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DESIGN OF AN EVENT-BASED FEEDFORWARD STRATEGY FOR SOPTD PROCESSES

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***Abstract*— An event-based method to perform the set-point following task for second order processes with time delay (SOPTD) is investigated in this work. The aim of the research is to extend an event-based controller that has been tested previously in processes responding to the first-order with time delay (FOPTD) model. The controller is based on feedforward and feedback actions to do the two main tasks of a controller, set-point following and disturbances rejection, with an event triggered paradigm. This work is mainly focused on the feedforward part of the controller, with the goal of establishing a method to calculate the control signal and the conditions that must be satisfied to update it.**

***Index Terms*— Event-based, PID control, feedforward**

I. INTRODUCTION

Event-based strategies are advantageous when the nature of the control problem impose restrictions over the number of control actions to be applied. This is due to the fact that, in periodic control systems, communications are done regardless of the state of the plant, and thus much of the information flow can be unnecessary. In [1] periodic and event-based sampling for first order stochastic systems are compared. From the implementation point of view, frequently the data acquisition hardware is designed to be used in periodic sampling systems. It is thus necessary in these cases to perform periodic sampling at high frequency and then to implement by software the event detection mechanism. Moreover, in many cases the energy efficiency is a key issue, as it occurs with wireless sensor network (see [2]), where the elements have autonomous power supply. In this cases the communication must be optimized to obtain the maximum life of the batteries.

Different types of event-based PID control algorithms have been proposed in recent works. In [3] and [4] a send-on-delta sampling is incorporated to the controller. In [5] a state-feedback approach with a disturbance estimator is investigated.

Our approach is similar to that described in [6], where the two main controller tasks, i.e. set-point following and disturbances rejection, are performed with decoupled event-based feedforward and feedback actions. The controller is extended to the case of second order processes, because although the FOPTD model is simple and describes the dynamics of many industrial processes, there are situations when it is convenient to consider other models, and the SOPTD model is a natural generalization of the FOPTD model. In these cases where the first order model is not enough, we can take advantage of the second order model capabilities while maintaining the same approach of the method. The idea is to use the feedforward actions to perform the set-point following task with the minimum number of control actions, and the feedback actions to cope with the disturbances. Thus it is necessary to solve different design problems: the first one is to find a method to generate the control signals, and since an event-based paradigm is considered, it is also needed to define the conditions that trigger the feedforward part events. The second problem is to describe the disturbances rejection task by defining the control law and the events conditions that determine how and when the control signal must be updated. In addition, it is necessary to perform the coupling of the two parts. Though both tasks are important, this work is mainly focused on the event-based feedforward design problem. Finally, with the purpose of illustrating the described algorithm and to show that the developed methods are applicable in real situations, several examples with simulations and real plants are presented. In these examples, as in the theory, the two problems are considered independently: set-point following and disturbances rejection tasks.

The organization of this paper is as follows: Section II describes the design of the feedforward algorithm, and Section III describes the feedback part. Section IV presents the results obtained both in simulation and in practice. Finally, Section V contains the conclusions obtained from the results and some future lines of research to improve the method.

II. FEEDFORWARD ACTION

The main idea of the algorithm is to apply a feedforward control signal that moves the process output to the set-point value, and then a feedback control action to reject disturbances. It is assumed that, starting from null initial conditions, a change in the process output from 0 to y_{sp} is required. In [6], a feedforward controller for FOPTD processes is designed, applying only two control actions to move the process to the reference. In this case, since we are considering a second order model, two actions are not enough to obtain the same response as in the FOPTD. To achieve the desired response three control actions are needed.

A. SOPTD process

We start from the second order plus dead time (SOPTD) model,

$$G(s) = \frac{K}{(1+Ts)(1+\eta Ts)}\mathrm{e}^{-Ls} \quad (1)$$

where T and ηT are the poles of the sistem, K the gain and L the dead time. It is assumed, without loss of generality, that $0 < \eta \leq 1$. Two different cases have to be considered separately: $0 < \eta < 1$, and $\eta = 1$. Applying the inverse Laplace transform to (1), the expression of the process step response in the time domain can be described as,

$$y(t) = \begin{cases} Ku(1 - \frac{1}{1-\eta}\mathrm{e}^{-\frac{t-L}{T}} - \frac{\eta}{\eta-1}\mathrm{e}^{-\frac{t-L}{\eta T}}) & \eta \neq 1 \\ \\ Ku(1 - \mathrm{e}^{-\frac{t-L}{T}} - \frac{t}{T}\mathrm{e}^{-\frac{t-L}{T}}) & \eta = 1 \end{cases} \quad (2)$$

where u is the amplitude of the input step.

The feedforward control signal desired is generated with the following scheme,

$$u_{ff}(t) = \begin{cases} K_{ff_1}\frac{y_{sp}}{K} & t \leq \tau_1 \\ \\ K_{ff_2}\frac{y_{sp}}{K} & \tau_1 < t < \tau_2 \\ \\ \frac{y_{sp}}{K} & t > \tau_2 \end{cases} \quad (3)$$

where control signal levels have been expressed as a function of y_{sp} in order to obtain the values of K_{ff_1} and K_{ff_2} for the controller independently of the amplitude of the set-point step. Initially, a first action $K_{ff_1}\frac{y_{sp}}{K}$ is applied to move the controlled variable to approach the desired set-point. Then, when the process output crosses a certain level Δ_1 (at a time $t = \tau_1$), the control signal is switched to the second value, $K_{ff_2}\frac{y_{sp}}{K}$. Since we are considering a second order process, after the initial phase the first derivative of the output cannot be reset instantaneously, and thus it is necessary to apply this second control action to decelerate the process output. Finally, when a level Δ_2 (at $t = \tau_2$) is crossed, a control action $\frac{y_{sp}}{K}$ is applied to maintain the output at the set-point value. Figure 1 shows a graphical representation of the described behaviour.

In summary, two are the constraints that must be satisfied,

1) The process output at $t = \tau_2 + L$ must be equal to the set-point, $y(\tau_2 + L) = y_{sp}$, and
2) At time $t = \tau_2 + L$, the first derivative of the process output must be null: $\dot{y}(\tau_2 + L) = 0$

The analysis of the two cases is detailed in the following paragraphs.

First case($\eta \neq 1$). Applying the control action (3) to the system, the process output can be described with a piecewise function composed of four segments. Due to the time delay, the process output is zero until time $t = L$. In the time interval $L \leq t \leq \tau_1 + L$, the process begins to be affected by the first control action, and the output (and its first derivative) can be obtained from the step response of the process starting from null initial conditions,

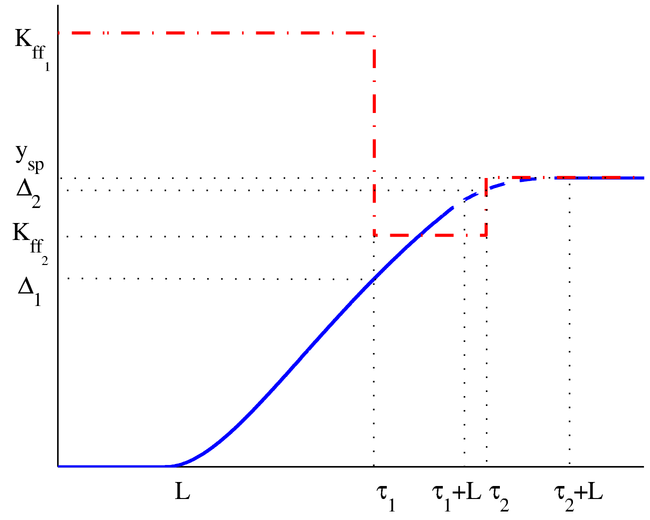

Fig. 1. Behaviour of the event-based feedforward algorithm for SOPTD processes

$$y(t) = K_{ff_1}(1 - \frac{1}{1-\eta}e^{-\frac{t-L}{T}} - \frac{\eta}{\eta-1}e^{-\frac{t-L}{\eta T}})y_{sp} \quad (4)$$

and

$$\dot{y}(t) = \frac{K_{ff_1}y_{sp}}{1-\eta}(e^{-\frac{t-L}{T}} - e^{-\frac{t-L}{\eta T}}) \ . \quad (5)$$

The third part of the output corresponds with the effect of the second control action in the time interval $\tau_1 + L < t \leq \tau_2 + L$. The expression is the same as in the previous interval, but with the initial conditions given by (4) and (5) at time $t = \tau_1 + L$,

$$y(t) = \Big(K_{ff_2} - \frac{K_{ff_2} - y_1 - \eta T\dot{y}_1}{1-\eta}\mathrm{e}^{-\frac{t-\tau_1-L}{T}} \\ -\eta\frac{K_{ff_2} - y_1 - T\dot{y}_1}{\eta-1}\mathrm{e}^{-\frac{t-\tau_1-L}{\eta T}}\Big)y_{sp} \quad (6)$$

where $y_1 = y(\tau_1 + L)$ and $\dot{y}_1 = \dot{y}(\tau_1 + L)$. Note that using $y_1 + \eta T\dot{y}_1 = K_{ff_1}(1 - \mathrm{e}^{-\frac{t}{T}})$ and $y_1 + T\dot{y}_1 = K_{ff_1}(1 - \mathrm{e}^{-\frac{t}{\eta T}})$. The first derivative of the output can be obtained from,

$$\dot{y}(t) = \Big(\frac{K_{ff_2} - y_1 - \eta T\dot{y}_1}{T(1-\eta)}\mathrm{e}^{-\frac{t-\tau_1-L}{T}} \\ -\frac{K_{ff_2} - y_1 - T\dot{y}_1}{T(1-\eta)}\mathrm{e}^{-\frac{t-\tau_1-L}{\eta T}}\Big)y_{sp} \ . \quad (7)$$

Finally, for all $t >= \tau_2 + L$ the process output is equal to y_{sp}. In order to simplify the notation, the variable changes $t' = \frac{t}{T}$, and $L' = \frac{L}{T}$ are introduced. Note that from now on, the time variable in the equations is normalized. However, this does not have to be considered further because t is used only as an intermediate variable to obtain the switching levels for the event-based controller. For the same reason, t' and the normalized switching time $\tau_1' = \frac{\tau_1}{T}$ will be represented as t and τ_1, respectively. After the changes mentioned before, the expression of the process output can be written as,

$$y(t) = \begin{cases} 0 & 0 < t < L \\ K_{ff_1}\Big(1 - \frac{1}{1-\eta}e^{-(t-L)} & L \le t \\ \quad - \frac{\eta}{\eta-1}e^{-\frac{t-L}{\eta}}\Big)y_{sp} & t < \tau_1 + L \\ \Big(K_{ff_2} - \frac{f(\bar{K}_{ff},\tau_1)}{1-\eta}e^{-(\tau-\tau_1-L)} & \tau_1 + L < t \\ -\frac{f(\bar{K}_{ff},\frac{\tau_1}{\eta})}{\eta-1}e^{-\frac{\tau-\tau_1-L}{\eta}}\Big)y_{sp} & t \le \tau_2 + L \\ y_{sp} & \tau_2 + L < t \end{cases} \tag{8}$$

where $f(\bar{K}_{ff}, t) = K_{ff_2} - K_{ff_1}(1 - \mathrm{e}^{-t})$, and $\bar{K}_{ff} = \big(K_{ff_1}\ K_{ff_2}\big)^T$.

The first derivative of the output (8) with respect to the time is,

$$\dot{y}(t) = \begin{cases} 0 & 0 < t < L \\ \frac{K_{ff_1}}{1-\eta}\Big(\mathrm{e}^{-(t-L)} - \mathrm{e}^{-\frac{t-L}{\eta}}\Big)y_{sp} & L \le t,\ t < \tau_1 + L \\ \Big(\frac{f(\bar{K}_{ff},\tau_1)}{1-\eta}\mathrm{e}^{-(t-\tau_1-L)} & \tau_1 + L \le t \\ -f(\bar{K}_{ff},\frac{\tau_1}{\eta})\mathrm{e}^{-\frac{t-\tau_1-L}{\eta}}\Big)y_{sp} & t < \tau_2 + L \\ 0 & \tau_2 + L < t \end{cases} . \tag{9}$$

Then the value of the process output at $t = \tau_2 + L$ is,

$$y(\tau_2 + L) = \Big(-\frac{K_{ff_2} - K_{ff_1}(1-e^{-\tau_1})}{1-\eta}e^{-(\tau_2-\tau_1)} - \frac{K_{ff_2} - K_{ff_1}(1-e^{-\frac{\tau_1}{\eta}})}{\eta-1}e^{-\frac{\tau_2-\tau_1}{\eta}} + K_{ff_2}\Big)y_{sp} \tag{10}$$

and its first derivative,

$$\dot{y}(\tau_2 + L) = \Big(-\frac{K_{ff_2} - K_{ff_1}(1-e^{-\tau_1})}{1-\eta}\mathrm{e}^{-(\tau_2-\tau_1)} - \frac{K_{ff_2} - K_{ff_1}(1-e^{-\frac{\tau_1}{\eta}})}{\eta-1}e^{-\frac{\tau_2-\tau_1}{\eta}} + K_{ff_2}\Big)y_{sp} . \tag{11}$$

In order to do a smooth transition to the stationary value, the first derivative of the output must be equal to zero at $\tau_2 + L$. Introducing this constraint in (11), and after reordering the equation, it can be expressed as,

$$\big(K_{ff_2} - K_{ff_1}(1 - \mathrm{e}^{-\tau_1})\big)\,\mathrm{e}^{-(\tau_2-\tau_1)} = \Big(K_{ff_2} - K_{ff_1}(1 - \mathrm{e}^{-\frac{\tau_1}{\eta}})\Big)\,\mathrm{e}^{-\frac{\tau_2-\tau_1}{\eta}} . \tag{12}$$

From (12) the value of τ_2 can be obtained as a function of τ_1,

$$\tau_2 = \tau_1 + \frac{\eta}{1-\eta}\ln\left(\frac{K_{ff_2} - K_{ff_1}(1 - \mathrm{e}^{-\tau_1})}{K_{ff_2} - K_{ff_1}(1 - \mathrm{e}^{-\frac{\tau_1}{\eta}})}\right) . \tag{13}$$

Substituting (13) in (10), we obtain the following expression,

$$\frac{y(\tau_2 + L)}{y_{sp}} = K_{ff_2} - \frac{\big(K_{ff_2} - K_{ff_1}(1 - \mathrm{e}^{-\tau_1})\big)^{\frac{2\eta-1}{\eta-1}}}{\Big(K_{ff_2} - K_{ff_1}(1 - \mathrm{e}^{-\frac{\tau_1}{\eta}})\Big)^{\frac{\eta}{\eta-1}}} \tag{14}$$

The expression (14) gives the value of the process output depending on the two control signal levels and the first switching time τ_1. As the reference must be reached at $t = \tau_2 + L$, thus combining (14) with $y(\tau_2 + L) = y_{sp}$,

$$K_{ff_2} - \frac{\big(K_{ff_2} - K_{ff_1}(1 - \mathrm{e}^{-\tau_1})\big)^{\frac{2\eta-1}{\eta-1}}}{\Big(K_{ff_2} - K_{ff_1}(1 - \mathrm{e}^{-\frac{\tau_1}{\eta}})\Big)^{\frac{\eta}{\eta-1}}} = 1 . \tag{15}$$

Here, the values of K_{ff_1} and K_{ff_2} are left as design parameters. After fixing these parameters, equation (15) can be solved numerically for τ_1 to find the switching time. Evaluating (13) with τ_1 yields the value of the time τ_2. Finally, the levels Δ_1 and Δ_2 can be obtained by evaluating $y(t)$ at the switching times τ_1 and τ_2.

As a particular case, the value of K_{ff_2} can be chosen to be zero. This leads to a simplification of the equations derived above. To see this, introducing $K_{ff_2} = 0$ in (10) and (13) gives,

$$y(\tau_2 + L) = K_{ff_1}(1 - \mathrm{e}^{-\frac{\tau_1}{\eta}})\left(\frac{1 - \mathrm{e}^{-\tau_1}}{1 - \mathrm{e}^{-\frac{\tau_1}{\eta}}}\right)^{\frac{1}{1-\eta}} y_{sp} \tag{16}$$

and

$$\tau_2 = \tau_1 + \frac{\eta}{1-\eta}\ln\left(\frac{1 - \mathrm{e}^{-\tau_1}}{1 - \mathrm{e}^{-\frac{\tau_1}{\eta}}}\right) . \tag{17}$$

Second Case($\eta = 1$). This is the case of a transfer function with two equal poles.

The same reasoning as in the previous case yields the expressions,

$$1 = K_{ff_2}\big(1 - (1 + \tau_2 - \tau_1)\mathrm{e}^{-(\tau_2-\tau_1)}\big) + K_{ff_1}\left(1 + \tau_1 - (1 + 2\tau_1)\mathrm{e}^{-\tau_1}\right)\mathrm{e}^{-(\tau_2-\tau_1)} \tag{18}$$

$$\tau_2 - \tau_1 = L + \frac{K_{ff_1}\tau_1\mathrm{e}^{-\tau_1}}{K_{ff_1}(1 - \mathrm{e}^{-\tau_1}) + K_{ff_2}} \tag{19}$$

where $\tau_2 + L$ is the time at which the process output reaches the set-point.

Introducing (19) in (18) leads to a complex expression. Thus it is proposed to simplify the problem by choosing $K_{ff_2} = K_{ff_1}$. Then, (19) reduces to $t = \tau_1$ which after substituting in (18) yields,

$$K_{ff_1}\left((1 - 2\tau_1)\,\mathrm{e}^{-2\tau_1} - 1\right) = 1 . \tag{20}$$

After fixing the value of K_{ff_2}, the equation (18) must be solved for τ_1 to find the switching time. Note that the previous discussion implies that the reference value can be reached with only two control actions as in the case of FOPTD processes.

B. Processes with a pure integrator

The method can also be applied to systems with a pure integrator which can be modeled by the transfer function,

$$P(s) = \frac{K}{s(Ts + 1)}\mathrm{e}^{-Ls} . \tag{21}$$

The expression of the process step response in the time domain can be expressed as,

$$y(t) = Ku\left(t - L - T(1 - \mathrm{e}^{-\frac{t-L}{T}})\right) \tag{22}$$

where u is the amplitude of the input step.

Applying the control signal (3), the output of the process is described by the following expression,

$$y(t) = \begin{cases} 0 & 0 \le t < L \\ K_{ff_1}(t - L - T(1 - \mathrm{e}^{-\frac{t-L}{T}}))y_{sp} & \begin{array}{l} L \le t \\ t < \tau_1 + L \end{array} \\ \begin{array}{l} \big(K_{ff_1}\tau_1 + K_{ff_2}(t_2 - T) - \\ T(K_{ff_2} - K_{ff_1}(1 - \mathrm{e}^{-\frac{\tau_1}{T}}))\mathrm{e}^{-\frac{t_2}{T}}\big)\, y_{sp} \end{array} & \begin{array}{l} \tau_1 + L \le t \\ t < \tau_2 + L \end{array} \\ y_{sp} & \tau_2 + L \le t \end{cases} \tag{23}$$

where $t_2 = t - \tau_1 - L$. And the first derivative of the process is $\dot{y}(t) =$

$$\begin{cases} 0 & 0 \le t < L \\ K_{ff_1}(1 - \mathrm{e}^{-\frac{t-L}{T}})y_{sp} & \begin{array}{l} L \le t \\ t < \tau_1 + L \end{array} \\ \begin{array}{l} \Big(K_{ff_1}(1 - \mathrm{e}^{-\frac{\tau_1}{T}})\mathrm{e}^{-\frac{t-\tau_1-L}{T}} \\ K_{ff_2}(1 + \mathrm{e}^{-\frac{t-\tau_1-L}{T}})y_{sp}\Big) \end{array} & \begin{array}{l} \tau_1 + L \le t \\ t < \tau_2 + L \end{array} \\ 0 & \tau_2 + L \le t \end{cases} . \tag{24}$$

Following the same reasoning as in previous cases, the first derivative of the process output is forced to be zero at $t = \tau_2 + L$ and the equation is solved for the variable τ_2, which is expressed as a function of the first switching time τ_1. Then the solution of τ_2 is introduced in the expression of the process output and equated to y_{sp}. This yields an equation with only τ_1 as unknown.

The expressions of the process output and its first derivative at $t = \tau_2 + L$ are,

$$\begin{aligned} y(\tau_2 + L) = K_{ff_1}\tau_1 + K_{ff_2}(\tau_2 - \tau_1 - T) \\ -T(K_{ff_2} - K_{ff_1}(1 - \mathrm{e}^{\frac{\tau_1}{T}}))\mathrm{e}^{\frac{\tau_2-\tau_1}{T}} \end{aligned} \tag{25}$$

and

$$\dot{y}(\tau_2 + L) = K_{ff_2} + (K_{ff_2} - K_{ff_1}(1 - \mathrm{e}^{\frac{\tau_1}{T}}))\mathrm{e}^{\frac{\tau_2-\tau_1}{T}} . \tag{26}$$

Then, the time corresponding to the second switch τ_2 is,

$$\tau_2 = \tau_1 + T \ln\left(1 - \frac{K_{ff_1}}{K_{ff_2}}(1 - \mathrm{e}^{-\frac{\tau_1}{T}})\right). \tag{27}$$

Substituting (27) in (23) we have,

$$\begin{aligned} y(\tau_2 + L) = \Big(K_{ff_2}T \ln\Big(1 - \tfrac{K_{ff_1}}{K_{ff_2}}(1 - \mathrm{e}^{-\frac{\tau_1}{T}})\Big) \\ + K_{ff_1}\tau_1\Big)\, y_{sp} \ . \end{aligned} \tag{28}$$

Forcing the process output (28) to be equal to y_{sp} at time $\tau_2 + L$ we have,

$$K_{ff_1}\tau_1 + K_{ff_2}T \ln\left(1 - \frac{K_{ff_1}}{K_{ff_2}}(1 - \mathrm{e}^{-\frac{\tau_1}{T}})\right) = 1 \ . \tag{29}$$

The equation (29) can be solved numerically for τ_1. Finally, evaluating the process output at the switching times, τ_1 and τ_2, yields the values of the switching levels Δ_1 and Δ_2.

C. Algorithm

The algorithm can be described as,

Precalculations.

1) Choose the values of the controller parameters, K_{ff_1} and K_{ff_2}.
2) Find the first switching time τ_1.
3) Use the value of τ_1 to calculate the second switching time τ_2.
4) Determine the switching levels Δ_1 and Δ_2 by evaluating $y(\tau_1)$ and $y(\tau_2)$.

Feedforward action.

1) if $y_{sp} \neq y_{sp_{prev}}$ (first event).
   a) ff_enabled = true (enable the feedforward part).
   b) $u_{ff}(t) = K_{ff_1}\frac{y_{sp}}{K}$ (first feedforward action).
2) if ff_enabled
   a) if $|e| = |y_{sp} - \Delta_1|$ (second event). g
      i) $u_{ff}(t) = K_{ff_1}\frac{y_{sp}}{K}$ (second ff. action).
   b) if $|e| = |y_{sp} - \Delta_2|$ (third event).
      i) $u_{ff}(t) = K_{ff_2}\frac{y_{sp}}{K}$ (final ff. action).
   c) else
      i) $u_{ff}(t) = u_{ff}(t - T_s)$ (hold the last action).
      ii) if e=0 then ff_enabled=false
3) else
   a) $u_{ff}(t) = u_{ff}(t - T_s)$ (hold the last action).

D. How to choose K_{ff_1} and K_{ff_2}

One important practical issue is how to choose appropriate values of K_{ff_1} and K_{ff_2}. A possible approach is to fix the gains considering the saturation of actuators. In real applications the maximum set-point change allowed is limited by $y_{sp_{max}}$. Then the gains can be chosen to always generate a control signal between the maximum and minimum values admitted by the actuator (as in the example of Section 3.3.).

Another different approach could be to use an optimization algorithm to choose the values that obtain the best performance, for example to minimize the impact of disturbances effects or model uncertainties in the process response.

III. FEEDBACK

The previous discussion and the experimental results presented in Section IV show that the algorithm described works well in absence of disturbance and model uncertainty. However, in real systems there exists uncertainties, unmodelled dynamics, external disturbances, etc. Therefore it is needed to consider the disturbance rejection task. The solution adopted is taken from [6], i.e., a PI controller with an event-triggered sampling. The control law is $u_p = K_p \cdot e\,(t) + K_i \cdot IE\,(t)$, where K_p, K_i are the proportional and integral gains, and $IE(t) = \int_0^t e\,(t)\,dt$ is the integrated error. The proportional part, $K_p \cdot e\,(t)$, is updated when $|e\,(t) - e\,(t_{prev})| > \delta_P$ and the integral part, $K_i \cdot IE\,(t)$ is updated when $|IE\,(t) - IE\,(t_{prev})| > \delta_I$, where δ_P and δ_I are the event triggering thresholds for proportional and integral events, respectively.

IV. RESULTS

To illustrate the methods discussed before, some examples are presented for the cases considered in the theory. Some of them have been tested in simulation and others with a DC Motor.

A. Identification

In order to apply the algorithm described, it is needed to obtain a representation of the plant as a SOPTD model. The method applied in this work is known as 123c (see [7]), and it is based on the determination of three characteristic points in the step response of the process.

B. Simulation

A fourth-order process with a time delay is modeled with the following transfer function,

$$F(s) = \frac{1}{(s+1)^4} e^{-0.2s}.$$

As mentioned before, it is necessary to obtain a SOPTD model of this process in order to apply the algorithm described. Thus, applying the model order reduction method of previous section the transfer function is,

$$F(s) = \frac{1}{(1.1227s+1)(2.0812s+1)} e^{-1s}.$$

We assume that the maximum and minimum values of the control signal admitted by the actuator of this plant are $u_{max} = 3.0$ and $u_{min} = 0.0$, and the reference is in the range (0,1). Then, the controller gains are fixed as $K_{ff_1} = u_{max}$ and $K_{ff_2} = u_{min}$. The model obtained with the identification method has two different poles, i.e. $\eta \neq 1$. Thus the method applied is that described on Section 2.1. Solving the equations of the method described above, the values of the switching levels can be obtained,

$$\Delta_1 = 0.1216, \Delta_2 = 0.6564 \ .$$

The simulation of the controller applied to the fourth order process can be seen in Figure 2. The FOPTD method is also applied to the fourth order process in order to compare the performance. Applying the identification method to obtain an FOPTD model, the process is approached with the following transfer function,

$$F(s) = \frac{1}{3.5434s+1} e^{-1s}.$$

The values of the controller parameters which have been calculated by the method described in [6] are,

$$K_p = 2.3272, \Delta = 0.5705 \ .$$

Figure 3 shows a comparison between the SOPTD method and a time-based PI, and Figure 4 shows the response of the SOPTD method applied to the reduced order model and to the original process. The time-based PI was tuned to obtain a minimum value of the integrated absolute error. It can be seen how the SOPTD method achieved a better performance with the use of only three control actions. The performance has been compared by calculating the Integrated Absolute Error (IAE) for the three controllers. The results are showed in Table I.

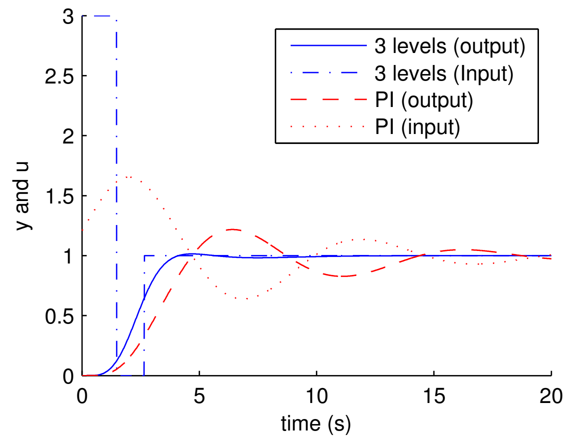

Fig. 2. Response obtained in simulation for the fourth-order process (solid line) with the three-levels feedforward controller and with a classic time based PI controller (dotted line) with $K_p = 1.2081$ and $T_i = 3.7482$.

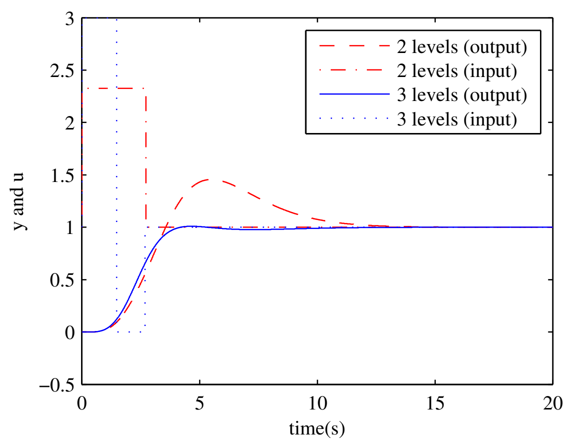

Fig. 3. Response obtained in simulation for the fourth-order process with the SOPTD event-based feedforward controller (solid line) an the FOPTD event-based controller (dash dotted line)

TABLE I

VALUE OF THE INTEGRATED ABSOLUTE ERROR (IAE) FOR THE FOPTD AND SOPTD METHODS

Method	IAE
Time-based PI	3.5503
Event-based FOPTD	4.3878
Event-based SOPTD	2.4841

C. Model uncertainty

To investigate the impact of the plant uncertainty, different simulations were realized varying the plant time constants and time delay at $\pm 40\%$, while the gain was constant. Figure 4 shows the simulation results.

D. Example

The plant consists of a DC Motor with two sensors which read periodically (with sampling period $T_s = 0.01s$) the position of the rotor and the angular velocity. The control

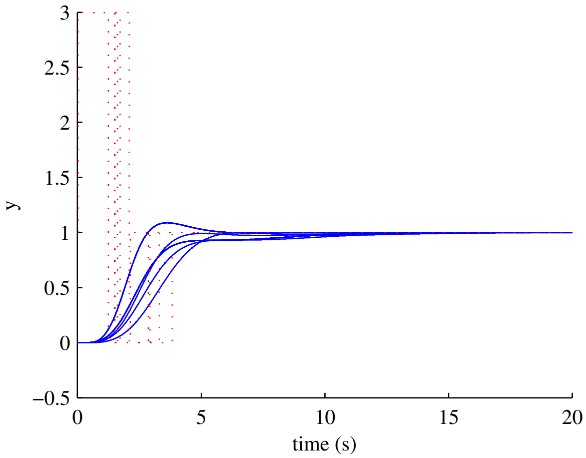

Fig. 4. Impact of the model uncertainty on the controller performance for the fourth-order process. Differents simulation are showed for values of the parameters varying at ±40%.

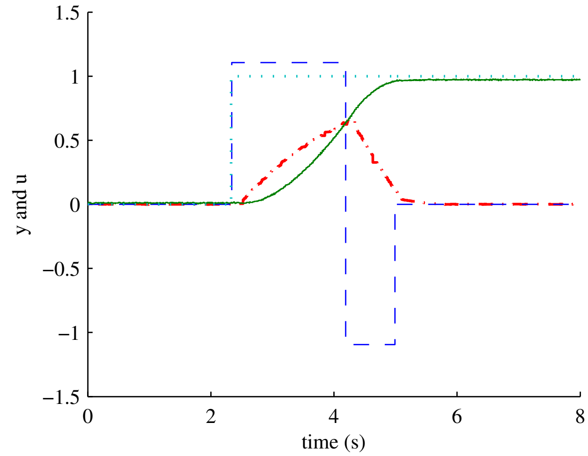

Fig. 5. Set-point step change applied to the DC motor. The graph shows the control signal (dashed line), the angle of the rotor (solid line) and the angular velocity (dash dotted line)

system is implemented as a wireless system, where the sensors and actuator are connected to a wireless module and controller is implemented over another module. The plant can be modeled with the following transfer function,

$$F(s) = \frac{0.91}{s(1.94s+1)} \mathrm{e}^{-0.2s} \ .$$

where the delay is due to the wireless communication modules. The actuator admits a control signal in the range: ($u_{min} = -3.5V$, $u_{max} = 3.5V$). Since we are controlling the rotor position, the maximum change in the set-point is limited to π. In order to not saturate the actuator, it must be verified that,

$$K_{ff_1} \leq \frac{K}{\pi} u_{max} \approx 1.01, K_{ff_2} \geq \frac{K}{\pi} u_{min} \approx -1.01 \ .$$

Then, choosing $K_{ff_1} = 1$ and $K_{ff_2} = -1$ and solving the equations (27), the values of the switching levels obtained are,

$$\Delta_1 = 0.6149, \Delta = 0.9903$$

and the control signal levels for $y_{sp} = 1$ are,

$$u_1 = K_{ff_1} \frac{y_{sp}}{K} = 1.0989, u_2 = K_{ff_1} \frac{y_{sp}}{K} = -1.0989 \ .$$

Figure 5 shows the response of the real plant with the controller, with a set-point step change $y_{sp} = 1$. Note also that the values of the switching levels have to be calculated only once, but keeping in mind that, as it occurs with the control actions, they must be scaled by the amplitude of the set-point step. As an example, for $y_{sp} = 0.5$ the switching levels must have been $\Delta_1 y_{sp} = 0.30745$ and $\Delta_2 y_{sp} = 0.49515$.

V. CONCLUSIONS

In this work an event-based algorithm has been developed to perform the set-point following task in second order processes with time delay with only two control actions, in absence of disturbance. Then, an event-based feedback is added to the controller to realize the disturbance rejection task. This algorithm was tested firstly in simulation, obtaining good results, and finally in a real plant, in order to compare and validate them. The motivations for the research in this algorithm were to extend the solution described in [6], and experimentally tested in [8], to a slightly more complicated model, with the purpose of having more flexibility in the representation of the process model under control. However, more research is still needed in some aspects. Though model uncertainties was considered in simulation, a thorough study of the robustness of the method is still required. It would be also convenient to generalize the solution to n-order models with time delay. Finally, the development of a tuning rule for the disturbances rejection task is still an open issue.

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