

Indirect continuous-time system identification—A subspace downsampling approach

P. Lopes dos Santos*, T-P Azevedo-Perdicoulis†, J. A. Ramos**,
G. Jank‡, J. L. Martins de Carvalho***

Abstract—This article presents a new indirect identification method for continuous-time systems able to resolve the problem of fast sampling. To do this, a Subspace Identification Down-Sampling (SIDDS) approach that takes into consideration the intermediate sampling instants of the input signal is proposed. This is done by partitioning the data set into m subsets, where m is the downsampling factor. Then, the discrete-time model is identified using a based subspace identification discrete-time algorithm where the data subsets are fused into a single one. Using the algebraic properties of the system, some of the parameters of the continuous-time model are directly estimated. A procedure that secures a prescribed number of zeros for the continuous-time model is used during the estimation process. The algorithm's performance is illustrated through an example of fast sampling, where its performance is compared with the direct methods implemented in Cotsid.

I. INTRODUCTION

Two different approaches to CT identification exist. A direct one, where the parameters of the CT model are directly identified from the discrete-time (DT) input-output (IO) data. Obviously, this type of methods always relies on some numerical skills. Its main difficulty comes from the fact that the successive derivatives are seldom available in practice, particularly in a noisy framework. Therefore, it is important to find other techniques able to evaluate the successive derivatives from measured data. Different methods to bypass this problem include: linear filters, modulating functions and integral methods [3]. Direct identification in CT is not as well developed for multiple-input multiple-output (MIMO) as for single-input single-output (SISO) systems [12]. Additional difficulties are the existence of many tuning parameters and the appearance of zeros in the integration methods. Subspace methods (SM) also have problems with low signal to noise ratio (SNR) levels.

In an indirect approach, the CT model is converted into a DT model and then the parameters are estimated with one of the several available DT identification methods [10]. Their advantage is to avoid computing the derivatives. Once

the parameters are obtained, the DT model needs to be converted back into CT, which may bring difficulties. To be able to recover the CT model, the sampling period needs to be chosen neither too big nor small [14]. Whenever the system to be identified possesses fast dynamics, a large sampling period may lead to a loss of information. Also the use of the matrix logarithm by some methods may produce complex arithmetics when the DT model matrices have negative eigenvalues [2], [12]. On the other hand, very fast sampling may raise numerical problems [16], coming from the clustering of the poles in a very small area of the z -plane, around the point $1 + 0j$. Also, may conduct to a loss of controllability/observability when the CT transfer function (TF) also has got zeros, since these zeros also tend to cluster at this small area [1]. Moreover, system parameters of DT models are functions of the sampling period [1], while those of CT models are always constant. Since a small modelling error may bring a larger error in the estimation of the parameters, differences in the error patterns may appear. This problem becomes more pressing when trying to identify parameters close to one, since even a small modelling error may make the estimated parameter completely unacceptable, and the DT model may become poor or even unstable as the sampling period decreases [16]. The numerical sensitivity of the DT model increases when the sampling period is small compared to the time constant of the system. For instance two poles located in the left-half s -plane may converge to one single pole (i.e., 1) in the z -plane. In this situation, the truncation error should be treated carefully.

Real systems many times require fast sampling in order to be replicated adequately. To avoid the problems arising from using a very small sampling interval, we propose an indirect CT-4sid alike method where the system can be sampled as fast as it needs and, using downsampling, the sampling cycles may be tuned to a convenient value. The parameters of a lower rate system are then estimated using a DT subspace identification (SID) approach [18], [15], [21], [9]. Our approach is distinct from decimation [6]. Moreover, some of the parameters of the CT model can be estimated directly and it has a mechanism able to control the number of zeros in the DT model, whenever this is known *a priori*.

In what follows, we derive the DT downsampled model in Section II. In Section III, the SID algorithm for the downsampled model, SIDDS, is described. In Section IV, we discuss the direct estimation of some parameters of the CT model. Also a procedure to limit the number of zeros of the CT state-space (SS) model is explained. In Section V,

This work is supported by FCT.

* Faculdade de Engenharia da Universidade do Porto, Portugal, Rua do Dr Roberto Frias s/n 4200-465 Porto, PORTUGAL pjsantos@fe.up.pt

†ISR—Coimbra & Departamento de Matemática, UTAD, 5001-801 Vila Real, Portugal, tazevedo@utad.pt

**Farquhar College of Arts and Sciences, Division of Mathematics, Science, and Technology, Nova Southeastern University, 3301 College Avenue, Fort Lauderdale, FL 33314, USA, jr1284@nova.edu

‡RWTH-University of Technology, Department of Mathematics, 52056 Aachen, Germany, jank@math2.rwth-aachen.de

***Faculdade de Engenharia da Universidade do Porto, Portugal, jmartins@fe.up.pt

the performance of SIDDS is assessed through a case study consisting of a fast sampled system very popular in the literature. Our approach is compared with the subspace CT identification methods implemented in Conssid, a toolbox for linear time-invariant SISO and MIMO systems from sampled IO data. This toolbox is a collection of the main approaches found in the literature, with principal emphasis on the estimation of parametric models [2], [5]. Monte Carlo simulations (MCS) are used in order to compare SIDDS with the direct methods, in the presence of noise.

II. DOWNSAMPLED DT SS MODEL

Consider the following state-space system

$$\dot{x}(t) = Ax(t) + Bu(t), \quad (1)$$

$$y(t) = Cx(t) + Du(t), \quad (2)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^{n_u}$, $y \in \mathbb{R}^{n_y}$ and $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times n_u}$, $C \in \mathbb{R}^{n_y \times n}$, $D \in \mathbb{R}^{n_y \times n_u}$. Thus, the system time response is given by

$$x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^t e^{A(t-\tau)}Bu(\tau)d\tau, \quad (3)$$

$$y(t) = Cx(t) + Du(t), \quad (4)$$

where t_0 is the initial instant.

Assume the input signal to be generated by a zero-order hold (ZOH) Digital Analog converter with sampling period T_s , and we can write

$$x[(k+1)T_s] = e^{AT_s}x(kT_s) + \int_0^{T_s} e^{A\tau}d\tau Bu(kT_s). \quad (5)$$

In order to drop the sampling period in the notation, define

$$u_1(k) = u(kT_s), \quad x_1(k) = x(kT_s), \quad y_1(k) = y(kT_s)$$

$$A_1 = e^{AT_s}, \quad B_1 = \int_0^{T_s} e^{A\tau}d\tau B, \quad (6)$$

$$C_1 = C, \quad D_1 = D.$$

At every sampling instant, the system is described by the DT SS model

$$x_1(k+1) = A_1x_1(k) + B_1u_1(k), \quad (7)$$

$$y_1(k) = C_1x_1(k) + D_1u_1(k). \quad (8)$$

The CT SS model (1)–(2) can be recovered from the DT model using (6). According to Sinha [14], the sampling period T_s should be such that the spectral radius of A is less than $\frac{1}{T_s}$. On the other hand, we also know that DT system identification algorithms have a poor performance when the sampling period is too small [14]. In order to overcome this difficulty, we start by modelling a low rate sampling DT model from high rate sampling data. Thus, consider the same problem as before but with a larger sampling time, i.e., mT_s , $m = 1, 2, \dots$. Thence, the discrete counterpart of (5):

$$x[(k+1)mT_s] = e^{AmT_s}x(kmT_s) + \sum_{i=0}^{m-1} \beta_i B u[(km+i)T_s], \quad (9)$$

$$\text{with } \beta_i = e^{A(m-i-1)T_s} \int_0^{T_s} e^{A\tau} d\tau.$$

Observe that we have m shifted sequences of the same type:

$$x[(k+1)m + \ell]T_s = e^{AmT_s}x[(km + \ell)T_s] + \sum_{i=0}^{m-1} \beta_i B u[(km + \ell + i)T_s], \quad (10)$$

with $\ell = 0, \dots, m-1$. For a more compact notation define

$$x[(km + \ell)T_s] := x_m(k, \ell), \quad (11)$$

$$u_m(k, \ell) := \begin{pmatrix} u((km + \ell)T_s) \\ u((km + 1 + \ell)T_s) \\ \dots \\ u(((k+1)m - 1 + \ell)T_s) \end{pmatrix} \quad (12)$$

$$y[(km + \ell)T_s] := y_m(k, \ell) \quad (13)$$

and

$$A_m := e^{AmT_s}, \quad (14)$$

$$B_{m_i} := \beta_i B, \quad (15)$$

$$B_m := (B_{m_0} \ \dots \ B_{m_{m-1}}), \quad (16)$$

$$C_m := C, \quad (17)$$

$$D_m = \begin{cases} D, & m = 1 \\ (D \ 0 \ \dots \ 0), & m > 1. \end{cases} \quad (18)$$

Hence, we end up with the following state-space model:

$$x_m(k+1, \ell) = A_m x_m(k, \ell) + B_m u_m(k, \ell), \quad (19)$$

$$y_m(k, \ell) = C_m x_m(k, \ell) + D_m u_m(k, \ell). \quad (20)$$

III. SID OF THE DOWNSAMPLED MODEL (SIDDS)

Solving (19)–(20) for $k = 0, 1, \dots, i-1$, we have

$$\mathbf{y}_i(0, \ell) = \Gamma_{mi} x_m(0, \ell) + \mathcal{H}_i \mathbf{u}_i(0, \ell), \quad (21)$$

where

$$\mathbf{y}_i(0, \ell) = (y_m^T(0, \ell) \ y_m^T(1, \ell) \ \dots \ y_m^T(i-1, \ell))^T,$$

$$\mathbf{u}_i(0, \ell) = (u_m^T(0, \ell) \ u_m^T(1, \ell) \ \dots \ u_m^T(i-1, \ell))^T,$$

$$\Gamma_{mi} := (C_m \ C_m A_m \ C_m A_m^2 \ \dots \ C_m A_m^{i-1})^T$$

is the extended observ. matrix and \mathcal{H}_i is the Töplitz matrix

$$\mathcal{H}_i := \begin{pmatrix} D_m & 0 & \dots & 0 & 0 \\ C_m B_m & D_m & \dots & 0 & 0 \\ C_m A_m B_m & C_m B_m & \dots & 0 & 0 \\ \vdots & & & & \\ C_m A_m^{i-2} B_m & \dots & \dots & C_m B_m & D_m \end{pmatrix}.$$

If we replace $\mathbf{y}_i(0, \ell)$, $\mathbf{u}_i(0, \ell)$ and $x_m(0, \ell)$ by $Y_p = Y_{0|i-1}$, $U_p = U_{0|i-1}$ and $X_p = X_0$, respectively, defined by

$$Y_{0|i-1} = (y_{i \times m}(0) \ y_{i \times m}(1) \ \dots \ y_{i \times m}(j-1)),$$

$$U_{0|i-1} = (u_{i \times m}(0) \ u_{i \times m}(1) \ \dots \ u_{i \times m}(j-1)),$$

$$X_0 = (x_m(0) \ x_m(1) \ \dots \ x_m(j-1)), \quad (22)$$

with

$$\begin{aligned} \mathbf{y}_{i \times m}(k) &= \left(\mathbf{y}_i(k, 0) \quad \mathbf{y}_i(k, 1) \quad \cdots \quad \mathbf{y}_i(k, m-1) \right), \\ \mathbf{u}_{i \times m}(k) &= \left(\mathbf{u}_i(k, 0) \quad \mathbf{u}_i(k, 1) \quad \cdots \quad \mathbf{u}_i(k, m-1) \right), \\ \mathbf{x}_m(k) &= \left(x_m(k, 0) \quad x_m(k, 1) \quad \cdots \quad x_m(k, m-1) \right). \end{aligned}$$

equation (21) also holds. In a similar manner, (21) also holds for $Y_f = Y_{i|2i-1}$, $U_f = U_{i|2i-1}$ and $X_f = X_i$, i.e., $Y_f = \Gamma_i X_f + \mathcal{H}_i U_f$. On the other hand, we can see that $X_f = A_m^i X_p + \mathcal{C}_i^R U_p$, where \mathcal{C}_i^R is the reversed accessibility matrix $\mathcal{C}_i^R = \begin{pmatrix} A_m^{i-1} B_m & \cdots & A_m B_m & B_m \end{pmatrix}$. From these equations, we can estimate the DT SS model parameters, A_m, B_m, C_m, D_m , using a SID algorithm [9], [18], [21].

IV. CT MODEL PARAMETERS ESTIMATION

The SID algorithm estimates the parameters of the DT model (19)–(20). Equations (14)–(18) relate the DT model parameters to the CT ones. Hence $A = \frac{1}{mT_s} \log(A_m)$ and $C = C_m$, where $\log(A_m)$ is the principal matrix logarithm of A_m . Since B and D are linear functions of B_m and D_m , the subspace algorithm can be changed to compute directly these CT model matrices. Moreover, a model with a prescribed number of zeros can be obtained if linear restrictions are imposed on the estimates of B and D .

In this section, it is shown how to estimate directly the CT model parameters B and D . Also, conditions to impose a certain number of zeros on the model are derived.

A. Estimation of B and D

In SM, B_m and D_m are the solution of a least squares (LS) problem $Z = M \begin{pmatrix} D_m \\ B_m \end{pmatrix}$, where $Z \in \mathbb{R}^{\times mn_u}$ and $M \in \mathbb{R}^{\times (n_y+n)}$. From (15)–(16) and (18), we have: $Z = \begin{pmatrix} M_1 & M_2 \end{pmatrix} \begin{pmatrix} D & 0 & \cdots & 0 \\ \beta_0 B & \beta_1 B & \cdots & \beta_{m-1} B \end{pmatrix}$, with $M_1 \in \mathbb{R}^{\times n_y}$, $M_2 \in \mathbb{R}^{\times n}$.

If Z is partitioned in a convenient way: $Z = \begin{pmatrix} Z_0 & Z_1 & \cdots & Z_{m-1} \end{pmatrix}$, with

$$\begin{aligned} Z_0 &= M_1 D + M_2 \beta_0 B \\ Z_\ell &= M_2 \beta_\ell B, \quad \ell = 1, \dots, m-1. \end{aligned}$$

Stacking together the columns of the matrices, hence

$$\mathcal{Z} = \mathcal{M} \begin{pmatrix} D \\ B \end{pmatrix}, \quad (23)$$

where

$$\mathcal{Z} = \begin{pmatrix} Z_0 \\ Z_1 \\ \vdots \\ Z_{m-1} \end{pmatrix} \quad \text{and} \quad \mathcal{M} = \begin{pmatrix} M_1 & M_2 \beta_0 \\ 0 & M_2 \beta_1 \\ \vdots & \vdots \\ 0 & M_2 \beta_{m-1} \end{pmatrix}.$$

Then, B and D can then be found by a LS estimator [23].

B. Restriction of the zeros of the CT model

Consider (1)–(2) as a SISO system ($n_y = n_u = 1$). Differentiate (2) k times and define $y_k = \frac{d^k y}{dt^k}$ and $u_k = \frac{d^k u}{dt^k}$. Then $y_k = CA^k x + Du_k + CBu_{k-1} + \cdots + CA^{k-1}Bu$. Assume

$$CA^{k-2}B = CA^{k-3}B = \cdots = CB = D = 0, \quad (24)$$

and thence $y_k = CA^k x + \underbrace{CA^{k-1}B}_{:=D_k} u$

$$\Rightarrow G_k(s) = \frac{Y_k(s)}{U(s)} = \frac{s^k Y(s)}{U(s)} = s^k G(s),$$

where $G(s)$ is the TF of (1)–(2). If $G(s)$ has n_z zeros, then they are also zeros of $G_k(s)$ and G_k has k additional zeros at the origin. Therefore, $G_k(s)$ has $n_{z_k} = n_z + k$ zeros. If $D_k = 0$ then $n_{z_k} \leq n - 1 \Rightarrow n_z \leq n - k - 1$. To restrict the number of zeros to z_M , we need to impose $z_M = n - k - 1 \Leftrightarrow k = n - z_M - 1$. Define

$$\Gamma_{n-z_M-1} := \begin{pmatrix} C & CA & \cdots & CA^{n-z_M-2} \end{pmatrix}^T$$

and consequently

$$n_z \leq n_{z_{max}} \Leftrightarrow \begin{pmatrix} 1 & 0 \\ 0 & \Gamma_{n-z_M-1} \end{pmatrix} \begin{pmatrix} B \\ D \end{pmatrix} = 0. \quad (25)$$

To simplify notation, define $\Psi := \begin{pmatrix} 1 & 0 \\ 0 & \Gamma_{n-z_M-1} \end{pmatrix}$ and $\theta := \begin{pmatrix} D \\ B \end{pmatrix}$. Then (25) is equivalent to $\theta \in \text{Im}(\Psi^T)^\perp \equiv \text{Ker}(\Psi)$. Next, define the orthogonal projection operator P on $\text{ker}(\Psi)$ $P := I - \Psi^T (\Psi \Psi^T)^{-1} \Psi$, and then $\theta \in \{P\phi, \forall \phi \in \mathbb{R}^n\}$, i.e., $\theta = \phi \setminus_{(\text{Ker}(\Psi))^T}$, where $V \setminus_W$ means the projection of the column space of V into the column space of W . Thence (23) can be written as $\mathcal{Z} = \mathcal{M}P\phi$. Given that $\mathcal{M}P$ is rank deficient, there is an infinite number of LS solutions for ϕ [8]:

$$\phi = (\mathcal{M}P)^\dagger \mathcal{Z} + \left(I - (\mathcal{M}P)^\dagger \mathcal{M}P \right) \xi. \quad (26)$$

Notice that $\vartheta = \left(I - (\mathcal{M}P)^\dagger \mathcal{M}P \right) \xi$ is the projection of ξ into the orthogonal complement of the column-space of $\mathcal{M}P$. Since $\mathcal{M}P$ is in the $\text{Im}(\Psi^T)^\perp$, then $\vartheta \in \text{Im}(\Psi^T)$ and consequently $\hat{\theta} = (\mathcal{M}P)^\dagger \mathcal{Z}$, i.e., $\hat{\theta}$ is the constrained LS estimator of θ .

When (1)–(2) is a MIMO ($n_u > 1$) and θ becomes:

$$\theta = (\theta_1 \cdots \theta_{n_u}) := \begin{pmatrix} D_1 & \cdots & D_{n_u} \\ B_1 & \cdots & B_{n_u} \end{pmatrix} \in \mathbb{R}^{(n_y+n) \times n_u}$$

Also, $G_{i,j}(s) = \frac{Y_j(s)}{U_i(s)}$ is the TF between input $u_i(t)$, $i = 1, \dots, n_u$, and output $y_j(t)$, $j = 1, \dots, n_y$. Likewise, the restrictions on $z_{M_{i,j}}$, the maximum number of zeros of $G_{i,j}$, can be represented by

$$\Psi_i \theta_i = 0, \text{ where } \Psi_i := \begin{pmatrix} I & 0 \\ 0 & \Gamma_{n-z_{M_{i,1}}-1}(1) \\ 0 & \Gamma_{n-z_{M_{i,2}}-1}(2) \\ \vdots & \vdots \\ 0 & \Gamma_{n-z_{M_{i,n_y}}-1}(n_y) \end{pmatrix},$$

$\Gamma_k(j) := (C(j,:) \ C(j,:)A \ \dots \ C(j,:)A^{k-1})^T$, where $C(j,:)$ is the j^{th} row of C . Once having defined $P_i := I - \Psi_i^T (\Psi_i \Psi_i^T)^{-1} \Psi_i$, the constrained LS estimator for each θ_i can be written as $\hat{\theta}_i = (\mathcal{M}P_i)^\dagger \mathcal{Z}$.

V. CASE STUDY

We analyse a fourth order non minimal phase system with high rate sampled data, used in the program *idcdemo* of ConSID [2], [5] and appearing often in the literature, e.g. [11], [13], [17]:

$$G(s) = -\frac{6400 \left(s - \frac{1}{4} \right)}{(s^2 + 4s + 400)(s^2 + s + 4)}. \quad (27)$$

The system has two resonance frequencies, 20 and 2 rad/sec, with the respective damping values of 0.01 and 0.25. As the magnitude of its modes is rather different, this is a stiff system. Since the poles and zeros are well apart, it should be feasible to identify the system.

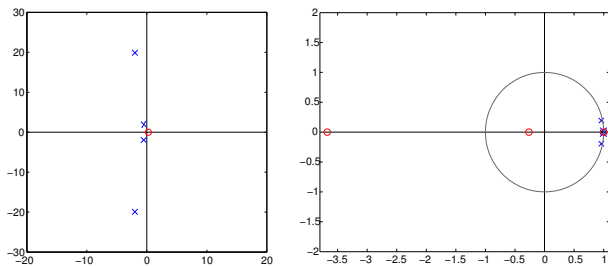


Fig. 1. Left: Pole zero map of $G(s)$. Right: Pole zero map of $G_d(s)$.

A TF of the discretised system for ZOH inter-sample behaviour and a sampling period of $T_s = 0.01$ sec is

$$G_d = -\frac{0.001051(z - 1.003)}{\left(z^2 - \frac{2487}{1250}z + \frac{98962873}{100000000} \right)} \times \left(\frac{\text{sampling zero}}{z + 3.675} \right) \left(\frac{\text{sampling zero}}{z + 0.2621} \right) \times \frac{1}{\left(z^2 - \frac{9609}{5000}z + \frac{3843549}{4000000} \right)}. \quad (28)$$

One may observe that the discrete-poles $p_{d1,1} = 0.9948 \pm j0.0113$ and the discrete zero $z_{d1} = 1.003$ cluster around the point $1 + j0$ and the zero almost cancels with one of the poles. This is due to a very small sampling period and translates into numerical difficulties for the DT identification algorithm. Also, two sampling zeros appear [1].

A. Comparative simulation results

The CT system (27) was simulated using $T_s = 0.01$ sec, a ZOH and a pseudo random binary sequence (PRBS) with 10 stages and a switching time $T_{sw} = 7T_s = 0.07$ sec as input.

1) *Influence of the sampling period:* The singular values (SV) pattern of an extended observability matrix estimated by a subspace algorithm is shown in Figure 2. For $m = 1$, as the fourth SV is very small, a small amount of noise to the output might be sufficient to hide it. For $m = 6$, there are 4 SV clearly different from zero, denoting significant degree of robustness to noise.

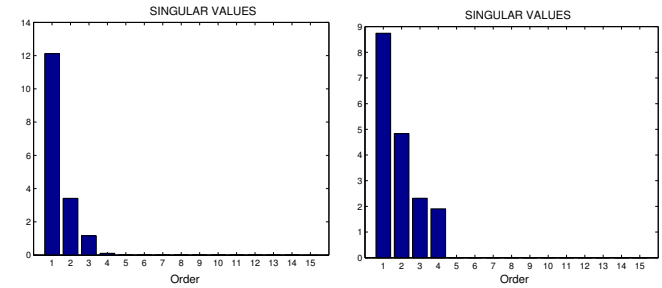


Fig. 2. Left: SV of SIDS(1). Right: SV plot of SIDS(6).

When a band limited white noise is added to the output (SNR= 20 dB/ 0 dB), extra SV appeared. For $m = 1$, the noise SV can be easily confused with the fourth process SV:

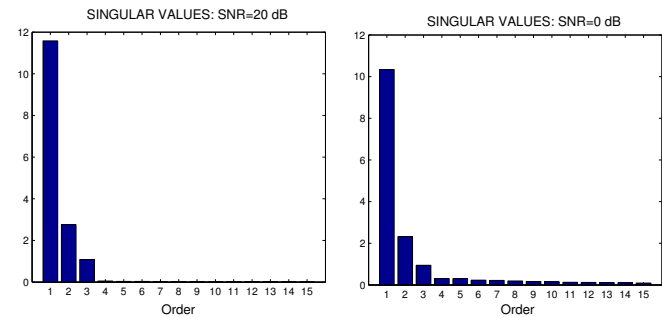


Fig. 3. SV of SIDS(1).

For $m = 6$, there is a clear distinction between the process and noise SV.

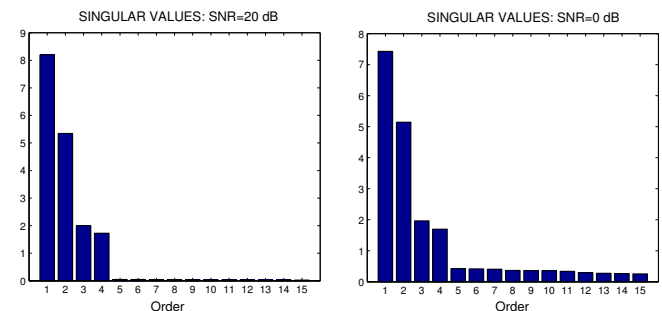


Fig. 4. SV of SIDS(6).

2) *Comparison with Direct methods of ConSID*: Since a choice of $m > 15$ results into aliasing in the exponential matrix e^{AmT_s} , the CT system (27) was identified with SIDDS for $m = 1, \dots, 15$, SIDDS(m). The obtained models were compared with the following ConSID system identification methods:

- SVRVC—Simplified Refined Instrumental Variable for Continuous Time Output Error Model
- SSBCGPMF—State-Space Biased Compensating Generalized Poisson Moment Functionals
- SSIVGPMF—State-Space Instrumental Variable Generalized Poisson Moment Functionals
- SSLSGPMF—State-Space Least Squares Generalized Poisson Moment Functionals
- SIDGPMF—Subspace State-Space Model Identification Generalized Poisson Moment Functionals
- SIDFMF—Subspace State-Space Model Identification Fourier Modulating Functions
- SIDRPM—Subspace State-Space Model Identification Reinitialized Partial Moments
- SIDHMF—Subspace State-Space Model Identification Hartley Modulating Functions
- SIDLIF—Subspace State-Space Model Identification Linear Integral Filter

The SRIVC, an IO identification method was used as a reference, because it produces optimal results under the conditions considered in the MCS. The last three methods were discarded due to its poor performance when compared with the others. For the deterministic case, all ConSID methods estimated an exact model, as well as the SIDDS(m), $m = 1, \dots, 15$.

To evaluate the effect of noise, we run MCS with 100 experiments for SNR of 20 dB and 0 dB. That is, we kept the same conditions as before and added a bandwidth limited noise at the Nyquist frequency to the output. The performance of the methods was assessed by the percentage simulation error (PSE), defined as:

$$E_p = \sqrt{\frac{\sum_{k=1}^N (y_{det}(k) - \hat{y}(k))^2}{\sum_{k=1}^N y_{det}^2(k)}} \times 100, \quad (29)$$

where N is the length of the IO data record, y_{det} is the deterministic output generated by the true model and \hat{y} is the output generated by the estimated model. This error was calculated for two data sets: (i) the data set generated by the input signal used in the identification experiment, E_{pi} and (ii) the data set generated considering a random binary sequence as input, E_{pv} . Figure 5 displays a box plot of both the E_{pi} and E_{pv} errors, for the different identified models with SRN=20 dB. On each box, the central mark is the median, the edges of the box are the 25th and 75th percentiles, and the whiskers extend to the most extreme data points. Figure 6 displays a box plot of both the E_{pi} and E_{pv} errors, for the different identified models with SRN=0 dB. From Figure 5 and 6, we can assess the superior performance of the SRIVC,

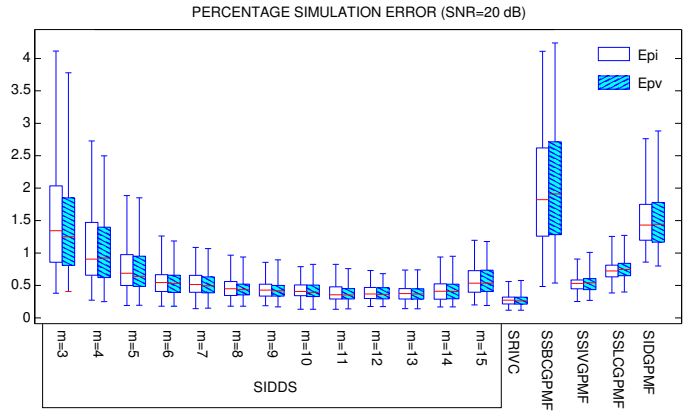


Fig. 5. Boxplot for the E_{pi} and E_{pv} for the different identification methods with SNR=20 dB.

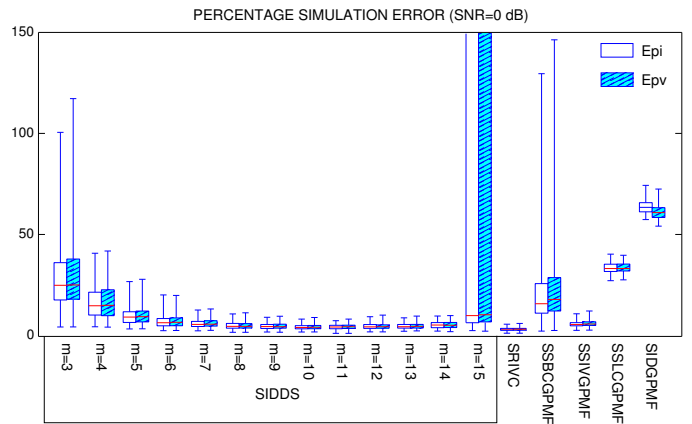


Fig. 6. Boxplot for the E_{pi} and E_{pv} for the different identification methods with SNR=0 dB.

followed from close by the SIDDS with an adequate choice of m , e.g. $m = 9, \dots, 13$. Consequently, SIDDS behaved better than the SS ConSID methods.

Figure 7 shows the scatter plot of the poles and zeros estimated by SRIVC, SIDDS(10) and SSIVGPMF, with SNR=0 dB. In this figure, one may observe that SIDDS and SRIVC have similar variability.

TABLE I

DC GAIN OF THE ESTIMATED MODELS WITH SNR=0 dB.

	SRIVC	SIDDS(10)	SSIVGPMF
K	1.0217 ± 0.3127	1.0190 ± 0.3553	1.0123 ± 0.5620

Table I displays the value and the 95 % confidence interval for the estimated DC gain of the DT model. Once again, SIDDS(10) is close to the SRIVC.

VI. CONCLUSION AND FUTURE WORK

A new SID method for CT systems have been proposed. This new approach is based on subspace DT identification methods, but takes into consideration the intermediate sampling

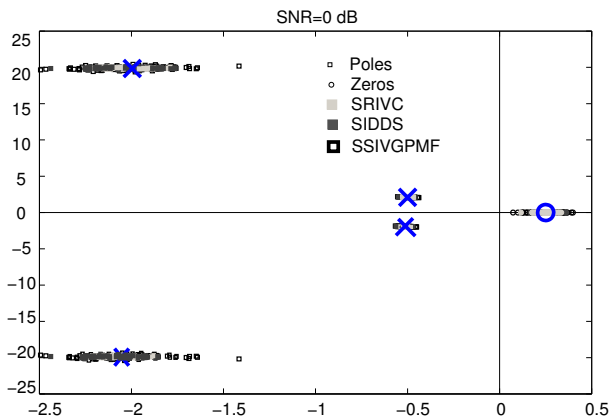


Fig. 7. Scatter plot of the zero-pole diagram using algorithms SIDDS(10), SRIVC, and SSVGPMF.

instants of the input signal of a downsampled data set. Some of the parameters of the CT are directly estimated, avoiding the problem of converting the parameters of a DT model into the CT ones. A procedure that secures an *a priori* number of zeros for the CT model is used during the estimation process. The algorithm performance is illustrated and compared using an example of fast sampling from Contsid.

The proposed algorithm has outperformed the SS methods described in Contsid and reached a performance very close to an optimal IO instrumental variable method, the SRIVC method.

Being a DT based SID algorithm our approach suffers from some problems that are inherent to these SM, namely when the row spaces defined by the past and future inputs are almost parallel. In the future, we would like to analyse this problem in order to make this algorithm able to cope with a wider range of input signals. We also intend to adapt SIDDS algorithm to the identification of CT linear parameter varying systems and to extend the downsampling approach to other classes of Linear Time Invariant Identification algorithms such as gradient based SS and prediction error methods for IO models.

VII. ACKNOWLEDGMENTS

The authors gratefully acknowledge the comments of the reviewers.

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