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Identification of a flexible robot manipulator using a linear parameter-varying descriptor state-space structure

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Abstract— **This paper presents a new approach for the identification of the dynamical model of flexible manipulators. The structure of the identified model, chosen as a descriptor LPV model, is derived from the original non-linear equations. A set of experiments around different configurations is involved, which is suitable for an accurate measurement of the tip of the manipulator by video camera. The final estimation step is global, allowing the direct identification of the global model based on the collection of local experimental data. In an output error context, a genetic algorithm is used for the minimization of the identification criterion. As a case study, a robotic arm with two flexible segments is considered. Identification results based on simulations including noise show the effectiveness of the approach.**

I. INTRODUCTION

A. Identification for robotics

Robotic systems generally have a non-linear behavior. Most of the time, their models are derived from the standard laws of physics (*e.g.*, Newton's laws of motion) [1], [2]. Their descriptions are mainly based on the Euler-Lagrange equations and the virtual work principle. Interesting from a theoretical point of view, the exclusive use of the standard laws of physics makes the final model quite complex and requires an accurate knowledge of the manipulator as well as high-level skills in robotics especially when different robot structures are handled. This is all the more true when the user wants to have access to physical parameters of the system which are imperfectly known. To get round this difficulty, efforts dedicated to robot identification (*i.e.*, parameter estimation from experimental data) are more and more made in the industry [3], [4]. However, a direct identification of a non-linear black-box model is often tricky because

- strong non-linearities can appear in particular working conditions,
- the development of a global non-linear model structure can rely on strong assumptions such as a uniform density of the manipulator segments or the nature of the deformations if any.

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Because a linear time-invariant model is often not sufficient when the system is used in a large robot workspace, linear parameter-varying (LPV) models are more and more introduced in robotics (see, *e.g.*, [5], [6], [7], [8]). The development of LPV model identification for the experimental modelling of robots is advocated for two main reasons. First, from an identification point of view, the introduction of such a structure allows the use of standard tools dedicated to LTI models for the estimation of models with a structural flexibility able to picture time-varying as well as non-linear dynamics. Second, the construction of a reliable LPV model is a standard initial step for many controller design techniques available in robotics [9].

B. LPV model identification

Historically, as far as LPV model identification is concerned, the first developments focused on a global procedure and assumed that one global experiment can be performed in which the control inputs as well as the scheduling parameters can be both excited (see, *e.g.*, [10], [11], [12]). On the other hand, recent methods are based on a multi-step procedure where (see, *e.g.*, [5], [6], [13], [14])

- a finite set of scheduling parameter values $\{\boldsymbol{\kappa}_i\}$, $i = \{1, q\}$, is handled,
- local experiments (corresponding to an almost constant scheduling parameter value) are carried out for each $\boldsymbol{\kappa}_i$,
- local LTI models are estimated from the sets of local I/O measurements, for each $\boldsymbol{\kappa}_i$,
- a global parameter-dependent model is built from the interpolation of the local LTI models.

This latter viewpoint is often considered for robots or mechanical systems identification (see, *e.g.*, [5], [6], [14], [15]). More precisely, in [5], [6], the derivation of robot LPV models is performed by interpolating local LTI state-space models calculated from frequency responses measured at different operating points. Then, in [14], [15], the identification of mechanical or vibro-acoustic applications is tackled by using the SMILE technique. This method is more particularly based on the interpolation of black-box state-space LTI models that are estimated for fixed operating conditions of the system. Interesting for controller design, these black-box techniques do not take into account the information available from the non-linear equations governing the behavior of the manipulator. Recently, this basic idea has been considered in [8]. In the aforementioned communication, the non-linear dynamic model of a CRS A465 robotic manipulator is used to derive a quasi-LPV model of the system. More precisely, a parameter-affine LPV model is identified by applying a global approach where typical trajectories of the scheduling signals are generated so that all of the expected operating regions of the plant are covered.

C. Topic of the paper

Inspired by the discussion in [16], the idea used in the current paper also consists in resorting to the knowledge available from the study of the non-linear equations governing the system behavior in order to fix the structure of the global LPV model. However, in the experimental framework considered hereafter, the global identification procedure carried out in [8] cannot be used. Indeed, for our robot manipulator, the position of the end-effector is measured by a video camera. This experimental condition makes the persistent excitation of the scheduling parameter not conceivable. Indeed, in the aimed setup, a single video camera is used to provide a measurement of the position of the end-effector equipped with visual markers. In this situation, it is recommended to zoom in on the markers in order to obtain an accurate measurement of the displacements due to the flexible modes which are of relatively small amplitude. When moving the arm to another configuration, the camera must be re-positioned in order to keep the markers in the image. Thus, a global procedure cannot be carried out. In order to circumvent this difficulty, an original approach is developed in the current paper. Based on local experiments (corresponding to an almost constant scheduling parameter value), the procedure introduced hereafter provides a global LPV model without requiring an interpolation stage. As explained in § II, this new approach is particularly adapted for our identification problem because

- a standard global procedure (requiring a persistent excitation of the scheduling parameter) is not conceivable in our experimental framework,
- a LPV descriptor structure of the non-linear system can be obtained quite easily from an analytical study of the non-linear equations governing the behavior of the robot.

Thus, the parameters of the global model are estimated from a collection of the different local experiments. This basic idea of realizing global functions by local actions is inspired from S. Hara's work dedicated to the design of "glocal controllers" [17]. For this reason, the identification procedure described hereafter will be also called "glocal" in the sequel. This approach is interesting from a theoretical as well as a practical point of view because

- it circumvents the standard problems related to the interpolation of local models such as [18], [19]
  - the numerical issues if an ill-conditioned parameterization for the model class is chosen (*e.g.*, use of canonical forms with coefficients with huge magnitude variations (see [5] for an illustration of this problem)),
  - the challenge of realizing all the models with respect to the same state variables which comes into play when dealing with the problem of interpolating black-box local state-space models,
- it does not require a persistent excitation of the scheduling parameters.

As a second originality, a descriptor representation of the LPV model (obtained from the laws of physics) is directly used. This feature reduces the complexity of the model in terms of order of the polynomial development of the state matrices.

This paper can be viewed as an initial study towards a handy methodology leading to a control-oriented LPV model for robotic manipulators. The final goal is to supply a procedure that would be useful for different kinds of manipulators with two joints and potentially including flexibilities. Hereafter, a planar manipulator with two DOF and with flexible segments is considered.

The paper is organized as follows. The non-linear model of the flexible manipulator is introduced in Section II. More precisely, the non-linear equations and the deduced analytical LPV model are provided. This study leads to a LPV model described with the help of a descriptor state-space structure. The different phases composing this identification procedure are described in Section III. The glocal procedure combining local experiments and a global LPV model is more precisely introduced. This procedure allows the estimation of physical parameters of the system such as the section of the arms composing the manipulator or the Young modulus of the material. This approach is validated in Section IV on simulation data obtained with a non-linear simulator of the flexible arm. Section V concludes the paper.

II. SYSTEM DESCRIPTION AND ANALYTICAL LPV MODELLING

A. Manipulator description

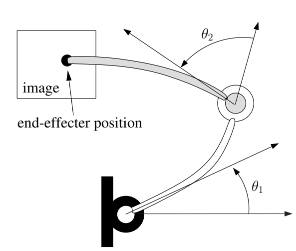

Fig. 1. Geometry of the flexible arm.

In this study, the system is a flexible robot composed of two segments as depicted in Figure 1. Such a flexible structure can be considered, for instance, when considering the two first rotoid joints of a SCARA manipulator. These flexible characteristics are satisfied by the SINTERS manipulator used in [20]. This robot is indeed lightweight as it was designed to attain fast dynamics in order to compensate the heart tissue motion for intra-cardiac surgery. As a result, it was observed that the bandwidth is restricted by flexible modes that can be attributed to small deformations of the segments.

Both joints are torque-controlled and the joint positions are measured by encoders. The aim is to control the position of the end-effector that can be measured by a video camera.

Deformations of the two segments are considered but are not measured. Both segments have the same length $l_1 = l_2 = 0.5\ m$ and respective masses of $m_1 = 7.5\ kg$ and $m_2 = 5\ kg$. Their sections are squares of width $d = 5\ cm$. The material has a Young modulus equal to $\varepsilon = 1\ GPa$. In the following, the parameters to be estimated are $\boldsymbol{\vartheta} = \begin{bmatrix}\varepsilon & d\end{bmatrix}$. The developed technique could be applied similarly for the estimation of other physical parameters such as the mass or the length of the segments. This choice is mainly motivated by the availability of reliable measurements of l_1, l_2, m_1 and m_2.

B. Non-linear dynamical model

Under the assumption of Euler-Bernoulli beam, the dynamical equations of a flexible arm can be derived by using the assumed-mode method where the deformation field is decomposed into a finite sum of elementary deformations [1]. In the current case, small deformations are considered and only one mode is chosen for the transverse deformation field. For segment #k, $k = 1, 2$, the deformation field writes $\delta_k(x,t) = x^2 v_k(t)$, where x represents the abscissa along the segment and $v_k(t)$ is the state of the deformation. Therefore, the resulting deformation at the end of the segment of length l_k is $\delta_k(l_k, t) = l_k^2 v_k(t)$. The dynamical model is derived from the Virtual Work Principle of the DynaFlex toolbox developed on Maple [21] (for more details on the basic ideas, refer to [2] and [22]). The resulting model relies on a generalized position vector $\mathbf{q} = \begin{bmatrix}\theta_1 & \theta_2 & v_1 & v_2\end{bmatrix}^\top$ and writes

$$\boldsymbol{\mathcal{M}}(\mathbf{q}(t))\ \ddot{\mathbf{q}}(t) = \boldsymbol{\mathcal{F}}(\mathbf{q}(t), \dot{\mathbf{q}}(t)) + \boldsymbol{\mathcal{G}}\,\mathbf{u}(t) \tag{1}$$

where $\boldsymbol{\mathcal{M}}(\mathbf{q})$ is the inertia matrix, $\boldsymbol{\mathcal{F}}(\mathbf{q}, \dot{\mathbf{q}})$ is a generalized force vector that accounts for the Coriolis and centrifugal effects (see [23] for details about the mathematical expressions of matrix $\boldsymbol{\mathcal{M}}$ and vector $\boldsymbol{\mathcal{F}}$). The torque vector $\mathbf{u} = \begin{bmatrix}u_1 & u_2\end{bmatrix}^\top$ has only effects on the dynamics of the rigid positions θ_1 and θ_2, corresponding to

$$\boldsymbol{\mathcal{G}} = \begin{bmatrix}\mathbf{I}_{n_\theta \times n_\theta} \\ \mathbf{0}_{n_v \times n_\theta}\end{bmatrix}. \tag{2}$$

The x and y positions of the end-effector can be written from the geometrical model, resulting in the non-linear measurement equation $\mathbf{z} = \mathbf{g}(\mathbf{q})$, *i.e.*,

$$\begin{aligned} z_1 =& (l_1 - \frac{2}{3} l_1^3 v_1^2)\cos(\theta_1) - l_1^2 v_1 \sin(\theta_1) \\ &+ (l_2 - \frac{2}{3} l_2^3 v_2^2)\cos(\theta_{12}) - l_2^2 v_2 \sin(\theta_{12}) \end{aligned} \tag{3a}$$

$$\begin{aligned} z_2 =& (l_1 - \frac{2}{3} l_1^3 v_1^2)\sin(\theta_1) + l_1^2 v_1 \cos(\theta_1) \\ &+ l_2^2 v_2 \cos(\theta_{12}) + (l_2 - \frac{2}{3} l_2^3 v_2^2)\sin(\theta_{12}) \end{aligned} \tag{3b}$$

where $\theta_{12} = \theta_1 + \theta_2 + 2 l_1 v_1$.

C. LPV descriptor model from a Jacobian linearization

The dynamical equations (1)-(3) are non-linear. The first step of this analytical study consists in applying a standard Jacobian linearization [24] in order to get a linear parameter-varying state-space representation of the generalized second order model (1). This LPV state-space form is obtained by considering a two-step approach. Firstly, the non-linear equation (1) is linearized for a set of working points $(\mathbf{q}_0, \dot{\mathbf{q}}_0)$, leading to[1]

$$\mathbf{M}(\boldsymbol{\kappa})\ddot{\mathbf{q}}(t) = -\mathbf{D}(\boldsymbol{\kappa})\dot{\mathbf{q}}(t) - \mathbf{K}(\boldsymbol{\kappa})\mathbf{q}(t) + \mathbf{G}\mathbf{u}(t) \tag{4}$$

with $\boldsymbol{\kappa} = \boldsymbol{\kappa}(\mathbf{q}_0, \dot{\mathbf{q}}_0)$ the scheduling parameter vector and where

$$\mathbf{M}(\boldsymbol{\kappa}) = \boldsymbol{\mathcal{M}}(\mathbf{q}_0) \qquad \mathbf{D}(\boldsymbol{\kappa}) = \frac{\partial \boldsymbol{\mathcal{F}}(\mathbf{q}_0, \dot{\mathbf{q}}_0)}{\partial \dot{\mathbf{q}}} \tag{5a}$$

$$\mathbf{K}(\boldsymbol{\kappa}) = \frac{\partial \boldsymbol{\mathcal{F}}(\mathbf{q}_0, \dot{\mathbf{q}}_0)}{\partial \mathbf{q}} \qquad \mathbf{G} = \boldsymbol{\mathcal{G}} \tag{5b}$$

are the inertia, the damping, the stiffness and the control matrices of the linearized model respectively. In this work, we focus on the identification of a model that includes the variability of the behavior with respect to the positions $\boldsymbol{\theta}_k$, $k = 1, 2$. Then, we neglect the other phenomena and choose $\mathbf{q}_0 = \begin{bmatrix}\theta_0^\top & \mathbf{0}_{1\times 2}\end{bmatrix}$ and $\dot{\mathbf{q}}_0 = \mathbf{0}_{4\times 1}$[2]. Notice that, unlike the model used in [8], the inertia matrix $\mathbf{M}(\boldsymbol{\kappa})$ is not inverted in the following. On the contrary, a descriptor model is extracted which is affine with respect to the scheduling parameter signal $\boldsymbol{\kappa}$ without any approximations. More precisely, by considering $\mathbf{x} = \begin{bmatrix}\mathbf{q}^\top & \dot{\mathbf{q}}^\top\end{bmatrix}^\top \in \mathbb{R}^8$ as the state vector, the following local linearized state equation can be easily deduced

$$\mathbf{E}(\boldsymbol{\kappa})\dot{\mathbf{x}}(t) = \mathbf{A}(\boldsymbol{\kappa})\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{6}$$

with

$$\mathbf{E}(\boldsymbol{\kappa}) = \begin{bmatrix}\mathbf{I}_{n_q \times n_q} & \mathbf{0}_{n_q \times n_q} \\ \mathbf{0}_{n_q \times n_q} & \mathbf{M}(\boldsymbol{\kappa})\end{bmatrix} \tag{7a}$$

$$\mathbf{A}(\boldsymbol{\kappa}) = \begin{bmatrix}\mathbf{0}_{n_q \times n_q} & \mathbf{I}_{n_q \times n_q} \\ -\mathbf{K}(\boldsymbol{\kappa}) & -\mathbf{D}(\boldsymbol{\kappa})\end{bmatrix} \tag{7b}$$

$$\mathbf{B} = \begin{bmatrix}\mathbf{0}_{n_q \times n_\theta} \\ \mathbf{G}\end{bmatrix}. \tag{7c}$$

Notice that the control matrix $\mathbf{B}$ is independent from the scheduling parameter $\boldsymbol{\kappa}$.

Let $\mathbf{J}(\boldsymbol{\theta})$ denote the Jacobian of the rigid geometric model, *i.e.*, $\mathbf{J}(\boldsymbol{\theta}) = \mathrm{d}\mathbf{g}([\boldsymbol{\theta}^\top\ \mathbf{0}_{1\times n_v}]^\top)/\mathrm{d}\boldsymbol{\theta}$ (see Eq. (3) for a definition of $\mathbf{g}$). Except for the singular positions, this Jacobian is invertible and it is possible to define a new measurement vector $\mathbf{y} = \mathbf{J}^{-1}(\boldsymbol{\theta})\,\mathbf{z}$. The entries of this new measurement vector $\mathbf{y}$ are the angular positions of a fictitious rigid arm that would have the same geometry and the same measurement $\mathbf{z}$. The use of $\mathbf{y}$ instead of $\mathbf{z}$ allows the simplification of the measurement equation, *i.e.*,

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{8}$$

[1] In the following, the equilibrium values are omitted in order to shorten the notations.

[2] This choice of the working point corresponds to neglect the Coriolis effects. The validity of this assumption will be evaluated simultaneously with the identified model.

where $\mathbf{C} = \begin{bmatrix}\mathbf{C}_1 & \mathbf{0}_{n_\theta \times n_q}\end{bmatrix}$ and

$$\mathbf{C}_1 = \begin{bmatrix} 1 & 0 & l_1 & 0 \\ 0 & 1 & l_1 & l_2 \end{bmatrix}. \tag{9}$$

Due to the lack of space and their relative complexity, the matrices $\boldsymbol{\mathcal{M}}$ and $\boldsymbol{\mathcal{F}}$ are not described in this paper. However, they are available in [23]. Looking closer at $\boldsymbol{\mathcal{M}}$ and $\boldsymbol{\mathcal{F}}$, it is clear that the matrices $\mathbf{E}$ and $\mathbf{A}$ are affine functions of $\cos(\theta_2)$ and $\sin(\theta_2)$ as well as linear with respect to the parameters d and ε. This property makes the developed model interesting and easy to be used from an identification point of view. As far as the choice of the scheduling parameter vector is concerned, it is obvious that $\boldsymbol{\kappa} = \begin{bmatrix}\cos(\theta_2) & \sin(\theta_2)\end{bmatrix}$ will suit. It is also important to notice that the angular position θ_2 is easy to be measured on a flexible robot as an encoder is generally located at the motor side of the joints. This availability is paramount when the experimental modelling of the LPV model is considered.

D. Velocity controlled model

Generally, industrial manipulators are equipped with low-level joint-velocity control loops in order to reduce the effects of the frictions that occur in the gear-boxes and therefore obtain a simpler behavior. Due to the co-location of the torque actuation and the position measurement at the motor side, a high bandwidth of this inner loop can be obtained with a standard static output feedback. More precisely,

$$\mathbf{u}(t) = \boldsymbol{\Lambda}\left(\dot{\boldsymbol{\theta}}^*(t) - \dot{\boldsymbol{\theta}}(t)\right) \tag{10}$$

where $\dot{\boldsymbol{\theta}}^*(t) = \begin{bmatrix}\dot{\theta}_1^*(t) & \dot{\theta}_2^*(t)\end{bmatrix}^\top$ is the vector of the speed references. Replacing Eq. (10) in Eq. (6) results in

$$\begin{aligned}\mathbf{E}(\boldsymbol{\kappa})\dot{\mathbf{x}}(t) = &\begin{bmatrix}\mathbf{0}_{n_q \times n_\theta} \\ \mathbf{G}\boldsymbol{\Lambda}\end{bmatrix}\dot{\boldsymbol{\theta}}^*(t) \\ &+ \begin{bmatrix}\mathbf{0}_{n_q \times n_q} & \mathbf{I}_{n_q \times n_q} \\ -\mathbf{K}(\boldsymbol{\kappa}) & -\mathbf{D}(\boldsymbol{\kappa}) - \mathbf{G}\boldsymbol{\Lambda}\mathbf{G}^\top\end{bmatrix}\mathbf{x}(t)\end{aligned} \tag{11}$$

where $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2)$.

Considering the position $\mathbf{y}$ as the output measurement has one drawback: the model is unstable due to the pure integration between speed and position. Therefore, it is generally recommended to use the velocity $\bar{\mathbf{y}} = \dot{\mathbf{y}}$ instead[3]. Using the state matrices introduced beforehand, the output equation rewrites

$$\bar{\mathbf{y}}(t) = \mathbf{C}\dot{\mathbf{x}}(t) = \bar{\mathbf{C}}\mathbf{x}(t) \tag{12}$$

where $\bar{\mathbf{C}} = \begin{bmatrix}\mathbf{0}_{n_\theta \times n_q} & \mathbf{C}_1\end{bmatrix}$.

III. IDENTIFICATION PROCEDURE OF THE LPV DESCRIPTOR MODEL

In order to estimate the parameter vector $\boldsymbol{\vartheta} = \begin{bmatrix}\varepsilon & d\end{bmatrix}$, a specific glocal approach is considered. By glocal, it is meant that a global LPV model is estimated from local experiments. This approach is interesting because

- it does not utilize the standard interpolation step used in the local approach,
- it does not require a persistent excitation of the scheduling parameters. Notice indeed that this constraint is often difficult to be fulfilled for many global experimental methods.

More precisely, as a local approach, several experiments are carried out where, for each experiment, the variations of the scheduling parameter are constrained to be very small and the speed references $\dot{\boldsymbol{\theta}}^*$ are excited. This approach is justified because the experiments with a video camera are easier to be performed when small variations of the position are considered. However, contrary to the standard local techniques, it is not intended to

- estimate local LTI models from the sets of local I/O measurements for each value of κ,
- build the global parameter-dependent model from the interpolation of the local LTI models.

Hereafter, all the local I/O data sets are used at the same time in order to estimate the parameters of the global LPV model. More precisely, considering the availability of δ local I/O data sets, the following cost function is used

$$V(\boldsymbol{\vartheta}) = \sum_{i=1}^{\delta} \|\boldsymbol{y}_i(t) - \mathbf{y}_i(t, \boldsymbol{\vartheta})\|_2^2 \tag{13}$$

where $\mathbf{y}(t, \boldsymbol{\vartheta})$ is the output of the LPV model and $\boldsymbol{y}(t)$ is the output of the flexible manipulator. By doing so, the parameters of the LPV model are estimated in one shot without requiring any interpolation step. Furthermore, by using such a cumulative cost function, the variance of the estimated parameters is smaller than by calculating the parameters as the mean value of δ estimates computed from each local experimental data set.

Because the criterion $V(\boldsymbol{\vartheta})$ can be highly non-linear with respect to (w.r.t.) the parameter vector $\boldsymbol{\vartheta}$, its optimization is performed by using a differential evolution algorithm [25]. As claimed by its authors, the differential evolution algorithm used in the following is “a very simple population-based stochastic function minimizer”. As a genetic algorithm, the basic idea of the differential evolution algorithm is a reliable scheme able to generate efficiently the trial parameter vectors. The main improvements of this approach (w.r.t. the literature dedicated to the genetic methods [26]) are mainly its good convergence properties and its suitability for parallelization [25]. Notice that the estimates obtained from this genetic algorithm can be used as initial estimates for a more standard non-linear optimization algorithm used by a prediction error or output error method.

IV. SIMULATION RESULTS

In this Section, the identification technique introduced beforehand is tested on data acquired from a non-linear simulator of the flexible manipulator. This simulator is built from the non-linear dynamical equations governing the behavior of the flexible arm robot presented in Section II-A. As said previously, δ local I/O data sets are firstly generated.

[3] In practice, the velocity is estimated from the discrete-time measurement of the position.

More precisely, for 7 constant θ_2 in the range $\left[\frac{\pi}{8} : \frac{\pi}{8} : \frac{7\pi}{8}\right]$, 7 noise-free I/O data sets are acquired. Remember that $\boldsymbol{\kappa} = \begin{bmatrix}\cos(\theta_2) & \sin(\theta_2)\end{bmatrix}$. For each run, the input signals are chosen as two uncorrelated pseudo-random binary sequences. Then, the parameter vector $\boldsymbol{\vartheta} = \begin{bmatrix}\varepsilon & d\end{bmatrix}$ is estimated by using

- the glocal technique described in § III,
- the same algorithm as the one applied for the glocal approach (*i.e.*, the differential evolution algorithm developed by K. Price *et al.* [25]) but now with the 7 local data set separately (so that 7 local models are estimated with a state-space structure satisfying Equations (11)-(12) but with a fixed value of $\boldsymbol{\kappa}$).

The length of each local data set is equal to 30000 samples with a sampling period of $0.1\ ms$. For the identification, a down-sampling is performed with a rate equal to 10.

As depicted in Tab. I, two signal-to-noise-ratios[4] are considered in this study (∞ and $20dB$ respectively). As far as the initialization of the evolution algorithm is concerned, the initial population members (for each new generation) is allowed to vary in the range $\begin{bmatrix}1\ GPa, & 5\ cm\end{bmatrix} \times \pm 20\%$. Notice also that, for the local approach as well as the glocal one, a Monte Carlo simulation of size 20 (corresponding to 20 different initial population members) has been carried out. For instance, for the local approach, 20 simulations (each with different initial population members) are performed for each operating point. These different initial points as well as the finite precision of the optimization procedure explain the reason why non-zero standard deviations are obtained when noise-free data are handled. For the noisy data, a different output noise is also generated with similar statistical properties for each run.

By analyzing the figures available in Tab. II, it is clear that the developed approach leads to reliable estimates of the parameters d and ε. As far as the noise-free case is concerned, both approaches give pretty much the same results. The figures in Tab. I and Tab. II for SNR = ∞ only highlight the effects of the finite precision of the optimization procedure as well as the different initial population members. When noisy data are considered, a comparison of Tab. I and Tab. II shows that the technique described in § III gives slightly better estimates than the local one. Indeed, the bias for the average of the local estimates is 2.4% (0.0512 vs 0.05) instead of 1.2% (0.0506 vs 0.05) for the glocal one. As far as the standard deviations are concerned, as shown in the rows entitled "mean" of Tab. I, a geometric propagation of the standard deviations for d and ε leads to values which are smaller than what is depicted in Tab. II. However, when the standard deviations of each local estimate are directly compared with the glocal ones, the standard deviation is generally smaller with the glocal technique. Of course, for the glocal approach, the noise effect is averaged (and the noise-free results are almost obtained) because all the data sets are used simultaneously. Thus, when noisy data are handled, the glocal method should be preferred than the local one.

[4]The signal-to-noise ratio is defined as follows: $SNR = 10\log\left(\frac{\mathrm{cov}\{\bar{y}_i\}}{\mathrm{cov}\{v\}}\right)$, $i \in [1, n_y]$ where v stands for the noise acting on the noise-free output $\bar{y}_i$.

	SNR = ∞			
	ε		d	
θ_2	mean	std	mean	std
$\frac{\pi}{8}$	1.0198e+09	1.0348e+08	0.0506	0.0015
$\frac{2\pi}{8}$	0.9976e+09	1.4689e+08	0.0509	0.0018
$\frac{3\pi}{8}$	1.0579e+09	1.3451e+08	0.0499	0.0017
$\frac{4\pi}{8}$	0.9622e+09	1.2826e+08	0.0509	0.0017
$\frac{5\pi}{8}$	1.0203e+09	0.9628e+08	0.0499	0.0012
$\frac{6\pi}{8}$	1.0767e+09	1.0749e+08	0.0490	0.0013
$\frac{7\pi}{8}$	0.9742e+09	1.2715e+08	0.0502	0.0017
mean	1.0155e+09	0.4603e+08	0.0503	0.000594
	SNR = 20 dB			
	ε		d	
θ_2	mean	std	mean	std
$\frac{\pi}{8}$	0.9156e+09	1.1052e+08	0.0527	0.0022
$\frac{2\pi}{8}$	1.0121e+09	1.3769e+08	0.0514	0.0026
$\frac{3\pi}{8}$	0.9652e+09	1.1691e+08	0.0513	0.0018
$\frac{4\pi}{8}$	1.0066e+09	1.3042e+08	0.0512	0.0021
$\frac{5\pi}{8}$	0.9646e+09	0.8966e+08	0.0506	0.0016
$\frac{6\pi}{8}$	1.0162e+09	1.8357e+08	0.0502	0.0030
$\frac{7\pi}{8}$	0.9599e+09	1.1275e+08	0.0508	0.0022
mean	0.9771e+09	0.4871e+08	0.0512	0.000853

TABLE I

ESTIMATED PARAMETERS FROM THE LOCAL EXPERIMENTS. THE TRUE VALUES ARE $\varepsilon = 1\ GPa$ AND $d = 5\ cm$ RESPECTIVELY. FOR THE "MEAN" STANDARD DEVIATION, A GEOMETRIC PROPAGATION IS USED, *i.e.*, $\mathrm{std}_{\mathrm{mean}} = \sqrt{\left(\sum_{\mathrm{i}=1}^{\delta} \mathrm{std}_{\mathrm{i}}^2\right)/\delta}$.

In order to validate the LPV descriptor model, the time responses of the system and the model are compared in the following. For the model, the estimated parameters are chosen equal to $d = 0.0506\ m$ and $\varepsilon = 9.83e+08\ Pa$, *i.e.*, the values obtained with the glocal method from noisy data. The system as well as the LPV model are excited so that the whole range of the scheduling parameter is smoothly visited (see Fig. 2 for a sample). The following measurement fit [5]

$$\mathrm{FIT} = 100 \times \max\left(1 - \frac{\|\bar{y}_i - \hat{\bar{y}}_i\|}{\|\bar{y}_i - \mathrm{mean}(\bar{y}_i)\|}, 0\right),\ i \in [1, 2]$$

is introduced in order to quantify the quality of the model on validation data (*i.e.*, a data set different from the one used for the estimation). Using 10 different sets of validation data, the average fit for the first and the second output of the system are 96.8% and 97.1% respectively. These figures prove the efficiency of the developed identification method.

[5]$\bar{y}_i$ stands for the ith output of the system and $\hat{\bar{y}}_i$ for its estimate.

SNR = ∞			
ε		d	
mean	std	mean	std
1.0098e+09	0.9240e+08	0.0502	0.0013

SNR = 20 dB			
ε		d	
mean	std	mean	std
0.9833e+09	0.9350e+08	0.0506	0.0013

TABLE II

ESTIMATED PARAMETERS WITH THE GLOCAL METHOD. THE TRUE VALUES ARE $\varepsilon = 1\ GPa$ AND $d = 5\ cm$ RESPECTIVELY.

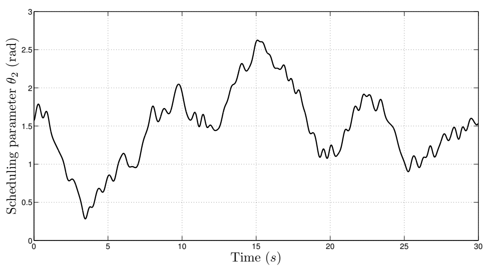

Fig. 2. Evolution of θ_2 during the LPV model validation.

V. CONCLUSION

In this paper, the construction of a descriptor linear parameter-varying model of a flexible robot manipulator is investigated. Based on the non-linear equations governing the behavior of such a system, a linear parameter-varying model structure is firstly extracted by applying a standard Jacobian linearization. A descriptor state-space linear parameter-varying model is more precisely derived. Then, in order to take advantage of this additional modeling effort during the estimation step of the parameters of the LPV model, a specific identification procedure is suggested in this article. Based on local experiments (where the scheduling parameter signal is fixed), a new global method (which herein does not require complex persistence of excitation assumptions for the scheduling parameters) is more precisely developed. Thus, a global linear parameter-varying model of the system is calculated from local experiments without requiring a standard interpolation step. This mixed analytical/experimental local/global technique is evaluated on data from a 2 degree-of-freedom flexible manipulator.

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