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Voronoi Coverage Control with Time-Driven Communication for Mobile Sensing Networks with Obstacles

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Abstract— **It is known that the optimal sensor coverage of a mission space is performed by the Voronoi coverage. Aside from the energy required to move, communication among sensors consumes a large amount of their limited energy. So, to reduce communication energy, we propose a distributed control method that deploys sensors avoiding the obstacles by means of time-driven communication. We apply a potential field method to avoid obstacles. We propose a control method that consists of two processes: *Voronoi update process* and *position update process*. In the Voronoi update process, the sensor communicates with the other sensors and exchange information about their positions, periodically. Then, the sensing area of the sensor is updated using their positions. On the other hand, in the position update process, each sensor moves to the optimal position to increase its sensing performance on its sensing area which is fixed in the process. Thus, it does not need to communicate with the other sensors in the process. We show that a locally optimal sensor position is a locally uniformly asymptotically stable equilibrium point by the proposed method.**

I. INTRODUCTION

The interest in the cooperative control of a group of mobile sensors has significantly increased over the last decades [1]. Coverage control of sensors on a mission space for accomplishing the optimal sensing performance has many applications such as surveillance, search, and target intercept. In a sensor network, each sensor communicates with its neighbor sensors locally and decides its optimal position based on the local information. A lot of attention has been paid to the distributed control for the optimal deployment of a group of the sensors. However, the coverage problem would require the sensors to exchange their current positions. Aside from the energy required to move, communication among sensors consumes a large amount of their limited energy. Therefore, it is an important issue to reduce communication among sensors while they move to the optimal position.

Cortés *et al.* proposed a coordination control approach for multisensor networks [2]. Kwok and Martínez extended this cooperative approach to power-aware coverage control[3]. Zhong and Cassandras proposed an event-driven optimization scheme for maximizing sensing performance and reducing communication costs [4]. Distributed coverage control for sensor networks has been extensively studied [5], [6], [7]. Guruprasad and Ghose proposed a deploy-and-search strategy for the Voronoi coverage problem [8]. In this strategy, the sensors search their optimal deployment first. Then, they gather information in their Voronoi cells. These operations are repeated until the required level for sensing performance is reached. Another recent topic of cooperative control is the avoidance problem of multiple obstacles in a mission space [9]. The coverage control can also be applied to a convex space with obstacles [10], [11], [12], [13].

In this paper, we focus on a Voronoi coverage problem in order to monitor the whole mission space with obstacles using a group of mobile sensors, where the sensing area of each sensor is represented as a Voronoi cell. We also consider a coverage control method to reduce communication costs. Thus, we propose a distributed control method to deploy the sensors, avoiding the obstacles with time-driven communication, and apply the potential field method to avoid obstacles. The potential field method has been extensively utilized for obstacle avoidance [14]. To reduce the costs of communication among sensors, we propose a control method that consists of two processes: *Voronoi update process* and *position update process*. These processes are activated alternatively. The Voronoi update process is activated periodically; it requires information about the other sensors' positions and provides the sensing area of each sensor as a Voronoi partition generated by sensors' positions. Thus, the sensors intermittently communicate with each other and exchange information about their positions so that the energy consumed for the communication is reduced. On the other hand, each sensor searches for his optimal deployment in his sensing area given by a Voronoi cell when it is in the position update process. This search is done independently so that no energy for communication is consumed. Then, we can obtain an optimal deployment to maximize the sensing performance with lower communication costs.

The rest of this paper is organized as follows. In Section II, we formulate the Voronoi coverage problem with multiple obstacles as an optimization problem with an objective function which evaluates the sensing performance avoiding the obstacles. Section III describes the proposed distributed Voronoi coverage control method and discusses its stability. In Section IV, we demonstrate its efficiency by simulation and discuss the communication costs and convergence times. Finally, we conclude the paper in Section V.

II. PROBLEM FORMULATION

In this section, we formulate the problem addressed in this paper. We consider a bounded planar search region with obstacles, which will be called a mission space. For simplicity, we assume that the mission space is represented

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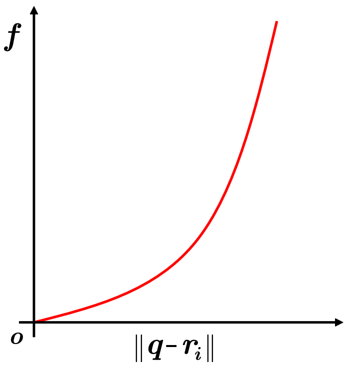


Fig. 1. Illustration of the sensing performance degradation function f.

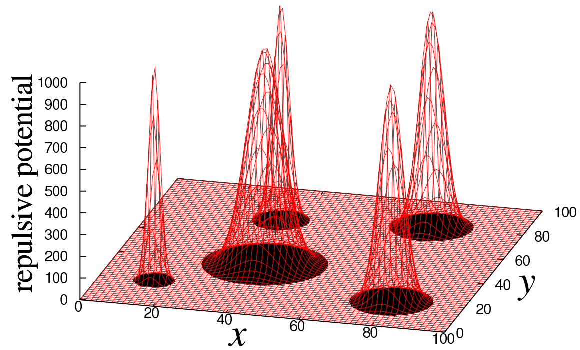


Fig. 2. Example of a convex mission space with obstacles colored by black and its repulsive potential.

by a convex polytope $Q \subset \mathbf{R}^2$. There are n mobile sensors collecting information about all points of the mission space. The collection at each point in the mission space is done by one mobile sensor. We introduce a function ϕ: $Q \rightarrow \mathbf{R}_+$ called a value function, which represents the (relative) importance of the information at each point in the mission space Q. The more important a point $q \in Q$ is, the larger the value $\phi(q)$ is. For example, the value function is a probability density function of the existence of a target object. We assume that ϕ is integrable and positive at every point in Q. Let $I = \{1, \ldots, n\}$ be a set of n mobile sensors which can move to any position in Q. Let $r_i(t) = (r_{i_1}(t), r_{i_2}(t))^T \in Q$ be the position of each sensor $i \in I$ at time t, where T denotes the transpose of a matrix or vector. $r(t) = ({r_1}^T(t), \ldots, {r_n}^T(t))^T$ denotes the deployment of n sensors.

Because of noise and loss of resolution, the sensing performance of sensor i at a point $q \in Q$ degrades with the distance $\|q - r_i\|$ between a point q and its current position r_i. The farther the distance $\|q - r_i\|$ is, the larger the cost of the sensing performance is. For simplicity, we assume that the degradation of the sensing performance of every sensor is identical and represented by a nondecreasing differentiable function $f : \mathbf{R}_+ \rightarrow \mathbf{R}_+$. Therefore, $f(\|q - r_i\|)$ provides a quantitative assessment of the degradation of the sensing performance. In general, we can assume that $f(\cdot)$ is a nondecreasing function as shown in Fig. 1.

Given the mission space $Q \subset \mathbf{R}^2$ and the deployment $p = ({p_1}^T, \ldots, {p_n}^T)^T \in \mathbf{Q}^n$ of n distinct points $p_1, \ldots p_n \in Q$, the Voronoi partition of Q generated by p is the collection of sets $\{\nu_1(p), \ldots, \nu_n(p)\}$, where $\nu_i(p) := \{q \in Q| \; ||q - p_i|| \leq ||q - p_j|| \text{ for all } i \neq j, \; j \in I\}$ [1]. The position p_i, the deployment p, and the set $\nu_i(p)$ are called the generating point, the generator, and the Voronoi cell, respectively.

We assume that the sensing area of the sensor i is the Voronoi cell $\nu_i(p)$. With the value function ϕ and the sensing performance degradation function f, we introduce an objective function $H_1(r,p)$ which evaluates the collective sensing performance of a group of all mobile sensors as follows:

$$H_1(r,p) = \sum_{i=1}^{n} \int_{\nu_i(p)} f(\|q - r_i\|)\, \phi(q) dq. \tag{1}$$

The smaller the value $H_1(r,p)$ is, the better the sensing performance in the mission space is. It is known that, for a given sensor deployment r, the optimal Voronoi partition minimizing $H_1(r,p)$ satisfies $p = r$.

In this paper, we assume that the mission space has several obstacles, which the deployment of the sensors must avoid. Inspired by the potential field method, we introduce a repulsive potential, which pushes the sensor away from the obstacles, as shown in Fig. 2. This repulsive potential creates a potential barrier around the obstacles and does not affect the trajectory of the sensor when it is sufficiently far from the obstacles. So, the sensors are able to avoid the obstacles. We define the repulsive potential as a differentiable function $g : Q \rightarrow \mathbf{R}_+$. Then, we define the objective function H_2, which evaluates the repulsive potential for sensors, as follows:

$$H_2(r) := \sum_{i=1}^{n} g(r_i). \tag{2}$$

Note that $H_2(r)$ is independent of the Voronoi partition.

To search a path to the optimal position for each mobile sensor while avoiding the obstacles, we introduce the following objective function $H(r,p)$:

$$\begin{aligned} H(r,p) \;&:=\; H_1(r,p) + H_2(r) \\ &=\; \sum_{i=1}^{n} \left\{ \int_{\nu_i(p)} f(\|q - r_i\|)\, \phi(q) dq + g(r_i) \right\}. \end{aligned} \tag{3}$$

The optimal sensor deployment r^* and the generator p^* minimize the objective function $H(r,p)$. Since r^* and p^* coincide, we formulate the Voronoi coverage problem with obstacles as the following optimization problem:

$$\begin{aligned} \underset{r}{\text{minimize}} \quad & H(r,r), \\ \text{subject to} \quad & r_i \in Q, \;\; \forall i \in I. \end{aligned} \tag{4}$$

We propose a distributed control method with intermittent exchange of positions among sensors to optimize the deployment of sensors in the next section.

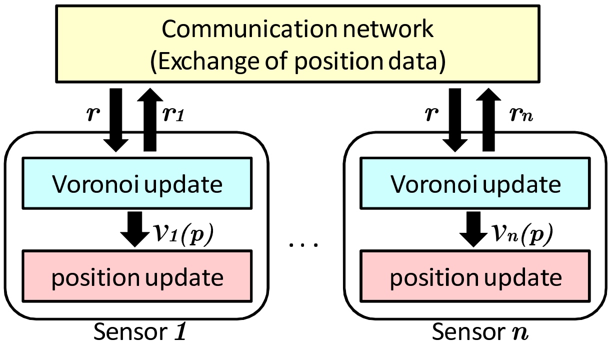


Fig. 3. Cooperative control of a mobile sensing network consisting of two processes: a Voronoi update process and a position update process. In each sensor, the former process sends its current position to the other sensors, receives their positions, and updates its Voronoi cell. The latter process controls the sensors' movements to reach its optimal position in the cell.

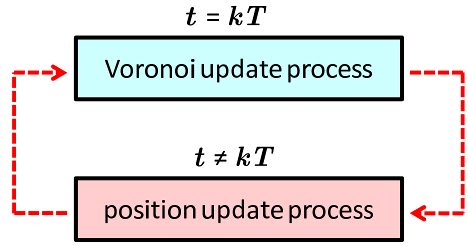


Fig. 4. The Voronoi update and the position update process. At time $t = kT$, the Voronoi update process communicates with the other sensors and updates the Voronoi cell. In every time interval $(kT,\ (k+1)T)$, the position update process makes a path to the optimal position in the cell on-line.

III. Cooperative control method

A. Architecture

To update a Voronoi partition, each sensor collects the current positions of its neighbor sensors by means of a communication network. To reduce the energy used for communication among sensors, we propose the cooperative control method for the Voronoi coverage problem with obstacles. The proposed control architecture consists of two processes as shown in Fig. 3: a *Voronoi update process* and a *position update process*. As shown in Fig. 4, in each mobile sensor, the Voronoi update process is activated periodically. Let T be its period. The process sends its current position to other sensors and receives their positions, periodically. Then, the Voronoi cell where the sensor collects information is updated such that the generator $p(kT)$ is set to the current deployment $r(kT)$ of sensors. For simplicity, we assume that the update is completed instantly since the process does not give any effect on the trajectory of sensors. The update is described as follows:

$$p_i(t) = r_i(kT),\ \ t \in [kT, (k+1)T)\,. \tag{5}$$

Equivalently,

$$\frac{dp_i(t)}{dt} = \sum_{k=0}^{\infty}(r_i(t) - p_i(t))\delta(t - kT), \tag{6}$$

where $\delta(\cdot)$ is the delta function and the initial condition is $p_i(0-) = 0$. After the updated Voronoi cell is computed, the position update process is activated and plans a path to the sensor's optimal position in the cell until the next activation of the Voronoi update process. So, the sensor moves along the path at every time interval $(kT,\ (k+1)T)$, where k is a nonnegative integer. It halts at time kT and the Voronoi update process updates its Voronoi cell. In the following, we discuss path planning in the position update process.

The position update process plans the sensor's trajectory to an optimal position in the fixed Voronoi cell while avoiding obstacles. In each time interval $(kT,\ (k+1)T)$, the generator of the Voronoi partition $p(t)$ is given by Eq. (5), so that the Voronoi cell $\nu_i(p(kT))$ where sensor i collects information is fixed. Eq. (3) is rewritten as

$$H(r,p) = \sum_{i=1}^{n} h_i(r_i, p), \tag{7}$$

where

$$h_i(r_i, p) = \int_{\nu_i(p)} f\left(\|q - r_i\|\right)\phi(q)dq + g(r_i). \tag{8}$$

For a given $\nu_i(p)$, Eq. (8) is independent of the positions of the other sensors. The generator $p(t)$ is constant in any interval $[kT, (k+1)T)$. Applying the gradient descent method, we introduce the following control law for sensor i: for any $t \in [kT, (k+1)T)$

$$\begin{aligned} \frac{dr_i(t)}{dt} &= -\alpha\frac{\partial H(r(t), p(t))}{\partial r_i} \\ &= -\alpha\frac{\partial h_i(r_i(t), p(kT))}{\partial r_i}, \end{aligned} \tag{9}$$

where α is a positive constant. Note that the trajectory of sensor i does not depend on the current state of other sensors, but on its Voronoi cell $\nu_i(p(kT))$, which is fixed in the time interval. Thus, the control law (9) is computed dispersively in the position update process of each sensor and makes each sensor move towards its optimal position in $\nu_i(p(kT))$.

Let it also be noted that the proposed control method is based on the same approach as the deploy-and-search strategy proposed by Guruprasad and Ghose [8]. They do not discuss obstacle avoidance or model a trajectory of the generator of the Voronoi partitions explicitly. Especially, using Eqs. (6) and (9), we will show in the next section that the generator of the Voronoi partition providing a locally optimal deployment is uniformly asymptotically stable.

B. Stability of optimal deployment

In this section, we discuss the stability of a locally optimal deployment of the objective function $H(r,p)$ by the proposed control law. Note that a pair $(r(t), p(t))$ of the current deployment and the Voronoi generator is a state of the considered sensing network. The state is governed by the time-dependent impulsive differential equations with periodic resetting (6) and (9). We assume that $H : Q^{2n} \to \mathbf{R}_+$ is continuously differentiable in Q^{2n}.

Let (r^*, r^*) be a locally optimal point of the optimization problem (4), and assume that:

(A1) (r^*, r^*) is in the interior of Q, which implies that

$$\frac{\partial H(r,p)}{\partial r}|_{(r,p)=(r^*,r^*)} = 0, \tag{10}$$

and

(A2) $\frac{\partial H(r,p)}{\partial r} \neq 0$ at any point $(r,p) \neq (r^*, r^*)$ in its sufficiently small neighborhood $B(r^*, r^*)$.

From the assumption (A1), (r^*, r^*) is an equilibrium point of Eqs. (6) and (9).

The sets $D(\gamma)$ and $M(\gamma)$ are defined for a real number γ greater than $H(r^*, r^*)$ as follows:

$$D(\gamma) = \{(r,p) \in B(r^*,r^*) \mid H(r,p) \leq \gamma\}, \tag{11}$$

$$M(\gamma) = \{(r,p) \in B(r^*,r^*) \mid H(r,p) = \gamma\}. \tag{12}$$

Let $\hat{\gamma} = \max\{\gamma \mid D(\gamma) \subseteq B(r^*, r^*)\}$. Then, we have $D(\hat{\gamma})$ is compact and $H(r,p) \geq H(r^*, r^*)$ for any $(r,p) \in D(\hat{\gamma})$.

We consider $H(r,p)$ to be a Lyapunov function candidate. Let $(r(t), p(t))$ be a trajectory of Eqs. (6) and (9) with $(r(0), p(0)) \in D(\hat{\gamma})$. Since the trajectory of the generator p is constant in each time interval $(kT, (k+1)T)$ for any $K \geq 0$, the derivative of $H(r,p)$ with respect to time along the trajectory exists in the time intervals:

$$\begin{aligned} \frac{dH(r(t),p(t))}{dt} &= \left(\frac{\partial H(r(t),p(t))}{\partial r}\right)^T \frac{dr(t)}{dt} \\ &= -\alpha \left\| \frac{\partial H(r(t),p(t))}{\partial r} \right\|^2 \leq 0. \end{aligned} \tag{13}$$

Thus, $D(\hat{\gamma})$ is positively invariant. According to Eq. (6), the generator $p(t)$ is discontinuous at each time kT and jumps to the current deployment. Since for a given sensor deployment, the optimal Voronoi partition of the mission space satisfies the condition that its generator is the deployment, we have:

$$\lim_{t \downarrow kT} H(r(t),p(t)) - \lim_{t \uparrow kT} H(r(t),p(t)) \leq 0. \tag{14}$$

The assumption (A2) implies that both Eqs. (13) and (14) hold only if $(r(t), p(t)) = (r^*, r^*)$. So, we have that $M(\gamma)$ does not contain any trajectory of Eqs. (6) and (9) for any γ with $H(r^*, r^*) < \gamma \leq \hat{\gamma}$. Thus, by applying Theorem 2.7 of [15], we conclude that, for any locally optimal point r^* of the optimization problem (4), the state (r^*, r^*) is a locally uniformly asymptotically stable equilibrium point of Eqs. (6) and (9).

Remark 1: If the assumption (A2) does not hold, the state (r^*, r^*) is a locally uniformly stable equilibrium point of Eqs. (6) and (9).

IV. Simulation Results

We consider that 10 sensors cover a square mission space $Q \in [0, 100] \times [0, 100]$ with 5 obstacles, as shown in Fig. 2. For simplicity, we assume that the obstacles are circles. Let s_j be a central coordinate of the obstacle j. We use the following repulsive potential corresponding to the obstacles:

$$g(q) = 1000 \sum_{j=1}^{5} \exp\left(-\frac{\|q - s_j\|}{d_j}\right), \tag{15}$$

where $(d_1, d_2, d_3, d_4, d_5) = (5, 48, 21, 12, 21)$. We consider the following value functions $\phi_1(q)$ and $\phi_2(q)$ for the mission space:

$$\phi_1(q) = 1, \tag{16}$$

$$\phi_2(q) = 2\exp\left(-\frac{\left\|q-(80,50)^T\right\|}{200}\right) + \exp\left(-\frac{\left\|q-(20,60)^T\right\|}{100}\right). \tag{17}$$

The value function $\phi_1(q)$ means that every point in the mission space has the same importance. Note that $\phi_2(q)$ is a two-hump function whose peaks are $(80, 50)^T$ and $(20, 60)^T$. We search the optimal sensor deployment to minimize the objective function (3) using the proposed control method.

We show the trajectories of sensors for the value function ϕ_1(q) in Figs. 5(a)–(d), where the symbol "$\times$" is the convergence position of each sensor and the dotted lines are the boundaries of the Voronoi partition when all sensors are at the convergence positions. At first, in the time interval $[0, T)$, each sensor monitors a fixed sensing area and moves controlled by the position update process as shown in Fig. 5(a). Next, the sensing area is updated at $t = T$ using the Voronoi update process and in the time interval $[T, 2T)$ the sensor moves in the updated sensing area as shown in Fig. 5(b). Each sensor repeats these processes until all sensors converge. Finally, the sensors converge to the positions marked by "$\times$" in Fig. 5(c), which shows that the sensors move to the convergence positions avoiding the obstacles. The Voronoi partition at the convergence positions is represented in Fig. 5(d). Since the value function $\phi_1(q)$ is uniform, the measure of each sensing area at the convergence position is almost the same.

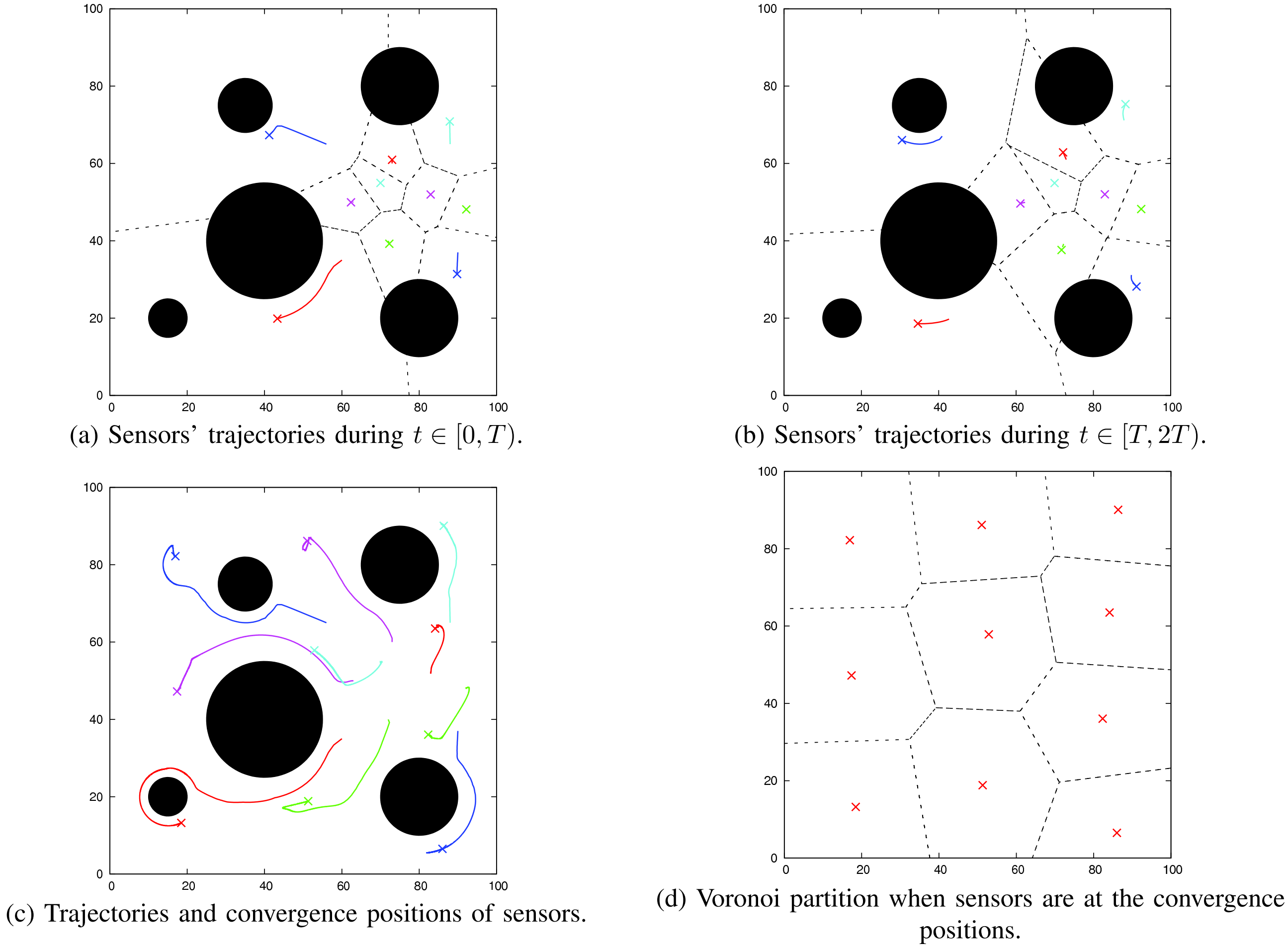
In this simulation, Eq. (9) is discretized with the step size $\Delta = (1/2)^{12}$ as follows:

$$r_i(t+\Delta) = r_i(t) + \Delta\alpha \frac{\partial H(r_i(t), p_i(kT))}{\partial r_i}. \tag{18}$$

Shown in Fig. 6 is a change of the objective function H along the sensors' trajectories for the following update period T:

- Ex1: $T = \Delta$
- Ex2: $T = 5\Delta$
- Ex3: $T = 10\Delta$
- Ex4: $T = 15\Delta$
- Ex5: $T = 20\Delta$
- Ex6: $T = 30\Delta$
- Ex7: $T = 40\Delta$
- Ex8: $T = 50\Delta$

As shown in Fig. 6, the objective function H is discontinuous when the Voronoi update process is activated at time $t = kT$. For case Ex1, each sensor updates its sensing area by the Voronoi update process every time because of $T = \Delta$. Thus, the sensors always communicate with each other. On the other hand, for case Ex8, each sensor updates its sensing area when the period $T = 50\Delta$ elapses. The larger the period T is, the longer it takes for the sensors to converge. Figure 7 shows the relationship between the period T and the convergence time. As T is smaller, the convergence time approaches to 0.55 while it increases linearly for a sufficiently large T. Figure 8 shows the relationship between the period T and the number of Voronoi updates. As T becomes smaller, the number increases exponentially. Since communication costs



(a) Sensors' trajectories during $t \in [0, T)$.

(b) Sensors' trajectories during $t \in [T, 2T)$.

(c) Trajectories and convergence positions of sensors.

(d) Voronoi partition when sensors are at the convergence positions.

Fig. 5. Sensors' trajectoris when the value function is ϕ_1.

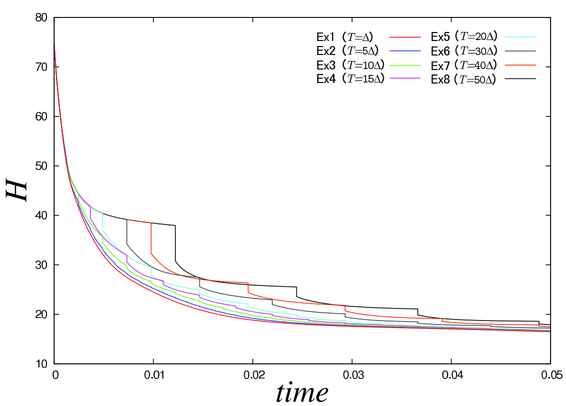


Fig. 6. For each period T, changes of the objective function H along sensors' trajectories.

are linearly dependent on the numbers of the update, Figs. 7 and 8 show the trade-off between the convergence time and the communication costs. In this paper, we also assume that the Voronoi update process is completed instantly. However, if the Voronoi update process takes a certain time, there exists an optimal period such that the convergence time is minimized under reduction of the communication costs.

On the other hand, Figs. 9(a) and (b) show the trajectories of sensors when the value function is $\phi_2(q)$, where the symbol "$\times$" and the dotted lines are used with the same meaning as in Fig. 5. Figure. 9(a) too shows that the sensors move to the convergence positions avoiding the obstacles. In Fig. 9(b), the value $\phi_2(q)$ is high at the red areas and low at the blue areas. As shown in Fig. 9(b), an area where $\phi_2(q)$ is high is partitioned into a smaller area than that where $\phi_2(q)$ is low. Since the ability of the sensor at a point q depends on the distance between the position of the sensor and the point q, the sensors tend to move into the more important area. Figure. 9(b) shows that 6 sensors converge to the area near the peak $(80, 50)^T$, rather than the area near peak $(20, 60)$, because the point $(80, 50)^T$ is more important than the point $(20, 60)$. We also confirmed that the objective function converges to a locally optimal value.

V. Conclusions

We proposed a distributed control method that deploys sensors avoiding obstacles and reduces communication costs by means of time-driven communication. We showed that the sensors converge to minimize the degradation of the sensing performance. We also indicated a trade-off between the convergence time and communication costs. As future

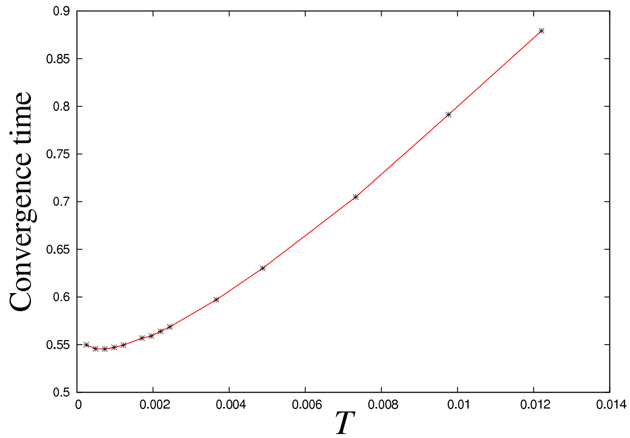


Fig. 7. Relationship between the period T and the convergence time.

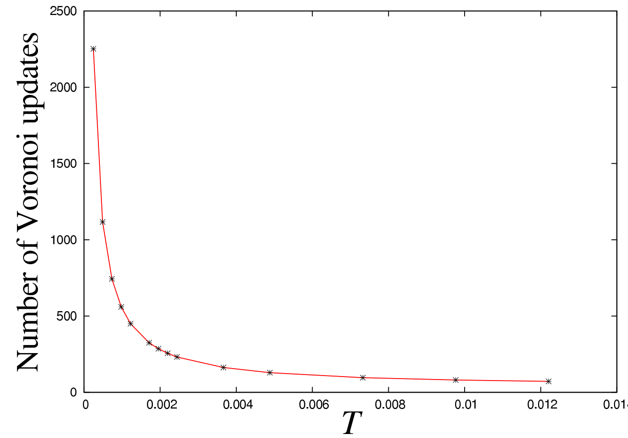
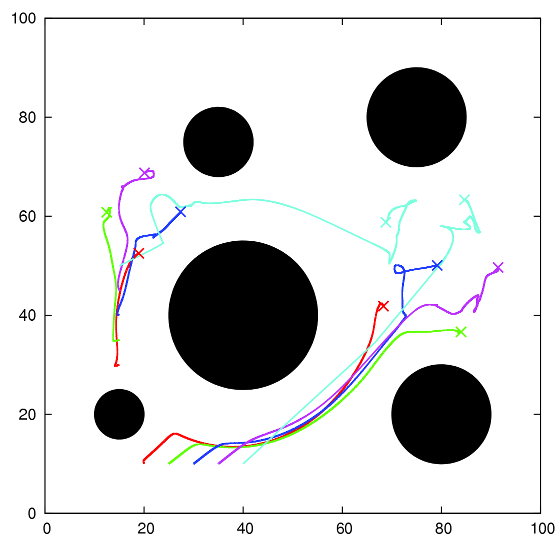
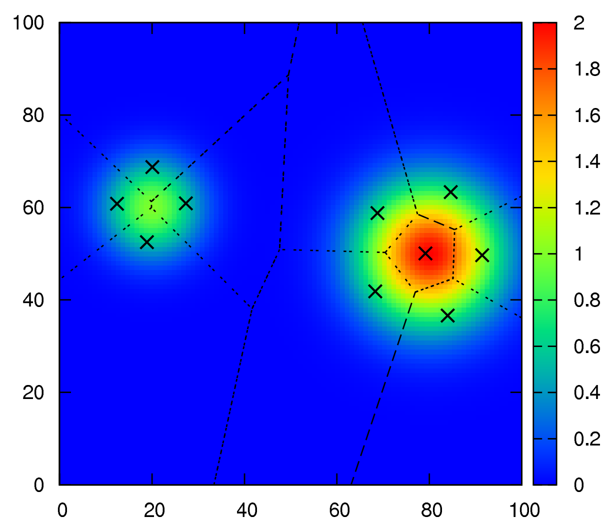


Fig. 8. Relationship between the period T and the number of Voronoi updates.



(a) Sensors' trajectories.



(b) Voronoi partition when sensors are at the convergence positions.

Fig. 9. Sensors' trajectories and optimal Voronoi coverage when the value function is ϕ_2.

work, we will consider the optimal Voronoi update period in order to minimize the convergence time and to reduce communication costs. It is also future work to investigate conditions of a repulsive potential in order for guaranteeing the obstacle avoidance.

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