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On the computation of ℓ_∞ gains of switched systems

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***Abstract*— In this paper we compute ℓ_∞ and ℓ_1 induced norm bounds of switched linear systems. In particular, we concentrate on certain types of such systems, namely output switching, input switching, and combinations thereof, and we provide exact gains for the worst case switching. As an application of the input output approach of the paper, we consider the problem of sensitivity minimization and show that it is convex in the Youla et.al.-Kucera parameter.**

Keywords: ℓ_∞ induced, Youla parameterization, switching

I. INTRODUCTION

Linear switched systems have been the focus of a lot of renewed research over at least the last decade under various contexts and several new results on stability and performance have been established by now. We refer to [1], [2], [3], [4] and references therein for some indicative samples of relevant works. Most of the stability results involve, in one way or another, state space, Lyapunov based approaches. The same holds true for performance aspects as well, as the works in the literature are concerned primarily with L_2 gains of these systems (e.g., [7], [8].) Semidefinite programming approaches have also been developed for analysis and synthesis of L_2-type of closed loop performance (e.g., [9].)

In this paper we consider ℓ_∞ (and ℓ_1) gains for linear, discrete-time, switched systems. We provide bounds of the ℓ_∞ gains in the general case which can be conservative (the upper bounds.) On the other hand, when we specialize to output-only, input-only, or input-output switched systems, we obtain exact results in terms of the underlying linear-time-invariant (LTI) systems associated with the specific switching structure. These developments rely on input-output point of view of the linear-time-varying (LTV) structure of the system and is along the lines of the author's earlier work in [5], [6]. In the paper we also consider designing controllers for stable, input, or output switching systems to optimize the sensitivity of the closed loop. These controllers are generated through a Youla et.al.-Kucera (Y-K) parametrization, restricted to output, or input switching Y-K parameter. The resulting problems are convex and can be solved by well developed methods [11].

II. BASIC SETUP

We consider switching systems G with respective input and output vectors u and y, state vector x, described in state space as

$$\begin{aligned} x^+ &= A_\sigma x + B_\sigma u \\ y &= C_\sigma x \end{aligned} \tag{1}$$

where $x^+(t) := x(t+1)$, and $\sigma = \{\sigma(t)\}_{t=0}^\infty$ is a switching sequence that, for every time instant t, takes values in a set of integers $J := \{1, 2, \dots, N\}$. Accordingly, the state-space matrices $A_{\sigma(t)}$, $B_{\sigma(t)}$, and $C_{\sigma(t)}$ take values respectively in the tuples $\{A_j\}_{j\in J}$, $\{B_j\}_{j\in J}$, and $\{C_j\}_{j\in J}$. We assume that G is internally stable for arbitrary switching which equivalently means [4] that there exists some integer q :

$$\rho := \max_{A_{i_k}\in\{A_j\}_{j\in J}} |A_{i_1} A_{i_2} \dots A_{i_q}| < 1$$

where $|\bullet|$ stands for the ∞-induced matrix norm[1]. Given a specific σ, we denote the corresponding LTV system as G_σ and the ℓ_∞-induced norm $\|G_\sigma\|$. Note that G_σ is associated with the time-varying, state space representation $G_\sigma \sim (A_{\sigma(t)}, B_{\sigma(t)}, C_{\sigma(t)})_{t=0}^\infty$ and the way G_σ relates to G is through the equation $(Gu)(t) = (G_\sigma u)(t)$. We are interested in characterizing the worst-case norm $\|G\| := \sup_\sigma \|G_\sigma\|$. Towards this end, let

$$\alpha := \max_{j\in J} |A_j|, \quad \beta := \max_{j\in J} |B_j|, \quad \gamma := \max_{j\in J} |C_j|$$

and let $\|G_j\|$ be the norm of the LTI system associated to the state space matrices of index j, i.e., $G_j \sim (A_j, B_j, C_j)$. The following proposition can be easily proved.

Proposition 2.1: If $\alpha < 1$ then

$$\max_j \|G_j\| \le \|G\| \le \frac{\gamma\beta}{1-\alpha}.$$

If $\alpha \ge 1$ then

$$\max_j \|G_j\| \le \|G\| \le \frac{\gamma\beta\bar\alpha}{1-\rho}$$

where $\bar\alpha := 1 + \alpha + \alpha^2 + \dots + \alpha^q$.

Proof: The lower bound follows trivially as the specific G_j correspond to a constant $\sigma(t) = j$ for all $t \ge 0$. For the case $\alpha < 1$ the result follows immediately as

$$\begin{aligned} |y(t)| &= |C_{\sigma(t)} \sum_{\tau=0}^{t-1} A_{\sigma(t)} \dots A_{\sigma(t-\tau-1)} B_{\sigma(\tau)} u(\tau)| \\ &\le \max_{j\in J} |C_j| (\sum_{\tau=0}^{t-1} \max_{j\in J} |A_j|^{t-\tau-1}) \max_{j\in J} |B_j| \max_{\tau\le t-1} |u(\tau)|. \end{aligned}$$

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[1]i.e., if $M = (M_{ij})$ is a matrix of real scalar elements M_{ij}, then $|M| = \max_i \sum_j |M_{ij}|$

The case $\alpha \geq 1$ follows in a similar pattern by bounding any product $A_{\sigma(t)} \ldots A_{\sigma(t-\tau-1)}$ in chunks of size q as

$$|A_{\sigma(t)} \ldots A_{\sigma(t-\tau-1)}| \leq \rho^{\lfloor t/q \rfloor} \alpha^{t-\tau-1-\lfloor t/q \rfloor}$$

if $\lfloor t/q \rfloor \leq t-1$.

■

The above upper bounds can be in general conservative and, in the case where $\alpha \geq 1$, finding the integer q and hence ρ is a combinatorial problem so these general bounds may not be very practical. Also, it is obvious that if we define an average system $\bar{G} := \frac{1}{N}\sum_{j=1}^{N} G_j$ then $\left\|\bar{G}_j\right\| \leq \max_j \|G_j\|$ for any system norm.

In the sequel we elaborate on specific classes of switched systems where exact expressions for $\|G\|$ can be obtained. These are systems with non-switching state dynamics, that is they have a constant A-matrix. We begin with the output switched systems. To avoid unnecessary complexity we will consider the case of two systems i.e., $J = \{1, 2\}$ to expose the developments. Generalizations are easy to obtain.

III. Output Switching

Herein we consider the case where the A and B-matrices are constant and the output matrix C switches between C_1 and C_2. Thus, the state space is of the form

$$\begin{aligned} x^+ &= Ax + Bu \\ y &= C_\sigma x \end{aligned} \tag{2}$$

Consider now the LTI two-output system $\begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$ and the output switching operator S defined by $y(t) = (S\begin{bmatrix} y_1 \\ y_2 \end{bmatrix})(t) = y_j(t)$ when $\sigma(t) = j$ with $j \in \{1, 2\}$. It should be clear that S represents a 1x2 time-varying gain matrix $S_{\sigma(t)}$ with takes the value $[1\ 0]$ if $\sigma(t) = 1$ and $[0\ 1]$ when $\sigma(t) = 2$. Then, G is the composition $G = S\begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$ as depicted in Figure 1. In this case, due to the definition of ℓ_∞-norm $\|y\|$ of a signal y,

$$\begin{aligned} \|y\| &= \max\{\|y_1\|, \|y_2\|\} \\ &\leq \max\{\|G_1 u\|, \|G_2 u\|\} \\ &\leq \max\{\|G_1\|, \|G_2\|\}\, \|u\| \end{aligned}$$

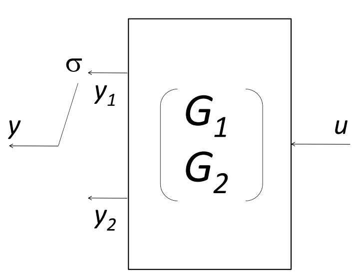

Fig. 1. Output switching

Based on the Proposition 2.1 it follows that

$$\|G\| = \max\{\|G_1\|, \|G_2\|\}.$$

The above assertion can also easily be seen from the infinite lower triangular representation of G_σ. Note that as G_σ represents a LTV system for a given σ, it can be thought of as an infinite lower triangular matrix. The elements of this matrix are the corresponding elements of the pulse response of either G_1 or G_2. For the output switching, G_σ will be made of rows that belong to either G_1 or to G_2 depending on what $\sigma(t)$ is. For example, for the sequence $\sigma = \{1, 1, 2, 1, 2, 2, \ldots\}$ the form of G_σ is as

$$G_\sigma = \begin{bmatrix} 1 & & & & & & \\ 1 & 1 & & & & & \\ 2 & 2 & 2 & & & & \\ 1 & 1 & 1 & 1 & & & \\ 2 & 2 & 2 & 2 & 2 & & \\ 2 & 2 & 2 & 2 & 2 & 2 & \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{bmatrix} \tag{3}$$

where the entries "1" and "2" correspond to the pulse response coefficients $\{g_1(t)\}_{t=0}^{\infty}$ and $\{g_2(t)\}_{t=0}^{\infty}$ of G_1 and G_2 respectively. That is, the (τ, ξ) entry will be $g_1(\tau - \xi)$ if it is "1" and $g_2(\tau - \xi)$ if it is "2." For example, the $(1, 0)$ entry which is "1" corresponds to $g_1(1)$; the $(4, 1)$ entry which is "2" corresponds to $g_2(3)$; the $(3, 3)$ entry which is "1" corresponds to $g_1(0)$, etc.

IV. Input Switching

Dual in a sense to the previous case is the input switching, where A and C are constant and the input matrix-B switches between B_1 and B_2. Thus, the state space is of the form

$$\begin{aligned} x^+ &= Ax + B_\sigma u \\ y &= Cx \end{aligned} \tag{4}$$

Consider now Figure 2 and the LTI two-input system $\begin{bmatrix} G_1 & G_2 \end{bmatrix}$ where the input switching operator S^* defined by $(S^* u)(t) = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}(t)$ with $u_j(t) = u(t)$ when $\sigma(t) = j$, otherwise $u_j(t) = 0$, with $j \in \{1, 2\}$. It should be clear that S^* represents a 2x1 time-varying gain matrix $S^*_{\sigma(t)}$ with takes the value $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ if $\sigma(t) = 1$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ when $\sigma(t) = 2$. Then, G is the composition $G = \begin{bmatrix} G_1 & G_2 \end{bmatrix} S^*$. We note that the infinite lower triangular representation of G_σ in this case will be made of columns that belong to either G_1 or to G_2 depending on what is $\sigma(t)$. For example, for the sequence $\sigma = \{1, 1, 2, 1, 2, 2, \ldots\}$ of the previous section, the form of G_σ is as

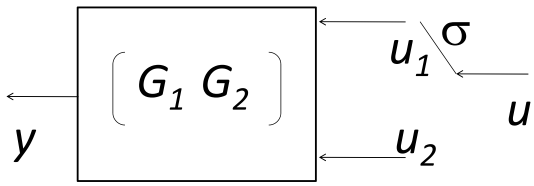

Fig. 2. Input switching

$$G_\sigma = \begin{bmatrix} 1 & & & & & & \\ 1 & 1 & & & & & \\ 1 & 1 & 2 & & & & \\ 1 & 1 & 2 & 1 & & & \\ 1 & 1 & 2 & 1 & 2 & & \\ 1 & 1 & 2 & 1 & 2 & 2 & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \tag{5}$$

where the entries are interpreted in a similar manner. Obtaining the worst ℓ_∞-induced norm of G_σ is not so clear, particularly for a MIMO system. Because of the duality however with the output switching, we can obtain exactly the worst ℓ_1-induced norm. Denoting $\|G_\sigma\|_*$ the ℓ_1-induced norm of G_σ, it is easy to see through the lower triangular representation above that

$$\|G\|_* := \sup_\sigma \|G_\sigma\|_* = \max\{\|G_1\|_*, \|G_2\|_*\}.$$

Note that, if G_j's are SISO systems, then $\|G_j\|_* = \|G_j\| = \sum_{t=0}^{\infty} |CA^t B_j|$. Also for SISO systems, it is possible to obtain the worst ℓ_∞-induced norm of G_σ as follows. Define

$$\begin{aligned} \bar{g} =& \{\bar{g}(t)\}_{t=0}^{\infty} := \{\max\{|g_1(0)|, |g_2(0)|\}, \\ & \max\{|g_1(1)|, |g_2(1)|\}, \max\{|g_1(2)|, |g_2(2)|\}, \dots\} \end{aligned}$$

Then, by using the same idea as in Theorem 5.1 of the following section, it can be shown that

$$\|G\| = \|\bar{g}\|_1 = \sum_{t=0}^{\infty} |\bar{g}(t)|.$$

Finally, we should mention that we can define a worst case $\mathcal{H}_2$ norm for input-switching systems and compute it exactly. For more details we refer to [6].

V. Input-Output Switching

We consider now the case of both output and input switching where both the C and B-matrices switch. In this case the state space description of G becomes

$$\begin{aligned} x^+ &= Ax + B_\sigma u \\ y &= C_\sigma x \end{aligned} \tag{6}$$

Considering the LTI systems $G_{ik} \sim (A, B_k, C_i)$, the system G can be represented (see Figure 3) as the composition a 2x2 LTI system with the switching operators S and S^* as

$$G = S \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} S^*.$$

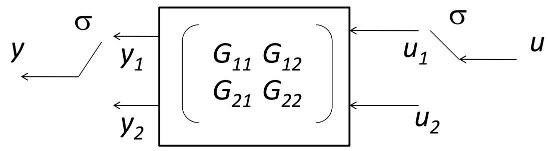

Fig. 3. Input-Output switching

A representation of G_σ in terms of an infinite lower triangular matrix is more elaborate than the one of the previous sections. Denoting $g_{ik} = \{g_{ik}(t)\}_{t=0}^{\infty}$ the unit pulse response of the LTI system G_{ik}, the (τ, ξ) entry will be one of the $g_{ik}(\tau - \xi)$; For example, for the sequence $\sigma = \{1, 1, 2, 1, 2, 2, \dots\}$ of the previous section, the form of G_σ is as

$$G_\sigma = \begin{bmatrix} 11 & & & & & & \\ 11 & 11 & & & & & \\ 21 & 21 & 22 & & & & \\ 11 & 11 & 12 & 11 & & & \\ 21 & 21 & 22 & 21 & 22 & & \\ 21 & 21 & 22 & 21 & 22 & 22 & \\ \dots\vdots & \dots\vdots & \dots\vdots & \dots\vdots & \dots\vdots & \dots\vdots & \dots\vdots \end{bmatrix} \tag{7}$$

where the entries "ik" correspond to $g_{ik}(\tau - \xi)$; so, the (3,3) entry is "11" and thus corresponds to $g_{11}(0)$, the (4,0) entry is "21" and thus corresponds to $g_{21}(4)$, etc. We note here that $SS^* = I$ and that for any norm it holds that $\|G\| \le \left\| \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \right\|$. However, this bound may be conservative. We will concentrate again on the ℓ_∞-induced norms case and assume for simplicity that G is a SISO system. Define the sequences

$$\begin{aligned} \bar{g}_1 =& \{\bar{g}_1(t)\}_{t=0}^{\infty} := \{|g_{11}(0)|, \max\{|g_{11}(1)|, |g_{12}(1)|\}, \\ & \max\{|g_{11}(2)|, |g_{12}(2)|\}, \dots\} \end{aligned}$$

and similarly

$$\begin{aligned} \bar{g}_2 =& \{\bar{g}_2(t)\}_{t=0}^{\infty} := \{|g_{22}(0)|, \max\{|g_{22}(1)|, |g_{21}(1)|\}, \\ & \max\{|g_{22}(2)|, |g_{21}(2)|\}, \dots\}. \end{aligned}$$

Let $\|\bar{g}_j\|_1 = \sum_{t=0}^{\infty} |\bar{g}_j(t)|$ be the ℓ_1-norm of the sequence $\bar{g}_j$. Then the following holds.

Theorem 5.1: With the above notations

$$\|G\| = \max_{j=1,2} \|\bar{g}_j\|_1$$

Proof: The proof relies on the matrix visualization of G_σ. To this end, one has to consider each row of G_σ and check its 1-norm, i.e., the absolute sum of its elements, to see what is its maximum possible value. For a given t let

$$g_{\sigma(t)} = [g_{\sigma(t)}(0), \dots, g_{\sigma(t)}(t)]$$

represent the t-th row of G_σ. We note that for the "last" element of the row we have $g_{\sigma(t)}(t) = g_{11}(0)$ if $\sigma(t) = 1$, or, $g_{\sigma(t)}(t) = g_{22}(0)$ if $\sigma(t) = 2$. We also note that $\sigma(t)$ does not affect any previous rows. Let's assume for the moment that $\sigma(t) = 1$ and so $g_{\sigma(t)}(t) = g_{11}(0)$. Then for the element $g_{\sigma(t)}(t-1)$ just before $g_{\sigma(t)}(t)$ we have $g_{\sigma(t)}(t-1) = g_{11}(1)$ if no switch occurred at $t-1$, i.e., $\sigma(t-1) = 1$, or, $g_{\sigma(t)}(t-1) = g_{12}(1)$ if a switch occurred at $t-1$, i.e., $\sigma(t-1) = 2$. Now, if $\max\{|g_{11}(1)|, |g_{12}(1)|\} = |g_{11}(1)|$ then it should clear that to maximize the 1-norm of the row we need not have a switch at $t-1$, that is we need $\sigma(t-1) = 1$. On the contrary, if $\max\{|g_{11}(1)|, |g_{12}(1)|\} = |g_{12}(1)|$ then maximize the 1-norm of the row we need to have a switch at $t-1$, that is we need $\sigma(t-1) = 2$. In this fashion, working backwards from $g_{\sigma(t)}(t)$ to $g_{\sigma(t)}(t-1)$, to $g_{\sigma(t)}(t-2)$, etc, we can find what is the "worst" possible 1-norm of row t, i.e., $\sup \sum_{\tau=0}^{t} |g_{\sigma(t)}(\tau)|$, and the corresponding switching sequence $\sigma(t), \sigma(t-1), \ldots$ when $\sigma(t) = 1$. This leads us to the definition of $\bar{g}_1$ of the theorem. Similarly, under the assumption that $\sigma(t) = 2$ and so $g_{\sigma(t)}(t) = g_{22}(0)$, using the same backwards approach we are led to the definition of $\bar{g}_2$ of the theorem. Therefore, the "worst" possible row will correspond to the $\max\{\|\bar{g}_1\|_1, \|\bar{g}_2\|_1\}$ as stated in the theorem. ∎

Generalizations to MIMO G are more complicated and will be presented in future publications. Generalizations to the case of N, SISO switching systems however, are immediate. For the SISO case regarding the ℓ_1-induced norm, dual results hold. That is,

$$\|G\|_* = \max_{j=1,2} \|\bar{g}_{j*}\|_1$$

where

$$\bar{g}_{1*} = \{\bar{g}_{1*}(t)\}_{t=0}^{\infty} := \{|g_{11}(0)|, \max\{|g_{11}(1)|, |g_{21}(1)|\}, \max\{|g_{11}(2)|, |g_{21}(2)|\}, \ldots\}$$

and similarly,

$$\bar{g}_{2*} = \{\bar{g}_{2*}(t)\}_{t=0}^{\infty} := \{|g_{22}(0)|, \max\{|g_{22}(1)|, |g_{12}(1)|\}, \max\{|g_{22}(2)|, |g_{12}(2)|\}, \ldots\}.$$

Finally, we mention that all the developments in sections do not rely at all on the strict causality of the systems, i.e., the fact that we did not include any feed-through term (D-matrix) in the state space descriptions; they rely only on the input output representations and so they still hold when there are nonzero D-matrices.

VI. Sensitivity Minimization

We consider the problem of sensitivity minimization when we have an output switching plant $P = S \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$ as in Figure 4. The LTI systems P_1 and P_2 are stable. We consider controllers K being parameterized by a Youla-Kucera parameter Q generated as an input switching of two stable LTI systems Q_1 and Q_2, i.e., $Q = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} S^*$. We remark here that these K are a subset of all possible stabilizing controllers due to the fact that we prescribe the

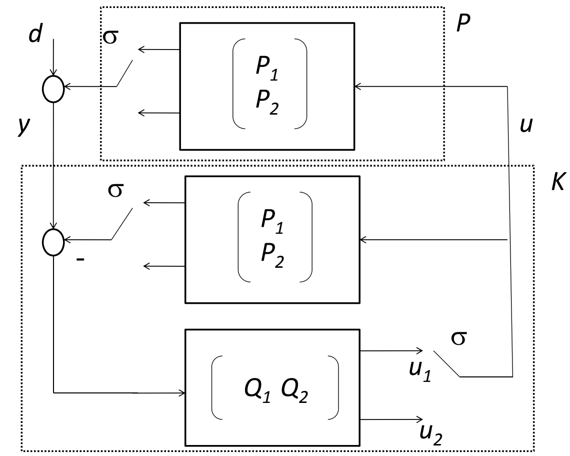

Fig. 4. Sensitivity minimization

structure of Q as an input switching system. More general parameterizations of (switching) stabilizing K can be found in [10]. At this point, it is not clear how much is missed by imposing this structure on Q, but we certainly search over a large class of Ks which lead to exact convex optimization problems. Indeed, the resulting K in Figure 4 is expressed as $K = Q(I + PQ)^{-1}$ and the resulting sensitivity map $\Phi : d \mapsto y = (I - PK)^{-1}$ becomes $\Phi = I + PQ$. Upon substitution of P and Q we obtain

$$\Phi = I + PQ = I + S \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} S^* = S \begin{bmatrix} I + P_1Q_1 & P_1Q_2 \\ P_2Q_1 & I + P_2Q_2 \end{bmatrix} S^*.$$

This shows that Φ is an input-output switching system. Based on the development in the previous section (Theorem 5.1), minimizing $\|\Phi\|$ is a convex problem, in fact a linear program. We should note here that in [5], among other things, the same problem was considered with Q being an output switching system. The resulted problem was, however, significantly more complicated as the composition of two output (input) switching systems generates more complex structures than an input-output switching system. Finally, we mention that if P and Q are input and output switching respectively and the input sensitivity map $\Psi := (I - KP)^{-1}$ is of interest it results in a similar expression as above, namely,

$$\Psi = I + QP = I + S \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \begin{bmatrix} P_1 & P_2 \end{bmatrix} S^* = S \begin{bmatrix} I + Q_1P_1 & Q_1P_2 \\ Q_2P_1 & I + Q_2P_2 \end{bmatrix} S^*$$

which is an input-output switching system. Minimizing $\|\Psi\|$ results in a convex problem. In fact, in this case the optimization of Q_1 and Q_2 decouples (as they are involved in separate rows.) With the same token, minimizing $\|\Phi\|_*$ decouples Q_1 and Q_2 (as they are involved in separate columns.) Furthermore, including control effort constraints in the two set ups of the form, $\|Q\|_* \leq \gamma$ or $\|Q\| \leq \gamma$, in minimizing $\|\Phi\|$ or $\|\Psi\|_*$, results in linear programs. These infinite dimensional, convex problems can be solved by the methods in [11] to obtain solutions within any pre-specified accuracy. It is also worth to point out that although the controller $K = Q(I + QP)^{-1}$ is a feedback interconnection of an input and an output switching system, it is not an input, or an output, or an input-output switching system.

VII. Conclusions

We presented exact expressions for the ℓ_∞ and ℓ_1 gains of certain types of switching systems. For the case of input-output switching of SISO systems we showed that certain sensitivity minimization problems are convex using the Y-K parametrization. Future work includes the full development of similar results for MIMO systems as well as coprime factorization of general switching systems in terms of specific structures.

VIII. Acknowledgments

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