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Robust Adaptive Failure Compensation of Hysteretic Actuators for Parametric Strict Feedback Systems

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Abstract— **It is well known that hysteresis often exists and failures may occur in practical actuators during system operation. The compensation for uncertainties caused by unknown hysteresis and unknown failures is an important problem in ensuring system stability and performances. However, available results based on adaptive approaches to address such a problem are still very limited. In this paper, an adaptive tracking control scheme is proposed to solve such a problem. Simulation results also illustrate the effectiveness of the control scheme.**

I. INTRODUCTION

Actuator failures seems inevitable in practical systems and such failures may lead to instability or even catastrophic accidents during their operations. So how to maintain acceptable system performance when a failure occurs has become more and more important in the design of control schemes. To address such an issue, several methods have been proposed in recent years, see for examples in [1]-[12]. Among these methods, adaptive control approach offers certain advantages. For example, it can handle system parametric uncertainties and avoid false alarms and delays possibly caused by failure detection. In the context of unknown failure compensation based on adaptive approach, several schemes have been proposed, see for examples in [4]-[12]. In [4] [5] adaptive control schemes were proposed to compensate for uncertainties caused by unknown actuator failures for linear systems and it was extended to strict feedback nonlinear systems with backstepping techniques in [6]. A result on MIMO systems was presented in [7]. In [8] an output feedback adaptive control law was designed for a class of nonlinear systems to address failure problem, but the previous assumption of identical relative degrees with respect to all the inputs was still needed. By using pre-filters, the relative degree condition was relaxed for linear systems in [9] and the result was extended to nonlinear systems in [10]. In [11] an adaptive control scheme based on a prescribed performance bound was proposed to guarantee transient performance.

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On the other hand, hysteresis nonlinearities exists in a wide range of practical systems especially in mechanical actuators, electro-magnetic system and so on. So the effect of hysteresis cannot be ignored in the design and analysis of control systems. In recent years, several schemes based on adaptive approach to compensate for hysteresis have been proposed, see for examples [13]-[21]. In [14] adaptive control law was proposed for a class of linear systems by constructing an inverse operator to cancel the effects of backlash hysteresis. In [15] and [16], a robust adaptive control scheme was proposed to compensate for backlash-like hysteresis for a class of nonlinear plants. In [14] [15], an assumption that all the system parameters must be in a known bounded set was imposed. It was relaxed in [16] by using backstepping design approaches. The result was extended to decentralized adaptive control systems in [17]. By introducing a smooth inverse function of backlash and with backstepping technique, an adaptive output feedback control law was designed to ensure the stability of system in [18] and [19].

Failures of such hysteretic actuators may occur in practice. However, the available results based on adaptive approaches to address such a problem are very limited. Recently in [20], we proposed an adaptive scheme for a simple class of systems studied in [15] - [17]. In [20] and [15] - [17], the control gains are constants. Thus the effects of approximating the actuator hysteresis will be bounded after multiplying these gains and they can be handled in similar ways to bounded external disturbances. In this paper, we will consider parametric strict feedback systems. For such systems, the control gains are nonlinear functions of system states and therefore the relevant effects in approximating the actuator hysteresis can no longer be assumed bounded. How to handle such effects is a challenging issue in the design and analysis of adaptive controllers and it becomes even more difficult in the presence of possible actuator failures. Our idea is to separate such effects into two parts by applying Young's inequality. One is bounded and it is combined with external disturbances. An estimator is designed to estimate their unknown upper bound. The other part is handled together with unknown nonlinear modeling errors by finding a suitable smooth function as their upper bound. This upper bounding function is in turn employed in the design of estimators and controllers. It is shown that the adaptive controllers designed in this way can ensure global stability and asymptotic tracking even in the presence of possible failures of hysteretic actuators. Results

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from simulation studies also verify the established theoretical results.

The paper is organized as follows. In Section 2, we present and discuss the models of the class of systems to be controlled and hysteretic actuators with possible failures. Adaptive control scheme is proposed and analyzed in Section 3. In Section 4, we use a simple system to illustrate the effectiveness of our control scheme. Finally, the paper is concluded in Section 5.

II. PROBLEM STATEMENT

We will consider the following class of uncertain nonlinear systems under the control of m actuators.

$$\begin{aligned} \dot{x}_1 &= x_2 + \varphi_1^T(x_1)\theta \\ \dot{x}_2 &= x_3 + \varphi_2^T(x_1,x_2)\theta \\ &\vdots \\ \dot{x}_n &= \varphi_0(x) + \varphi_n^T(x)\theta + \sum_{i=1}^m b_i\beta_i(x)u_i + \bar{\bar{d}}(t) + \eta(x,t) \\ y &= x_1 \end{aligned} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]$ is system state, $y \in R$ is the output, $u_i \in R(i = 1,2,\ldots,m)$ are inputs, $\varphi_0 \in R$, $\varphi_i \in R^p$ and $\beta_i(x) \in R(i = 1,2,\ldots,m)$ are known and sufficient smooth functions, $\theta \in R^p$ and $b_i \in R(i = 1,2,\ldots,m)$ are unknown constant parameters, $\bar{\bar{d}}(t) \in R$ is bounded disturbance and $\eta(x,t) \in R$ is a unknown nonlinear function representing system modeling errors. There exists a known function $\delta(x)$ such that $|\eta(x,t)| \le \delta(x)$.

We now consider the ith hysteretic actuator which may fail during its operation. It exhibits backlash-like hysteresis behavior denoted as $v_i = B(u_{ci})$ and u_{ci} is the designed control signal and v_i represents the output. As shown in [15] [16] [17], a backlash-like hysteresis model is given as

$$\frac{dv_i}{dt} = \alpha_i\left|\frac{du_{ci}}{dt}\right|(c_i u_{ci} - v_i) + B_{i1}\frac{du_{ci}}{dt} \tag{2}$$

where α_i and c_i and B_{i1} are constants, $c_i > 0$ and $c_i > B_{i1}$. It can be solved as

$$\begin{aligned} v_i &= c_i u_{ci} + \bar{d}_{1i}(u_{ci}) \\ \bar{d}_{1i}(u_{ci}) &= (v_i - c_i u_{ci}(0))e^{-\alpha_i(u_{ci}-u_{ci}(0))sign(\dot{u}_{ci})} \\ &\quad + e^{-\alpha_i u_{ci} sign(\dot{u}_{ci})}\int_{u_{ci0}}^{u_{ci}} (B_{i1} - c_i)e^{\alpha_i \xi sign(\dot{u}_{ci})} d\xi \end{aligned} \tag{3}$$

where $\bar{d}_{1i}$ is bounded as shown in [15].

Similar to [11], failure of the ith actuator at time instant t_{if} is modeled as

$$\begin{aligned} u_i &= \rho_i v_i + u_{ki}, \quad (\forall t \ge t_{if}) \\ \rho_i u_{ki} &= 0 \end{aligned} \tag{4}$$

where $0 \le \rho_i \le 1$, u_{ki} and t_{if} are unknown constants. For different values of ρ_i, the actuator operates under the following situations:

- $\rho_i = 1$, the actuator works normally, namely $u_i = v_i$.
- $0 < \rho_i < 1$,
  It indicates $u_i = \rho_i v_i$. The ith actuator is called partial loss of effectiveness.
- $\rho_i = 0$,
  It indicates $u_i = u_{ki}$. The ith actuator is called total loss of effectiveness.

With (3) and (4), the model of ith hysteretic actuator can be re-written as follows

$$\begin{aligned} u_i &= \rho_i c_i u_{ci} + u_{ki} + \bar{d}_i, \quad (\forall t \ge t_{if}) \\ \rho_i u_{ki} &= 0 \end{aligned} \tag{5}$$

where $\bar{d}_i = \rho_i \bar{d}_{1i}(u_{ci})$ is bounded.

From (1) and (5), the system can be expressed as

$$\begin{aligned} \dot{x}_1 &= x_2 + \varphi_1^T(x_1)\theta \\ \dot{x}_2 &= x_3 + \varphi_2^T(x_1,x_2)\theta \\ &\vdots \\ \dot{x}_n &= \varphi_0(x) + \varphi_n^T(x)\theta + \sum_{i=1}^m b_i\beta_i(x)(\rho_i c_i u_{ci} + u_{ki} + \bar{d}_i) \\ &\quad + \bar{\bar{d}}(t) + \eta(x,t) \\ y &= x_1 \end{aligned} \tag{6}$$

To derive a suitable adaptive control scheme, the following Assumptions are made.

Assumption 1: The number of totally failed actuators is up to $m-1$ and the control objectives can be achieved by the remaining normal actuators. Also any actuator can change only from normal to partial failure or total failure.

Remark 1: In adaptive failure compensation problem, the stability of closed-loop system and desirable system performance are achieved by the remaining actuations. Therefore, as explained in [6] and [11] the above assumption is indispensable. Note that all actuators are allowed to have partial loss of effectiveness simultaneously.

Remark 2: The ith actuator becomes faulty at an uncertain time instant t_{if}. Furthermore, any actuator fails only once. Hence, There exists a finite time instant T_f after which no new failure will occur.

Assumption2: $\beta_i(x) \ne 0$ and b_i is in a known bounded interval which does not include zero, namely $b_{imin} \le b_i \le b_{imax}(i = 1,2,\cdots,m)$ for some known constants b_{imin} and b_{imax}. Without loss of generality, we suppose $0 < b_{imin}$.

Remark 3: The class of model is much more general than that in [20]. Due to the nonlinear functions $\beta_i(x)$, the effects of $\bar{d}_i$ caused by approximating the actuator hysteresis will be $b_i\beta_i(x)\bar{d}_i$ and cannot be assumed bounded as in [20] and [15] - [17]. How to handle such effects is a challenging issue in the design and analysis of adaptive controllers and it becomes even more difficult in the presence of possible actuator failures.

Assumption 3: Reference signal $y_r(t)$ and its i-order $(i = 1,2,\ldots,n-1)$ derivatives are known and bounded.

III. DESIGN OF CONTROLLERS

Our objective is to design adaptive controllers to guarantee the global stability of closed loop system. To obtain suitable control law and update laws for controller parameters based on the backstepping approach, the following change of coordinates are introduced.

$$\begin{aligned} z_1 &= x_1 - y_r \\ z_i &= x_i - \alpha_{i-1} - y_r^{(i-1)}, (i = 2,\ldots,\rho) \end{aligned} \tag{7}$$

where variable z_1 is tracking error, α_{i-1} $(i=2,\ldots,n)$ is a virtual control to be designed at step $i-1$.

A. Control design

In this subsection, we will carry out the design recursively as in [21] and [22]. The first $n-1$ steps are rather standard and thus no details are given. However, the last step is totally different and will be elaborated.

Step 1: From (6) and (7) the derivative of tracking error can be rewritten as

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_r = z_2 + \alpha_1 + \varphi_1^T\theta \tag{8}$$

where α_1 is the virtual control. We define a positive definite Lyapunov function as follows

$$\bar{V}_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{9}$$

where $\tilde{\theta} = \theta - \hat{\theta}$, $\hat{\theta}$ is an estimate of unknown parameters θ and Γ is a positive definite matrix. Virtual control α_1 can be chosen as

$$\alpha_1 = -K_1 z_1 - \varphi_1^T\hat{\theta} \tag{10}$$

where K_1 is positive constant. From (8) (9) (10), the derivative of $\bar{V}_1$ is

$$\dot{\bar{V}}_1 = -K_1 z_1^2 + z_1 z_2 - \tilde{\theta}^T\Gamma^{-1}(\dot{\hat{\theta}} - \tau_1) \tag{11}$$

where τ_1 is a turning function chosen as

$$\tau_1 = \Gamma\varphi_1 z_1 \tag{12}$$

Step $i(i=2,3,\ldots,n-1)$: In this step, the following Lyapunov function $\bar{V}_i$ is considered

$$\bar{V}_i = \bar{V}_{i-1} + \frac{1}{2}z_i^2 \tag{13}$$

We choose the virtual control as

$$\begin{aligned}\alpha_i \;=\;& -K_i z_i - z_{i-1} - (\varphi_i^T - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\varphi_k^T)\hat{\theta}\\ &+\sum_{k=1}^{i-1}(\frac{\partial\alpha_{i-1}}{\partial x_k}x_{k+1} + \frac{\partial\alpha_{i-1}}{\partial y_r^{(k-1)}}y_r^{(k)}) + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\tau_i\\ &+\sum_{k=2}^{i-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma(\varphi_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\varphi_k)z_k \end{aligned}\tag{14}$$

where K_i is a positive design parameter, turning function τ_i is given as

$$\tau_i = \tau_{i-1} + \Gamma(\varphi_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\varphi_k)z_i \tag{15}$$

Step n: From (6) and (7), the derivative of z_ρ can be expressed as follows

$$\begin{aligned}\dot{z}_n \;=\;& \varphi_0 + \varphi_n^T\theta + \sum_{i=1}^{m}b_i\beta_i(x)(\rho_i c_i u_{ci} + u_{ki} + \bar{d}_i) + \bar{\bar{d}}(t)\\ &+\eta(x,t) - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}(x_{k+1} + \varphi_k^T\theta) - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\\ &-\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(k-1)}}y_r^{(k)} - y_r^{(n)}\end{aligned}\tag{16}$$

In the stability analysis given later, we need to consider the derivative of $\frac{1}{2}z_n^2$, a term included in the Lyapunov function. Certain parameters in its bound need to be estimated for designing our control law. Thus we now study $\frac{1}{2}z_n^2$. From (16), we have

$$\begin{aligned}\frac{1}{2}(z_n^2)' \;&=\; z_n(\Xi + \sum_{i=1}^{m}b_i\beta_i(x)\bar{d}_i + \bar{\bar{d}}(t) + \eta(x,t)) \qquad (17)\\ &\leq\; z_n\Xi + |z_n||\sum_{i=1}^{m}b_i\beta_i(x)\bar{d}_i| + z_n\bar{\bar{d}}(t) + z_n\eta(x,t)\end{aligned}$$

where

$$\begin{aligned}\Xi \;=\;& \varphi_0 + \varphi_n^T\theta + \sum_{i=1}^{m}b_i\beta_i(x)(\rho_i c_i u_{ci} + u_{ki}) - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(k-1)}}y_r^{(k)}\\ &-\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}(x_{k+1} + \varphi_k^T\theta) - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} - y_r^{(n)}\end{aligned}\tag{18}$$

Using Young's inequality, we can get

$$\begin{aligned}|\sum_{i=1}^{m}b_i\beta_i(x)\bar{d}_i| \;&\leq\; \frac{1}{2}(\|\,(b_1\beta_1(x),\cdots,b_m\beta_m(x))\,\|^2\\ &\quad +\|(\bar{d}_1,\cdots,\bar{d}_m)\|^2)\\ &\leq\; b\beta(x) + \bar{d}(t)\end{aligned}\tag{19}$$

where

$$b = max\{|b_i|^2,(i=1,2,\cdots,m)\}$$

$$\beta(x) = \frac{1}{2}\|(\beta_1(x),\cdots,\beta_m(x))\|^2$$

$$\bar{d}(t) = \frac{1}{2}\|(\bar{d}_1,\cdots,\bar{d}_m)\|^2$$

Therefore

$$\frac{1}{2}(z_n^2)' \leq z_n\Xi + |z_n|b\beta(x) + z_n\eta(x,t) + z_n d(t) \tag{20}$$

where $d(t) = sign(z_n)\bar{d}(t) + \bar{\bar{d}}(t)$ and it is bounded.

The unknown bound D of $d(t)$ will be estimated. b is unknown and will also be estimated with its estimate denoted as $\hat{b}$. From Assumption 2, we have $b \in [b_{min}, b_{max}]$ where

$$b_{min} = min\{b_{imin}^2\}; b_{max} = max\{b_{imax}^2\}, (i=1,2,\cdots,m)$$

A projection operation can guarantee that $\hat{b}$ Is always in the bounded interval $[b_{min}, b_{max}]$. To compensate for the effects of $|\sum_{i=1}^{m}b_i\beta_i(x)\bar{d}_i|$ and modeling error $\eta(x,t)$ in the design of adaptive controllers we also need a smooth function to bound $-\tilde{b}\beta(x) + sign(z_n)\eta(x,t)$ where $\tilde{b} = \hat{b} - b$. Note that $|\tilde{b}| \leq b_{max} - b_{min}$. Then

$$\begin{aligned}|-\tilde{b}\beta(x) + sign(z_n)\eta(x,t)| \;&\leq\; |-\tilde{b}\beta(x)| + |sign(z_n)\eta(x,t)|\\ &\leq\; (b_{max} - b_{min})\beta(x) + \delta(x)\end{aligned}\tag{21}$$

Such a bounding function, denoted as h_n, can thus be chosen as

$$h_n(x) = \sqrt{((b_{max} - b_{min})\beta(x) + \delta(x))^2 + \varepsilon} \tag{22}$$

where ε is any positive constant. Clearly h_n is differentiable.

We now return to our design problem. Different from the standard backstepping approach, a virtual control α and turning function τ_n at this step are designed as follows

$$\begin{aligned}\alpha &= -K_n z_n - z_{n-1} - \varphi_0 + y_r^{(n)} - (\varphi_n^T - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} \varphi_k^T)\hat{\theta} \\ &\quad + \sum_{k=1}^{n-1} (\frac{\partial \alpha_{n-1}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{n-1}}{\partial y_r^{(k-1)}} y_r^{(k)}) + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \tau_n \\ &\quad - sign(z_n)\hat{D} + \sum_{k=2}^{n-1} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma(\varphi_n - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} \varphi_k) \\ &\quad - sign(z_n)\hat{b}\beta(x) - sign(z_n) h_n(x) \end{aligned} \tag{23}$$

$$\tau_n = \tau_{n-1} + \Gamma(\varphi_n - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} \varphi_k) z_n \tag{24}$$

where $\hat{\theta}$ is the estimate of unknown θ.

The virtual control in (23) together with the control law and parameter update laws below is obtained based on the control Lyapunov function approach which will become clear in the stability analysis of the next subsection.

Similar to [20], if knowing the system parameters and failures, we could choose the control law as

$$u_{ci} = \frac{1}{\beta_i(x)} \kappa^T \omega \tag{25}$$

where κ is a desired parametric vector and ω is a known vector to be specified in the stability analysis. Both are $m+1$ dimensional vectors denoted as

$$\kappa = (\kappa_1, \kappa_{21}, \ldots, \kappa_{2m})^T, \omega = (\omega_1, \omega_{21}, \ldots, \omega_{2m})^T \tag{26}$$

However, owing to the unknown parameters and failures, κ is unknown and thus needs to be estimated. With κ replaced by its estimate $\hat{\kappa}$, the control law is obtained as

Control Law:

$$u_{ci} = \frac{1}{\beta_i(x)} \hat{\kappa}^T \omega \tag{27}$$

Based on the consideration of control Lyapunov function approach, particularly from (33) in the next subsection, the parameter update laws are also obtained

Update Laws:

$$\begin{aligned} &\dot{\hat{D}} = \eta_d |z_n| \\ &\dot{\hat{\kappa}} = -\Gamma_\kappa \omega z_n \\ &\dot{\hat{\theta}} = \tau_n \\ &\dot{\hat{b}} = proj(\Gamma_b |z_n| \beta(x)) \end{aligned} \tag{28}$$

where Γ_b, η_d are positive constants, Γ_κ is a positive define matrix and $proj(\cdot)$ denotes a projection operator.

B. Stability analysis

We now analyze the stability of closed loop system with control law and update laws in (27) and (28). Suppose that $p_j (0 \le p_j \le m)$ actuators are faulty and no new normal actuator fails in time interval $(T_j, T_{j+1}), (j = 0, 1, \ldots, f)$. Let the set Q_{jT} denote the actuators of total failure in interval $(T_j, T_{j+1}), (j = 0, 1, \ldots, f)$ and use the set $\bar{Q}_{jT}$ to represent other actuators but total failure. It is clear $Q_{jT} \cup \bar{Q}_{jT} = \{1, 2, \ldots, m\}$.

Remark 4: Let $T_0 = 0, T_{f+1} = \infty$ and $p_0 = 0$. So we can get all actuators work normally in time interval $[T_0, T_1)$ and no new failure will occur after time instant T_f. Furthermore, the set $\bar{Q}_{0T} = \{1, 2, \cdots, m\}$ and the set $\bar{Q}_{fT}$ are not empty according to Assumption 1.

Now consider the following Lyapunov function in time interval (T_j, T_{j+1})

$$V_j = \bar{V}_{n-1} + \frac{1}{2} z_n^2 + \sum_{i \in \bar{Q}_{jT}} \frac{\rho_i c_i b_i}{2} \tilde{\kappa}^T \Gamma_\kappa^{-1} \tilde{\kappa} + \frac{1}{2\eta_d} \tilde{D}^2 \tag{29}$$

Using (20) gives

$$\begin{aligned} \dot{V}_j &\le \dot{\bar{V}}_{n-1} + z_n \Xi + |z_n| b \beta(x) + z_n \eta(x,t) + z_n d(t) \\ &\quad - \sum_{i \in \bar{Q}_{jT}} \rho_i c_i |b_i| \tilde{\kappa}^T \Gamma_\kappa \dot{\hat{\kappa}} - \frac{1}{\eta_d} \tilde{D} \dot{\hat{D}} \end{aligned} \tag{30}$$

Vector κ and ω in (26) should be such that

$$\sum_{i=1}^{m} c_i b_i \kappa^T \omega = \alpha \tag{31}$$

This gives

$$\begin{aligned} &\kappa_1 = \frac{1}{\sum_{i \in \bar{Q}_{jT}} \rho_i c_i b_i} \\ &\kappa_{2i} = 0, (i \in \bar{Q}_{jT}) \\ &\kappa_{2i} = -\frac{b_i u_{ki}}{\sum_{i \in \bar{Q}_{jT}} \rho_i c_i b_i} (i \in Q_{jT}) \\ &\omega_1 = \alpha \\ &\omega_{2i} = \beta_i(x), (i = 1, 2, \ldots, m) \end{aligned} \tag{32}$$

With (23), (24), (30) and (31),we have

$$\begin{aligned} \dot{V}_j &\le -\sum_{k=1}^{n} K_k z_k^2 - \tilde{\theta}^T \Gamma^{-1} (\dot{\hat{\theta}} - \tau_n) - \sum_{k=2}^{n-1} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} (\dot{\hat{\theta}} - \tau_n) \\ &\quad - z_n \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} (\dot{\hat{\theta}} - \tau_n) - \sum_{i \in \bar{Q}_{jT}} c_i b_i \tilde{\kappa}^T \Gamma_\kappa^{-1} (\dot{\hat{\kappa}} + \Gamma_\kappa \omega z_n) \\ &\quad + |z_n| [| - \tilde{b} \beta(x) + sign(z_n) \eta(x,t)| - h_n] \\ &\quad - \frac{1}{\eta_d} \tilde{D} (\dot{\hat{D}} - \eta_d |z_n|) \end{aligned} \tag{33}$$

With the update laws in (28), we obtain

$$\begin{aligned} \dot{V}_j &\le -\sum_{k=1}^{n} K_k z_k^2 - \tilde{\theta}^T \Gamma^{-1} (\dot{\hat{\theta}} - \tau_n) - \sum_{k=2}^{n} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} (\dot{\hat{\theta}} - \tau_n) \\ &\quad - \sum_{i \in \bar{Q}_{jT}} c_i |b_i| \tilde{\kappa}^T \Gamma_\kappa^{-1} (\dot{\hat{\kappa}} + \Gamma_\kappa \omega z_n) - \frac{1}{\eta_d} \tilde{D} (\dot{\hat{D}} - \eta_d |z_n|) \\ &\le -\sum_{k=1}^{n} c_k z_k^2 \end{aligned} \tag{34}$$

It is clear that V_j is non increasing. So we have $V_j(T_{j+1}^-) \le V_j(T_j^+)$. Let $j = 0$, then we can get $V_0(T_1^-) \le V_0(0)$. It can be concluded that all signals z_i, $\tilde{\theta}$, $\tilde{\kappa}, \hat{D}$ are bounded in the time interval $[0, T_1)$. Note that the difference between $V_1(T_1^+)$ and $V_0(T_1^-)$ is only the coefficients in front of the term $\kappa^T \Gamma_\kappa \kappa$. Since all the possible jumping on κ are bounded, $V_1(T_1^+)$

is bounded, then $V_1(T_2^-)$ is bounded. Similar to the above analysis, we can get $V_j(T_{j+1}^-)$ is bounded from the bound of $V_j(T_j^+)$. According to Assumption 1, there is a finite time instant T_f such that in the time interval (T_f, ∞), the operation of all actuators remain unchanged, namely no new failures occur. Also in time interval (T_f, ∞), it can be shown that $V_f(t) \le V_f(T_f^+)$. Then we have $z, \tilde{\theta}, \tilde{\kappa}, \tilde{D}$ bounded in $[0, \infty]$. From this, we have the following result.

Theorem 1: Consider the closed loop system consisting of nonlinear plant (1), m hysteretic actuators modeled in (2) with possibly unknown failures described by (4) and adaptive controllers with the control law in (27) and the update laws in (28). Under Assumptions 1 to 3, all signals are bounded. In addition, asymptotic tracking is achieved, i.e. $lim_{t\to\infty}(y - y_r) = 0$.

Proof: As analyzed above, signals z_i, $\tilde{\theta}$, $\tilde{D}$ and $\tilde{\kappa}$ are bounded. Then all the virtual control $\alpha_i, i = 1, 2, \cdots, n-1$, α and states $x_i, i = 1, 2, \cdots, n$ are bounded. From the control law (27), u_{ci} is ensured bounded. In addition, by applying the Lasalle-Yoshizzawa Theorem, it follows that $lim_{t\to\infty}(y - y_r) = 0$.

IV. SIMULATION STUDIES

To illustrate the effectiveness of the proposed scheme, we use the aforementioned methodology on a simple system described as follows

$$\dot{x} = \varphi(x)^T\theta + b_1\beta_1(x)u_1(t) + b_2\beta_2(x)u_2(t) + \eta(x,t) \quad (35)$$

where $u_1(t)$ and $u_2(t)$ are the outputs of the two hysteretic actuators. The known function $\varphi(x) = \frac{1-e^{-x}}{1+e^{-x}}$, $\beta_1(x) = 1.9 + 0.1sin(x)$, $\beta_2(x) = 2$. $\theta, b_1, b_2 \in R$ are unknown constants. $\eta(x,t)$ is unknown nonlinear functions represents system modeling errors.

In simulation, The actual parameter values are $\theta = 2$ and $b_1 = b_2 = 1$. The uncertain nonlinear function $\eta(x,t)$ is $0.1sin(t)$. The reference signal is $sin(t)$.

The backlash-like hysteresis is described by (2). The parameters in the model (2) are $\alpha_2 = \alpha_1 = 1$, $c_2 = c_1 = 3.1635$ and $B_{21} = B_{11} = 0.345$.

We choose $b_{1max} = b_{2max} = 1.5, b_{1min} = b_{2min} = 0.5$. Then with (21) we choose $h_n(x) = 8 + 0.38sin(x)$ and the designed parameters $\Gamma_\theta = 0.1$, $\Gamma_\kappa = 1, \Gamma_b = 0.1$, $\eta_d = 0.1$.The initial value are chosen as follows: $z(0) = 0.5$, $u_1(0) = u_2(0) = 0$, $\hat{\theta}(0) = 0$, $\hat{\kappa}(0) = 0$, $\hat{D}(0) = 0, \hat{b}(0) = 0.6$. When all actuators work normally, Fig.1 shows the tracking error, Fig.2 and Fig. 3 present the inputs $u_{c1}(t)$ and $u_{c2}(t)$, and outputs $u_1(t)$ and $u_2(t)$ of the actuators, respectively. Suppose that at $t = 2$ second actuator u_2 is stuck at an unknown value 4.5. Fig.4 gives the tracking error under the above failure. In this case, the inputs $u_{c1}(t)$ and $u_{c2}(t)$, and outputs $u_1(t)$ and $u_2(t)$ of the actuators are shown in Fig.5 and Fig.6, respectively. Clearly the proposed scheme has been verified effective by these simulation results.

V. CONCLUSIONS

A robust adaptive control scheme is proposed by using backstepping techniques to compensate for uncertain failures

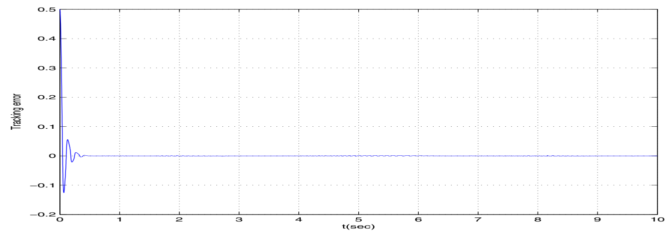

Fig. 1. Tracking error(no failure)

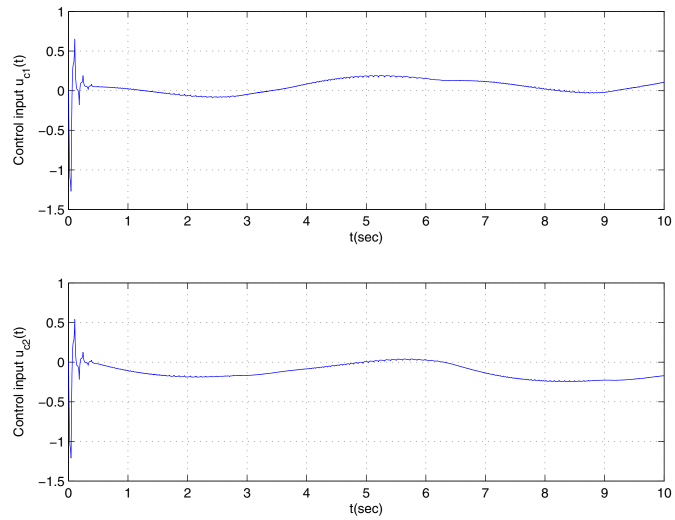

Fig. 2. Backlash inputs(no failure)

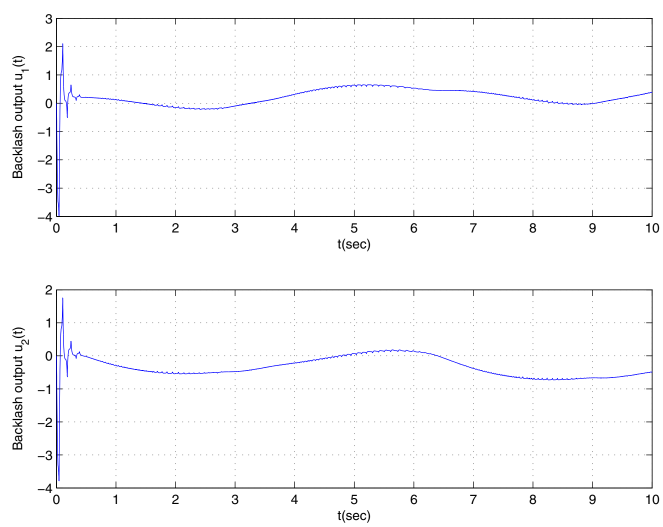

Fig. 3. Backlash outputs(no failure)

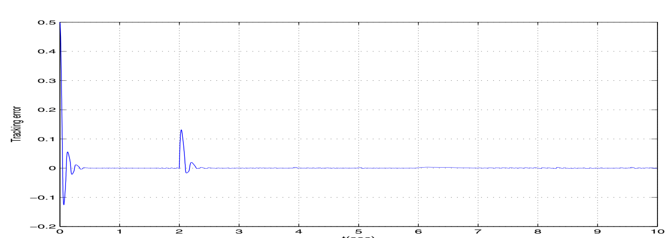

Fig. 4. Tracking error(failure)

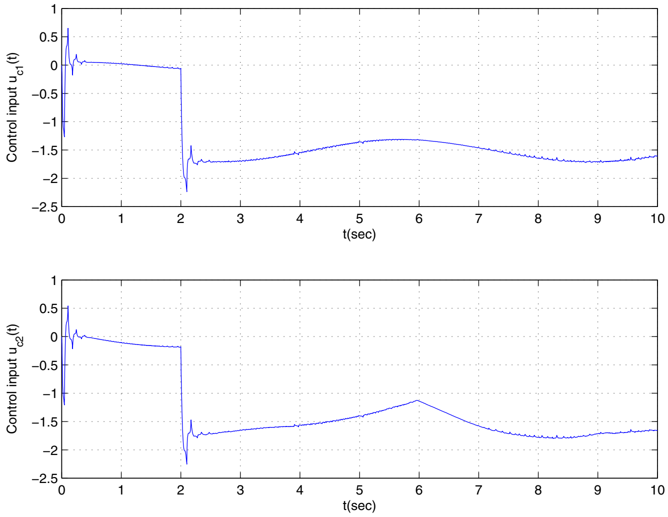

Fig. 5. Backlash inputs(failure)

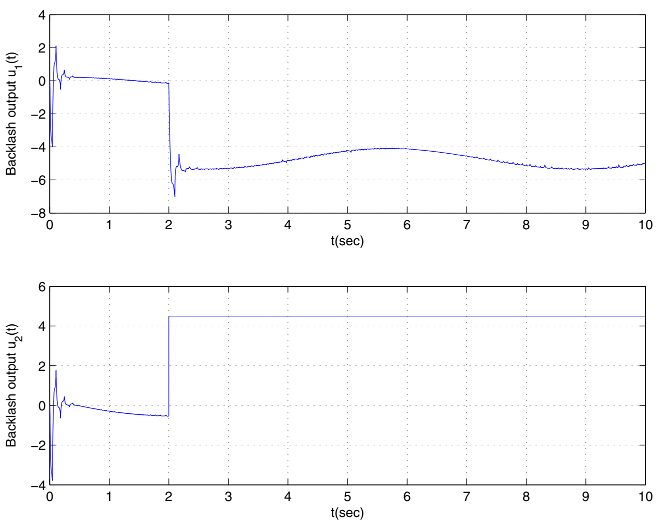

Fig. 6. Backlash outputs(failure)

of hysteretic actuators for a class of nonlinear systems with uncertainties including unknown parameters, unknown external disturbance and unknown system modeling errors. The stability of closed-loop system and output tracking performance can be ensured by the adaptive controllers. Simulation results also verify the effectiveness of the proposed control scheme.

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