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On the Optimal Search Neighborhood in Mixed Wireless Sensor Networks

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Abstract— **This paper considers the problem of improving the monitoring capability of a sparse stationary sensor network with mobile sensor nodes. The main idea is that the mobile sensors should sample the areas that are least covered (monitored) by the static sensors. Thus, a simple path planning strategy is presented that decides the next point to be visited using only "local" information. For a simplified scenario, the paper derives the optimal path strategy and extrapolates some of the properties of the scenario to a more general instance of the problem. Furthermore, the paper proposes a surrogate metric that can be used in order to determine the optimal searching neighborhood and presents extensive simulation results which indicated that this approach can achieve very good results.**

I. INTRODUCTION

Mixed Wireless Sensor Networks (WSNs) are sensor networks that consist of both static and mobile sensor nodes. In this paper we consider the use of a mixed wireless sensor network for improving the area coverage (monitoring capability) of the network. The main idea is to use mobile sensor nodes that will collaborate with the static ones in order to sample the coverage holes. The main objective of this work is to determine the near optimal path that the mobile node (or a group of nodes) should follow in order to better cover the monitored area. In general, this is a difficult problem and it is not possible to guarantee optimality for a given instance of the problem.

Our approach in addressing this problem is to use a dynamic search strategy where the mobile determines the biggest coverage hole in an area (neighborhood) around it which constitutes its target location (i.e., the area that needs to be sampled next) [1]. An interesting question that needs to be addressed is the size of the neighborhood that the mobile needs to consider when determining its target. Clearly, that neighborhood cannot be very small since this will lead to myopic strategies where the mobile will search for very small holes ignoring much bigger holes that are a little further away. On the other hand, what we show in this paper is that the neighborhood should not be very big either which is a rather counter-intuitive result. This result indicates that the mobile should look for a "medium" size coverage hole located in the mobile's immediate neighborhood and ignore the possibly larger holes that are located further away. This strategy is justified because the mobile will waste valuable time traveling towards a bigger hole when it can sample the smaller coverage holes that are located much closer to it.

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As indicated above, the searched neighborhood should not be very small but it should not be very big either, therefore, there must be an optimal size. Formulating the problem to determine the optimal neighborhood size is not straightforward, thus we resort to a surrogate metric that can lead us to the optimal neighborhood size. The surrogate metric used is the *variance of vacancy* used in coverage processes [2], [3]. In this context, the computation of the variance of vacancy is a function of the size of the area that is used for the computation. The main idea in this paper is to associate the neighborhood where a mobile is going to search for the biggest coverage hole with the area that maximizes the variance of vacancy. The justification behind this approach is that the mobile needs to consider as much new information as possible when it will decide where it will go next. Thus, the mobile uses the neighborhood that maximizes variance of vacancy. As will be presented in the sequel, this choice achieves very good results.

The contributions of this paper are the following. In the context of mixed wireless sensor networks, it shows that it is not optimal to first search the largest coverage hole in the entire field; rather searching a big enough hole close to the current mobile location can yield faster coverage. Furthermore, the paper proposes a surrogate metric that can be used in order to determine the optimal size of the search neighborhood. Even though the proposed search approach cannot guarantee an optimal solution, the obtained solutions are satisfactory considering the difficulty of the problem.

II. RELATED WORK

Searching for targets in unknown environments is an area that has been investigated in [4], [5] which especially study the moving target problem. However, in [4], [5], the focus is on how to allocate search effort across the environment instead of finding the best search path to follow. In the context of sensor networks, various algorithms have been proposed for maximizing area coverage. Several algorithms are based on the notion of potential field and virtual forces [6], [7], while others are based on the structure of a Voronoi diagram in which nodes are relocated to fill up coverage holes [8], [9]. In addition, a distributed coverage control scheme has been proposed in [10] where mobile sensors move collaboratively in order to reposition and organize themselves in response to the events distribution in the environment and also incorporate communication costs into the coverage control problem. However, the underlying idea of our approach is to use mobile sensors that move using a path planning algorithm in order to enhance the dynamic coverage of a sparse stationary sensor network.

In [11], the authors focus on the dynamic coverage capabilities that result from the continuous random movement of mobile sensors. Polycarpou et al. [12] propose a general framework for directing a group of agents to cooperatively search a dynamic and uncertain environment. Along similar lines, a receding horizon approach with dynamic search is proposed in our previous work, [1]. Finally, dynamic multi-vehicle routing is addressed in [13] where the objective is to minimize the delay of servicing certain demands. Dynamic routing is also presented in [14] but the objective is to minimize event detection time and also the problem of optimal neighborhood is not considered.

III. Models and Assumptions

We consider a mixed wireless sensor network made of a large number of static sensor nodes deployed in a large square area $\mathcal{A}$ as shown in Fig. 1. For the purposes of this paper we make the following assumptions:

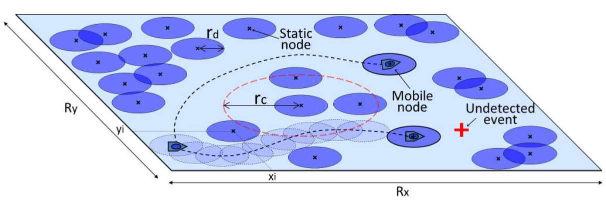

Fig. 1. Mixed sensor network model.

- A set $\mathcal{S}$ with $S = |\mathcal{S}|$ static sensor nodes are randomly placed in $\mathcal{A}$ at positions $\mathbf{x}_i = (x_i, y_i)$, $i = 1, \cdots, S$.
- A set $\mathcal{M}$ of $M = |\mathcal{M}|$ mobile sensor nodes are available and their position after the k-th time step is $\mathbf{x}_i(k) = (x_i(k), y_i(k))$, $i = 1, \cdots, M$, $k = 0, 1, \cdots$.
- All nodes have a common (known) sensing range r_d and communication range $r_c > r_d$ (see Fig. 1).
- All nodes know their position through a combination of GPS and localization algorithms.

For notational convenience we also define the set of *all* sensor nodes $\mathcal{N} = \mathcal{S} \cup \mathcal{M}$ where $N = S + M$. The objective of the WSN is to detect a static event that may occur at a random position $\mathbf{e} = (x^e, y^e)$ in $\mathcal{A}$. If the event occurs in the coverage area of at least one static sensor it is immediately detected by the network. However, if the event occurs at a point that is not covered by any sensor, it will remain undetected. Thus the objective of all mobile nodes is to sample the uncovered regions efficiently such that an event that has occurred in an uncovered region is detected as fast as possible.

Next, we define the *dynamic area coverage* $\mathcal{C}$ which will serve as an objective function to be maximized by the mobile sensors. At any instant τ, let $I(\mathbf{x}, \tau)$ be an indicator function that takes the value 1 if point $\mathbf{x} \in \mathcal{A}$ has been covered by at least one sensor (static or mobile) in the interval $[0, \tau]$, and 0 otherwise. In other words, $I(\mathbf{x}, \tau) = 1$ if there exist a sensor $s \in \mathcal{S}$ such that $\|\mathbf{x}_s - \mathbf{x}\| \leq r_d$ or if a sensor $s \in \mathcal{M}$ has passed from a point such that $\mathbf{x}$ was covered, i.e., $\|\mathbf{x}_s(k) - \mathbf{x}\| \leq r_d$. Thus, the coverage achieved by the network at τ is given by

$$C(\tau) = \frac{1}{A} \int_{\mathcal{A}} I(\mathbf{x}, \tau) d\mathbf{x}. \quad (1)$$

As mobile nodes move, they cover new areas, thus a reasonable objective function is

$$\mathcal{C}(t) = \int_0^t C(\tau) d\tau \quad (2)$$

which is the objective that needs to be maximized by the mobiles. Next, we present the algorithm used by the mobile to navigate through the sensor field in order to maximize (2).

IV. The Two Hole Problem

In this section we investigate what happens if there are only two coverage holes in order to gain some insight that can be used in other heuristic approaches for efficiently solving the problem.

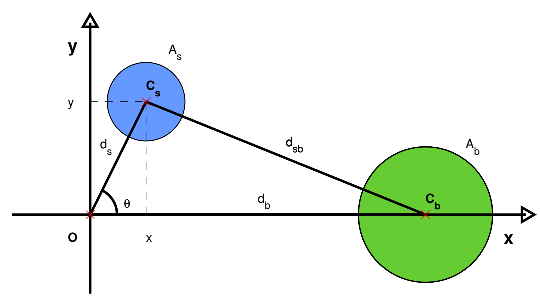

Fig. 2. Problem geometry

Assume that the field has only two coverage holes with areas A_b and A_s ($A_b \geq A_s$) and centroids, $\mathbf{C_b}$ and $\mathbf{C_s}$ respectively (see Fig. 2). For simplicity, it is also assumed that there is no overlap between the two holes. A mobile node is initially placed at position $\mathbf{O}$ at distance $d_b = \|\mathbf{C_b} - \mathbf{O}\|$ from hole A_b and at distance $d_s = \|\mathbf{C_s} - \mathbf{O}\|$ from hole A_s. The distance between the two coverage holes is indicated by $d_{sb} = \|\mathbf{C_s} - \mathbf{C_b}\|$. The objective of the mobile is to maximize $\mathcal{C}(T)$ where $\mathcal{C}(t)$ is given by (2) and T is some time instant such that in all of the paths considered, the mobiles achieve full coverage.

Given that there are only two holes, the mobile has only two options. First go to A_b, search A_b and then go to A_s or first go to A_s, search A_s and then go to A_b. Fig. 3 shows $C(t)$ under the two different paths thus $\mathcal{C}(T)$ for each path is the area under the corresponding curve from 0 until $T \geq t_{b4}$. In this figure, we assume that when the mobile travels over covered regions $\dot{C}(t) = 0$ while when it searches in coverage holes the coverage improvement is constant at rate $\dot{C}(t) = 2r_d v/(A_s + A_b)$ where v is the constant mobile speed.

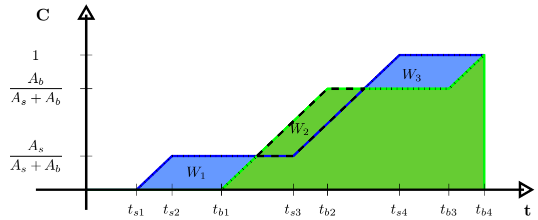

Fig. 3. Coverage over time

When the mobile follows the path from $\mathbf{O} \to \mathbf{C_s} \to \mathbf{C_b}$, $\mathcal{C}_{sb}(T)$ is given by

$$\begin{aligned}\mathcal{C}_{sb}(T) \quad = \quad & \frac{1}{2}\frac{A_s}{2r_d v}\frac{A_s}{A_s+A_b} + \frac{d_{sb}}{v}\frac{A_s}{A_s+A_b} + \\ & \frac{A_b}{2r_d v}\frac{A_s}{A_s+A_b} + \frac{1}{2}\frac{A_b}{2r_d v}\frac{A_b}{A_s+A_b} + \frac{d_b-d_s}{v}\end{aligned} \tag{3}$$

Similarly, if the mobile follows the path from $\mathbf{O} \to \mathbf{C_b} \to \mathbf{C_s}$, $\mathcal{C}_{bs}(T)$ can be computed. Comparing $\mathcal{C}_{sb}(T)$ (3) and $\mathcal{C}_{bs}(T)$ or simply observing Fig. 3, the decision of the mobile is to follow the path that maximizes $\mathcal{C}(T)$ which is equivalent to comparing the three areas W_1, W_2 and W_3 in Fig. 3. Thus

$$\mathcal{C}_{sb}(T) \lesseqgtr \mathcal{C}_{bs}(T) \Leftrightarrow W_1 + W_3 \lesseqgtr W_2 \tag{4}$$

which in turn is equivalent to

$$\mathcal{C}_{sb}(T) \lesseqgtr \mathcal{C}_{bs}(T) \Leftrightarrow d_{sb}\left(A_b - A_s\right) \gtreqless \left(d_b - d_s\right)\left(A_b + A_s\right) \tag{5}$$

Next, we consider the following cases (proofs are omitted due to space limitations):

C1 $\{A_b = A_s = A\}$: The decision problem $\mathcal{C}_{sb}(T) \lesseqgtr \mathcal{C}_{bs}(T)$ reduces to $d_s \gtreqless d_b$, i.e., the mobile should go to its nearest coverage hole first.

C2 $\{d_b = d_s = d\}$: The decision problem $\mathcal{C}_{sb}(T) \lesseqgtr \mathcal{C}_{bs}(T)$ reduces to $A_b \gtreqless A_s$, i.e., the mobile should go to the biggest hole A_b first (since by assumption $A_b \geq A_s$).

C3 $\{A_b > A_s \textit{ and } d_b < d_s\}$: The decision is to always go to the biggest hole which is also located nearer to the mobile.

C4 $\{A_b > A_s \textit{ and } d_b > d_s\}$: The decision depends on the relative position (d_{sb}) and the area ratio ($\varrho = A_b/A_s$) of the the two holes. Specifically, if the smaller hole is located inside an "egg shaped" area defined by (6) (in polar coordinates) then the decision is to search the smaller hole first, otherwise, it is better to search the larger hole first.

$$\begin{aligned} d_s &= \frac{d_b\left((\varrho+1)^2-(\varrho-1)^2cos(\theta)-\sqrt{\left((\varrho+1)^2-(\varrho-1)^2cos(\theta)\right)^2-(4\varrho)^2}\right)}{4\varrho} \\ \theta &= [0, 2\pi) \end{aligned} \tag{6}$$

This holds true when $\mathbf{O} = (\mathbf{0}, \mathbf{0})$. One can use (6) to draw the region in polar coordinate system as illustrated in Fig. 4.

Concluding, the analysis above demonstrates that a mobile should not go immediately to the largest hole in the field but it should first search smaller holes that are closer to the mobile (areas in the egg shaped region). However, note that the precise size of the egg, depends on the relative size of the two coverage holes (ϱ). If for example the smaller hole is significantly smaller than the larger one, then the egg will be significantly narrower, implying that the smaller one should be visited first only if it is exactly in the straight path to the big hole. Furthermore, in many scenarios it may be difficult to clearly identify two holes (some holes may be connected) and as already mentioned there may be more than two holes which makes it impractical to determine the optimal path of the mobile. Thus, the implementation of such an algorithm is rather difficult, however, the insight from the analysis is clear: "Large enough holes close to the mobile should be searched first, before moving towards the biggest holes of the field". The simplest way to implement this "insight" is by searching for the biggest coverage hole in a neighborhood around the mobile. This is the approach of the algorithm presented next (see also [1]).

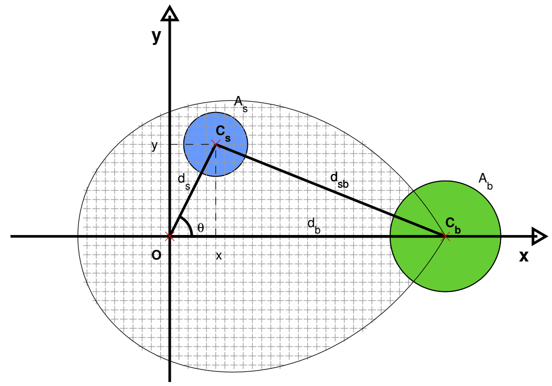

Fig. 4. The Egg-shaped region for $\varrho = 3$ and $\mathbf{O} = (\mathbf{0}, \mathbf{0})$. If $\mathbf{C_s}$ is located inside the shaded region then a mobile should follow the path $\mathbf{O} \to \mathbf{C_s} \to \mathbf{C_b}$ to maximize coverage over time.

V. Mobile Path Planning Strategy

The mobile's path planning strategy is based on a Receding-Horizon approach where at each step the mobile's controller evaluates the cost of moving to a finite set of candidate positions and moves to the one that minimizes an overall cost. Suppose that during the kth step, the mobile node is at position $\mathbf{x}(k)$ and is heading to a direction θ. The next candidate positions are the ν points $\mathbf{y}_1, \cdots, \mathbf{y}_\nu$ that are uniformly distributed on the arc that is ρ meters away from $\mathbf{x}(k)$ and are within an angle $\theta - \phi$ and $\theta + \phi$. The mobile node evaluates a cost function $J(\mathbf{y}_i)$ for all candidate locations $(\mathbf{y}_1, \cdots, \mathbf{y}_\nu)$ and moves to the location $\mathbf{x}(k+1) = \mathbf{y}_{i^*} = \mathbf{x}(k) + \rho.e^{\mathrm{i}(\theta+\varphi_{i^*})}$ where i is the imaginary unit and i^* is the index that minimizes $J(\mathbf{y}_i)$.

$$J(y_{i^*}) = \min_{1 \leq i \leq \nu} \{J(\mathbf{y}_i)\} \tag{7}$$

In this model, θ is the direction that the mobile is heading, ρ is the distance that the mobile can cover in one time step, ϕ is the maximum angle that the mobile can turn in a single step, and ν is the number of candidate positions that is being evaluated for the next step.

The cost function that each mobile is trying to minimize is of the form

$$J(\mathbf{y}) = \sum_j w_j J_j(\mathbf{y}) \tag{8}$$

where $J_j(\cdot)$ is a specific objective and w_j's are non-negative constant weights such that $\sum_j w_j = 1$.

After extensive investigation [1], two specific normalized functions have been selected: $J_t(\cdot)$ which penalizes positions that are away from its target (medium term goal) position and $J_s(\cdot)$ which penalize positions that are close to static or mobile sensors. Assuming that the mobile has a target destination point $\mathbf{x}_t$, the $J_t(\mathbf{y})$ is a function that pulls the mobile towards its target. The objective of $J_s(\cdot)$ function is to push the mobile away from areas covered by other sensors, thus the $J_s(\mathbf{y})$ used involves a repulsion force that pushes the mobile away from its closest neighbor (see [1] for more details).

Given the intuition gained from the 2-hole problem, the mobile's target position $\mathbf{x}_t$ is set as the centroid of the biggest coverage hole in a "local" neighborhood of radius r_z around its current location $\mathbf{x}(k)$ This target point is dynamic as it is updated at every moving step and is determined using the zoom algorithm [1], [15]. An important question that arises is the size of the neighborhood. If r_z is too small, then the mobile may waste time searching insignificant holes missing much larger holes. On the other hand, if the neighborhood is too big, then the mobile will move straight towards much larger holes avoiding significant holes that are located close to it. Therefore, there is an optimal neighborhood size. In the next section we investigate a surrogate function that we can use to perform this optimization.

VI. Vacancy

Given the difficulty in directly finding the optimal value for r_z, the objective of this section is to derive a surrogate function that can be used to approximate the solution of this problem.

A. Preliminaries on coverage processes

In this subsection we use tools from the theory of coverage processes [2], [3] in order to analyze the coverage holes that are generated from the random deployment of sensors in $\mathcal{A}$. Consider a two-dimensional point process where a collection of N random points is thrown in a square area $\mathcal{A}$ according to the probability density $f(\mathbf{x}) = \frac{1}{A}$. Let the countable collection of randomly distributed points be $\mathcal{P} \equiv \{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N\}$. Assume that there exists a disc around each point of radius r (in our case $r = r_d$ the detection range) thus all points in the union of all N discs are considered as covered while all non-covered points are considered as vacant. Vacancy is the collection of all vacant points within an arbitrary area $\mathcal{R} \subset \mathcal{A}$ which constitutes a random variable with mean and variance that are defined in the sequel [16]. Let $\overline{I(\mathbf{x})}$ be the indicator function of uncovered points such that $\overline{I(\mathbf{x})} = 1 - I(\mathbf{x}) = 1$ if $\mathbf{x} \in \mathcal{A}$ is not covered by any disk of radius r_d or $\overline{I(\mathbf{x})} = 0$ otherwise. The vacancy within and arbitrary area $\mathcal{R} \subset \mathcal{A}$, $V_{\mathcal{R}} = V(\mathcal{R})$ is given by

$$V_{\mathcal{R}} = V(\mathcal{R}) \equiv \int_{\mathcal{R}} \overline{I(\mathbf{x})} d\mathbf{x} \tag{9}$$

and the mean of vacancy (expected uncovered area) is

$$\begin{aligned} E(V_{\mathcal{R}}) &= \int_{\mathcal{R}} E\{\overline{I(\mathbf{x})}\} d\mathbf{x} = \int_{\mathcal{R}} P(\mathbf{x} \ \textit{not covered}) \ d\mathbf{x} \\ &= \int_{\mathcal{R}} \Big(1 - \frac{a}{A}\Big)^N d\mathbf{x} = R\Big(1 - \frac{a}{A}\Big)^N \end{aligned} \tag{10}$$

where $p = \frac{a}{A}$ is the probability that a point $\mathbf{x} \in \mathcal{A}$ is covered by a disk of area $a = \pi r_d^2$ and $(1-p)^N$ is the probability that the point $\mathbf{x}$ is not covered by any of the N disks (sensor positions are independent). Also R is the area of $\mathcal{R}$.

The variance of vacancy is

$$Var(V_{\mathcal{R}}) = E\left(V_{\mathcal{R}}^2\right) - \left(E(V_{\mathcal{R}})\right)^2 \tag{11}$$

where the mean square of vacancy is

$$\begin{aligned} E(V_{\mathcal{R}}^2) &= \int\int_{\mathcal{R}^2} E\{\overline{I(\mathbf{x}) I(\mathbf{y})}\} d\mathbf{x} d\mathbf{y} \\ &= \int\int_{\mathcal{R}^2} P(\mathbf{x}, \mathbf{y} \ \textit{both not covered}) d\mathbf{x} d\mathbf{y} \end{aligned} \tag{12}$$

Thus, $Var(V_{\mathcal{R}})$ can be computed numerically by performing an integration of the probability $P(\mathbf{x}, \mathbf{y} \ \textit{both not covered})$.

Let the density $\lambda \equiv \frac{N}{A}$ of points per unit area of $\mathcal{A}$ converges to a constant value as $\mathcal{A}$ increases. Hence, for N large and $\frac{a}{A}$ small, by (10) the mean of vacancy in a region $\mathcal{R} \subseteq \mathcal{A}$ is approximated by

$$E(V_{\mathcal{R}}) \approx R e^{-\lambda a} \tag{13}$$

An approximation of the variance of vacancy in a subregion $\mathcal{R} \subseteq \mathcal{A}$ is derived in [16] and is given by

$$Var(V_{\mathcal{R}}) \approx R a e^{-2\lambda a} \left(8 \int_0^1 x \Big(e^{\lambda \frac{a}{\pi} \mathrm{B}(x,1)} - 1\Big) dx - R a \lambda^2\right) \tag{14}$$

where $\mathrm{B}(x, r)$ is the intersection area of two disks with radius r and which are centered $2x$ apart. This area is given by

$$\mathrm{B}(x, r) = \begin{cases} 4r^2 \int_{x/r}^1 \sqrt{1 - y^2} dy & \text{if } 0 \leq x \leq r \\ 0 & \text{if } x > r \end{cases} \tag{15}$$

Hence $\mathrm{B}(x, 1) = 2\arccos(x) - 2x\sqrt{1 - x^2}$. Even though (14) cannot be computed analytically, it can be computed numerically (see [16]). Let

$$Q(\lambda, r_d) = \int_0^1 x \left(e^{\lambda r_d^2 \mathrm{B}(x,1)} - 1\right) dx \tag{16}$$

independent of R, then the $Var(V_{\mathcal{R}})$ can be written as

$$Var(V_{\mathcal{R}}) \approx R \pi r_d^2 e^{-2\pi r_d^2 \lambda} \left(8 Q(\lambda, r_d) - R \lambda^2 \pi r_d^2\right) \tag{17}$$

which is a polynomial of R with a maximum at

$$R^* = \frac{4 Q(\lambda, r_d)}{\pi \lambda^2 r_d^2} \tag{18}$$

B. An Approximation of Optimal Search Neighborhood

Next, we use the optimal area size R^* in order to determine the optimal neighborhood size that the mobile node should use in order to determine the biggest coverage hole to visit next. Recall that the conjecture is that the neighborhood size should be large enough such that the new information considered by the mobile in making this decision is maximized. Assuming that at time k the mobile is at position $\mathbf{x}(k)$, then it should search for the biggest hole in a circular area $\mathcal{R}_1$ with radius r_z. During the next step, the mobile will move to a new location $\mathbf{x}(k+1) = \mathbf{x}(k) + \boldsymbol{\rho}$, $\boldsymbol{\rho} \in \mathbb{R}^2$, where the region that the mobile will search for a coverage hole will be $\mathcal{R}_2$. Thus, the *new* information that the mobile will consider from one step to the next is $\Delta\mathcal{R} = \mathcal{R}_2 \setminus \mathcal{R}_1$ (i.e $\mathcal{R}_1^c \cap \mathcal{R}_2$). The objective then is to choose the size of the areas $\mathcal{R}_1$ and $\mathcal{R}_2$ (the radius r_z) such that the variance of vacancy in $\Delta\mathcal{R}$ is maximized. As the variance of vacancy in $\Delta\mathcal{R}$ is maximized between two consecutive steps, the mobile can exploit, on average, the "maximum" difference in vacancy at each step k in order to take, on average, the optimal local decision. The new region $\Delta\mathcal{R}$ depends on the current position of the mobile and the next candidate position. This means that by maximizing the variance of vacancy in the new region $\Delta\mathcal{R}$ (between two consecutive steps) one also maximizes the amount of new information that is used by the mobile to decide its next target.

Given the result of (18), the optimal radius r_z^* is the solution to the equation

$$\Delta R = \frac{4Q(\lambda, r_d)}{\pi\lambda^2 r_d^2} \tag{19}$$

where ΔR is the area of $\Delta\mathcal{R}$.

Lemma 1: The solution to (19) is approximated by

$$r_z^* \approx \frac{64Q^2(\lambda, r_d) + \pi^2(\rho\lambda r_d)^4}{32\pi\rho\lambda^2 r_d^2 Q(\lambda, r_d)} \tag{20}$$

where $\rho = \|\boldsymbol{\rho}\|$ is the distance traveled by the mobile in one step.

VII. Simulation Results

In this section, we present some numerical results that support the conjecture of this paper, i.e., that the optimal search radius r_z is given by Lemma 1. For the mobility strategy the following parameters were used: the mobile evaluates $\nu = 10$ candidate next positions which are uniformly distributed on an arc with radius $\rho = 2.5m$ and extends $\phi = 35°$ above and below of the current direction of the mobile. Unless otherwise stated, all experiments refer to a square sensor field of area $A = 40000m^2$. A set of $S = 200$ static sensors are deployed and their coordinates are generated according to a uniform distribution. The detection radius of all sensors is $r_d = 5m$ and the communication range $r_c = r_z + r_d$. The radius r_z defines the radius of the search area where the mobile is searching for its target (largest coverage hole center). All simulations performed in MATLAB and the outcomes are the averages of 100 independent random deployments.

In the first simulation experiment we investigate the effect of the sensor detection range r_d on the optimal neighborhood size. Using Lemma 1, the optimal neighborhood size for different r_d is presented in Table I. As shown in Table I

r_d	S	ρ	$r_z{}^*$	$Var(V_{\Delta\mathcal{R}})$
2	200	2.5	20.3	35.9
5	200	2.5	21.9	847
8	200	2.5	25.7	2232.1
10	200	2.5	30.1	2417.6

TABLE I
THE OPTIMAL SEARCH NEIGHBORHOOD r_z^* FOR DIFFERENT r_d VALUES

as the detection radius r_d increases, the r_z^* radius, where $Var(V_{\Delta\mathcal{R}})$ is maximized, also increases but remains small compared to the field size (e.g. 200m). This is reasonable because as the sensing radius of each sensor increases (and given that the number of sensors is fixed $S = 200$) it is possible to generate deployments with higher variation in the achieved coverage.

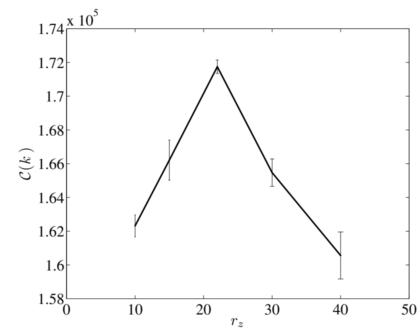

Fig. 5. The average dynamic coverage accomplished over 100 sensor fields by a mobile node after k=2000 moving steps for different r_z values when $r_d = 5m$

Fig. 5 presents the average dynamic coverage $\mathcal{C}(k)$ achieved by the path planning algorithm after $k = 2000$ time steps accomplished over 100 sensor fields by one mobile node when $r_d = 5m$. The figure indicates that coverage is maximized when $r_z = 22m$ which is what was also predicted by Lemma 1 (see Table I).

In the next simulation experiment we investigate how the optimal r_z value is affected by the density $\lambda \equiv \frac{S}{A}$ of the static sensors. First, using Lemma 1 we compute the optimal r_z^* as shown in II.

r_d	S	ρ	$r_z{}^*$	$Var(V_{\Delta\mathcal{R}})$
5	100	2.5	41.9	1141.3
5	200	2.5	21.9	847
5	300	2.5	15.4	630.4
5	400	2.5	12.1	470.7

TABLE II
THE OPTIMAL SEARCH NEIGHBORHOOD r_z^* FOR DIFFERENT S VALUES

Fig. 5 shows that for $S = 200$ the optimal $r_z^* = 22m$ which is in agreement with the results of Table II. Furthermore, Fig. 6 presents the coverage achieved by the path planning algorithm when $S = 300$ sensors are deployed. The maximum coverage is achieved when $r_z = 15m$ which is again consistent with the Lemma 1 prediction as indicated in Table II.

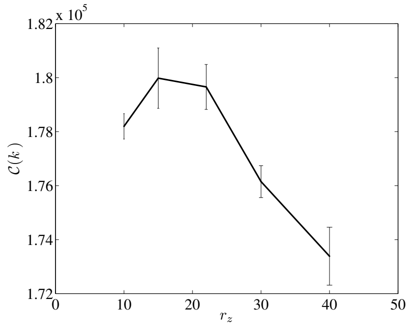

Fig. 6. The average dynamic coverage accomplished over 100 sensor fields by a mobile node after k=2000 moving steps for different r_z values when $N = 300$

The next simulation considers how the optimal r_z value is affected by ρ, the distance that the mobile can move in one time step. Distance ρ also indicates how frequently the target (biggest coverage hole centroid) in the searching neighborhood is computed with respect to the distance moved. Again, we evaluate the optimal radius r_z^* using Lemma 1 as shown in Table III.

r_d	S	ρ	$r_z{}^*$	$Var(V_{\Delta\mathcal{R}})$
5	200	1	54.8	846.99
5	200	2.5	21.9	846.99
5	200	4	13.8	846.98
5	200	5	11.1	846.97

TABLE III

THE OPTIMAL SEARCH NEIGHBORHOOD r_z^* FOR DIFFERENT ρ VALUES

Fig. 5 indicated that the optimal r_z for $\rho = 2.5m$ is about $22m$ while Fig. 7 indicated that for $\rho = 4m$ the optimal r_z is about $15m$. Both of these results are consistent with the Lemma 1 predictions shown in Table III.

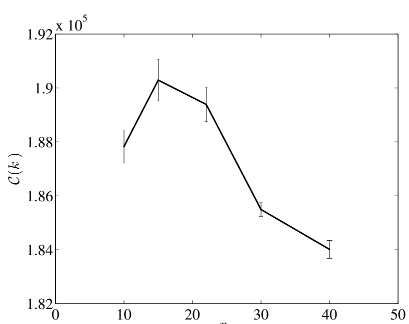

Fig. 7. The average dynamic coverage accomplished over 100 sensor fields by a mobile node after k=2000 moving steps for different r_z values when $\rho = 4m$

In the previous simulations we have investigated the single mobile case, but we point out that the approximation method for obtaining r_z^* also remains valid for the case of multiple mobiles given that the coverage process is mainly governed by the initial distribution of stationary nodes. In other words, when the number of mobiles as well as their coverage rate are small enough, however simulations are omitted due to space limitations.

VIII. CONCLUSION

In this paper we propose a method to approximate the optimal searching neighborhood that enhances the dynamic coverage performance in a mixed sensor network architecture in conjunction with other parameters used in the path planning method presented in previous work. This approximation is based on the variance of vacancy of the binomial coverage process. Obtained results from numerical evaluations of the mathematical approximations have been verified by Monte Carlo simulation outcomes of the dynamic coverage performance of the path planning method.

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