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Identification and Fault Diagnosis for LPV Uncertain Systems

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Abstract— **In this paper, identification and fault diagnosis methods for uncertain Multiple Input Multiple Output (MIMO) Linear Parameters Varying (LPV) models is presented. The fault detection methodology is based on checking if measurements are inside the prediction bounds provided by a MIMO LPV model identified using real data and the parity equations approach. The proposed approach takes into account existing coupling between the different measured outputs. Modeling and prediction uncertainty bounds are handled using zonotopes. An identification algorithm that provides model parameters and their uncertainty such that all measured data free of faults will be inside the predicted bounds is also proposed. The fault isolation and estimation algorithm is based on the use of residual fault sensitivity. Finally, a case study based on a four tank system is used to illustrate the effectiveness of the proposed approach.**

I. Introduction

MODEL-BASED fault diagnosis is based on the use of mathematical models of the monitored system. Currently, most of the existing approaches that have been investigated and developed over the last few years are based on linear models (see, e.g, [1], [2] and [3]). However, physical systems are inherently non-linear. This has motivated the interest of researchers in the development and application of non-linear *Fault Detection and Isolation* (FDI) methods [4]. LPV models can be used to efficiently represent some non-linear systems. This is the reason why they have recently attracted the attention of the FDI research community to develop model-based methods using this kind of models. But, even with the use of LPV models, modeling errors are present. Reliability and performance of fault diagnosis algorithms depend on the quality of the model used. Thus, since modeling errors introduce uncertainty in the model, they interfere with the fault detection. A fault detection algorithm able to handle uncertainty is called *robust* and its *robustness* is the degree of sensitivity to faults compared to the degree of sensitivity to uncertainty. In this work, the description of the noise is based on what is known as "*unknown but bounded noise*" description [5]. Moreover, not only noise but also bounded modeling errors are considered. Fault detection methods that only require knowledge about bounds in noise and parameters are known as *set-membership* and follow the passive robust approach [2] by enhancing the fault detection robustness at the decision-making stage using an adaptive threshold.

The contribution of this paper is to propose new identification and fault diagnosis approaches for systems that can be described by MIMO LPV models with uncertainty. The identification approach provides model parameters and their uncertainty such that all measured data free of faults will be inside the predicted bounds. The fault detection methodology is based on extending the *parity equations approach* proposed by [1] to this kind of systems. The fault detection procedure consists in comparing on-line the real behavior of the monitored system obtained by means of sensors with the estimated behavior using the uncertain MIMO LPV model. In case of a significant discrepancy (residual) is detected between the model and the measurements obtained by the sensors, the existence of a fault is assumed. In particular, in this work the parametric uncertainty is bounded by a zonotope and propagated to the residuals determining their alarm limits bounded by a zonotope as well. When the residuals are outside of the zonotope that define the alarm limits, it is argued that model uncertainty alone can not explain the residual and therefore the presence of a fault is proved.

Finally, a fault isolation approach based on the residual fault sensitivity [6] that provides an estimation of the fault magnitude is proposed.

The structure of this paper is the following: *Section II* introduces LPV parity equations with uncertainty and the consistency test used for fault detection. In *Section III*, a parameter estimation procedure that allows bounding the parametric uncertainty is described. *Section IV* presents a fault detection methodology based on the use of LPV parity equations and zonotopes. *Section V* presents the fault isolation and estimation methodology. Finally, in *Section VI*, a four tank system is used to assess the validity of the proposed approach.

II. LPV Parity Equations with Uncertainty

A. LPV parity equations with uncertainty

In this paper, the system to be monitored is assumed that can be described by a MIMO LPV model

$$\boldsymbol{y}(k)=\boldsymbol{\Phi}(k)\boldsymbol{\theta}(\mathbf{p}_k)+\boldsymbol{e}(k)=\hat{\boldsymbol{y}}(k)+\boldsymbol{e}(k) \quad (1)$$

where

- $\boldsymbol{y}(k)$ is the output vector of dimension $n_y \times 1$.
- $\boldsymbol{\Phi}(k)$ is the regressor matrix of dimension $n_y \times n_\theta$ which can contain any function of inputs $\boldsymbol{u}(k)$ and outputs $\boldsymbol{y}(k)$.
- $\mathbf{p}_k \triangleq \mathbf{p}(k)$ is a vector of measurable process variables of dimension $n_p \times 1$ that defines the system operating point.

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- $\boldsymbol{\theta}(\mathbf{p}_k) \in \boldsymbol{\Theta}_k$ is the LPV parameter vector of dimension $n_\theta \times 1$ whose values can vary according to the system operating point following some known function $g(\mathbf{p}_k)$, usually named as *scheduling function*.
- $\boldsymbol{\Theta}_k$ is the set that bounds parameter values that can vary according to the system operating point as well.
- $\boldsymbol{e}(k)$ is a vector of dimension $n_y \times 1$ that contains the sensor additive noises whose components have known bounds $|e_i(k)| \le \sigma_i$, $i = 1,...,n_y$.

In this paper, the uncertain parameter set $\boldsymbol{\Theta}_k$ is described by a zonotope centered in the nominal LPV model

$$\boldsymbol{\Theta}_k = \boldsymbol{\theta}^0(\mathbf{p}_k) \oplus \mathbf{H}\mathbb{B}^n = \left\{\boldsymbol{\theta}^0(\mathbf{p}_k) + \mathbf{Hz} : \mathbf{z} \in \mathbb{B}^n\right\} \quad (2)$$

where
- $\boldsymbol{\theta}^0(\mathbf{p}_k) \in \mathbb{R}^{n_\theta}$ is the nominal LPV model.
- $\mathbf{H} \in \mathbb{R}^{n_\theta \times n}$ is the shape of the zonotope.
- $\mathbb{B}^n \in \mathbb{R}^{n \times 1}$ is a unitary box composed by n unitary ($\mathbb{B} = [-1,1]$) interval vectors.
- $\oplus$ denotes the Minkowski sum.

B. Consistency Test

Given a MIMO LPV model (1), the output measurement vector $\boldsymbol{y}(k)$ will be consistent with the output predicted by the model when

$$\boldsymbol{y}(k) \in \boldsymbol{\Upsilon}(k) \quad (3)$$

where $\boldsymbol{\Upsilon}(k)$ is the output predicted set. When the uncertain parameter set $\boldsymbol{\Theta}_k$ is described by means of a zonotope, as in (2), then $\boldsymbol{\Upsilon}(k)$ can be expressed as follows

$$\boldsymbol{\Upsilon}(k) = \hat{\boldsymbol{y}}^0(k) \oplus \hat{\boldsymbol{\Gamma}}(k) \quad (4)$$

with

$$\hat{\boldsymbol{y}}^0(k) = \boldsymbol{\Phi}(k)\boldsymbol{\theta}^0(\mathbf{p}_k) \quad (5)$$

and

$$\hat{\boldsymbol{\Gamma}}(k) = \begin{pmatrix}\boldsymbol{\Phi}(k)\mathbf{H} & \mathbf{E}\end{pmatrix} \mathbb{B}^{n+n_y} \quad (6)$$

where $\mathbf{E} = diag\left(\sigma_1, \cdots, \sigma_{n_y}\right)$.

Notice that $\boldsymbol{\Upsilon}(k)$ is a zonotope centered in the nominal output estimation $\hat{\boldsymbol{y}}^0(k)$ and with a shape defined by $\hat{\boldsymbol{\Gamma}}(k)$. Thus, condition (3) can be rewritten as

$$\boldsymbol{r}^0(k) \in \hat{\boldsymbol{\Gamma}}(k) \quad (7)$$

where $\boldsymbol{r}^0(k)$ is the MIMO nominal residual defined by

$$\boldsymbol{r}^0(k) = \boldsymbol{y}(k) - \hat{\boldsymbol{y}}^0(k) \quad (8)$$

III. Uncertain Parameter Estimation

A. Problem definition

The problem of the uncertain parameter estimation can be formulated as follows: Let us consider a sequence of M regressor matrix values $\boldsymbol{\Phi}(k)$, output measurements $\boldsymbol{y}(k)$ in a fault free scenario, the model of the system to be monitored parameterized as in (1) and the parameter set $\boldsymbol{\Theta}_k$ as in (2). The aim is to estimate a nominal parameter vector $\boldsymbol{\theta}^0(\mathbf{p}_k)$ and their uncertainty (model set) defined by the matrix $\mathbf{H}$ in such a way that all measured data in a fault free scenario satisfy condition (7).

Nominal LPV parameters $\boldsymbol{\theta}^0(\mathbf{p}_k)$ could be obtained using LPV identification algorithms as the ones proposed by [7] using real data. As a result of this process, some modeling uncertainty (2) in the LPV parameters appears. Then, once the nominal LPV parameters $\boldsymbol{\theta}^0(\mathbf{p}_k)$ have been calibrated the problem of computing the uncertainty in the parameters, defined by matrix $\mathbf{H}$, can be formulated as an optimization problem. In order to maximize the model sensitivity to faults, the objective function of the optimization problem can be the volume of the output predicted set (4)

$$J = \sum_{k=1}^{M}(vol(\boldsymbol{\Upsilon}(k))) \quad (9)$$

and whose constraints are defined by condition (7). This optimization problem with no assumptions about the knowledge of matrix $\mathbf{H}$ is in general very hard to solve even in the single output case [8]. In order to reduce the complexity, the zonotope that bounds $\boldsymbol{\Theta}_k$ can be parameterized such that $\mathbf{H} = \lambda\mathbf{H}_0$, as proposed in [8] for the single output case, that corresponds to a zonotope with predefined shape (determined by $\mathbf{H}_0$) and a scalar $\lambda \ge 0$. $\mathbf{H}_0$ can be obtained using physical knowledge of the system or extracting this information from the identification.

B. Uncertainty shape $\mathbf{H}_0$

Matrix $\mathbf{H}_0$ determines the weight and relations between the different parameter uncertainties. In this section, a data-based procedure to find a suitable $\mathbf{H}_0$ is presented. In the following, it will be described how to find a convex set $\Delta\boldsymbol{\Theta}$ considering consistency test condition (7) for all the identification data with model (1) and parameter vector $\boldsymbol{\theta}(\mathbf{p}_k) \in \boldsymbol{\Theta}_k$ with

$$\boldsymbol{\Theta}_k = \boldsymbol{\theta}^0(\mathbf{p}_k) + \Delta\boldsymbol{\Theta} \quad (10)$$

Notice that this parameterization of the parametric uncertainty is more general than the parametric uncertainty bounded by a zonotope given in (2), as a zonotope is a particular case of convex set.

Then, the matrix $\mathbf{H}_0$ will be computed in such a way that the zonotope centered in the origin defined by $\mathbf{H}_0$ include the set $\Delta\boldsymbol{\Theta}$

$$\Delta\boldsymbol{\Theta} \subseteq \mathbf{H}_0\,\mathbb{B}^n \quad (11)$$

Step 1. Finding convex set $\Delta\boldsymbol{\Theta}$

At every instant k, the regressor matrix $\boldsymbol{\Phi}(k)$ and the measured output $\boldsymbol{y}(k)$ define two half-spaces $\Delta\overline{\boldsymbol{\Theta}}_k$ and $\Delta\underline{\boldsymbol{\Theta}}_k$ in $\mathbb{R}^{n_\theta}$ that fulfill

$$\Delta\bar{\Theta}_k = \left\{\Delta\theta \in \mathbb{R}^{n_\theta} : \varphi_i(k)\Delta\theta \geq y_i(k) - \hat{y}_i^0(k) - \sigma_i, \forall i = 1,..,n_y\right\}$$
$$\Delta\underline{\Theta}_k = \left\{\Delta\theta \in \mathbb{R}^{n_\theta} : \varphi_i(k)\Delta\theta \leq y_i(k) - \hat{y}_i^0(k) + \sigma_i, \forall i = 1,..,n_y\right\} \quad (12)$$

where $\varphi_i(k)$ denotes the row i of the regressor matrix $\Phi(k)$. Then, the sets $\Delta\bar{\Theta}$ and $\Delta\underline{\Theta}$ that fulfill conditions (12), for all the instants k, are defined by

$$\Delta\bar{\Theta} = \bigcap_{k=1}^{M} \Delta\bar{\Theta}_k \text{ and } \Delta\underline{\Theta} = \bigcap_{k=1}^{M} \Delta\underline{\Theta}_k \quad (13)$$

Notice that: $\Delta\bar{\Theta}$ and $\Delta\underline{\Theta}$ are $\mathcal{H}$-polyhedrons defined by the intersection of half-spaces represented by linear inequalities [9]. And finally, the set $\Delta\Theta$ that fulfills condition (7) $\forall k = 1,...,M$ satisfies

$$\Delta\Theta \cap \Delta\bar{\Theta} \neq \varnothing \text{ and } \Delta\Theta \cap \Delta\underline{\Theta} \neq \varnothing \quad (14)$$

Condition (14) implies that at least one point of every $\mathcal{H}$-polyhedron defined in (13) belongs to the uncertain set $\Delta\Theta$. These two points will be denoted as $\Delta\bar{\theta}$ and $\Delta\underline{\theta}$ belonging to the $\mathcal{H}$-polyhedrons $\Delta\bar{\Theta}$ and $\Delta\underline{\Theta}$, respectively. In addition to condition (14), the set $\Delta\Theta$ has to minimize the volume of the output predicted set defined in (9). $\Delta\bar{\theta}$ and $\Delta\underline{\theta}$ can be computed solving two linear optimization problems that minimize the uncertainty of the output estimation subject to (12).

Step 2. Bounding convex set $\Delta\Theta$ using a zonotope

Once $\Delta\bar{\theta}$ and $\Delta\underline{\theta}$ have been calculated, the zonotope centered in the origin that contains these two points can be determined as the box (particular case of zonotope) whose opposite vertex are $\Delta\bar{\theta}$ and $\Delta\underline{\theta}$. This zonotope fulfills (11) and is defined by $\mathbf{H}_0$ with

$$\mathbf{H}_0 = \mathrm{diag}\left(\mathrm{H}_{1,1} \quad \cdots \quad \mathrm{H}_{n_\theta,n_\theta}\right) \quad (15)$$

where $\mathrm{H}_{i,i} = \max\left(abs(\Delta\bar{\theta}_i), abs(\Delta\underline{\theta}_i)\right) \quad i = 1,..,n_\theta$

In order to benefit of the richness of the zonotope representation of the uncertain parameter set, that allows to take into account possible dependencies between the different components of the parametric uncertainty, the data can be divided depending on the direction of the regressor matrix in n_D groups. Then, the parameter sets (13) can be obtained for every set of data ($\Delta\bar{\Theta}^j$ and $\Delta\underline{\Theta}^j \quad j = 1,..,n_D$). And, in the same way, optimal parameters ($\Delta\bar{\theta}^j$ and $\Delta\underline{\theta}^j \quad j = 1,..,n_D$) can be calculated. Then, the zonotope centered in the origin that contains these $2n_D$ points can be calculated considering

$$\mathbf{H}_0 = \left(\alpha_1\mathbf{d}_1, \cdots, \alpha_{nD}\mathbf{d}_{nD}\right) \quad (16)$$

where $\mathbf{d}_j \quad j = 1,..,n_D$ are the unitary vectors (dimension $n_\theta \times 1$) that define the directions of the data sets, and $\alpha_j \geq 0 \quad j = 1,..,n_D$ are the coefficients to be computed. The problem of computing $\alpha_j \quad j = 1,..,n_D$ can be formulated as a linear programming optimization problem.

Once the matrix $\mathbf{H}_0$ has been determined, the problem of uncertainty identification can be reformulated as an optimization problem that minimizes (9) subject to (7) considering $\mathbf{H} = \lambda\mathbf{H}_0$. This problem is a non-linear polynomial optimization problem that can be solved globally using *GloptiPoly* tool developed by [10].

IV. Fault Detection Methodology

The fault detection methodology is based on the residual evaluation obtained from the difference between measurements and LPV model prediction using (1)

$$\boldsymbol{r}(k) = \boldsymbol{y}(k) - \hat{\boldsymbol{y}}(k) - \boldsymbol{e}(k) = \boldsymbol{y}(k) - \Phi(k)\theta(\mathbf{p}_k) - \boldsymbol{e}(k) \quad (17)$$

Residual (17) corresponds to a *Moving Average* (MA) parity equation [1]. Ideally, when modeling errors and noise are neglected, residual (17) should be zero in a fault-free scenario and different from zero, otherwise. However, because of modeling errors and noise, residual can be different from zero in a non faulty scenario. In order to take into account uncertainty in parameters and additive noise, the effects of these uncertainties will be propagated to the residual, defining the region of admissible residuals $\Gamma(k)$. Then consistency test (3) is equivalent to check

$$\mathbf{0} \in \Gamma(k) \quad (18)$$

where $\mathbf{0}$ is a vector ($n_y \times 1$) of zeros $\mathbf{0} = (0 \cdots 0)^t$.

Taking into account (17), (8) and (4), $\Gamma(k)$ can be parameterized as a zonotope

$$\Gamma(k) = \boldsymbol{r}^0(k) \oplus \hat{\Gamma}(k) \quad (19)$$

where $\boldsymbol{r}^0(k)$ and $\hat{\Gamma}(k)$ are defined as in (8) and (6). Then, test (18) involves checking if the point $\mathbf{0}$ belongs to the zonotope $\Gamma(k)$ and can be formulated as the following system of n_y equalities

$$\begin{array}{c} y_1(k) - \hat{y}_1^0(k) - \varphi_1(k)\mathbf{H}\mathbf{z}(k) - e_1(k) = 0 \\ \vdots \\ y_{n_y}(k) - \hat{y}_{n_y}^0(k) - \varphi_{n_y}(k)\mathbf{H}\mathbf{z}(k) - e_{n_y}(k) = 0 \end{array} \quad (20)$$

Checking the satisfaction of conditions (20) can be formulated as the feasibility of a linear programming problem without objective function.

V. Fault Isolation and Estimation Methodology

A. Fault Sensitivity

Residuals (17) can be expressed in a compact form using the fault sensitivity transfer function matrix, defined as

$$\boldsymbol{r}(k) = \mathbf{S}(q^{-1}, \theta(\mathbf{p}_k), \boldsymbol{f})\boldsymbol{f}(k) + \boldsymbol{e}(k) \quad (21)$$

where

- $\boldsymbol{f}(k)$ is the vector of possible faults of dimension $n_f \times 1$.
- q^{-1} is the shift operator.

- $\mathbf{S}(q^{-1},\boldsymbol{\theta}(\mathbf{p}_k),\boldsymbol{f})$ is the fault sensitivity matrix of dimension $n_y \times n_f$ that can be calculated as

$$\mathbf{S}(q^{-1},\boldsymbol{\theta}(\mathbf{p}_k),\boldsymbol{f}) = \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{f}} = \begin{pmatrix} \frac{\partial \boldsymbol{r}}{\partial f_1} & \cdots & \frac{\partial \boldsymbol{r}}{\partial f_{n_f}} \end{pmatrix} \tag{22}$$

On the other hand, nominal multi-output residual (8) can be expressed as

$$\boldsymbol{r}^0(k) \simeq \mathbf{S}(q^{-1},\boldsymbol{\theta}^0(\mathbf{p}_k),\boldsymbol{f})\boldsymbol{f}(k) \tag{23}$$

where $\mathbf{S}(q^{-1},\boldsymbol{\theta}^0(\mathbf{p}_k),\boldsymbol{f})$ is the sensitivity matrix (22) particularized for the nominal parameter vector $\boldsymbol{\theta}^0(\mathbf{p}_k)$.

B. Fault Isolation and Estimation Algorithm

Considering (23), the problem of fault isolation and estimation can be formulated as a least squares problem. Assuming abrupt faults,

$$\boldsymbol{f}(k) = \begin{cases} \mathbf{0}, & k < k_{fault} \\ \boldsymbol{f}_0, & k \ge k_{fault} \end{cases} \tag{24}$$

i.e, faults have appeared at instant k_{fault} and $\boldsymbol{f}_0 \in \mathbb{R}^{n_f}$. This problem can be implemented by solving *Problem 1*, once the fault has been detected.

Problem 1: "Fault isolation and estimation (general case)"

$\boldsymbol{f}_0(k) = \arg\min_{\boldsymbol{f}} \{J(\boldsymbol{f},k)\}$

subject to

$$J(\boldsymbol{f},k) = \sum_{i=\max\{k_{fault},k-\ell+1\}}^{k} \left\| \boldsymbol{r}^0(i) - \mathbf{S}\left(q^{-1},\boldsymbol{\theta}^0(\mathbf{p}_i),\boldsymbol{f}\right)\boldsymbol{f} \right\|^2$$

where ℓ is a time moving horizon

The role of the time moving horizon ℓ is to minimize the noise and parameter uncertainty effects: since the longer the moving horizon is, the smaller these effects will be. However, increasing the moving horizon will lead to slower fault isolation.

Problem 1 can be simplified when single faults are considered, this is $\boldsymbol{f}_0(k) = f_0\begin{pmatrix}0 & \cdots & 1 & \cdots & 0\end{pmatrix}^{\mathrm{t}}$ in (24). Then the problem of fault isolation and estimation can be solved through *Algorithm 1* that implies solving n_f least squares error optimization problem, one for every possible n_f single faults. The most probable fault $I(k)$ at time k is determined as the fault that gives the minimum cost function $J_j(f,k)$ after solving the set of least squares problems for the set of considered single faults.

Algorithm 1:"Fault isolation and estimation (single faults)"

1: ***for*** $j = 1,\cdots,n_f$ ***do***

2: $\left(J_j^{opt}(k), f_j^{opt}(k)\right) \leftarrow \min_f J_j(f,k)$

subject to

$$J_j(f,k) = \sum_{i=\max\{k_{fault},k-\ell+1\}}^{k} \left\| \boldsymbol{r}^0(i) - \boldsymbol{s}^0_{\_,j} f \right\|^2$$

where

$\boldsymbol{s}^0_{\_,j} = \partial \boldsymbol{r}^0 / \partial f_j$ is the j^{th} column of $\mathbf{S}\left(q^{-1},\boldsymbol{\theta}^0(\mathbf{p}_k),\boldsymbol{f}\right)$

3: ***end for***

4: $I(k) = \arg\min_{j=\{1,\cdots,n_f\}} \left\{J_j^{opt}(k)\right\}$

5: $f_0(k) = f_{I(k)}^{opt}(k)$

VI. Case Study: Four Tank System

A. Description of the system

A quadruple tank process, proposed by [11], will be used to illustrate the results presented in this paper. The process inputs are v_1 and v_2 (input voltages to the pumps) and the outputs are the tank levels h_1, h_2, h_3 and h_4. The equations that describe the system are

$$\begin{aligned} \frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \\ \frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \\ \frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 \\ \frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1 \end{aligned} \tag{25}$$

where $a_1 = a_3 = 0.071cm^2$, $A_1 = A_3 = 28cm^2$, $\gamma_1 = 0.7$, $k_1 = 3.33cm^3/Vs$, $a_2 = a_4 = 0.057cm^2$, $A_2 = A_4 = 32cm^2$, $\gamma_2 = 0.6$ $k_2 = 3.35cm^3/Vs$, $g = 981cm/s^2$ and assumed constants.

Eq. (25) can be discretized by the Euler method with sampling time $\Delta t = 1s$ and it can be expressed as in (1) through the following parameterization

$\boldsymbol{y}(k) = \begin{pmatrix}h_1(k) & h_2(k) & h_3(k) & h_4(k)\end{pmatrix}^{\mathrm{t}}$, $\boldsymbol{\Phi}(k) = \begin{pmatrix}\boldsymbol{\Phi}_1(k) & \boldsymbol{\Phi}_2(k)\end{pmatrix}$

$\boldsymbol{\theta}(\mathbf{p}_k) = \begin{pmatrix}\boldsymbol{\theta}_1(\mathbf{p}_k) \\ \boldsymbol{\theta}_2\end{pmatrix}$, $\boldsymbol{e}(k) = \begin{pmatrix}e_1(k) & e_2(k) & e_3(k) & e_4(k)\end{pmatrix}^{\mathrm{t}}$

with

$$\boldsymbol{\Phi}_1(k) = \begin{pmatrix} h_1(k-1) & h_3(k-1) & 0 & 0 & 0 & 0 \\ 0 & 0 & h_2(k-1) & h_4(k-1) & 0 & 0 \\ 0 & 0 & 0 & 0 & h_3(k-1) & 0 \\ 0 & 0 & 0 & 0 & 0 & h_4(k-1) \end{pmatrix}$$

$$\boldsymbol{\Phi}_2(k) = \begin{pmatrix} v_1(k-1) & 0 & 0 & 0 \\ 0 & v_2(k-1) & 0 & 0 \\ 0 & 0 & v_2(k-1) & 0 \\ 0 & 0 & 0 & v_1(k-1) \end{pmatrix}, \quad \boldsymbol{\theta}_2 = \begin{pmatrix} b_{1,1} \\ b_{2,2} \\ b_{3,1} \\ b_{4,2} \end{pmatrix}$$

$$\boldsymbol{\theta}_1(\mathbf{p}_k) = \begin{pmatrix} a_{1,1}(\mathbf{p}_k) & a_{1,3}(\mathbf{p}_k) & a_{2,2}(\mathbf{p}_k) & a_{2,4}(\mathbf{p}_k) & a_{3,3}(\mathbf{p}_k) & a_{4,4}(\mathbf{p}_k) \end{pmatrix}^{\mathrm{t}}$$

where $\mathbf{p}_k = \begin{pmatrix} h_1(k-1) & h_2(k-1) & h_3(k-1) & h_4(k-1) \end{pmatrix}^{\mathrm{t}}$,

$$b_{1,1} = \frac{\gamma_1 k_1}{A_1}, b_{2,2} = \frac{\gamma_2 k_2}{A_2}, b_{3,1} = \frac{(1-\gamma_2)k_2}{A_3}, b_{4,2} = \frac{(1-\gamma_1)k_1}{A_4}$$

$$a_{i,i}(\mathbf{p}_k) = 1 - \frac{a_i\sqrt{2g}}{A_i} g_i(\mathbf{p}_k) \quad i = 1,\dots,4 \quad a_{1,3}(\mathbf{p}_k) = \frac{a_3\sqrt{2g}}{A_1} g_3(\mathbf{p}_k)$$

and $$a_{2,4}(\mathbf{p}_k) = \frac{a_4\sqrt{2g}}{A_2} g_4(\mathbf{p}_k) \tag{26}$$

Additive errors $e_i(k)$ contain the noise effect and discretization error of the level sensors ($\sigma_i = 0.05\text{cm}$, $i = 1,\dots,4$). The parameter uncertainty is assumed to be located in parameters $a_{j,i}$ because are functions of the measurements through the scheduling functions $g_i(\mathbf{p}_k)$, $i = 1,\dots,4$

$$g_i(\mathbf{p}_k) = g_i^0(\mathbf{p}_k) + \Delta g_i(k) \tag{27}$$

where

$$g_i^0(\mathbf{p}_k) = \frac{1}{\sqrt{h_i(k-1)}} \tag{28}$$

and $\Delta g_i(k)$ is the uncertainty.

Then, the LPV parameter vector $\boldsymbol{\theta}(\mathbf{p}_k)$ can be expressed as

$$\boldsymbol{\theta}(\mathbf{p}_k) = \begin{pmatrix} \boldsymbol{\theta}_1^0(\mathbf{p}_k) \\ \boldsymbol{\theta}_2 \end{pmatrix} + \begin{pmatrix} \Delta\boldsymbol{\theta}_1(k) \\ 0 \end{pmatrix} \tag{29}$$

with nominal parameters

$$\boldsymbol{\theta}_1^0(\mathbf{p}_k) = \begin{pmatrix} a_{1,1}^0(\mathbf{p}_k) & a_{1,3}^0(\mathbf{p}_k) & a_{2,2}^0(\mathbf{p}_k) & a_{2,4}^0(\mathbf{p}_k) & a_{3,3}^0(\mathbf{p}_k) & a_{4,4}^0(\mathbf{p}_k) \end{pmatrix}^{\mathrm{t}}$$

where $a_{j,i}^0(\mathbf{p}_k)$ are obtained with (26) considering $g_i(\mathbf{p}_k) = g_i^0(\mathbf{p}_k)$.

Thus, parametric uncertainty $\Delta\boldsymbol{\theta}_1(k)$ can be expressed using a zonotope as in (2) as follows

$$\Delta\boldsymbol{\theta}_1(k) \in \mathbf{H}\,\mathbb{B}^n \tag{30}$$

B. Identification

In order to apply identification techniques presented in *Section III*, input/output data in a fault free scenario applying a pseudorandom binary sequence (PRBS) as pump input that sweep all the operation range ($v_1 \in [2.4, 3.8]$ V and $v_2 \in [2.3, 3.5]$ V) has been used.

In order to take into account physical relations in parametric uncertainties and for the sake of simplicity the following parameterization of matrix $\mathbf{H}$ in (30) is used

$$\mathbf{H} = \mathbf{H}_t \mathbf{H}_g \tag{31}$$

with

$$\mathbf{H}_t = \sqrt{2g} \begin{pmatrix} \frac{-a_1}{A_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-a_2}{A_2} & 0 & 0 & 0 \\ 0 & \frac{a_3}{A_1} & 0 & 0 & \frac{-a_3}{A_3} & 0 \\ 0 & 0 & 0 & \frac{a_4}{A_2} & 0 & \frac{-a_4}{A_4} \end{pmatrix}^{\!\mathrm{t}} \tag{32}$$

and $\mathbf{H}_g \in \mathbb{R}^{4\times n}$ is the matrix to be identified that defines the zonotope that bounds parameters $\Delta\boldsymbol{g}(k)$, that is

$$\Delta\boldsymbol{g}(k) = \begin{pmatrix} \Delta g_1(k) & \Delta g_2(k) & \Delta g_3(k) & \Delta g_4(k) \end{pmatrix}^{\mathrm{t}} \in \mathbf{H}_g\,\mathbb{B}^n \tag{33}$$

The parametric uncertainty $\Delta\boldsymbol{\theta}_1(k)$ in (34) can be viewed a linear transformation of $\Delta\boldsymbol{g}(k)$.

$$\Delta\boldsymbol{\theta}_1(k) = \mathbf{H}_t \Delta\boldsymbol{g}(k) \tag{34}$$

considering $\mathbf{H}_g = \lambda\mathbf{H}_0$, with $\mathbf{H}_0$ is obtained as it is described in *Section III.B* by dividing the data in four different directions. Once $\mathbf{H}_0$ has been calculated, λ is computed as described in *Section III*.

C. Fault detection, isolation and estimation

In order to illustrate the fault detection, isolation and estimation procedures described in *Sections IV* and *V* in several fault scenarios, two different kinds of faults have been considered: additive faults (in input and output sensors: $\boldsymbol{f_u}$ and $\boldsymbol{f_y}$) and multiplicative faults (in parameters: $\boldsymbol{f}_\theta$).

Only single faults have been considered. The time moving horizon used in the fault isolation and estimation procedure is $\ell = 20\text{s}$. This horizon has been selected as a trade-off to minimize the noise effect and to follow possible fault changes.

Fault isolation and estimation method presented in *Section V* is based on the fault sensitivity transfer function matrix (22). In our case study, with 4 residuals and 12 possible different faults to be detected, the dimension of the matrix (22) is 4×12.

In the following, a fault scenario has been simulated and the results of the fault detection, isolation and estimation procedures are presented.

Fault scenario: *" y_1 sensor additive fault of $f_{y_1} = 0.8cm$ at t=9500s"*

Figures 1a) and b) show the result of the detection test in this fault scenario. The fault is detected at the appearance time (*t*=9500s). At this time, the residual admissible space $\boldsymbol{\Gamma}$ does not contain the origin in the $r_1 - r_3$ projection, thus condition (18) is not fulfilled what proves the existence of a fault.

Once the fault has been detected, the fault isolation and estimation procedure is activated by solving optimization *Algorithm 1* considering the 12 different possible single faults. Figure 2 shows the inverse of the optimization cost function for the different considered faults, obtained solving *Algorithm 1* since the fault detection time (t=9500s).

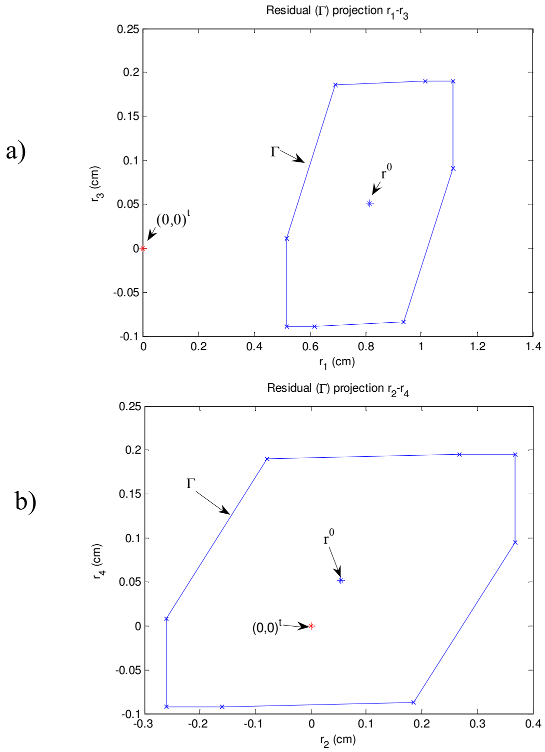

Figure 1: Residual admissible space $\boldsymbol{\Gamma}$ projections a) $r_1 - r_3$ and b) $r_2 - r_4$, at the fault time detection (t=9500s).

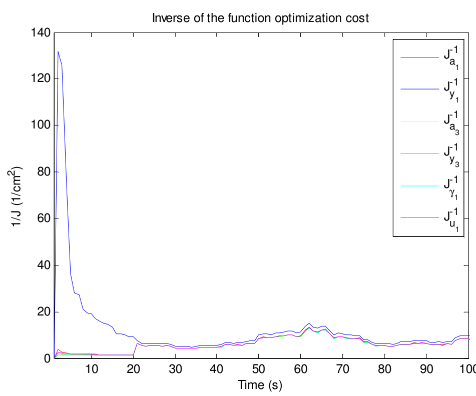

Figure 2. Inverse of optimization cost function (J^{-1}) of different considered faults.

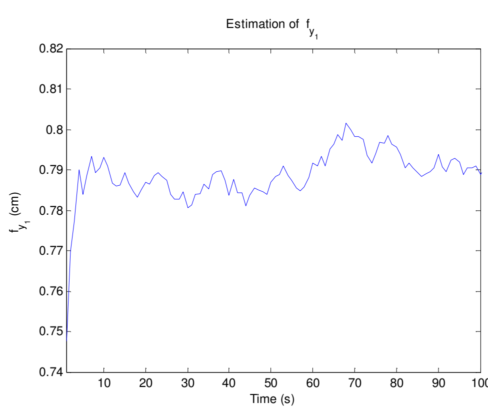

Figure 3. Fault estimation of f_{y_1}

As the objective function (J) corresponding to f_{y_1} is smaller than the objective function corresponding to the other considered faults, the fault isolation algorithm determines that the fault is an additive fault affecting output sensor y_1. The fault magnitude estimation corresponding to f_{y_1}, determined also as result of the solution of *Algorithm 1*, is presented in Figure 3. The time origin of Figures 2 and 3 corresponds with the fault detection time (t=9500s).

8. CONCLUSIONS

In this paper, identification and fault diagnosis methods for systems that can be modeled by uncertain MIMO LPV models have been presented. The identification procedure is formulated as an optimization problem that determines a zonotope that encloses the parametric uncertainty given a model structure and additive error bounds. The fault detection methodology is based on checking if measurements are inside the prediction bounds provided by the LPV model, parametric uncertainty and additive error. It has been formulated as the feasibility problem that can be solved using linear programming techniques. The fault isolation and estimation algorithm is based on residual fault sensitivity analysis. This methodology allows increasing fault isolability by considering the residual fault sensitivity as additional information to the relationship between residuals and faults. Moreover, it allows obtaining fault estimation. Finally, satisfactory results have been obtained using a case study based on a four tank system.

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