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An Actuator Failure Tolerant Robust Control Approach for an Underwater Remotely Operated Vehicle

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Abstract— **This paper proposes an actuator failure tolerant robust control scheme for underwater Remotely Operated Vehicles (ROVs). A reduced order observer has been introduced first, for estimating the ROV velocities. In order to solve the control problem for the ROV positions, a sliding mode control law has been developed using the available position measurements and the velocity estimates provided by the observer. A thruster failure is shown to be detectable simply checking the presence of any deviation of the observed sliding surfaces. Moreover, an isolation policy for the failed thruster is proposed. Finally, control reconfiguration is performed exploiting the inherent redundancy of actuators. An extensive simulation study has been performed, supporting the effectiveness of the proposed approach.**

I. INTRODUCTION

In the last decades Unmanned Underwater Vehicles (UUVs) have increased their popularity, especially as a cost-effective solution for performing complex tasks in the underwater environment without risking human life (e.g. environmental data gathering, transportation of assembling modules for submarine installations, inspection of underwater structures). On the other hand, underwater environment introduces numerous challenges in control, navigation and communication of such vehicles.

With increasing mission durations in complex marine applications, one of the primary concerns is the failure occurrence on the actuators [1], [2]. When actuator failures occur and result in abnormal operations, the only present solution is to abort the mission, and use a damage control to make UUVs surface [3]. Therefore, the problem of reliability and security of UUVs, especially their ability of actuator fault tolerance, has become a major concern. Even though most UUVs use adaptive control systems, the response of the controller is reactive, and no consideration is given to the source or extent of the failures. It is desirable to incorporate a function of actuator fault detection and isolation into the control system, so that it is possible to detect and identify actuator fault and/or failures, and design compensation control laws.

This paper presents a fault tolerant control scheme in the framework of underwater robotics, specifically addressing an underwater Remotely Operated Vehicle (ROV) [4] used by SNAMprogetti (Fano, Italy) in the exploitation of combustible gas deposits at great water depths. The vehicle is equipped with four thrusts propellers, controlling its position and orientation in planes parallel to the sea surface, and is connected with the surface vessel by a supporting cable which controls the vehicle depth and provides power and communication facilities. The control system is composed of two independent parts: the first part, placed on the surface vessel, monitors the vehicle depth, and the second part controls the position and orientation of the vehicle in the dive plane. In this paper the attention has been focused on this second part of the control system: the ROV is supposed to move on a plane, with three degrees of freedom. Due to this assumption, the thrusters configuration is redundant, as in general happens with UUV's. This redundancy can be exploited to enhance the ROV ability to achieve the mission objective in the presence of a thruster fault.

The actuator failure tolerant control scheme presented in this paper is composed by the usual modules performing detection, isolation, accommodation of failures by control reconfiguration [5]. A reduced order observer has been specifically designed for the ROV exploiting the features of the underwater vehicle, permitting to estimate the underwater vehicle velocities, which are in general difficult to be gathered and poorly reliable. These velocity estimations and the available position measurements, have been then used for developing a robust sliding mode control law [6], which is able to solve the regulation problem for the ROV positions, with respect to the reference ones. The developed sliding surfaces have been used both for designing a robust ROV control algorithm ensuring plant regulation, and for detecting the thruster failures. Failure detection is here performed simply checking the presence of any deviation of the observed sliding surfaces, which can be due only to the occurrence of a thruster failure. Exploiting the ROV structure, faulty thruster can be successfully isolated. Once the failed actuator has been identified, control reconfiguration is performed using the redundant healthy actuators. In other words, the control activity is redistributed among the actuators still working such that the failed actuator is compensated for, and control performances are this way preserved.

II. MATHEMATICAL MODEL OF THE ROV

A. ROV nonlinear model

The equations describing the ROV dynamics have been obtained from classical mechanics [4], [7], [8]. The ROV considered as a rigid body can be fully described with six degrees of freedom, corresponding to the position and orientation with respect to a given coordinate system. Let

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us consider the inertial frame $\boldsymbol{R}(0, x, y, z)$ and the body reference frame $\boldsymbol{R}_a(0_a, x_a, y_a, z_a)$ [7] shown in Fig. 1.

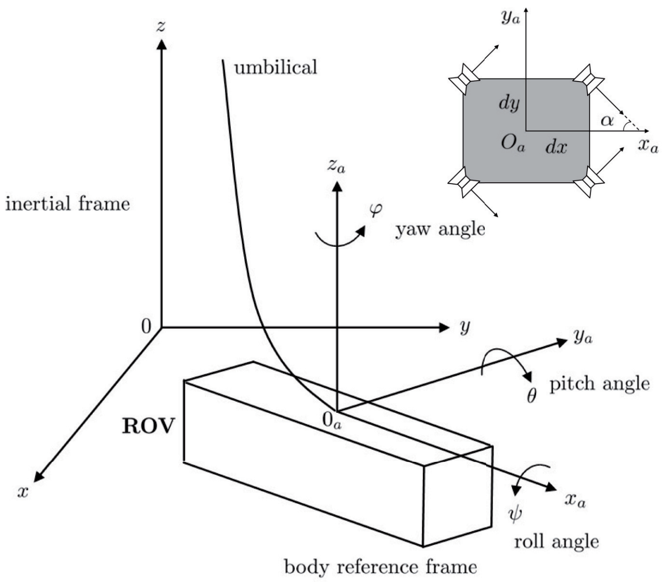

Fig. 1. ROV operational configuration.

The ROV position with respect to $\boldsymbol{R}$ is expressed by the origin of the system while its orientation by the roll, pitch, and yaw angles ψ, θ, and ϕ, respectively. Being the depth controlled by the surface vessel, the ROV is considered to operate on surfaces parallel to the $x-y$ plane. Accordingly the controllable variables are x, y, and the yaw angle ϕ. It should be noticed that the roll and pitch angles ψ and θ will not be considered in the dynamic model: their amplitude, in fact, has been proved to be negligible in a wide range of load conditions, and with different intensities and directions of the underwater current as well [4], [7]. Therefore, the ROV model is described by the following system of differential equations [4], [7], [8], [9] :

$$\begin{cases} p_1\ddot{x} + (p_2|\cos(\phi)| + p_3|\sin(\phi)|)V_x|\boldsymbol{V}| + \\ \quad p_4x - p_5V_{cx}|\boldsymbol{V}_c| = T_x \\ p_1\ddot{y} + (p_2|\sin(\phi)| + p_3|\cos(\phi)|)V_y|\boldsymbol{V}| + \\ \quad p_4y - p_5V_{cy}|\boldsymbol{V}_c| = T_y \\ p_6\ddot{\phi} + p_7\dot{\phi}\left|\dot{\phi}\right| + p_8|\boldsymbol{V}_c|^2\sin\left(\frac{\phi-\phi_c}{2}\right) + p_9 = M_z \end{cases} \tag{1}$$

where $V_c = \begin{bmatrix} V_{cx} & V_{cy} \end{bmatrix}^T$ is the submarine current velocity, $\boldsymbol{V} = \begin{bmatrix} V_x & V_y \end{bmatrix}^T = \begin{bmatrix} (\dot{x} - V_{cx}) & (\dot{y} - V_{cy}) \end{bmatrix}^T$, and the expressions of coefficients p_i $(i = 1, \ldots, 9)$ are reported in [8], [9], where M is the vehicle mass, m is the addition mass, I_z is the vehicle inertia moment around the z axis, i_z is the addition inertia moment, M_c is the resistance moment of the cable, L is the cable length, T_v the vehicle weight in the water, W the weight for length unit of the cable, ρ_w the water density, C_{dc} is the drag coefficient of the cable, and D_c is the cable diameter, C_{di} is the drag coefficient of the $i-$th side wall $(i = 1, 2)$, C_{ri} the packing coefficient (depending on the geometrical characteristics of the $i-$th side wall $(i = 1, 2)$), S_i is the area of the $i-$th side wall $(i = 1, 2)$, C_d is the drag coefficient of rotation, C_r is the packing coefficient of rotation, S is the equivalent area of rotation, r is the equivalent arm of action, d_i $(i = 1, 2, 3)$ are the vehicle dimensions along the x_a, y_a and z_a axes, respectively, and ϕ_c is the angle between the x axis and the velocity direction of the current. This model is in agreement with models usually proposed in literature for underwater ROV's moving in the dive plane [1].

The quantities T_x, T_y and M_z appearing in (1) are the decomposition of the thrust and the torque provided by the four vehicle propellers along the axes of $\boldsymbol{R}$:

$$\begin{cases} T_x = \cos(\phi)(T_1 + T_2 + T_3 + T_4)\cos(\alpha) - \\ \quad \sin(\phi)(-T_1 - T_2 + T_3 + T_4)\sin(\alpha) \\ T_y = \sin(\phi)(T_1 + T_2 + T_3 + T_4)\cos(\alpha) + \\ \quad \cos(\phi)(-T_1 - T_2 + T_3 + T_4)\sin(\alpha) \\ M_z = (-T_1 + T_2 - T_3 + T_4)d_a \end{cases} \tag{2}$$

with $\alpha = \dfrac{\pi}{4}$, $d_a = (d_x\sin(\alpha) + d_y\cos(\alpha))$ (see Fig. 1).

B. State Space ROV model

Define the vectors $\boldsymbol{z}_p = [z_1 \ z_2 \ z_3]^T = \begin{bmatrix} x & y & \phi \end{bmatrix}^T$, $\boldsymbol{z}_v = [z_4 \ z_5 \ z_6]^T = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\phi} \end{bmatrix}^T$, the state vector:

$$\boldsymbol{z} = \begin{bmatrix} \boldsymbol{z}_p^T & \boldsymbol{z}_v^T \end{bmatrix}^T = \begin{bmatrix} z_1 & z_2 & z_3 & z_4 & z_5 & z_6 \end{bmatrix}^T$$

and introduce the input vector $\boldsymbol{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T = \begin{bmatrix} T_x & T_y & M_z \end{bmatrix}^T$. Moreover, since model parameters and submarine current are not exactly known, bounded uncertainties are taken into account as follows: $p_i = \hat{p}_i + \Delta p_i$, $|\Delta p_i| \le \rho_{p_i}$, $i = 1, \ldots, 9$, $V_{cx} = \hat{V}_{cx} + \Delta V_{cx}$, $|\Delta V_{cx}| \le \rho_{V_{cx}}$, $V_{cy} = \hat{V}_{cy} + \Delta V_{cy}$, $|\Delta V_{cy}| \le \rho_{V_{cy}}$, being $\hat{p}_i$, $\hat{V}_{cx}$, $\hat{V}_{cy}$ the nominal values of the parameter and of the submatine current components, respectively, and $|\Delta p_i|$, $|\Delta V_{cx}|$, $|\Delta V_{cy}|$ the corresponding uncertainties, bounded by ρ_{p_i}, $\rho_{V_{cx}}$, $\rho_{V_{cy}}$, respectively. Considering the above definitions and equations (1), the following state space model is obtained:

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{f}(\boldsymbol{z}(t)) + \boldsymbol{\Delta f}(\boldsymbol{z}(t), \boldsymbol{u}(t)) + \boldsymbol{g}\boldsymbol{u}(t) \tag{3}$$

where

$$\boldsymbol{f}(\boldsymbol{z}) = \begin{bmatrix} z_4 \\ z_5 \\ z_6 \\ -f_4(z_3)\bar{z}_4N_z + \varphi_4(z_1) \\ -f_5(z_3)\bar{z}_5N_z + \varphi_5(z_2) \\ -\dfrac{\hat{p}_7}{\hat{p}_6}z_6|z_6| + \varphi_6(z_3) \end{bmatrix} \tag{4}$$
$$\boldsymbol{g} = \begin{bmatrix} \boldsymbol{0}_{3\times3} \\ diag\left\{\frac{1}{\hat{p}_1}, \frac{1}{\hat{p}_1}, \frac{1}{\hat{p}_6}\right\} \end{bmatrix}$$

being

$$f_4(z_3) = \frac{1}{\hat{p}_1}\left(\hat{p}_2\,|c_3| + \hat{p}_3\,|s_3|\right)$$
$$f_5(z_3) = \frac{1}{\hat{p}_1}\left(\hat{p}_2\,|s_3| + \hat{p}_3\,|c_3|\right)$$
$$\varphi_4(z_1) = \frac{1}{\hat{p}_1}\left(-\hat{p}_4 z_1 + \hat{p}_5 \hat{V}_{cx}|\hat{\boldsymbol{V}}_c|\right)$$
$$\varphi_5(z_2) = \frac{1}{\hat{p}_1}\left(-\hat{p}_4 z_2 + \hat{p}_5 \hat{V}_{cy}|\hat{\boldsymbol{V}}_c|\right)$$
$$\varphi_6(z_3) = -\frac{1}{\hat{p}_6}\left(\hat{p}_8|\hat{\boldsymbol{V}}_c|^2 \sin\left(\frac{z_3-\hat{\phi}_c}{2}\right) + \hat{p}_9\right)$$

with $\bar{z}_4 = z_4 - \hat{V}_{cx}$, $\bar{z}_5 = z_5 - \hat{V}_{cy}$, $N_z = \sqrt{\bar{z}_4^2 + \bar{z}_5^2}$, $c_3 = \cos(z_3)$, $s_3 = \sin(z_3)$, $\hat{\boldsymbol{V}}_c = \sqrt{\hat{V}_{cx}^2 + \hat{V}_{cy}^2}$, $\hat{\phi}_c = \arctan\left(\frac{\hat{V}_{cy}}{\hat{V}_{cx}}\right)$, and the term $\Delta\boldsymbol{f}(\boldsymbol{z},\boldsymbol{u}) = [0,0,0,\Delta_4(z_1,z_3,z_4,T_x),\Delta_5(z_2,z_3,z_5,T_y),\Delta_6(z_3,z_6,M_z)]^T$ can be easily computed considering the uncertainties $|\Delta p_i|$ $(i = 1\dots 9)$, ΔV_{cx}, ΔV_{cy}.

III. Robust Control Design

A. Robust Design of the Reduced Order Observer

In this section, a reduced order observer, specifically designed for the ROV, will be proposed. In fact, while position measurements are usually available and are in general sufficiently reliable, velocity measurements are either difficult to be gathered and poorly reliable.

The observer design will be carried out exploiting the following features of the ROV:

- in the model (3), (4), positive scalars γ_{14}, γ_{24}, γ_{15}, γ_{25} can be found such that: $0 < \gamma_{14} \le f_4(z_3) \le \gamma_{24}$, $0 < \gamma_{15} \le f_5(z_3) \le \gamma_{25}$;
- there is a maximal (known) velocity $z_4^{max}, z_5^{max}, z_6^{max}$ achievable by the vehicle, as a consequence of the existence of a maximal power supply by thrusters. Therefore it exists a bound $M = \sqrt{(z_4^{max} - V_{cx})^2 + (z_5^{max} - V_{cy})^2}$ such that $N_z \le M$.
- as a consequence of the existence of a maximal power supply by thrusters, control variables T_x, T_y, M_z are bounded, too, and bounds can be easily computed also for the uncertain terms. Therefore, it can be assumed that: $|\Delta_4(z_1,z_3,z_4,T_x)| \le \rho_4$; $|\Delta_5(z_2,z_3,z_5,T_y)| \le \rho_5$; $|\Delta_6(z_3,z_6,M_z)| \le \rho_6$.

Define $\boldsymbol{\xi} = [\xi_1\ \ \xi_2\ \ \xi_3]^T$, and consider the following reduced state observer:

$$\begin{aligned}\dot{\xi}_1 &= -\,f_4(z_3)\hat{N}_z\bar{\xi}_1 + \varphi_4(z_1) + v_1\\ \dot{\xi}_2 &= -\,f_5(z_3)\hat{N}_z\bar{\xi}_2 + \varphi_5(z_2) + v_2\\ \dot{\xi}_3 &= -\,\alpha_6\bar{M}\xi_3 + \varphi_6(z_3) + v_3\end{aligned} \tag{5}$$

with $\bar{\xi}_1 = \xi_1 - V_{cx}$, $\bar{\xi}_2 = \xi_2 - V_{cy}$, $\alpha_6 = p_7/p_6$, $\bar{M} = \sup\{|z_6^{max}|, M\}$ and $\hat{N}_z = \sqrt{(\xi_1 - V_{cx})^2 + (\xi_2 - V_{cy})^2}$.

A robust control law, coupled with the above observer, will be here presented aimed at solving the regulation problem for the variables z_1, z_2, z_3 with respect to reference variable $\boldsymbol{z}_d = [z_{1d}\ z_{2d}\ z_{3d}]^T$. Define the following sliding surface:

$$\tilde{\boldsymbol{s}} = \begin{bmatrix} \tilde{s}_1 & \tilde{s}_2 & \tilde{s}_3 \end{bmatrix}^T = (\boldsymbol{\xi} - \dot{\boldsymbol{z}}_d) + \boldsymbol{z}_p - \boldsymbol{z}_d = \boldsymbol{0}. \tag{6}$$

with $\boldsymbol{\Lambda} = diag\{\lambda_i\}$, $\lambda_i > 0$, $i = 1,\dots,3$, and being $\boldsymbol{\epsilon} = \boldsymbol{z}_p - \boldsymbol{z}_d$ the tracking error.

Lemma 1: Consider the uncertain ROV model (3). The control law $\boldsymbol{u} = \boldsymbol{u}_{eq} + \boldsymbol{u}_n$, with:

$$\boldsymbol{u}_{eq} = \boldsymbol{g}^{-1}\begin{bmatrix} f_4(z_3)\bar{\xi}_1\hat{N}_z - \varphi_4(z_1) - \lambda_1(z_4 - \dot{z}_{1d}) + \ddot{z}_{1d} - v_1\\ f_5(z_3)\bar{\xi}_2\hat{N}_z - \varphi_5(z_2) - \lambda_2(z_5 - \dot{z}_{2d}) + \ddot{z}_{2d} - v_2\\ \alpha_6\bar{M}\xi_3 - \varphi_6(z_3) - \lambda_3(z_6 - \dot{z}_{3d}) + \ddot{z}_{3d} - v_3\end{bmatrix}$$
$$\boldsymbol{u}_n = -\boldsymbol{g}^{-1}\begin{bmatrix}\bar{\rho}_4 sign(\tilde{s}_1)\\ \bar{\rho}_5 sign(\tilde{s}_2)\\ \rho_6 sign(\tilde{s}_3)\end{bmatrix} \tag{7}$$

guarantees the asymptotical achievement of a sliding motion on (6).

Proof: The achievement of a sliding motion on (6) is guaranteed by the following condition:

$$\begin{aligned}\tilde{\boldsymbol{s}}^T\dot{\tilde{\boldsymbol{s}}} &= \tilde{s}_1\left(\dot{\xi}_1 - \ddot{z}_{1d} + \lambda_1(z_4 - \dot{z}_{1d})\right) + \\ &\tilde{s}_2\left(\dot{\xi}_2 - \ddot{z}_{2d} + \lambda_2(z_5 - \dot{z}_{2d})\right) + \tilde{s}_3\left(\dot{\xi}_3 - \ddot{z}_{3d} + \lambda_3(z_6 - \dot{z}_{3d})\right)\\ &< 0\end{aligned} \tag{8}$$

which can be fulfilled imposing separately three inequalities, i.e. $s_i\left(\dot{\xi}_i - \ddot{z}_{id} + \lambda_i(z_{3+i} - \dot{z}_{id})\right) < 0$, $i = 1,2,3$. The first inequality gives, e.g.:

$$\tilde{s}_1\left(-f_4(z_3)\hat{N}_z\bar{\xi}_1 + \varphi_4(z_1) + v_1 - \ddot{z}_{1d} + \lambda_1(z_4 - \dot{z}_{1d})\right) < 0$$

and one gets immediately the controller (7). ■

Define the observation error as $\boldsymbol{e} = [e_1\ e_2\ e_3]^T = \boldsymbol{z}_v - \boldsymbol{\xi}$. The following result can be given omitting the proof for brevity:

Corollary 2: Consider the uncertain plant model (3) driven by the control law (7). The reduced order observer (5) ensures the robust asymptotical vanishing both of the observation error and of the tracking error designing $\boldsymbol{v}$ as follows:

$$\begin{aligned}\boldsymbol{v} = -\theta &\begin{bmatrix}[f_4(z_3)M(\bar{z}_4^{max} + |\xi_1|) + \rho_4]\\ [f_5(z_3)M(\bar{z}_5^{max} + |\xi_2|) + \rho_5]\\ [\alpha_6(\bar{z}_6^{max})^2 + M|\xi_3|) + \rho_6]\end{bmatrix}\cdot\\ &\big[\ sign(\dot{\xi}_1 + \xi_1 - \ddot{z}_{1d} - \dot{z}_{1d}) \quad sign(\dot{\xi}_2 + \xi_2 - \ddot{z}_{2d} - \dot{z}_{2d})\\ &\quad sign(\dot{\xi}_3 + \xi_3 - \ddot{z}_{3d} - \dot{z}_{3d})\ \big]\end{aligned} \tag{9}$$

with $\theta > 1$.

IV. Fault Detection, Isolation and Accommodation

In the *scenario* considered in this paper, each thruster is an actuator potentially affected by faults. The basic idea is that, whenever a failure is detected and identified, a supervisor performs a control reconfiguration exploiting thrusters redundancy (three propellers are enough to control the ROV trajectory). In this framework, it is convenient to rewrite the model (3) as follows:

$$\dot{\boldsymbol{z}} = \boldsymbol{f}(\boldsymbol{z}) + \Delta\boldsymbol{f}(\boldsymbol{z},\boldsymbol{u}) + \bar{\boldsymbol{g}}(\boldsymbol{z}_p)\begin{bmatrix} T_1+T_2+T_3+T_4 \\ -T_1-T_2+T_3+T_4 \\ -T_1+T_2-T_3+T_4 \end{bmatrix} \tag{10}$$

with:

$$\bar{\boldsymbol{g}}(\boldsymbol{z}_p) = \begin{bmatrix} \boldsymbol{0}_{3\times 3} \\ \bar{\boldsymbol{g}}_1(z_3) \\ \bar{\boldsymbol{g}}_2(z_3) \\ \bar{\boldsymbol{g}}_3(z_3) \end{bmatrix} = \begin{bmatrix} & \boldsymbol{0}_{3\times 3} & \\ \frac{c_3\cos(\alpha)}{p_1} & \frac{-s_3\sin(\alpha)}{p_1} & 0 \\ \frac{s_3\cos(\alpha)}{p_1} & \frac{c_3\sin(\alpha)}{p_1} & 0 \\ 0 & 0 & \frac{d_a}{p_6} \end{bmatrix}$$

A. *Class of potential faults*

The addressed potential faults belong to a wide class. First, the so called *abrupt fault* [10], [11], [12] are considered. They are described by a step function, modeling the case when the faulty variable is instantaneously stuck to an unknown but bounded value. Such fault may occur when a failure of a component produce a sudden deviation of the actuator dynamics (as for example a valve completely failing to open or close, a short circuit in the motor circuitry [13]). This type of thruster faults has the following model:

$$|T_j(t)| = \bar{T} \leq T_{max} \quad \forall t \geq t_j, \quad j \in \{1,2,3,4\} \tag{11}$$

where the failure times t_j are unknown. In other words, when a fault does occur on a thruster, this causes the complete and permanent unavailability of the considered actuator at unknown time instant. This means that, from the unknown time istant t_j, it is not possible to recover the thruster functionality and thus only the remaining working thrusters can be used to control the vehicle. The case when a thruster undergoes a failure occurs when $\bar{T} = 0$.

Also, the behavior of a faulty device can be consequence of deterioration, obsolescence or cumulation phenomena, producing a small instantaneous deviation of the actuator behavior, but it cumulates in time; as a result, these faults can result in a loss of efficiency within the system. A usual way to mathematically describe such temporal behavior is assuming that the development of the fault is given by

$$T_j(t) = \begin{cases} T_j(t) & \text{for } t < t_j \\ T_j(t_j^-) + \left(\bar{T} - T_j(t_j^-)\right)\left(1 - e^{-\left(\frac{t-t_j}{\theta_j}\right)}\right) \\ \quad \text{for } t \geq t_j; \quad \theta_j > 0 \quad j \in \{1,2,3,4\} \end{cases} \tag{12}$$

where $|T_j(t)| \leq \bar{T} \leq T_{max}$, i.e. the loss of effectiveness slowly changes from zero (i.e. no fault is present) to a steady-state value $\bar{T}$ [12].

Assumption 3: Only one of the four thrusters can undergo a fault, i.e. multiple thruster faults cannot be admitted. Moreover, it is assumed that any fault does not compromise controllability of the plant driven by the remaining healthy thrusters.

Assumption 4: In the case of the fault model (12), it is assumed that the loss of effectiveness occurs slowly enough (see subsection IV-C).

Assumption 5: In view of the fact that the reaching phase can be made arbitrarily short, it is assumed that the fault can occur only after a sliding motion has been achieved on (6).

B. *Fault Detection*

The eventual occurrence of a fault and the identification of the failed thruster can be performed by means of simple considerations exploiting the ROV model. To this purpose, it is important to notice that the control law is computed imposing the achievement of a sliding motion of the observed surface (6). Therefore, once the sliding motion is established (i.e. when $\tilde{s}_i = 0$ after the reaching phase), it is straightforward to verify that any deviation of the sliding surface is due to the occurrence of a fault. For instance, if a fault occurs at the time t_f on the thruster T_1 such that the control input actually supplied undergoes a deviation $\Delta T_1(t-t_f)$ for $t > t_f$ with respect to its theoretical value, for the first sliding surface it holds:

$$\tilde{s}_1(t) - \tilde{s}_1(t_r) = \tilde{s}_1(t) = \frac{1}{p_1}\int_{t_f}^{t} \Delta T_1(\tau - t_f)\cos(z_3 - \alpha)d\tau$$

having denoted by t_r the time when the sliding motion is achieved (therefore $\tilde{s}_1(t) = 0$ for $t \geq t_r$).

Proposition 6: Consider the uncertain ROV model (3) driven by the robust controller (7). Suppose that the thruster T_k, $k \in \{1,\ldots,4\}$ undergoes a fault at time t_f, thus causing a deviation of $\Delta T_k(t-t_f)$ of the control input supplied with respect to its theoretical value. Then one has, for $t > t_f$:

$$\begin{aligned} \tilde{s}_1(t) &= \frac{1}{p_1}\int_{t_f}^{t} \Delta T_k(\tau - t_f)\cos(z_3 - (-1)^{k\div 3}\alpha)d\tau \\ \tilde{s}_2(t) &= \frac{1}{p_1}\int_{t_f}^{t} \Delta T_k(\tau - t_f)\sin(z_3 - (-1)^{k\div 3}\alpha)d\tau \\ \tilde{s}_3(t) &= \frac{d_a}{p_6}\int_{t_f}^{t} (-1)^k \Delta T_k(\tau - t_f)d\tau \end{aligned} \tag{13}$$

where the symbol $\div$ denotes the operator of division between integers.

Proof: The statement follows directly from Assumption 5 and from the observers (5). ■

From the previous proposition, a fault detection rule immediately follows.

Proposition 7: Consider the uncertain ROV model (3) driven by the robust controller (7) under Assumptions 3,4,5. Suppose that the thruster T_k, $k \in \{1,\ldots,4\}$ underwent a fault at time t_f. The fault can be detected checking the variables $\tilde{s}_i(t)$, $i = 1,2,3$, at $t < t_f$, i.e. if

$$(\tilde{s}_1(t) \neq 0) \; OR \; (\tilde{s}_2(t) \neq 0) \; OR \; (\tilde{s}_3(t) \neq 0)$$

then a fault has occurred.

Proof: The proof is straightforward. It simply consists in checking the eventual violation of the sliding mode existence condition, according to (13). It is worth recalling that, according to Assumption 5, the sliding motion has been established, and the sliding surface (6) should be zero in the absence of a fault affecting the actuators. ■

Remark 8: It should be noticed that the previous Proposition gives a sufficient condition, therefore the occurrence of a fault could not necessarily produce the variation of all sliding surfaces $\tilde{s}_i$ $(i = 1, 2, 3)$. Nevertheless, also in view of (13), one should consider the deviation from zero of the surfaces as symptomatic of the occurrence of a fault, since whenever the occurred fault were not severe enough to cause any deviation of $\tilde{s}_i$, $i = 1, 2, 3$, from zero, this would simply mean that the controller is still able to guarantee the achievement of the required performances in face of the fault itself.

C. Failed Thruster Isolation

The identification of the thruster which underwent the fault can be performed by means of simple considerations exploiting the ROV structure. Assume a fault has occurred at time t_f, and define for $t > t_f$:

$$I_{c1}(t) = \int_{t_f}^{t} \cos(z_3 - \alpha)d\tau; \quad I_{c2}(t) = \int_{t_f}^{t} \cos(z_3 + \alpha)d\tau;$$
$$I_{c3}(t) = \int_{t_f}^{t} \sin(z_3 - \alpha)d\tau; \quad I_{c4}(t) = \int_{t_f}^{t} \sin(z_3 + \alpha)d\tau.$$

Proposition 9: Consider the uncertain ROV model (3) driven by the robust controller (7), under Assumptions 3,4,5. Suppose that the thruster T_k, $k \in \{1, \ldots, 4\}$ underwent a fault at time t_f, then detected at time $t_d > t_f$. Compute the quantities for $t > t_d$:

$$\begin{aligned} \mu_{11}(t) =& \frac{\tilde{s}_1(t)}{I_{c1}(t)} p_1 \quad \mu_{13}(t) = \frac{\tilde{s}_1(t)}{I_{c3}(t)} p_1 \\ \mu_{22}(t) =& \frac{\tilde{s}_2(t)}{I_{c2}(t)} p_1 \quad \mu_{24}(t) = \frac{\tilde{s}_2(t)}{I_{c4}(t)} p_1 \\ \mu_3(t) =& \tilde{s}_3(t) \frac{p_6}{d_a} \end{aligned} \tag{14}$$

The failed thruster T_f can be isolated according to the following rule. Fix a time $\bar{t} > t_d$.

- If $sign(\mu_{11}(\bar{t})) \neq sign(\mu_3(\bar{t}))$ then
  - if $\mu_{22}(\bar{t}) = \mu_{11}(\bar{t})$ then $T_f = T_1$ else $T_f = T_3$;
- else (if $sign(\mu_{11}(\bar{t})) = sign(\mu_3(\bar{t}))$ then)
  - if $\mu_{22}(\bar{t}) = \mu_{11}(\bar{t})$ then $T_f = T_2$ else $T_f = T_4$;

Proof: The statement follows directly from Proposition 6. In fact, under Assumption 4 and for short intervals $(t_f, \bar{t})$, the term $\Delta T_k(\tau - t_f)$ in (13) can be moved outside the integral signs. It follows that comparing signs of the quantities (14) one can identify the failed actuator, exploiting the model (10). Just as an example, suppose that the thruster T_2 underwent a fault of intensity $\Delta T_2(\tau - t_f)$. According to the model (10), one has

$$\begin{aligned} \tilde{s}_1(t) &= \frac{1}{p_1} \Delta T_2 \int_{t_f}^{t} \cos(z_3 - \alpha)d\tau \\ \tilde{s}_2(t) &= \frac{1}{p_1} \Delta T_2 \int_{t_f}^{t} \sin(z_3 - \alpha)d\tau \\ \tilde{s}_3(t) &= \frac{d_a}{p_6} \Delta T_2(t - t_f) \end{aligned} \tag{15}$$

therefore $\mu_{11} = \Delta T_2$, $\mu_3 = \Delta T_2(t - t_f)$, and μ_{11}, μ_3 have the same sign. The same would anyway have occurred if the fault had undergone in the thruster T_4, in view of the structure of the matrix $\bar{\boldsymbol{g}}(z_3)$ of the model (10). To discriminate between T_2 and T_4, it is enough to consider that, from the first two equalities of (15):

$$\frac{1}{p_1} \Delta T_2 = \frac{\tilde{s}_1(t)}{\int_{t_f}^{t} \cos(z_3 - \alpha)d\tau} = \frac{\tilde{s}_2(t)}{\int_{t_f}^{t} \sin(z_3 - \alpha)d\tau}$$

This approach can be generalized to the remaining possible cases. In particular, in the case when $sign(\mu_{11}(\bar{t})) \neq sign(\mu_3(\bar{t}))$, only thrusters T_1 or T_3 could have experienced a fault. Moreover, if $\mu_{22}(\bar{t}) = \mu_{11}(\bar{t})$, then the failed actuator is T_1, otherwise is T_3. An analogous argument holds for the case when $sign(\mu_{11}(\bar{t})) = sign(\mu_3(\bar{t}))$, for which the candidate failed actuators are T_2 or T_4. ■

D. Control reconfiguration

After a failure has been detected and isolated by the FD and FI module respectively, the supervisor has to perform a control reconfiguration to preserve the desired performances in face of the failure occurrence. In particular, the inherent redundancy of the considered ROV can be exploited for fault accommodation. Consider the model (10), and suppose that a fault has been detected and isolated. The plant model can be rearranged separating the failed and the active thrusters, e.g. if the failed thruster was T_1, one would get:

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{f}(\boldsymbol{z}) + \Delta \boldsymbol{f}(\boldsymbol{z}, \boldsymbol{u}) + \begin{bmatrix} \mathbf{0}_{3\times3} \\ \bar{\boldsymbol{g}}_1(z_3) \\ -\bar{\boldsymbol{g}}_2(z_3) \\ -\bar{\boldsymbol{g}}_3(z_3) \end{bmatrix} T_1 +$$
$$\begin{bmatrix} \mathbf{0}_{3\times3} \\ \bar{\boldsymbol{g}}_1(z_3) \\ \bar{\boldsymbol{g}}_2(z_3) \\ \bar{\boldsymbol{g}}_3(z_3) \end{bmatrix} \begin{bmatrix} T_2 + T_3 + T_4 \\ -T_2 + T_3 + T_4 \\ T_2 - T_3 + T_4 \end{bmatrix} \tag{16}$$

To accommodate the fault, it is now easy to rewrite the sliding mode controller using this model and using the bound T^{max} available for the failed actuator, following the lines of Lemma 1. In view of the robustness properties of sliding-mode control, this procedure guarantees the robust asymptotical vanishing of the tracking errors also in the presence of a faulty actuator (with known upper bound), i.e. that robust regulation is achieved asymptotically.

V. Simulation results

The proposed actuator fault tolerant control scheme has been validated by simulation. Tests have been performed in the following operative condition: 1) Parameters variations of 10% with respect to their nominal value [8], [9]; 2) In the simulation tests, the plant initial condition has been chosen as $x(0) = 0$, $y(0) = 0$, $\phi(0) = 0$, $\dot{x}(0) = 0$, $\dot{y}(0) = 0$, $\dot{\phi}(0) = 0$, and the set point as $\boldsymbol{y}_d = \begin{bmatrix} 1 \text{ m} & 1 \text{ m} & 30° \end{bmatrix}^T$. 3) Favorable submarine current has been considered, with $V_c = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T$ m/s. Notice that such marine currents have been considered constant since they are very slowly time-varying due to the fact that they model submarine currents at great sea depth.

4) The actuator T_2 has been supposed to undergo an abrupt fault at $t = 70$ s of the form: $T_2(t) = 2000N \quad for\ t \geq 70\ s$. Before fault occurrence, the control action applied to the ROV is subdivided among the four thrusters (one of which is redundant). The thruster T_1 is used as the redundant one, and its value is set to $500N$ before the fault being detected and identified, in order to check that the value of the redundant thruster does not affect the performances of the overall control architecture. Of course, when the fault is detected and isolated, control reconfiguration is performed including the (previously redundant) actuator T_1 in the triple of actuators needed to control the ROV.

Results about the abrupt fault case have been reported in Figs. 2-5. It can be verified that, before the fault occurrence on T_2, actuators T_2, T_3 and T_4 are able to effectively control the ROV (see Figs. 2-3). Moreover, simulations results show that satisfactory performances are maintained also in the faulty situation, since the ROV controlled outputs effectively follow the reference values (see Fig. 2) also after fault occurrence, and that observation errors are bounded (see Fig. 4). It is interesting to verify that detection of the fault is correctly performed by the sliding surfaces at $t = 70$ s (see Fig. 5), since the sliding surfaces noticeably deviate from zero. After fault isolation, reconfiguration is performed, and the previously redundant thruster T_1 is activated, along with T_3 and T_4, for ensuring output regulation and for maintaining the required control performances.

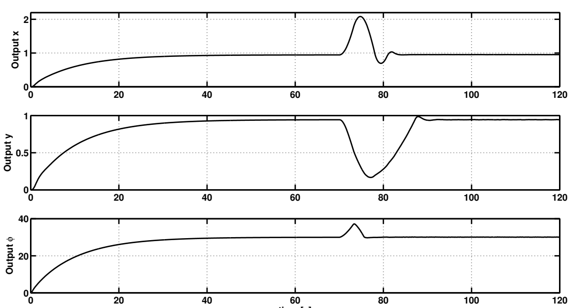

Fig. 2. Outputs.

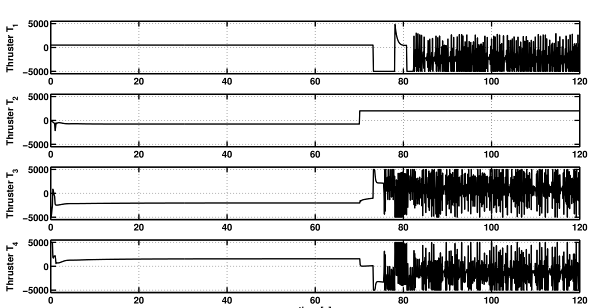

Fig. 3. Thrusters.

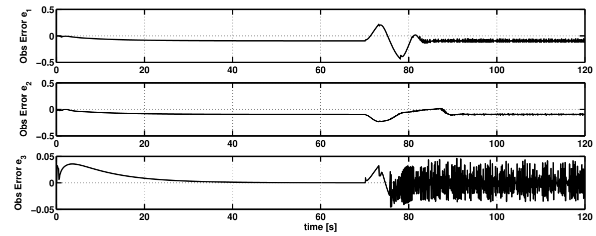

Fig. 4. Observation Errors.

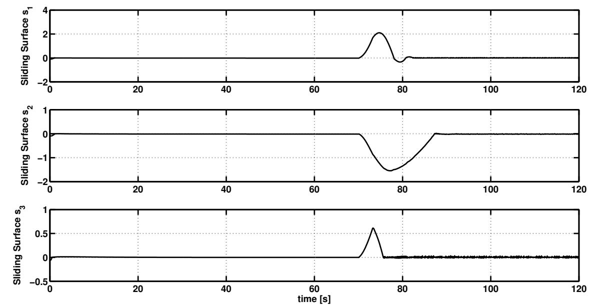

Fig. 5. Sliding Surfaces.

REFERENCES

[1] T. Fossen, *Guidance and control of ocean vehicles*. J.Wiley & Sons, New York., 1994.

[2] N. Sarkar, T. Podder, and G. Antonelli, "Fault-accommodating thruster force allocation of an AUV considering thruster redundancy and saturation," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 2, pp. 223–233, 2002.

[3] R. Isermann and P. Ballé, "Trends in the application of model-based fault detection and diagnosis of technical processes," *Control Engineering Practice*, vol. 5, no. 5, pp. 709–719, 1997.

[4] S. Longhi and A. Rossolini, "Adaptive control for an underwater vehicle: Simulation studies and implementation details," *Proc. IFAC Wkshp. Expert Syst. Signal Processing Marine Automa, Denmark*, pp. 271–280, 1989.

[5] J. Gertler, "Survey of model-based failure detection and isolation in complex plants," *Control Systems Magazine, IEEE*, vol. 8, no. 6, pp. 3–11, 2002.

[6] V. Utkin, *Sliding modes in control optimization*. Berlin: Springer Verlag, 1992.

[7] A. Conter, S. Longhi, and C. Tirabassi, "Dynamic model and self-tuning adaptive control of an underwater vehicle," *Proceedings Eighth International Conference on Offshore Mechanics and Arctic Engineering, The Hague, The Netherlands*, pp. 139–146, 1989.

[8] M. Corradini and G. Orlando, "A discrete adaptive variable structure controller for mimo systems, and its application to an underwater ROV," *IEEE Transactions on Control System Technology*, vol. 5, no. 3, pp. 349–359, 1997.

[9] M. Corradini, A. Monteriú, and G. Orlando, "A sliding mode switching controller for actuator failure compensation," *IEEE Transactions on Control systems Technology*, 2011, in press.

[10] R. Isermann, "Supervision, fault-detection and fault-diagnosis methods - an introduction," *Control Engineering Practice*, vol. 5 no. 5, pp. 639–652, 1997.

[11] J. Boskovic, S. Yu, and R. Mehra, "A stable scheme for automatic control reconfiguration in the presence of actuator failures," *Proc. 1998 Amer. Control Conf., Philadelphia, Pennsylvania*, pp. 2455–2459, 1998.

[12] T. Parisini and X. Z. M. Polycarpou, "A robust detection and isolation scheme for abrupt and incipient faults in nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 47 no. 4, pp. 576 – 593, 2002.

[13] J. Chen and R. Patton, "Fault-tolerant control using lmi design," *Control Engineering Practice*, vol. 5 no. 5, pp. 671–682, 1997.