substack{J=1 \\ J\neq I}}^{N} h_{I,J}^{(k)}(y_I, y_J, u_I)\Big). \end{aligned} \tag{20}$$

Prior to the fault, equation (20) can be written as:

$$\sum_{i=0}^{p} c_i D_i \epsilon_I^{(k)} = \alpha^p \chi_I^{(k)}(t), \tag{21}$$

where

$$\chi_I^{(k)}(t) \triangleq \Delta f_I^{(k)}(t) + \Delta h_{I,J}^{(k)}(t) + \eta_I^{(k)}(x_I, u_I, t) \tag{22}$$

$$\Delta f_I^{(k)}(t) \triangleq f_I^{(k)}(x_I, u_I) - f_I^{(k)}(y_I, u_I) \tag{23}$$

$$\Delta h_{I,J}^{(k)}(t) \triangleq \sum_{\substack{J=1 \\ J\neq I}}^{N}\Big(h_{I,J}^{(k)}(x_I, x_J, u_I) - h_{I,J}^{(k)}(y_I, y_J, u_I)\Big). \tag{24}$$

According to (21):

$$\epsilon_I^{(k)}(t) = H_p(s)\left[\chi_I^{(k)}(t)\right], \tag{25}$$

where $\chi_I^{(k)}(t)$ is the total uncertainty term given by (22). In this case the initial conditions are nonzero. Seen differently, (21) is an ODE with initial conditions:

$$D_i\epsilon_I^{(k)}(0) = D_{i+1} y_{I,p}^{(k)}(0),\ i = 0, 1, \ldots, p-1. \tag{26}$$

These initial conditions can be calculated based on the following lemma:

Lemma 1: Let $y_{I,j}^{(k)}(t) = H_j(s)\left[y_I^{(k)}(t)\right]$. Then, the following recursive equation holds:

$$D_i y_{I,j}^{(k)} = -\alpha D_{i-1} y_{I,j}^{(k)} + \alpha D_{i-1} y_{I,j-1}^{(k)} \tag{27}$$

Proof: Using the definition of $y_{I,j}^{(k)}$, we obtain:

$$\begin{aligned} y_{I,j}^{(k)}(t) &= \left(\frac{\alpha}{s+\alpha}\right)^j \left[y_I^{(k)}(t)\right] \\ &= \frac{\alpha}{s+\alpha}\left(\frac{\alpha}{s+\alpha}\right)^{j-1}\left[y_I^{(k)}(t)\right] \\ &= \frac{\alpha}{s+\alpha}\left[y_{I,j-1}^{(k)}(t)\right]. \end{aligned}$$

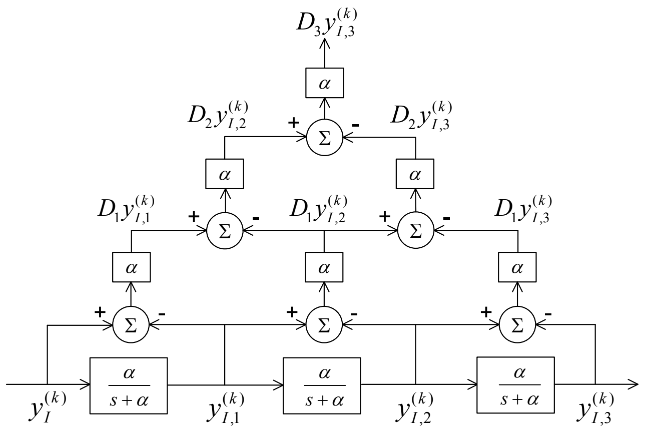

Fig. 1. Filtering architecture and calculation of $D_i y_{I,j}^{(k)}$ signals for $p = 3$

Therefore,

$$D_1 y_{I,j}^{(k)} = -\alpha y_{I,j}^{(k)} + \alpha y_{I,j-1}^{(k)}.$$

By successively differentiating the last equation we obtain the recursive equation (27). ■

To simplify the notation and without loss of generality, the initial conditions of all the first order filters are chosen to be zero, that is:

$$y_{I,j}^{(k)}(0) = 0,\ j = 1, \ldots, p \tag{28}$$

and thus the initial values of the signals $D_i y_{I,j}^{(k)}$ become

$$D_i y_{I,j}^{(k)}(0) = \begin{cases} \alpha^i y_I^{(k)}(0) & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \tag{29}$$

where $i = 1, \ldots, p,\ j = i, \ldots, p$.

Therefore, the initial conditions (26) of the ODE (21) become:

$$D_{i-1}\epsilon_I^{(k)}(0) = \begin{cases} \alpha^p y_I^{(k)}(0) & \text{if } i = p. \\ 0 & \text{if } i = 1, 2, \ldots, p-1. \end{cases} \tag{30}$$

Figure 1 illustrates how the equations (27), (28) and (29) come together for the case $p = 3$. This "full version" of the filtering scheme will be compacted later on in the implementation of the residual signal $\epsilon_I^{(k)}(t)$ (by decomposing it into a series of a $p-1$ and a first order filter; see Figure 2) but it is considered important at this point to be demonstrated in a complete way to make things more clear.

Now, the solution of (21) can be written as the sum of the zero input response $\epsilon_{zi_I}^{(k)}(t)$ (due to initial conditions) and the zero state response $\epsilon_{zi_I}^{(k)}(t)$ (due to input):

$$\epsilon_I^{(k)}(t) = \epsilon_{zi_I}^{(k)}(t) + \epsilon_{zs_I}^{(k)}(t). \tag{31}$$

The zero input response term is obtained from the solution of the homogeneous equation $\sum_{i=0}^{p} c_i D_i \epsilon_I^{(k)} = 0$. Therefore

$$\epsilon_{zi_I}^{(k)}(t) = \sum_{i=0}^{p-1} A_i t^i e^{-\alpha t}, \tag{32}$$

where the coefficients A_i are obtained by solving a linear system of equations so that the initial conditions (30) are satisfied. After some algebraic manipulation, we obtain that the coefficients A_i are given by

$$A_i = \begin{cases} 0 & \text{if } i = 0, 1, \ldots, p-2. \\ \frac{1}{(p-1)!}\alpha^p y_I^{(k)}(0) & \text{if } i = p-1. \end{cases} \tag{33}$$

The zero state response term is simply the response of an LTI system with transfer function $H_p(s)$ (and zero initial conditions) to the input $\chi_I^{(k)}(t)$. Mathematically:

$$\epsilon_{zs_I}^{(k)}(t) = \frac{\alpha^p}{(s+\alpha)^p}\left[\chi_I^{(k)}(t)\right]. \tag{34}$$

Finally, by combining equations (31)-(34) the residual becomes:

$$\begin{aligned} \epsilon_I^{(k)}(t) = & \frac{1}{(p-1)!}\alpha^p y_I^{(k)}(0) t^{p-1} e^{-\alpha t} \\ & + \frac{\alpha^p}{(s+\alpha)^p}\left[\chi_I^{(k)}(t)\right]. \end{aligned} \tag{35}$$

By using the triangle inequality the previous equation becomes:

$$\begin{aligned} \left|\epsilon_I^{(k)}(t)\right| \leq & \frac{1}{(p-1)!}\alpha^p \left|y_I^{(k)}(0)\right| t^{p-1} e^{-\alpha t} \\ & + \frac{\alpha^p}{(s+\alpha)^p}\left[\left|\chi_I^{(k)}(t)\right|\right]. \end{aligned} \tag{36}$$

Based on (36), a suitable detection threshold $\bar{\epsilon}_I^{(k)}(t)$ is given by:

$$\begin{aligned} \bar{\epsilon}_I^{(k)}(t) = & \frac{1}{(p-1)!}\alpha^p \left|y_I^{(k)}(0)\right| t^{p-1} e^{-\alpha t} \\ & + \frac{\alpha^p}{(s+\alpha)^p}\left[\bar{\chi}_I^{(k)}(t)\right], \end{aligned} \tag{37}$$

where $\bar{\chi}_I^{(k)}(t)$ is the bound on the total uncertainty term $\chi_I^{(k)}(t)$; i.e.,

$$0 \leq \left|\chi_I^{(k)}(t)\right| \leq \bar{\chi}_I^{(k)}(t). \tag{38}$$

Using Assumption 2, the bound $\bar{\chi}_I^{(k)}(t)$ is defined as:

$$\begin{aligned} \bar{\chi}_I^{(k)}(t) \triangleq & \sup_{\xi_I \in \mathcal{R}^{n_I}} \left|\Delta f_I^{(k)}(t)\right| + \sup_{\xi_I \in \mathcal{R}^{n_I}} \sup_{\xi_J \in \mathcal{R}^{n_J}} \left|\Delta h_{I,J}^{(k)}(t)\right| \\ & + \bar{\eta}_I^{(k)}(y_I, u_I). \end{aligned} \tag{39}$$

Following the preceding mathematical analysis, in the absence of any faults the absolute value of the residual signal $\epsilon_I^{(k)}(t)$ is always bounded by the detection threshold $\bar{\epsilon}_I^{(k)}(t)$ given by (37). Therefore, this guarantees that there will be no false alarms, which is stated formally in the following lemma.

Lemma 2: Consider a distributed system comprised of N subsystems Σ_I given by (1). In the absence of any faults, the absolute values of the residual signals $\epsilon_I^{(k)}(t)$ given by (12), where the signals $D_1 y_{I,p}^{(k)}(t)$ and $z_I^{(k)}(t)$ are given by (27) and (13) respectively, are bounded by the detection thresholds $\bar{\epsilon}_I^{(k)}(t)$, given by (37), thus guaranteeing no false alarms.

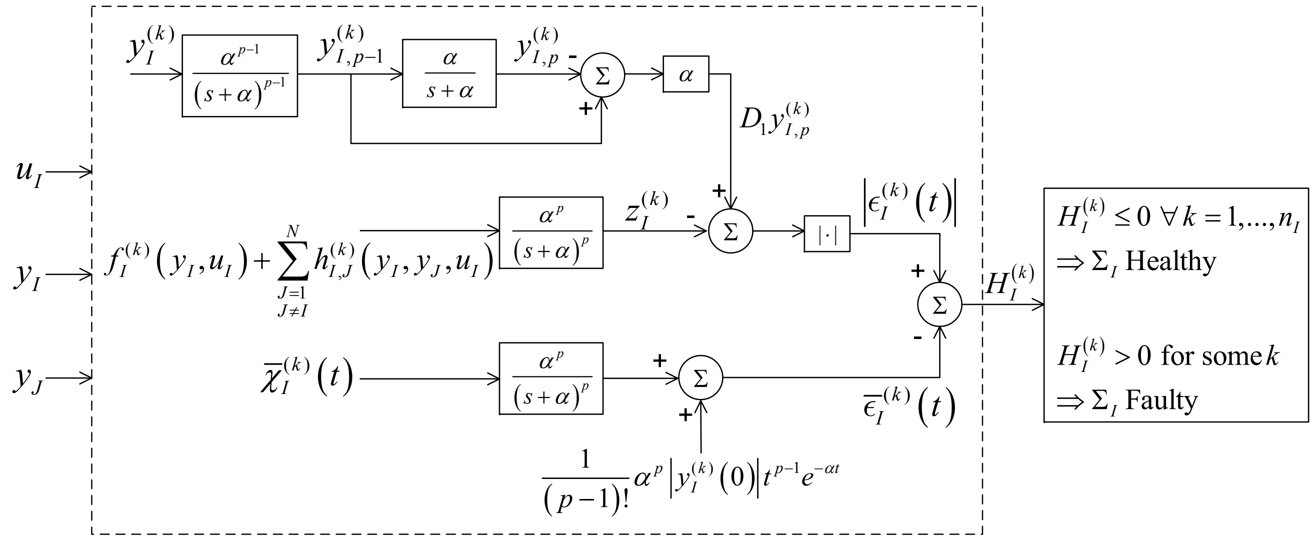

Fig. 2. Local Filtered Fault Detection Scheme

IV. Implementation of Distributed Fault Detection Scheme

Next we proceed with the implementation of the residual signals and thresholds and the discussion of certain practical issues. Figure 2 illustrates the implementation of the local filtered fault detection scheme for the I-th subsystem. It is important to note that we do not need the whole filtering structure as in Figure 1 but rather a simplified filtering scheme using a series of a $p-1$ and a first order filter as shown in Figure 2.

In general, the distributed fault detection scheme will comprise of N local filtered fault detection modules, one for each subsystem Σ_I. The implementation of the I-th fault detection module requires the measurements y_J of all subsystems Σ_J that are influencing Σ_I. Therefore, there is the need of communication between the fault detection modules depending on their interconnections. Figure 3 illustrates the distributed fault detection scheme for the case of two subsystems Σ_1, Σ_2 where Σ_1 influences Σ_2.

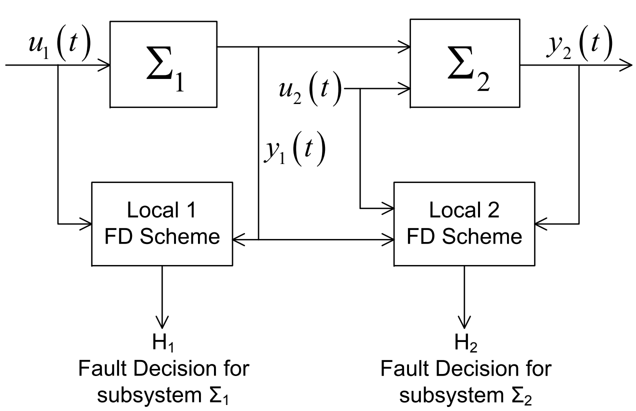

Fig. 3. Distributed fault detection scheme for the case of two subsystems Σ_1, Σ_2 where Σ_1 influences Σ_2.

A practical issue that requires consideration is the derivation of the bound $\bar{\chi}_I^{(k)}(t)$ given in (39). Specifically, the derivation of $\bar{\chi}_I^{(k)}(t)$ requires the bounds on $\Delta f_I^{(k)}(t)$ and $\Delta h_{I,J}^{(k)}(t)$. Using (23), the bound on $\Delta f_I^{(k)}(t)$ is given by

$$\sup_{\xi_I \in \mathcal{R}^{n_I}} \left|\Delta f_I^{(k)}(t)\right| = \sup_{\xi_I \in \mathcal{R}^{n_I}} \left|f_I^{(k)}(x_I,u_I) - f_I^{(k)}(y_I,u_I)\right|$$

An approach for deriving the bound is to consider a local Lipschitz assumption:

$$\left|f_I^{(k)}(x_I,u_I) - f_I^{(k)}(y_I,u_I)\right| \le L_{f_I^{(k)}} |x_I - y_I| = L_{f_I^{(k)}} |\xi_I| \tag{40}$$

where $L_{f_I^{(k)}}$ is the Lipschitz constant for the function $f_I^{(k)}(x_I,u_I)$ with respect to x_I. Therefore, if we have a bound $\bar{\xi}_I$ on the measurement uncertainty, then we can obtain a bound on $\Delta f_I^{(k)}(t)$. A similar approach can be followed for the interconnection functions $\Delta h_{I,J}^{(k)}(t)$.

V. Fault Detectability

The design and analysis in the previous two sections was based on the derivation of suitable thresholds $\bar{\epsilon}_I^{(k)}(t)$ such that the absolute values of the residual signals $\epsilon_I^{(k)}(t)$ are bounded by $\bar{\epsilon}_I^{(k)}(t)$ in the absence of any fault. In the presence of a fault, an important question is what type/magnitude of fault can be detected. This is referred to as *fault detectability analysis*. In this section the detectability condition of the aforementioned fault detection scheme is presented. This condition constitutes a theoretical result that characterizes quantitatively the class of faults detectable by the proposed scheme.

Theorem 1: Consider the distributed fault detection scheme described in (12), (14), (37). A fault in the I-th subsystem is detectable if the fault function $\phi_I(x_I,u_I)$

satisfies the following inequality:

$$\left| \int_{T_0}^{t} \frac{1}{(p-1)!}(t-\tau)^{p-1} e^{-\alpha(t-\tau)} \alpha^p \left(1 - e^{-b_I(\tau - T_0)}\right) \phi_I^{(k)}(x_I, u_I)\,\mathrm{d}\tau \right| > 2\bar{\epsilon}_I^{(k)}(t) \quad (41)$$

Proof: In the presence of a fault that occurs at $t = T_0$, equation (21) becomes:

$$\sum_{i=0}^{p} c_i D_i \epsilon_I^{(k)} = \alpha^p \chi_I^{(k)}(t) + \alpha^p \beta_I(t - T_0) \phi_I^{(k)}(x_I, u_I). \quad (42)$$

Following a similar procedure as in the derivation of (35), the solution of (42) satisfies

$$\begin{aligned} \epsilon_I^{(k)}(t) =& \frac{1}{(p-1)!} \alpha^p y_I^{(k)}(0) t^{p-1} e^{-\alpha t} \\ &+ \frac{1}{(s+\alpha)^p} \left[\alpha^p \chi_I^{(k)}(t)\right] \\ &+ \frac{1}{(s+\alpha)^p} \left[\alpha^p \beta_I(t-T_0) \phi_I^{(k)}(x_I, u_I)\right]. \end{aligned} \quad (43)$$

By using the triangle inequality and equation (38), equation (43) becomes:

$$\begin{aligned} \left|\epsilon_I^{(k)}(t)\right| \geq& - \frac{1}{(p-1)!} \alpha^p \left|y_I^{(k)}(0)\right| t^{p-1} e^{-\alpha t} \\ &- \frac{1}{(s+\alpha)^p} \left[\alpha^p \bar{\chi}_I^{(k)}(t)\right] \\ &+ \left|\frac{1}{(s+\alpha)^p} \left[\alpha^p \beta_I(t-T_0) \phi_I^{(k)}(x_I, u_I)\right]\right| \\ \geq& - \bar{\epsilon}_I^{(k)}(t) \\ &+ \left|\frac{1}{(s+\alpha)^p} \left[\alpha^p \beta_I(t-T_0) \phi_I^{(k)}(x_I, u_I)\right]\right|. \end{aligned} \quad (44)$$

For fault detection $\left|\epsilon_I^{(k)}(t)\right| > \bar{\epsilon}_I^{(k)}(t)$ must hold, so the final fault detectability condition is obtained:

$$\left|\frac{1}{(s+\alpha)^p} \left[\alpha^p \beta_I(t-T_0) \phi_I^{(k)}(x_I, u_I)\right]\right| > 2\bar{\epsilon}_I^{(k)}(t). \quad (45)$$

This can be rewritten based on (3) in the integral form (41) of the Theorem. ■

The above fault detectability theorem provides a measure of the type of faults that can be detected with the proposed distributed fault detection scheme. Clearly, the fault functions $\phi_I(x_I, u_I)$ are typically unknown and therefore this condition cannot be checked apriori. However, it provides useful intuition about the type of faults that are detectable.

VI. Conclusion

In this paper a distributed fault detection filtering approach for a class of interconnected, continuous-time, nonlinear systems with modeling and measurement uncertainty is presented. Under certain assumptions, a distributed estimation scheme is designed, suitable adaptive detection thresholds are derived analytically and the fault detectability condition is obtained that characterizes quantitatively the class of faults that can be detected by the proposed scheme. The implementation requires each subsystem to be monitored by a distinct module called local fault detection scheme. Each module requires the input and output measurements of the subsystem that is monitoring and also the measurements of all the subsystems that are influencing the subsystem that the specific module is monitoring. Further on, in order to deal with a practical issue regarding the bound of the total uncertainty term a method was proposed, considering a Lipschitz assumption. Appropriate choices should be made regarding the filters' coefficients α and order p since they have a great effect in the fault detection scheme. Future research will be devoted to the investigation of the higher order filtering effects, to the extension of the methodology in the case of unknown interconnection functions by utilizing adaptive approximation and also to the application of the results demonstrating the effectiveness of the proposed scheme.

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