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Hierarchical Control with Partial Observations: Sufficient Conditions

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Abstract— **In this paper, hierarchical control of both monolithic and modular discrete-event systems under partial observations is studied. Two new conditions, called observation consistency and local observation consistency, are proposed. These conditions are sufficient for the preservation of observability between the original and the abstracted plant. Moreover, it is shown that both conditions are compositional, that is, they are preserved by the synchronous product. This property makes it possible to use hierarchical and decentralized supervisory control for discrete-event systems with partial observations.**

I. INTRODUCTION

In supervisory control of discrete-event systems (DES), the main issue is the combinatorial explosion of the state space complexity inherent to large systems, which renders the standard approaches that compute and use the whole system model very difficult and often impossible to use.

Therefore, particular techniques are needed to decrease the computational complexity of supervisory control. Among these approaches, *decentralized* (often called *modular*) *control* and *hierarchical control* are the most successful. These two approaches are complementary, because the decentralized approach can be seen as a horizontal modularity, while the hierarchical approach can be seen as a vertical modularity. The best results are achieved when these perpendicular approaches are combined, cf. [10].

During the last two decades, hierarchical control of discrete-event systems with complete observations has been widely investigated. Two important concepts, *observer property* [11], and *output control consistency* (OCC) [13] or its weaker version *local control consistency* (LCC) [8] have been proposed. These concepts are sufficient conditions under which the high level synthesis of a nonblocking (respectively optimal, it means, the least restrictive) supervisor has a low level implementation. It has to be noted that these conditions are applicable for DES with full observations.

The basic supervisory control theorem under partial observations [2] states that a specification language must be controllable, observable, and $L_m(G)$-closed in order to be achievable as the language of the closed-loop system in a nonblocking manner. This means that from all the reachable states in the resulting generator, a marked state can be reached.

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There are only very few results concerning hierarchical control of partially observed discrete-event systems, although it is important to decrease the complexity of the supervisor synthesis procedure, which is exponential for discrete-event systems with partial observations.

The authors of [3] are the first to consider hierarchical control with partial observations, but using a different framework of Moore automata and different concepts of controllable and observable events based on vocalization. Moreover, their approach is monolithic and requires a specific definition of the low-level supervisor, while our approach allows distributed hierarchical synthesis using standard synchronous composition of the plant with the supervisor.

In [5], we have presented sufficient conditions for preservation of high level supremal controllable and normal sublanguages at the low level, which ensure that the optimal high level supervisor with partial observations is implementable in the original plant (the low level). However, that paper imposes the restrictive condition that all observable events must be included in the high level.

It is hence our goal to find a weaker condition that is useful in hierarchical control with partial observations. Since the hierarchical control synthesis is done in the abstracted (high level) plant, the major problem is how to ensure that the high level supervisor is implementable at the low level, i.e., in the original plant. This amounts to showing that observability and controllability are preserved in the original (low level) plant in both directions from the high level to the low level and vice-versa.

As the first result of this paper, we introduce two new structural conditions called *local observation consistency* (LOC) and *observation consistency* (OC) for projections. The latter one (OC) addresses a certain consistency property regarding the observations of strings on the high level and the low level. The former one (LOC) can be considered as a specialization of the observer property under partial observations. We show that projections that satisfy OC, LOC, and LCC, and that are observers are also suitable for the nonblocking least restrictive hierarchical control under partial observation. As the second main result of the paper, we prove that both LOC and OC are compositional in the sense that these properties are preserved after applying the synchronous product. Hence, they are particularly useful in the setting of modular discrete-event systems. Due to composability, our novel conditions need not be verified for a large global discrete-event system but can be checked for its smaller modular components.

The paper is organized as described below. In the next section, preliminary results from supervisory control with

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partial observations are briefly recalled. Then, Section III presents the hierarchical control with partial observations, where new sufficient conditions for the preservation of observability between the high level and the low level are presented. In Section IV, it is shown that these conditions are compositional, which allows for a modular framework.

II. Preliminaries

A *generator* is a quintuple $G = (Q, A, f, q_0, Q_m)$, where Q is a finite set of *states*, A is a finite *event set*, $f : Q \times A \to Q$ is a *partial transition function*, $q_0 \in Q$ is the *initial state*, and $Q_m \subseteq Q$ is the set of *marked states*. As usual, f can be extended to a function $f : Q \times A^* \to Q$. The behaviors of G are defined in terms of languages. The *language* generated by G is defined as $L(G) = \{s \in A^* \mid f(q_0, s) \in Q\}$, and the *marked language* generated by G is defined as $L_m(G) = \{s \in A^* \mid f(q_0, s) \in Q_m\}$.

A string $s \in A^*$ is a *prefix* of a string $w \in A^*$ if $w = st$, for some $t \in A^*$. The prefix closure $\overline{L} = \{w \in A^* \mid \exists v \in A^* \text{ such that } wv \in L\}$ of a language $L \subseteq A^*$ is the set of all prefixes of all its elements. A language L is prefix-closed if $L = \overline{L}$. Note that, by definition, $L(G)$ is always prefix-closed.

Let L_m and L be languages over an event set A with the uncontrollable event set $A_u \subseteq A$, whereby L is prefix-closed. A language $K \subseteq A^*$ is *controllable* with respect to L and A_u if

$$\overline{K}A_u \cap L \subseteq \overline{K}\,.$$

Moreover, K is L_m-closed if $K = \overline{K} \cap L_m$.

A *projection* $P : A^* \to B^*$, for some $B \subseteq A$, is a homomorphism defined so that $P(a) = \varepsilon$, for $a \in A \setminus B$, and $P(a) = a$, for $a \in B$. The *inverse image* of P, denoted by $P^{-1} : B^* \to 2^{A^*}$, is defined as $P^{-1}(a) = \{s \in A^* \mid P(s) = a\}$. These definitions can naturally be extended to languages.

Definition 1 (Observability): Let K and $L = \overline{L}$ be languages over an event set A. Let $A_c \subseteq A$ be the subset of *controllable* events, and let $A_o \subseteq A$ be the set of *observable* events with P as the corresponding projection from A^* to A_o^*. The specification language $K \subseteq L$ is said to be *observable* with respect to L, A_o, and A_c if for all $s, s' \in L$ such that $P(s) = P(s')$ and for all $e \in A_c$

$$se \in L \wedge s'e \in \overline{K} \wedge s \in \overline{K} \Rightarrow se \in \overline{K}.$$

Unfortunately, observability is not closed under union, but another stronger property called normality, that implies observability, can be used in that case [2, Section 3.7.5]. Consider a prefix-closed language $L = \overline{L} \subseteq A^*$ and a projection $P : A^* \to A_o^*$. A language $K \subseteq L$ is said to be *normal* with respect to L and P if

$$\overline{K} = P^{-1}[P(\overline{K})] \cap L\,.$$

Let G be a generator over an event set A. Let $A_u \subseteq A$ be the set of its uncontrollable events, $A_c = A \setminus A_u$ be the set of its controllable events, and $A_o \subseteq A$ be the set of its observable events. Given a specification language $K \subseteq L_m(G) \subseteq A^*$, the aim of supervisory control theory is to find a nonblocking supervisor S such that $L_m(S/G) = K$ and $\overline{L_m(S/G)} = L(S/G)$. It is known that such a supervisor exists if and only if K is controllable with respect to $L(G)$ and A_u, $L_m(G)$-closed, and observable with respect to $L(G)$, A_o, and A_c, see [2].

A formula for calculating supremal controllable sublanguages of K can be found in [1], in case K does not comply with the previous properties.

Recall that the synchronous product of languages $L_1 \subseteq A_1^*$ and $L_2 \subseteq A_2^*$ is defined by

$$L_1 \| L_2 = P_1^{-1}(L_1) \cap P_2^{-1}(L_2) \subseteq A^*\,,$$

where $A = A_1 \cup A_2$ and $P_i : A^* \to A_i^*$, for $i = 1, 2$, are projections to local event sets. The synchronous product can also be defined in terms of generators (the reader is referred to [2] for more details). In this case, for two generators G_1 and G_2, it is well known that $L(G_1 \| G_2) = L(G_1) \| L(G_2)$ and $L_m(G_1 \| G_2) = L_m(G_1) \| L_m(G_2)$.

We finally cite the definition of the *observer property* and *synchronously nonconflicting languages* and state a related condition that is beneficial for the further discussion.

Definition 2 (Observer property [11]): Let A be an event set. A projection $Q : A^* \to A_{hi}^*$, where $A_{hi} \subseteq A$, is an *L-observer* for a language $L \subseteq A^*$ if the following holds: for all strings $t \in Q(L)$ and $s \in \overline{L}$, if $Q(s) \leq t$, then there exists $u \in A^*$ such that $su \in L$ and $Q(su) = t$.

Two languages $L_1 \subseteq A_1*$ and $L_2 \subseteq A_2^*$ are said to be *synchronously nonconflicting* if

$$\overline{L_1 \| L_2} = \overline{L_1} \| \overline{L_2}\,.$$

It is shown in [4, Theorem 1] that this condition is preserved after using abstraction projections that have the observer property.

Lemma 3: Let $L_i \subseteq A_i^*$, $i = 1, 2$, be languages and $A_{hi} \supseteq (A_1 \cap A_2)$ with the projections $Q_i : A_i^* \to (A_{hi} \cap A_i)^*$, $i = 1, 2$. If Q_i, $i = 1, 2$, is an L_i-observer, then L_1 and L_2 are synchronously nonconflicting if and only if $Q_1(L_1)$ and $Q_2(L_2)$ are synchronously nonconflicting.

III. Local observation consistency

In this section, we study the problem of supervisor existence under partial observation based on the computation of a plant abstraction. In this setting, the plant is given by a generator G over an event set A and it is desired to realize a specification $K \subseteq A_{hi}^*$ that is formulated over a subset $A_{hi} \subseteq A$ of the plant event set. Moreover, it is assumed that the controllable/uncontrollable event sets are given as A_c/A_u and that only the subset $A_o \subseteq A$ is observable. In what follows, we use the following notation for projections between the respective event sets: $P : A^* \to A_o^*$, $Q : A^* \to (A_{hi})^*$, $P_{hi} : (A_{hi})^* \to (A_{hi} \cap A_o)^*$, and $Q_o : A_o^* \to (A_{hi} \cap A_o)^*$ as illustrated in the commutative diagram in Fig. 1. Finally, we define the plant abstraction as G_{hi} over the event set A_{hi} such that

$$L(G_{hi}) = Q(L(G)) \quad \text{and} \quad L_m(G_{hi}) = Q(L_m(G)).$$

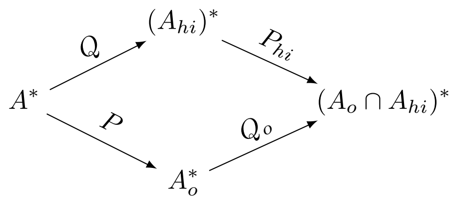

Fig. 1. Commutative diagram of abstraction projections.

Given this set-up, the main goal of this section is to determine the existence of a nonblocking supervisor S such that $L_m(S/G) = K||L_m(G)$. However, instead of using the original (usually large) plant for this computation, our goal is to verify the supervisor existence based on the (potentially smaller) abstraction G_{hi}. Precisely, we want to identify conditions on the projections Q, P, and P_{hi}, and the relevant event sets A_u and A_c, such that controllability, $L_m(G)$-closure, and observability of $K||L_m(G)$ for the original model G are equivalent to controllability, $L_m(G)$-closure, and observability of K for the abstracted model G_{hi}.

In the first step, we study observability and introduce *observation consistency* and *local observation consistency* as two novel conditions for partial observations in the abstraction-based supervisory control.

Definition 4 (Observation consistency): A language $L = \overline{L} \subseteq A^*$ is said to be *observation consistent* with respect to projections Q, P, and P_{hi}, if for all strings $t, t' \in Q(L)$ such that $P_{hi}(t) = P_{hi}(t')$ there exist strings $s, s' \in L$ such that $Q(s) = t$, $Q(s') = t'$, and $P(s) = P(s')$.

That is, observation consistency requires that any two strings that have the same observation in the abstracted language $Q(L)$ must have corresponding strings in the original plant with the same observation as well.

Definition 5 (Local observation consistency): A language $L = \overline{L} \subseteq A^*$ is said to be *locally observation consistent* with respect to projections Q, P, and the set of controllable events A_c if for all strings $s, s' \in L$ and events $e \in A_c \cap A_{hi}$ such that $Q(s)e \in Q(L)$, $Q(s')e \in Q(L)$ and $P(s) = P(s')$, there are $u, u' \in (A \setminus A_{hi})^*$ such that $P(u) = P(u')$ and $sue \in L$, $s'u'e \in L$.

This condition states that whenever we remain within the abstracted plant by continuing two observationally equivalent high level strings by the same controllable event, then the corresponding low level observationally equivalent strings can be continued by this same event within the original plant in the future (after some possible empty low level strings that show the same observations). This condition can be seen as a specialization of the observer property for partially observed DES and controllable events in the abstraction alphabet.

As the main novel contribution of this section, we establish that observation consistency and local observation consistency together imply the bidirectional preservation of observability. In the following, we identify the plant language $L(G)$ with the prefix-closed language L in Definitions 4 and 5.

Theorem 6: Let G be a generator over an event set A, and let $K \subseteq Q(L)$ be a (high-level) specification. Assume that L is observation consistent with respect to projections Q, P, and P_{hi}, that K and L are synchronously nonconflicting, and that L is locally observation consistent with respect to Q, P, and A_c. Then, the language K is observable with respect to $Q(L)$, $A_{hi} \cap A_o$, and $A_{hi} \cap A_c$ if and only if $K \| L$ is observable with respect to L, A_o, and A_c.

Proof: Assume that K is observable with respect to $Q(L)$, $A_{hi} \cap A_o$, and $A_{hi} \cap A_c$, and let us show that $\overline{K \| L}$ is observable with respect to L, A_o, and A_c. Let $s, s' \in \overline{K \| L}$, and let $e \in A_c$ be such that $se \in L$, $s'e \in \overline{K \| L}$, and $P(s) = P(s')$. We need to prove that $se \in \overline{K \| L}$. From $P(s) = P(s')$ we have $(Q_o \circ P)(s) = (Q_o \circ P)(s')$, hence using the commutative diagram in Fig. 1 we obtain that $P_{hi}Q(s) = Q_oP(s) = Q_oP(s') = P_{hi}Q(s')$. As $Q(s), Q(s') \in \overline{K}$, we have the following two cases: (i) If $Q(e) = \varepsilon$, then $se \in L$ implies $se \in \overline{K \| L}$ as well. (ii) If $Q(e) = e$, then $Q(s)e = Q(se) \in Q(L)$, $Q(s')e = Q(s'e) \in \overline{K}$. Since we have shown that $P_{hi}Q(s) = P_{hi}Q(s')$, observability of K with respect to $Q(L)$ and P_{hi} implies that $Q(s)e \in \overline{K}$. Since K and L are synchronously nonconflicting, this means that $se \in \overline{Q^{-1}(K)} \cap L = \overline{K} \| L = \overline{K \| L}$, which was to be shown.

Now, the opposite implication is shown. Let $\overline{K} \| L = \overline{Q^{-1}(K)} \cap L$ be observable with respect to L, A_o, and A_c. It will be shown that K is observable with respect to $Q(L)$ and P_{hi}. Assume that $t, t' \in \overline{K} \subseteq Q(L)$, $te \in Q(L)$, for some $e \in A_c \cap A_{hi}$, $t'e \in \overline{K}$, and $P_{hi}(t) = P_{hi}(t')$. We have to show that $te \in \overline{K}$. Since $t, t' \in Q(L)$ and $P_{hi}(t) = P_{hi}(t')$, observation consistency implies that there are $s, s' \in L$ such that $Q(s) = t$, $Q(s') = t'$, and $P(s) = P(s')$. Moreover, since $Q(s)e, Q(s)e' \in Q(L)$ and $P(s) = P(s')$, we know from local observation consistency that there are $u, u' \in (A \setminus A_{hi})^*$ such that $sue \in L$ and $s'u'e \in L$, while $P(u) = P(u')$. Hence, we have $Q(su) = t$, $Q(s'u') = t'$, $sue, s'u'e \in L$, and $P(su) = P(s'u')$. Moreover, since $t'e \in \overline{K}$, we have that $s'u'e \in Q^{-1}(t'e) \subseteq Q^{-1}(\overline{K})$. Also from $t \in \overline{K} \cap Q(L)$, we conclude $su \in Q^{-1}(\overline{K})$. Hence, $su, s'u' \in Q^{-1}(\overline{K}) \cap L$, $P(su) = P(s'u)$, $sue \in L$, and $s'u'e \in Q^{-1}(\overline{K}) \cap L$. Then, because L is prefix-closed by definition, observability of $Q^{-1}(\overline{K}) \cap L$ with respect to L, A_o, and A_c implies that $sue \in Q^{-1}(\overline{K}) \cap L$. Thus, according to Definition 1, $te = Q(sue) \in \overline{K} \cap Q(L) \subseteq \overline{K}$. ■

Let us remark that the assumption of synchronous nonconflictingness is only needed to cover the case where the specification is not prefix-closed. If Q is an L-observer then Lemma 3 implies that K and L are always synchronously nonconflicting, hence synchronous nonconflictingness is not needed in Corollary 8 below, where the observer property is required. In addition, it has to be emphasized that our conditions of local observation consistency and observation consistency are both structural and as such holds for any specification once the plant is fixed.

In addition to observability, the preservation of controllability and $L_m(G)$-closure for the original plant and its abstraction has to be addressed. At this point, it has to be noted that such a result has been previously stated in the literature. Precisely, the $L_m(G)$-observer property is needed together with *local control consistency* [8], [9].

Definition 7 (Local control consistency): Let G be a generator, G_{hi} its hierarchical abstraction with the corresponding high-level alphabet A_{hi} and projection $Q : A^* \to (A_{hi})^*$, and A_u the set of uncontrollable events. We say that Q is *locally control consistent* (LCC) for a string $s \in L(G)$ if for all $\hat{e} \in A_{hi} \cap A_u$ such that $Q(s)\hat{e} \in L(G_{hi})$, it holds that either $\nexists u \in (A \setminus A_{hi})^*$ such that $su\hat{e} \in L(G)$ or there is a $u \in (A_u \setminus A_{hi})^*$ such that $su\hat{e} \in L(G)$. Furthermore, we call Q LCC for a language $M \subseteq L(G)$ if Q is LCC for all $s \in M$.

The following result follows by combining Theorem 6 with the results of [8], [9].

Corollary 8: Let G be a generator over an event set A, and let $K \subseteq Q(L_m(G))$ be a (high level) specification language. Assume that Q is locally control consistent for $L(G)$ and A_u, observation consistent with respect to Q, P, P_{hi}, and locally observation consistent with respect to Q, P, and A_c. Furthermore, let the abstraction projection Q be an $L_m(G)$-observer. Then, K is $L_m(G_{hi})$-closed, controllable with respect to $Q(L(G))$ and $A_u \cap A_{hi}$, and observable with respect to $Q(L(G))$, $A_o \cap A_{hi}$, and $A_c \cap A_{hi}$ if and only if $K \| L_m(G)$ is $L_m(G)$-closed, controllable with respect to $L(G)$ and A_u, and observable with respect to $L(G)$, A_o, and A_c.

The benefit of the stated theorem is that it allows to verify the existence of a supervisor that realizes a high-level specification K for a given DES G, bearing the aforementioned properties, based on the abstraction G_{hi}. Whenever a nonblocking supervisor S_{hi} exists for the smaller abstracted model such that $L_m(S_{hi}/G_{hi}) = K$, then a nonblocking supervisor S exists such that $L_m(S/G) = L_m(G)||K$. In particular, a generator realization C of K such that $L_m(C) = K$ can be used to implement the supervisor in the form $C||G$.

Finally, let us mention another interesting aspect. In the special case, where $A_c \subseteq A_o$ (and, henceforth, observability is equivalent to the stronger property called normality), observation consistency is sufficient for preservation of observability, i.e., local observation consistency is not needed any more.

Corollary 9: Let G be a nonblocking generator over an event set A, and let $K \subseteq Q(L_m(G))$ be a high-level specification. Assume that $L(G)$ is observation consistent with respect to projections Q, P, and P_{hi}, and that K and $L_m(G)$ are synchronously nonconflicting. Then the language K is normal with respect to $Q(L(G))$ and P_{hi} if and only if $K \| L_m(G)$ is normal with respect to $L(G)$ and P.

Proof: To simplify the notation, $L(G)$ will be denoted by L and $L_m(G)$ by L_m from now on. One implication, namely that normality of K with respect to $Q(L)$ and P_{hi} implies normality of $K \| L_m$ with respect to L and P holds without any assumption similarly as for observability in the proof of Theorem 6.

On the other hand, let $K \| L_m$ be normal with respect to L and P. It will be shown that K is normal with respect to $Q(L)$ and P_{hi}. Assume that $t' \in \overline{K} \subseteq Q(L)$, $t \in Q(L)$, and $P_{hi}(t) = P_{hi}(t')$. We have to show that $t \in \overline{K}$ as well. Since $t, t' \in Q(L)$ and $P_{hi}(t) = P_{hi}(t')$, observation consistency implies that there are $s, s' \in L$ such that $Q(s) = t$, $Q(s') = t'$, and $P(s) = P(s')$. Since $t' \in \overline{K}$ we have also $s' \in Q^{-1}(\overline{K})$, which gives $s' \in Q^{-1}(\overline{K}) \cap L = \overline{K} \| L$. Then, normality of $K \| L_m$ with respect to L and P implies that $s \in Q^{-1}(\overline{K}) \cap L$. Thus, $t = Q(s) \in \overline{K} \cap Q(L) \subseteq \overline{K}$, which was to be shown. ■

Next, we illustrate the proposed conditions by a small example.

Example 1: Let $A = \{a, b, c, e, f, g\}$, $A_o = \{c, e, f\}$, $A_u = \{a, b, g\}$, $A_c = \{e\}$, and $A_{hi} = \{a, b, e, f\}$ be event sets. The plant generator G is given in Fig. 2.

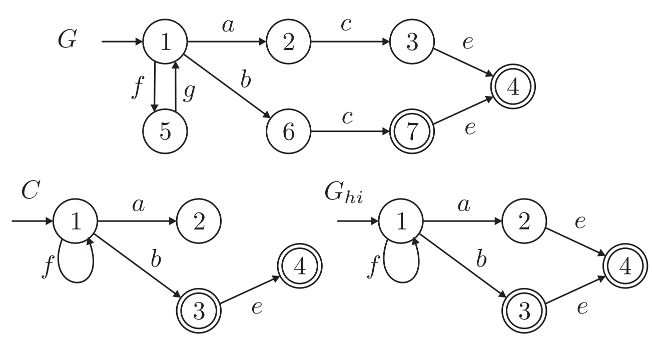

Fig. 2. Plant G, specification $K = L_m(C)$, and abstraction G_{hi}.

Then, the projection $Q : A^* \to (A_{hi})^*$ is an $L_m(G)$-observer and also locally control consistent. Note that $A_o \not\subseteq A_{hi}$, hence the strong condition proposed in [5] is not applicable. However, it can be verified that $L(G)$ is observation consistent with respect to Q, P, and P_{hi}. For example, the abstracted strings $t = a$ and $t' = b$ with the observation $P_{hi}(t) = P_{hi}(t') = \epsilon$ correspond to the strings $s = ac$ and $s' = bc$ with the same observation $P(s) = P(s') = c$. Furthermore, $L(G)$ is locally observation consistent with respect to Q, P, and A_c. For example, the strings $s = a$ and $s' = b$ with $Q(s)e = ae \in L(G_{hi})$ and $Q(s')e = be \in L(G_{hi})$ and $P(s) = P(s') = \epsilon$ both have an extension $u = c$ and $u' = c$ with $P(u) = P(u') = c$ and both $ace \in L(G)$ and $bce \in L(G)$. Since both sufficient conditions are fulfilled, observability of any language K for the abstracted model G_{hi} translates to observability for the original model G.

To further illustrate this result, we look at the specification K that is recognized by C in Fig. 2. It can be verified that K is not observable with respect to $L(G_{hi})$, $A_o \cap A_{hi}$, and $A_c \cap A_{hi}$ since the controllable event e has to be disabled after the string a and enabled after the string b with the same observation. In accordance with Theorem 6, $L(G)||K$, the expression of the specifications at the plant level, is not observable with respect to $L(G)$, A_c, and A_o. It also holds that if some hypothesis of Theorem 6 are missing, observability of $Q^{-1}(\overline{K}) \cap L(G)$ for the low-level model G is not enough to infer observability of K for the abstracted model G_{hi}.

This implication is supported by the plant G' in Fig. 3, where the transition from state 6 to state 7 is now labelled with a new event $d \in A_o$ instead of c. Now, G' is neither observation consistent nor locally observation consistent. For example, the abstracted strings $t = ae$ and $t' = be$ have the same observation but do not correspond to strings $s, s' \in L(G')$ with the same observation. Consequently, the specification K is not observable with respect to $L(G'_{hi})$, $A_o \cap A_{hi}$, and $A_c \cap A_{hi}$ although $K||L(G')$ is observable with respect to $L(G')$, A_o, and A_c.

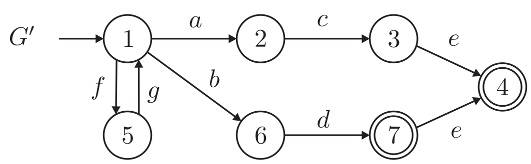

Fig. 3. Example generator G' that violates observation consistency.

IV. Modular Partially Observed Discrete-Event Systems

In this section, we show that the concepts of observation consistency and local observation consistency are also applicable in the modular setting. This is a crucial result for our novel conditions since, in combination with existing results for local control consistency and the $L_m(G)$-observer property [9], it allows to compute the abstracted model G_{hi} in a modular way. In particular, the generally very large plant G will not need to be evaluated explicitly. Modular control of discrete-event systems with partial observations has been studied in the past as well (cf., e.g., [6], [7]). However, the results proposed in the latter reference are quite restrictive and for large systems it is more beneficial to combine both horizontal and vertical abstractions to achieve the maximum of computational saving.

Let $G = G_1 \| G_2 \| \dots \| G_n$ be a modular discrete-event system with the language $L = L(G) = L_1 \| L_2 \| \dots \| L_n$, where L_i is a short notation for $L(G_i)$, and the marked language $L_m = L_m(G) = L_m(G_1) \| L_m(G_2) \| \dots \| L_m(G_n)$ for $i = 1, \dots, n$.

There are many notations for projections and event sets needed. In addition to the high level event set A_{hi} and the set of observable events A_o, the local event sets are denoted by A_i, $i = 1, \dots, n$. The intersections of these event sets are mostly denoted by adding the two subscripts, e.g., locally observable events of A_i are denoted by $A_{i,o} = A_i \cap A_o$, the high level local events by $A_{hi,o} = A_{hi} \cap A_o$. The various projections are then denoted as shown in Fig. 4. We assume that the high level event set contains all shared events. Namely, $A_s \subseteq A_{hi}$, where $A_s = \cup_{i \neq j}(A_i \cap A_j)$ is the set of all events that are shared by two or more components. In addition, we assume that the modular components agree on the controllability and observability status of the shared events. This is a standard assumption in hierarchical decentralized control.

We now observe that the conditions of observation consistency and local observation consistency, proposed in this paper, can only be useful in the modular setting if it is shown that they are compositional, i.e., that they are preserved by the synchronous product. The following statement shows that our main condition of observation consistency used in Theorem 6 is compositional.

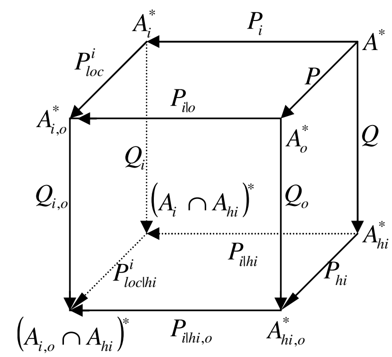

Fig. 4. Various projections: our notation.

Theorem 10: Assume that all shared events are included in the high level and that they are observable, i.e., $A_s \subseteq A_{hi}$ and $A_s \subseteq A_o$. Let L_i, for $i = 1, \dots, n$, be observation consistent with respect to projections Q_i, P^i_{loc}, and $P^i_{loc|hi}$. Then, $L = \|_{i=1}^n L_i$ is observation consistent with respect to projections Q, P, and P_{hi}.

To prove the theorem, an auxiliary result from [12] is needed. Consider the projections Q and Q_i, $i = 1, \dots, n$.

Lemma 11 ([12]): Let $A_s \subseteq A_{hi}$, and let $L_i \subseteq A_i^*$ be languages. Then, projection Q satisfies $Q(\|_{i=1}^n L_i) = \|_{i=1}^n Q_i(L_i)$.

We can now prove Theorem 10.

Proof: Let L_i, for $i = 1, \dots, n$, be observation consistent with respect to projections Q_i, P^i_{loc}, $P^i_{loc|hi}$ and let $t, t' \in Q(L)$ be such that $P_{hi}(t) = P_{hi}(t')$. It must be shown that there exist strings $s, s' \in L$ such that $Q(s) = t$, $Q(s') = t'$, and $P(s) = P(s')$. Let us recall that $A_s \subseteq A_{hi}$ and Lemma 11 imply that $Q(L) = Q(\|_{i=1}^n L_i) = \|_{i=1}^n Q_i(L_i)$. By projecting to local alphabets we get $P_{i|hi}(t) \in Q_i(L_i)$ and $P_{i|hi}(t') \in Q_i(L_i)$, for $i = 1, \dots, n$.

Also note that the equality $P_{hi}(t) = P_{hi}(t')$ implies that $P_{i|hi,o}(P_{hi}(t)) = P_{i|hi,o}(P_{hi}(t'))$. Indeed, by applying the commutative diagram of Fig. 4 we get $P^i_{loc|hi} P_{i|hi}(t) = P^i_{loc|hi} P_{i|hi}(t')$, by $P_{i|hi,o} P_{hi}(t) = P_{i|hi,o} P_{hi}(t')$. By abuse of notation we write the projection to A_i^* of $t \in (A_{hi})^*$ simply $P_i(t)$ instead of the more rigorous notation $P_{i|hi}(t)$.

Therefore, observation consistency of L_i, $i = 1, \dots, n$, with respect to projections Q_i, P^i_{loc}, $P^i_{loc|hi}$ implies that there exist strings $s_i, s'_i \in L_i$ such that $Q_i(s_i) = P_i(t)$, $Q(s'_i) = P_i(t')$, and $P^i_{loc}(s_i) = P^i_{loc}(s'_i)$. We claim that there exist strings $s \in \|_{i=1}^n s_i$ and $s' \in \|_{i=1}^n s'_i$ that satisfy the condition of observation consistency of L. First of all, consider any $s \in \|_{i=1}^n s_i$ and $s' \in \|_{i=1}^n s'_i$. Then, $s, s' \in L$ because $s_i, s'_i \in L_i$ for any $i = 1, 2, \dots, n$. Also, $P^i_{loc}(s_i) = P^i_{loc}(s'_i)$ for any $i = 1, 2, \dots, n$ means that also $P(s) \in P(\|_{i=1}^n s_i) = \|_{i=1}^n P^i_{loc}(s_i) = \|_{i=1}^n P^i_{loc}(s'_i) = P(\|_{i=1}^n s'_i) \ni P(s')$ due to Lemma 11 and the assumption that $A_s \subseteq A_o$. Hence, we can choose s and s' so that $P(s) = P(s')$ provided the languages above are nonempty, which is shown below.

Note that the existence of at least one such pair s, s', i.e., that both synchronous products $\|_{i=1}^n s_i$ and $\|_{i=1}^n s_i'$ are nonempty, follows from the assumption that $A_s \subseteq A_{hi}$.

Indeed, it suffices to prove that $Q(\|_{i=1}^n s_i)$ is nonempty, because then $\|_{i=1}^n s_i$ itself must be nonempty as well. Let us recall from above that there exist $t \in (A_{hi})^*$ such that for any $i = 1, 2, \ldots, n$ we have $Q_i(s_i) = P_i(t)$. Now, since $A_s \subseteq A_{hi}$ we get by Lemma 11 that $Q(\|_{i=1}^n s_i) = \|_{i=1}^n Q_i(s_i) = \|_{i=1}^n P_i(t)$. But it is clear that $t \in P_i^{-1} P_i(t)$ for all $i = 1, 2, \ldots, n$. Therefore, $t \in \|_{i=1}^n P_i(t) = \cap_{i=1}^n P_i^{-1} P_i(t)$ for all i. Thus, $\cap P_i^{-1} P_i(t)$ is nonempty. This completes the proof. ∎

Finally, we show that also local observation consistency is compositional.

Theorem 12: Assume that all shared events are included in the high level, i.e., $A_s \subseteq A_{hi}$. Let L_i, for $i = 1, \ldots, n$, be locally observation consistent with respect to projections Q_i and P_{loc}^i. Then, $L = \|_{i=1}^n L_i$ is locally observation consistent with respect to projections Q and P.

Proof: Let $s, s' \in L$ and $e \in A_c \cap A_{hi}$ be such that $Q(s)e \in Q(L)$, $Q(s')e \in Q(L)$, and $P(s) = P(s')$. We have to show that there are $u, u' \in (A \setminus A_{hi})^*$ such that $P(u) = P(u')$ and $sue \in L$, $s'u'e \in L$. Define $s_i := P_i(s)$ and $s_i' := P_i(s')$, and write $\mathcal{I}_e := \{i \mid e \in A_i\}$. For all $i \in \mathcal{I}_e$, we have that $Q_i(s_i)e = Q_i P_i(s)e = P_{i|hi} Q(s)e \in P_{i|hi} Q(L) \subseteq P_{i|hi} P_{i|hi}^{-1} Q_i(L_i) = Q_i(L_i)$ and, similarly, $Q_i(s_i')e \in Q_i(L_i)$ with $P_{loc}^i(s_i) = P_{loc}^i(s_i')$. Because of local observation consistency, it holds for all such i that there are $u_i, u_i' \in (A_i \setminus A_{hi})^*$ such that $P_{loc}^i(u_i) = P_{loc}^i(u_i')$ and $s_i u_i e \in L_i$ and $s_i' u_i' e \in L_i$. For the remaining $i \notin \mathcal{I}_e$, we choose $u_i = \epsilon$. Considering that u_i and u_j (u_i' and u_j') do not share events for $i \neq j$, we know that $\|_{k=1}^n u_k \neq \emptyset$ and $\|_{k=1}^n u_k' \neq \emptyset$. In particular, the string $u := u_1 u_2 \cdots u_n \in \|_{k=1}^n u_k$ and $u' := u_1' u_2' \cdots u_n \in \|_{k=1}^n u_k'$. Hence, $P(u) = P(u')$. Finally, since $s_i u_i e \in L_i$ for all $i \in \mathcal{I}_e$, we also know that $sue \in (\|_{i \in \mathcal{I}_e} s_i u_i e) \| (\|_{i \notin \mathcal{I}_e} s_i) \subseteq \|_{i=1}^n L_i = L$ and $s'u'e \in L$ with the same construction. ∎

An immediate consequence of this theorem is that under the above assumptions observation consistency can be checked in a compositional way with an obvious gain in the computational complexity. The first condition $A_s \subseteq A_{hi}$ might seem restrictive, but it is particularly useful for loosely coupled subsystems, where the interaction between the subsystems (via synchronization) is not too strong. In the general case multilevel hierarchy approach should be adopted, where the subsystems are aggregated into groups that are only loosely coupled. The second condition is also needed for compositionality. This means that all unobservable events should be private.

Finally, let us mention that our condition of observation consistency is structural, because it does not depend on the specification, which makes it possible to combine decentralized and hierarchical control synthesis in order to achieve even greater saving on complexity.

V. Conclusion

In this paper, hierarchical control of discrete-event systems has been extended to partially observed systems. The major issue in hierarchical control, that is to say the consistency between the original (low level) plant and the abstracted (high level) plant, has been studied. Both monolithic and modular plants have been considered. The main conditions we have proposed for observability to be preserved between the low level and the high level are structural and have been shown to be compositional. Therefore, the proposed conditions are applicable to large modular plants, where the hierarchical and the decentralized approaches should be combined in order to achieve considerable savings in the computational complexity.

In the future work, it is our plan to combine decentralized control of discrete-event systems with partial observations with the hierarchical approach proposed in this paper. At the same time we are working on effective algorithms for checking the new conditions we have presented.

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