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Synchronization of Linear Object Networks by Output Feedback

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***Abstract*— For a network of identical linear objects an output synchronization problem is considered. Problem of synchronization is solved under conditions of incomplete measurements, incomplete control and without constructing observers. Parameters of of static controller and sufficient synchronization conditions are obtained by means of passification method.**

I. Introduction

The problems of network systems control have a broad area of important applications: formation control of mobile robots [5], [14], control of power networks, control of physical, mechanical networks, etc. Also network systems are subject of study of a new field called "Cybernetical Physics" [**?**].

Synchronization is a one of the network control problems. Graph theory is often used for description and analysis of a network systems since informational flows in such systems can be described by the graphs. Design of a controller providing a synchronization in directed networks (i.e. described by directed graphs) is a more important and challenging problem than same problem in undirected networks since decrease of information exchange (traffic) [12].

Laplace matrices of graphs play important role when consensus control is used [3], [13], [12].

The aim of this paper is to design controllers providing convergence of a solutions of linear dynamical systems between themselves under conditions of incomplete measurements and incomplete control for a different cases applying consensus control. Mentioned dynamical systems form a network of objects which aren't dynamically interconnected, i.e. objects doesn't have a direct influence on each other.

Using passification method parameters of static controller and conditions of synchronization by output in directed networks consisting of linear objects are obtained for different informational graphs under conditions of incomplete measurements and incomplete control. Dynamical controller which is more complex considered in [5]. In [14] problem solved by constructing observers, it leads to a growth of system order if number of agents in network is growing. Moreover, conditions of operability is quite hard to analyze. In [9] network consisting of linear objects with incomplete measurements and incomplete control is considered. It is assumed that zero eigenvalue of Laplace matrix is simple. Problem of synchronization by output (consensus problem) is solved using observers and conditions of synchronization is formulated using LMI.

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II. Auxiliary results

A. Graph theory

In this section some terms of graph theory, definition and properties of Laplace matrix (Laplacian) needed in this paper are listed.

A pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ – set of vertices $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ – set of arcs, is called digraph (directed graph). Let $\mathcal{V}$ have d elements, $d \in \mathbb{N}$. If for any arc $(\alpha, \beta) \in \mathcal{E}$, where $\alpha, \beta \in \mathcal{V}$, arc $(\beta, \alpha) \in \mathcal{E}$, then graph called undirected. It is assumed hereafter that graphs doesn't have a self-loops, i.e. for any vertex $\alpha \in \mathcal{V}$ arc $(\alpha, \alpha) \notin \mathcal{E}$. If for any vertex there exists path to any other vertex then digraph is called strongly connected and undirected graph is called connected. In this case (di)graph has one connected component. It is known that undirected graph is connected iff it has a spanning tree.

Let's introduce a directed spanning tree as in [1], [13]. Digraph is called directed tree if all its vertices except one (called root) have exactly one parent. Directed spanning tree of a digraph $\mathcal{G}$ is a directed tree formed of by digraph $\mathcal{G}$ arcs such that there exists path from root to any other vertex of $\mathcal{G}$ in this tree.

An undirected graph is called weighted if for any pair of vertices $\alpha, \beta \in \mathcal{E}$ a number (weight) $w(\alpha, \beta) \geq 0$ with following properties is assigned:

1) $w(\alpha, \beta) > 0$ if $(\alpha, \beta) \in \mathcal{E}$ and $w(\alpha, \beta) = 0$ if $(\alpha, \beta) \notin \mathcal{E}$,
2) $w(\alpha, \beta) = w(\beta, \alpha)$.

A digraph is called weighted if for any pair of vertices $\alpha, \beta \in \mathcal{E}$ a such number $w(\alpha, \beta) \geq 0$ is assigned that first property holds. An adjacency matrix $A(\mathcal{G}) = [a_{ij}]$ is $d \times d$ matrix whose $i-$th, $j-$th entry is equal to $w(\alpha_i, \alpha_j), i, j = 1, \ldots, d$. Let us define in-degree of vertex α_i:

$$d_{\text{in}}(\alpha_i) = \sum_{j=1}^{d} a_{ji},$$

and out-degree:

$$d_{\text{out}}(\alpha_i) = \sum_{j=1}^{d} a_{ij}.$$

A (di)graph $\mathcal{G}$ is called balanced if for any vertex $\alpha_i \in \mathcal{V}$ in-degree is equal to out-degree ([3], [12]):

$$d_{\text{in}}(\alpha_i) = d_{\text{out}}(\alpha_i).$$

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Introduce $d \times d$ matrix

$$D(\mathcal{G}) = \mathrm{diag}\{d_{\mathrm{out}}(\alpha_1), d_{\mathrm{out}}(\alpha_2), \ldots, d_{\mathrm{out}}(\alpha_d)\}.$$

Laplace matrix (Laplacian) of (di)graph $\mathcal{G}$ is defined as follows:

$$L(\mathcal{G}) = D(\mathcal{G}) - A(\mathcal{G}).$$

Denote $\mathbf{1}_d$ column vector of order d cosisting of ones. It is known [3], [11], [13], [12], that introduced Laplacian L has following properties:

1) Matrix $L(\mathcal{G})$ has zero eigenvalue with corresponding right eigenvectorvector $\mathbf{1}_d : L(\mathcal{G})\mathbf{1}_d = 0$.
2) For undirected graph $\mathcal{G}$ the multiplicity of the zero value as an eigenvalue of $L(\mathcal{G})$ is equal to the number of connected components of $\mathcal{G}$.
3) Zero eigenvalue of L is simple if corresponding digraph is strongly connected.
4) Zero eigenvalue of L is simple iff corresponding digraph has a directed spanning tree.
5) All eigenvalues of Laplacian matrix have a nonnegative real parts.
6) If graph is balanced then $\mathbf{1}_d$ is left eigenvector corresponding to zero eigenvalue:

$$\mathbf{1}_d^{\mathrm{T}} L(\mathcal{G}) = 0.$$

B. Passification method

In this section some information about method of linear systems passification is listed [6], [8].

Let A, B, C, G, R be complex valued matrices of sizes $n \times n, n \times m, n \times l, l \times m, n \times n$ accordingly ($m \leq n, l \leq n$), and $R = R^* \geq 0$. Asterisk stands for transposition of the matrix and complex conjugation of its elements. Consider the following problem. Find conditions of existence Hermitian $n \times n$ matrix $H = H^* > 0$ and complex $l \times m$ matrix θ such that

$$HA(\theta) + A(\theta)^* H + R < 0, \tag{1}$$

$$HB = CG, \tag{2}$$

where

$$A(\theta) = A + B\theta^* C^*. \tag{3}$$

The case when matrices A, B, C, G, R are real valued is called real case. Let I_n stand for identity matrix of order n. Denote

$$\delta(s) = \det(sI_n - A), \chi(s) = C^*(sI_n - A)^{-1}B.$$

Let $W(s)$ - $m \times m$-matrix consisting of proper fractional rational functions, $\alpha(s)$ be least common multiple of $W(s)$ elements denominators. Let

$$\varphi(s) = \delta(s)\mathrm{det}W(s), \Gamma = \lim_{s \to \infty} sW(s).$$

Definition 1: Matrix is called minimum-phase if $\varphi(s)$ - is Hurwitz polynomial. Matrix $W(s)$ is called strictly minimum phase if it is minimum phase and matrix Γ is nonsingular: $\det \Gamma \neq 0$. Matrix is called hyper minimum phase if it is minimum phase and Γ is Hermitian and positive definite.

Solution to the problem provides the *Passification lemma*.

Lemma 1: For the existence of the matrices $H = H^* > 0$ and θ satisfying relations (1)-(3) and being real valued in the real case, it is sufficient (and when $\mathrm{rank}B = m$ it is also necessary) that the matrix $G^*\chi(s)$ be hyper minimum phase.

The definition simplifies for SISO systems.

Definition 2: Let $W(s) = \beta(s)/\alpha(s), z \in \mathbb{C}$ be proper fractional rational function, $\beta(s)$, $\alpha(s)$ are real polynomials. If numerator $\beta(s)$ of $W(s)$ is Hurwitz polynomial then $W(s)$ is called *minimum phase*. If $W(s)$ is minimum phase and number $\lim_{s \to +\infty} sW(s)$ is positive then $W(s)$ is called *hyper minimum phase*.

Remark 1: Vector θ in (3) can be chosen in form

$$\theta = -\varkappa \cdot G,$$

where number

$$\varkappa = \inf_{\omega \in \mathbb{R}^1} \mathrm{Re}\big(\mathrm{det}W(i\omega)\big),$$

see [7], [8].

III. Problem statement

Let the network S consist of d agents $S_i, i = 1, \ldots, d$. Each agent $S_i, i = 1, \ldots, d$ is modeled as a linear controlled system:

$$\dot{x}_i = Ax_i + Bu_i, \quad y_i = C^{\mathrm{T}} x_i, \tag{4}$$

where $x_i(t) \in \mathbb{R}^n$ is state vector, $u_i(t) \in \mathbb{R}^1$ is controlling input (control), $y_i(t) \in \mathbb{R}^l$ is the vector of measurements (output), time $t \in [0, +\infty)$.

Consider digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with the set of vertices $\mathcal{V}$ and the set of arcs $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Let us associate vertex v_i with agent S_i for each $i = 1, \ldots, d$.

The arc (v_i, v_j) will be considered to belong to the set of arcs $\mathcal{E}$ if an information from agent S_j is received by agent S_i. In addition let the weigh of each arc be equal to 1.

Let control law for agent S_i be

$$u_i = K \sum_{j \in \mathcal{N}_i} (y_i - y_j) = KC^{\mathrm{T}} \sum_{j \in \mathcal{N}_i} (x_i - x_j), \quad K \in \mathbb{R}^{1 \times l}, \tag{5}$$

where $\mathcal{N}_i = \{k = 1, \ldots, d | (v_i, v_k) \in \mathcal{E}\}$ is the set of neighbor vertices to v_i. It is assumed that graph has no selfloops i.e. $(v_i, v_i) \notin \mathcal{E}$ for all $i = 1, \ldots, d$. Such control is called "consensus control" in some papers.

Let the control goal be in a form of state synchronization:

$$\lim_{t \to \infty} (x_i(t) - x_j(t)) = 0, \qquad i, j = 1, \ldots, d. \tag{6}$$

The problem is to find K from (5) ensuring the goal (6).

In the case of goal (6) achievement asymptotic behaviour of all network S agents will be same. By denoting $c(t)$ this asymptotic synchronous behaviour of agents the control goal (6) can be reformulated as follows:

$$\lim_{t \to \infty} (x_i(t) - c(t)) = 0, i = 1, \ldots, d. \tag{7}$$

IV. Conditions of goal achievement in the case of balanced graph

Denote $\chi(s) = C^{\mathrm{T}}(sI_n - A)^{-1}B, s \in \mathbb{C}$ and make following assumptions.

A1) *Digraph $\mathcal{G}$ has an oriented spanning tree.*

This assumption ensures simplicity of Laplace matrix $L(\mathcal{G})$ zero eigenvalue (see section II-A).

A2) *There exists a vector $g \in \mathbb{R}^l$ such that function $g^{\mathrm{T}}\chi(s)$ is hyper minimum phase.*

According to lemma 1 from section II-B the latter assumption ensures existence of such matrix $H = H^{\mathrm{T}} > 0$ and vector $\theta \in \mathbb{R}^l$ that:

$$HA_* + A_*^{\mathrm{T}}H < 0, \quad HB = Cg, \quad A_* = A + B\theta^{\mathrm{T}}C^{\mathrm{T}}. \quad (8)$$

and vector θ can be taken as follows:

$$\theta = -\varkappa \cdot g, \quad (9)$$

where number $\varkappa > \inf\limits_{\omega\in\mathbb{R}^1} \mathrm{Re}\big(g^{\mathrm{T}}\chi(i\omega)\big)$ (see remark 1 section II-B).

Let us take feedback gain row vector K of control law (5) as follows:

$$K = -k \cdot g^{\mathrm{T}}, \quad k \in \mathbb{R}^1. \quad (10)$$

Associate graph $\widehat{\mathcal{G}}$ with digraph $\mathcal{G}$ such that $A(\widehat{\mathcal{G}}) = A(\mathcal{G}) + A(\mathcal{G})^{\mathrm{T}}$. Laplace matrix $L(\widehat{\mathcal{G}})$ of constructed graph $\widehat{\mathcal{G}}$ is symmetric and has simple zero eigenvalue. Matrix $L(\widehat{\mathcal{G}})$ has the following eigenvalues $0 = \lambda_1 < \lambda_2 \le \ldots \le \lambda_d$. Let $x = \mathrm{col}(x_1, \ldots, x_d)$, and $\otimes$ stand for Kronecker product of matrices.

Theorem 1: Let assumptions A1, A2 hold and graph $\mathcal{G}$ be balanced. Then for k such that

$$k \ge \frac{2\varkappa}{\lambda_2}, \quad (11)$$

where number $\varkappa = \inf\limits_{\omega\in\mathbb{R}^1} \mathrm{Re}\big(g^{\mathrm{T}}W(i\omega)\big)$, control (5) with feedback gain (10) ensures achievement of the goals (6) and (7) with function $c(t) = d^{-1/2}e^{At}(\mathbf{1}_d^{\mathrm{T}} \otimes I_n)x(0)$.

Proof. Some properties of matrix Kronecker product will be used (see [2], [10]). For the sake of brevity denote $L = L(\mathcal{G}), \widehat{L} = L(\widehat{\mathcal{G}})$. Let P be real orthogonal matrix such that

$$P^{\mathrm{T}}\widehat{L}P = \mathrm{diag}(0, \lambda_2, \ldots, \lambda_d)$$

with first column equal to $d^{-1/2}\mathbf{1}_d$. First column of product LP is zero vector since $\mathbf{1}_d$ is right eigenvector of matrix L. Therefore first column of $P^{\mathrm{T}}LP$ is also zero vector. According to assumption A1 digraph $\mathcal{G}$ is balanced, therefore L has left eigenvector consisting of ones corresponding to zero eigenvalue. Consequently first row of $P^{\mathrm{T}}LP$ is zero row. Thus:

$$P^{\mathrm{T}}LP = \begin{pmatrix} 0 & 0 & \ldots & 0 & \ldots & 0 \\ 0 & & & & & \\ \vdots & & & & & \\ 0 & & & \Lambda_e & & \\ \vdots & & & & & \\ 0 & & & & & \end{pmatrix}, \quad (12)$$

where $\Lambda_e \in \mathbb{R}^{(d-1)\times(d-1)}$.

Let $u = \mathrm{col}(u_1, \ldots, u_d)$. Then (5) can be rewritten as follows

$$u = (L \otimes KC^{\mathrm{T}})x. \quad (13)$$

Let us rewrite (4) using (13):

$$\dot{x} = \left((I_d \otimes A) + (L \otimes BKC^{\mathrm{T}})\right)x. \quad (14)$$

The idea of coordinate transformation was taken from [14].

Let $z = (P^{\mathrm{T}} \otimes I_n)x, z \in \mathbb{R}^{dn}$ and $z = \mathrm{col}(z_1, z_e), z_1 \in \mathbb{R}^n, z_e \in \mathbb{R}^{(d-1)n}$. Taking in account (12), whole system dynamics (14) can be represented as follows

$$\dot{z}_1 = Az_1, \quad (15)$$

$$\dot{z}_e = \left((I_{d-1} \otimes A) + (\Lambda_e \otimes BKC^{\mathrm{T}})\right)z_e. \quad (16)$$

If solution $z_e(t) \equiv 0$ of comined equations (16) is asymptotically stable then goal (7) is achieved with function $c(t) = d^{-1/2}e^{At}(\mathbf{1}_d^{\mathrm{T}} \otimes I_n)x(0)$.

Let us take following Lyapunov function:

$$V_e = z_e^{\mathrm{T}}(I_{d-1} \otimes H)z_e,$$

where matrix H is defined from (8). Derivative of V_e along the trajectories of (16):

$$\begin{aligned}\dot{V}_e =& z_e^{\mathrm{T}}(I_{d-1} \otimes (A^{\mathrm{T}}H + HA) + \Lambda_e^{\mathrm{T}} \otimes CK^{\mathrm{T}}B^{\mathrm{T}}H + \\ & \Lambda_e \otimes HBKC^{\mathrm{T}})z_e = z_e^{\mathrm{T}}(I_{d-1} \otimes (A^{\mathrm{T}}H + HA) + \\ & \Lambda_e^{\mathrm{T}} \otimes CK^{\mathrm{T}}g^{\mathrm{T}}C^{\mathrm{T}} + \Lambda_e \otimes CgKC^{\mathrm{T}})z_e.\end{aligned}$$

Equality $HB = Cg$ was used here, see (8). Denote

$$\mathcal{P} = I_{d-1} \otimes (A^{\mathrm{T}}H + HA) + \Lambda_e^{\mathrm{T}} \otimes CK^{\mathrm{T}}g^{\mathrm{T}}C^{\mathrm{T}} + \Lambda_e \otimes CgKC^{\mathrm{T}}.$$

If $\mathcal{P} < 0$, then zero solution of (16) is asymptotically stable, since $\dot{V}_e = z_e^{\mathrm{T}}\mathcal{P}z_e$.

Denote

$$\begin{aligned}\mathcal{K} =& -I_{d-1} \otimes (C\theta g^{\mathrm{T}}C^{\mathrm{T}} + Cg\theta^{\mathrm{T}}C^{\mathrm{T}}) + \\ & \Lambda_e^{\mathrm{T}} \otimes CK^{\mathrm{T}}g^{\mathrm{T}}C^{\mathrm{T}} + \Lambda_e \otimes CgKC^{\mathrm{T}}.\end{aligned}$$

Note that

$$A^{\mathrm{T}}H + HA = A_*^{\mathrm{T}}H + HA_* - (C\theta g^{\mathrm{T}}C^{\mathrm{T}} + Cg\theta^{\mathrm{T}}C^{\mathrm{T}}).$$

Taking in account the last equality, $\mathcal{P}$ can be represented in the following way

$$\mathcal{P} = I_{d-1} \otimes (A_*^{\mathrm{T}}H + HA_*) + \mathcal{K}.$$

If $\mathcal{K} \le 0$, then $\mathcal{P} < 0$. Using (9), let us rewrite expression for $\mathcal{K}$: (10):

$$\begin{aligned}\mathcal{K} =& -I_{d-1} \otimes (-\varkappa Cgg^{\mathrm{T}}C^{\mathrm{T}} - \varkappa Cgg^{\mathrm{T}}C^{\mathrm{T}}) - \\ & \Lambda_e^{\mathrm{T}} \otimes Cgkg^{\mathrm{T}}C^{\mathrm{T}} - \Lambda_e \otimes Cgkg^{\mathrm{T}}C^{\mathrm{T}} = \\ & (2\varkappa I_{d-1} - k(\Lambda_e + \Lambda_e^{\mathrm{T}})) \otimes (Cgg^{\mathrm{T}}C^{\mathrm{T}}).\end{aligned}$$

We conclude that $\mathcal{K} \le 0$, since $Cgg^{\mathrm{T}}C^{\mathrm{T}} \ge 0$ and $2\varkappa I_{d-1} - k(\Lambda_e + \Lambda_e^{\mathrm{T}}) \le 0$ by conditions of theorem.

V. Conditions of goal achievement in the case of unbalanced graph

Let assumption A1 hold, then zero eigenvalue of Laplace matrix is simple.

Let us represent Laplace matrix L in Jordan form

$$\Lambda = \begin{pmatrix} 0 & 0 \\ 0 & \Lambda_e \end{pmatrix} = P^{-1}LP,$$

assuming that first column of nonsingular matrx P is equal to $d^{-1/2}\mathbf{1}$. Denote by l_1 left eigenvector of Laplace matrix L corresponding to zero eigenvalue such that $l_1^{\mathrm{T}} d^{-1/2}\mathbf{1} = 1$.

Let us choose feedback gain row vector K of control law (5) in following form:

$$K = k \cdot \theta^{\mathrm{T}}, \quad k \in \mathbb{R}^1. \tag{17}$$

Theorem 2: Let assumptions A1, A2 hold and $\Lambda_e + \Lambda_e^* > 0$. Then for k such that

$$I_{d-1} + \frac{k}{2}(\Lambda_e + \Lambda_e^*) \leq 0$$

control (5) with feedback gain (17) ensures achievement of the goal (7) with function $c(t) = d^{-1/2}e^{At}(l_1^{\mathrm{T}} \otimes I_n)x(0)$.

Proof is similar to the proof of Theorem 1.

VI. Conditions of goal achievement in the case of undirected graph

In this section conditions of goal achievement in the case of undirected graph $\mathcal{G}$ is formulated. Let us make next assumption

A3) *Undirected graph $\mathcal{G}$ has a spanning tree.*

It is known that this assumption is equivalent to connectivity of graph $\mathcal{G}$. Thus, if this assumption holds, then zero eigenvalue of Laplace matrix $L = L(\mathcal{G})$ is simple. Denote eigenvalues of matrix L as follows: $0 = \lambda_1(L) < \lambda_2(L) \leq \ldots \leq \lambda_d(L)$.

Theorem 3: Let assumptions A2, A3 hold. Then for k such that

$$k \geq \frac{\varkappa}{\lambda_2(L)},$$

where number $\varkappa = \inf_{\omega \in \mathbb{R}^1} \mathrm{Re}\big(g^{\mathrm{T}} W(i\omega)\big)$, control (5) with feedback gain (10) ensures achievement of the goal (7) with function $c(t) = d^{-1/2}e^{At}(\mathbf{1}_d^{\mathrm{T}} \otimes I_n)x(0)$.

Proof is similar to the proof of Theorem 1.

VII. Example. Network of double integrators

A. System description and theoretical study

Consider network S, consisting of four agents $S_i, i = 1, \ldots, 4$. Each agent $S_i, i = 1, \ldots, 4$ is modelled as follows:

$$\dot{x}_i = Ax_i + Bu_i, \quad y_i = C^{\mathrm{T}}x_i,$$

where $x_i(t) \in \mathbb{R}^2$ is state vector, $u_i(t) \in \mathbb{R}^1$ is control, $y_i(t) \in \mathbb{R}^1$ is vector of measurements.

Let

$$A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, C = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}.$$

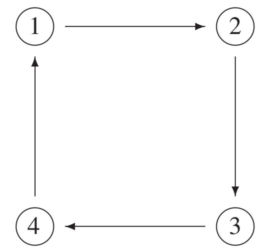

Fig. 1. Digraph $\mathcal{G}$.

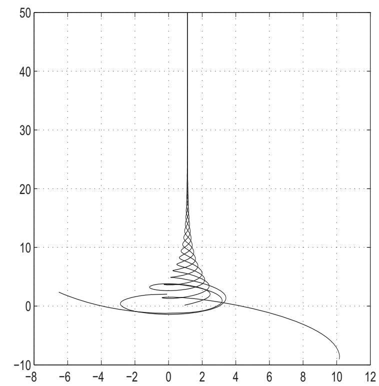

Fig. 2. Phase plane, $k = 1$.

Let digraph $\mathcal{G}$ describing informational flows in the network be balanced and such as illustrated on Fig. 1. Then Laplace matrix $\widehat{\mathcal{G}}$ has following eigenvalues: 0,2,2,4.

Let us apply Theorem 1. Transfer function

$$\chi(s) = C^{\mathrm{T}}(sI_2 - A)^{-1}B = \frac{s+1}{s^2},$$

is hyper minimum phace when $g = 1$. It is not difficult to show that number $\varkappa$ from (9) can be taken as follows: $\varkappa > 1$.

Thus, according to Theorem 1, if $k \geq 1$, then the controller (5) with feedback gain (10) ensures achievement of the goal (7).

First component of a single agent's state vector can be interpreted as a velocity, second component can be interpreted as a position on a straight line. Achievement of control goal means convergence of four points on straigh line and their motion with constant non-zero velocity.

B. Simulation results

Let agents have following initial conditions

$$x_1(0) = \mathrm{col}(0.5, 2), \qquad x_2(0) = \mathrm{col}(-7, 3),$$
$$x_3(0) = \mathrm{col}(1, 0), \qquad x_4(0) = \mathrm{col}(10, -10).$$

When $k = 1$ achievement of the control goal is illustrated by results of 50 second modelling. Trajectories of agents on a same phace plane with different values of k are shown on figures 2-5.

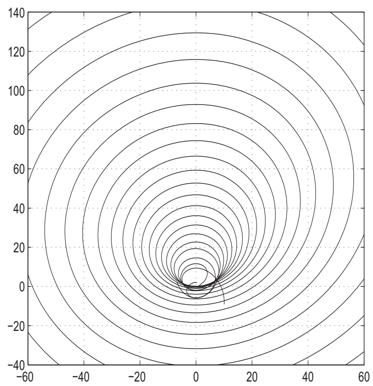

Fig. 3. Phase plane, $k = 0.3$.

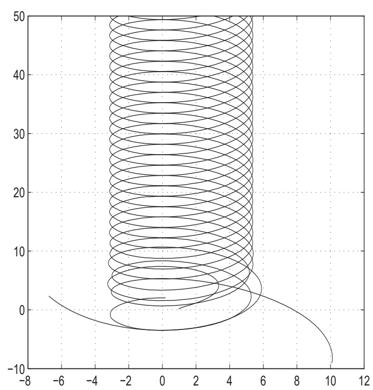

Fig. 4. Phase plane, $k = 0.5$.

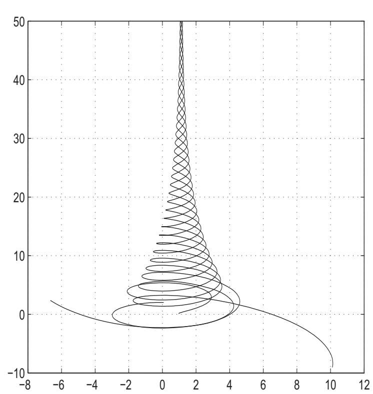

Fig. 5. Phase plane, $k = 0.7$.

VIII. CONCLUSION

By means of passification method sufficient conditions of state synchronization in linear object networks using static output feedback without constructing observers are found. Theoretical study is illustrated by example of synchronization in network consisting of four double integrators.

The obtained results can be extended to the case on nonlinear Lurie type dynamics of agents with sector bounded nonlinearities.

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