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Limited-Thrust Relative Position Holding for Adjacent Spacecraft with Thruster Nonlinearity

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Abstract— **The problem of spacecraft relative position holding with limited thrust and thruster nonlinearity is investigated in this paper. The relative dynamic model with sampled-data measurement is established based on the Clohessy-Wiltshire (C-W) equations. Thruster nonlinearity and limited thrust constraint are taken into consideration. Then, the relative position holding problem is regarded as a sampled-data output tracking control problem. Based on Lyapunov stabilization theory, the sampled-data controller is designed by solving a convex optimization problem. Simulations show the effectiveness of the proposed method.**

I. INTRODUCTION

Relative position holding of two adjacent spacecraft (Target and Chaser) means that the chaser holding at a position near to the target. During the holding period, the chaser can prepare docking actions with the target or complete other special missions such as monitoring, networking or formation flying. Thus, study of relative position holding problem for adjacent spacecraft is significant for many future astronautic missions. Due to the very short distance between the spacecraft, the relative motion during the holding process should be analyzed carefully, and it is a big challenge to utilize some advanced closed-loop control laws to enhance the accuracy and safety of this process [1]–[3].

Due to the unexpected factors such as actuator abrasion, friction and incomplete or excessive fuel combustion, it is difficult to make the actual thrust produced by the thruster exactly equal to the theoretical needed thrust. The unpredictable facts bring strong thruster nonlinear behaviors which are difficult to describe exactly. However, it is reasonable to assume that the actual thrust is bounded in a domain around the theoretical thrust, and the compounded thrust errors can be described as sector nonlinearity [4]–[6]. To the best of the authors' knowledge, few attempts have been made to tackle the problem of thruster sector nonlinearity for the spacecraft relative motion control. This motivates our present study.

As is well known, digital controllers have been widely used in spacecraft. For a system with digital controller, there exist both continuous-time and discrete-time signals in the continuous-time framework. Thus, the corresponding problem can be referred to as sampled-data control problem [7], [8]. However, for most of the previous studies on spacecraft orbital control, the aim is to design continuous-time controller for continuous-time system by assuming that the exact real-time measurements and the control input thrust can be obtained or produced immediately. This assumption may heavily degrade the performance or even cause instability of the closed-loop system due to the sampling intervals existing in digital controllers. Thus, study on the sampled-data control method for spacecraft orbital transfer process is also significant for practical spacecraft engineering.

According to above discussions, the sampled-data control problem is studied for relative position holding of two adjacent spacecraft in this paper. The relative dynamic model of chaser and target is established based on Clohessy-Wiltshire (C-W) equations [9], which has been widely used to study the relative motion between two neighboring spacecraft [10],[11]. The sampled-data measurements and sampled-data control signals are modelled, and the limited thrust constraint and the sector nonlinearity of thruster are taken into consideration. Then, the relative position holding problem is regarded as an output tracking problem. According to Lyapunov stabilization theory, the sampled-data controller is designed by solving a convex optimization problem. Some simulations are provided to show the effectiveness of the proposed control design method.

II. PROBLEM FORMULATION

A. Relative Motion Model

For two spacecraft, assume the orbital coordinate frame is a right-handed Cartesian coordinate, the origin attaches to the mass center of the target, x-axis is along the vector from earth center to the origin, y-axis is along the target orbit circumference, and z-axis completes the right-handed frame, r_0 is the radius of the target circular orbit, n is the angle velocity of the target which is equal with $(\mu_e/r_0^3)^{\frac{1}{2}}$, where μ_e is the gravitational parameter of the earth. Then, the relative motion of chaser and target can be described by C-W's equations as

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2 x = \frac{1}{m}T_x, \\ \ddot{y} + 2n\dot{x} = \frac{1}{m}T_y, \\ \ddot{z} + n^2 z = \frac{1}{m}T_z, \end{cases} \tag{1}$$

where x, y and z are the components of the relative position in corresponding axes, T_i $(i = x, y, z)$ is the i^{th} component of the control input force acting on the relative motion dynamics, m is the mass of the chaser. Then, by defining $\mathbf{x}(t) = [x,\ y,$

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$z,\ \dot{x},\ \dot{y},\ \dot{z}]^T$, $\mathbf{u}(t) = [T_x,\ T_y,\ T_z]^T$ and $\mathbf{y}(t) = [x,\ y,\ z]^T$, and by adopting the proper corresponding matrices according to (1), the relative orbital control system can be described as

$$\begin{cases} \dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t), \\ \mathbf{y}(t) = C\mathbf{x}(t). \end{cases} \tag{2}$$

Thus, we can see the specific relative motion can be realized by designing proper control input thrust $\mathbf{u}(t)$.

In this paper, we consider the sampled-data state-feedback control law. The system can be shown by Fig. 1. As shown

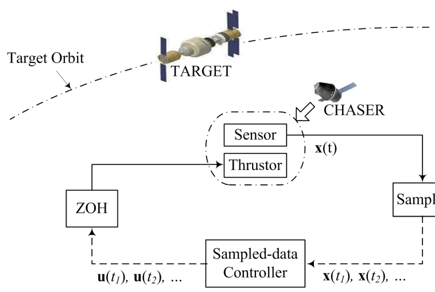

Fig. 1. Sampled-data control system for relative positional holding.

in Fig. 1, continuous-time measurement signal is sampled by a sampler, and the corresponding thrust are generated by the sampled-data controller and a zero-order hold (ZOH). We can see that the relative states are measured at the time instants $t_1,\ t_2,\ \ldots,\ t_k,\ t_{k+1},\ \ldots$, and only the signals at these instants are available for interval $t_k \le t < t_{k+1}$. Thus, the sampled-data state-feedback control law we need to determine in this paper can be described as

$$\mathbf{u}(t_k) = K\mathbf{x}(t_k). \tag{3}$$

where $\mathbf{u}(t_k) = [u_x(t_k), u_y(t_k), u_z(t_k)]^T$, and the thrust constraint can be described as

$$|u_i(t_k)| \le u_{i,\max}, \quad (i = x,\ y,\ z), \tag{4}$$

where $u_i(t_k)$ is the control input thrust along i^{th} axis at the sampling instant k, $u_{i,\max}$ is the maximum thrust the chaser's thruster can product along the i^{th} axis.

At each sampling instant, it also should be noted that, the thrust produced by the thruster may not accurately correspond with the calculated control input signals due to the complex thruster nonlinearities. Let u_r and u_d denote the real thrust and desired thrust respectively, and their relationship can be described as $u_r = \sigma u_d$, where σ is a scalar. Although it is hard to determine the exact σ at each sampling instant, we can assume it is always bounded in a sector domain $[\sigma_l,\ \sigma_h]$. Then, we introduce a scalar sector nonlinear function $\mathrm{sec}_i(u_i(t_k))$, $(i = x,\ y,\ z)$ satisfying

$$\sigma_{li}u_i(t_k) \le \mathrm{sec}_i(u_i(t_k)) \le \sigma_{hi}u_i(t_k), \quad 0 \le\ \sigma_{li} \le \sigma_{hi} < \infty.$$

Thus, the real thrust vector produced by the chaser's thruster can be described as

$$S(\mathbf{u}(t_k)) = [\mathrm{sec}_x(u_x(t_k)),\ \mathrm{sec}_y(u_y(t_k)),\ \mathrm{sec}_z(u_z(t_k))]^T, \tag{5}$$

and the system state function is rewritten as

$$\begin{cases} \dot{\mathbf{x}}(t) = A\mathbf{x}(t) + BS(\mathbf{u}(t_k)), \\ \mathbf{y}(t) = C\mathbf{x}(t). \end{cases} \tag{6}$$

The system in (6) describes the sampled-data position holding dynamic motion with thruster sector nonlinearity.

B. Problem Formation

In this paper, the relative positional holding problem is studied for the spacecraft. We note that the desired holding position can be regarded as a reference output of (6). In order to eliminate the steady-state tracking error, we define the output error $\mathbf{y}_e(t) = \mathbf{y}(t) - y_r$ and introduce the error integral action as

$$\mathbf{e}(t) = \int_0^t \mathbf{y}_e(t)dt. \tag{7}$$

To deal with the tracking error, we consider the following augmented system

$$\dot{\zeta}(t) = \bar{A}\zeta(t) + \bar{B}S(\mathbf{u}(t_k)) + \bar{E}\upsilon(t), \tag{8}$$

where

$$\zeta(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{e}(t) \end{bmatrix},\ \upsilon(t) = \begin{bmatrix} 0 \\ y_r \end{bmatrix},\ \bar{A} = \begin{bmatrix} A & 0_{6\times3} \\ C & 0_{3\times3} \end{bmatrix},$$
$$\bar{B} = \begin{bmatrix} B \\ 0_{3\times3} \end{bmatrix},\quad \bar{E} = \begin{bmatrix} 0_{6\times3} & 0_{6\times3} \\ 0_{3\times3} & -I_{3\times3} \end{bmatrix}.$$

Then, the augmented state-feedback controller can be described as

$$\bar{\mathbf{u}}(t_k) = \bar{K}\zeta(t_k) = \begin{bmatrix} K_{\mathbf{x}} & K_{\mathbf{e}} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t_k) \\ \mathbf{e}(t_k) \end{bmatrix}. \tag{9}$$

Obviously, if the augmented system in (8) is asymptotically stable, the relative position error integral action $\mathbf{e}(t) \to 0$, which means that $\lim_{t\to\infty} \mathbf{y}(t) = y_r$. Thus, the output tracking control problem can be further transformed into a stabilization problem of the augmented system (8). Thus, our aim in this paper can be formulated as:

Design a sampled-data control law as (9), such that the closed-loop system is asymptotically stable, which means that the chaser holds at a predetermined relative position which is described as y_r in spit of the sector thruster nonlinearity, and the needed thrust along each axis satisfies the bounded thrust requirement in (4);

III. Sampled-data Controller Design

To deal with the thruster sector nonlinearity in the further developments, we define the following two matrices

$$M = \frac{1}{2}diag[(\sigma_{l1} + \sigma_{h1}),\ (\sigma_{l2} + \sigma_{h2}),\ (\sigma_{l3} + \sigma_{h3})],$$
$$N = \frac{1}{2}diag[(\sigma_{h1} - \sigma_{l1}),\ (\sigma_{h2} - \sigma_{l2}),\ (\sigma_{h3} - \sigma_{l3})].$$

Then, if we define a vector $\eta(t_k) = S(\bar{\mathbf{u}}(t_k)) - M\bar{\mathbf{u}}(t_k)$, the actual thrust produced by the thrusters can be described as

$$S(\bar{\mathbf{u}}(t_k)) = \eta(t_k) + M\bar{\mathbf{u}}(t_k). \tag{10}$$

Furthermore, we assume that the sampling intervals between any two sequent sampling instants are bounded by h, that is $t_{k+1}-t_k \le h$. By defining $d(t)=t-t_k \le h$, the sampling intervals can be written as $t_k = t-(t-t_k) = t-d(t)$. Then, the control input vector can be transformed into

$$\bar{\mathbf{u}}(t_k) = \bar{K}\zeta(t_k) = \bar{K}\zeta(t-d(t)). \tag{11}$$

Thus, for (10) and (11), the augmented closed-loop system can be written by

$$\dot{\zeta}(t) = \bar{A}\zeta(t) + \bar{B}\eta(t_k) + \bar{B}M\bar{K}\zeta(t-d(t)) + \bar{E}\upsilon(t). \tag{12}$$

A. Stabilization

We first analyze the stabilization problem of the system in (12) with assumption of $\upsilon(t)=0$. Consider the following Lyapunov function

$$V(t) = V_1(t) + V_2(t), \tag{13}$$

where $V_2(t)=\int_{-h}^{0}\int_{t+\beta}^{t}\dot{\zeta}(\alpha)Q\dot{\zeta}(\alpha)d\alpha d\beta$ and $V_1(t) = \zeta^T(t)P\zeta(t)$. According to Lemma 1 in [10], and by defining $\varphi(t) = [\zeta^T(t),\ \zeta^T(t-d(t))]^T$, $\Pi = P\bar{A} + \bar{A}^T P + \varepsilon_1 P\bar{B}\bar{B}^T P$ and a positive scalar ε_1, we have

$$\dot{V}_1(t) \le \varphi^T(t)\Gamma_1\varphi(t), \tag{14}$$

where

$$\Gamma_1 = \begin{bmatrix} \Pi & P\bar{B}M\bar{K} \\ * & \varepsilon_1^{-1}\bar{K}^T NN\bar{K} \end{bmatrix}. \tag{15}$$

On the other hand, the derivative of $V_2(t)$ can be obtained as

$$\dot{V}_2(t) = h\dot{\zeta}^T(t)Q\dot{\zeta}(t) - \int_{t-h}^{t}\dot{\zeta}^T(\alpha)Q\dot{\zeta}(\alpha)d\alpha. \tag{16}$$

According to Lemma 2 in [5], by defining $\Theta = Q - \varepsilon_2^{-1}Q\bar{B}\bar{B}^T Q$ with a positive scalar ε_2, we have $h\dot{\zeta}^T(t)Q\dot{\zeta}(t) \le h\varphi^T(t)\Gamma_2\varphi(t)$, where

$$\begin{aligned} \Gamma_2 \ &= \ [Q\bar{A},\ Q\bar{B}M\bar{K}]^T\Theta^{-1}[Q\bar{A},\ Q\bar{B}M\bar{K}] \\ &\quad +\varepsilon_2[0,\ N\bar{K}]^T[0,\ N\bar{K}]. \end{aligned} \tag{17}$$

For the sampling interval is bounded by h, we have $d(t) = t-t_k \le t_{k+1}-t_k \le h$, then $t-h \le t-d(t)$. Thus, by Jensen inequality, the second term of (16) is less than $-h^{-1}\varphi^T(t)\Gamma_3\varphi(t)$, where

$$\Gamma_3 = [I,\ -I]^T Q[I,\ -I]. \tag{18}$$

Thus, for (14)–(18), we have

$$\dot{V}(t) \le \varphi^T(t)\left(\Gamma_1 + h\Gamma_2 - h^{-1}\Gamma_3\right)\varphi(t).$$

Therefore, if $\Gamma_1 + h\Gamma_2 - h^{-1}\Gamma_3 < 0$, then $\dot{V}(t) < 0$, which means that the augmented closed-loop system in (12) is asymptotic stable with the augmented sampled-data state-feedback controller $\bar{K}$. With this controller, as the analysis in the above section, the relative motion system in (6) is stable and its output $\mathbf{y}(t)$ tracks the reference signal y_r without steady-state error.

In order to deal with each input constraint along each axis, which is described in (4), we introduce three matrices R_i to divide the vector $\bar{\mathbf{u}}(t_k)$ into $u_x(t_k)$, $u_y(t_k)$ and $u_z(t_k)$ for corresponding axis. By defining $R_x = [1,\ 0,\ 0]^T[1,\ 0,\ 0]$, $R_y = [0,\ 1,\ 0]^T[0,\ 1,\ 0]$, $R_z = [0,\ 0,\ 1]^T[0,\ 0,\ 1]$, the thrust constraint in (4) can be written as $|R_i\bar{\mathbf{u}}(t_k)| \le u_{i,\max}$ for $i = x$, y, z. Then, we can readily have $(R_i\bar{\mathbf{u}}(t_k))^T(R_i\bar{\mathbf{u}}(t_k)) \le u_{i,\max}^2$, which equals to

$$\zeta^T(t-d(t))\bar{K}^T R_i^T R_i\bar{K}\zeta(t-d(t)) \le u_{i,\max}^2. \tag{19}$$

Based on the analysis of the stabilization, we have known that $\dot{V}(t) < 0$ if the proper $\bar{K}$ and other matrices exist. Then, we have $V(t) < V(0)$ with the proper $\bar{K}$. It is reasonable to assume that there exists a scalar ρ satisfying $V(0) \le \rho$. Noting that the second term of $V(t)$ is positive, we have $\zeta^T(t)P\zeta(t) < V(t) < V(t_0) \le \rho$ for any time t during the position holding process. Thus, for $t > d(t)$, it can also be true that $\zeta^T(t-d(t))P\zeta(t-d(t)) < \rho$. Then, we can see that (19) can be guaranteed by

$$\rho\bar{K}^T R_i^T R_i\bar{K} < u_{i,\max}^2 P, \quad (i = x,\ y,\ z). \tag{20}$$

This means that the thrust constraint can be ensured if the matrix P and $\bar{K}$ also satisfy (20).

Obviously, the proper sampled-data controller exists if the inequalities $\Gamma_1 + h\Gamma_2 - h^{-1}\Gamma_3 < 0$ and (20) are satisfied simultaneously. However, these two inequalities are just theoretical conditions and there is not yet any existing method can deal with them directly. Thus, in order to design the controller, we need to transform these two conditions into the forms which can be dealt with by existing methods.

B. Controller Design

To simplify the formulations, we define $\digamma_1 = P\bar{A} + \bar{A}^T P - h^{-1}Q$, $\digamma_2 = [P\bar{B}M\bar{K} + h^{-1}Q,\ P\bar{B},\ 0,\ 0]$, $\digamma_3 = [1,\ 0,\ 0,\ 0]^T\bar{K}^T N[0,\ 0,\ 1,\ 0]$ and $\digamma_4 = [1,\ 0,\ 0,\ 0]^T\bar{K}^T N[0,\ 0,\ 0,\ 1]$. Then, by Schur complements, it can be readily obtained that the inequality $\Gamma_1 + h\Gamma_2 - h^{-1}\Gamma_3 < 0$ holds if

$$\begin{bmatrix} -\varepsilon_2 I & \Xi \\ * & \Omega \end{bmatrix} < 0, \tag{21}$$

where, with the definition $\Delta = diag\{-h^{-1}Q,\ -\varepsilon_1^{-1}I,\ -\varepsilon_1 I,\ -(h\varepsilon_2)^{-1}I\}$, Ξ and Ω are given by $\Xi = [\bar{B}^T Q,\ 0,\ 0,\ 0,\ 0,\ 0]$,

$$\Omega = \begin{bmatrix} -Q & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix},\ \ \Omega_{22} = \begin{bmatrix} \digamma_1 & \digamma_2 \\ * & \Delta + \digamma_3 + \digamma_4 \end{bmatrix},$$
$$\Omega_{12} = \sqrt{h}\times[Q\bar{A},\ Q\bar{B}M\bar{K},\ 0,\ 0,\ 0].$$

Pre- and post-multiplying (21) by $diag\{I,\ Q^{-1},\ P^{-1},\ P^{-1},\ I,\ I,\ I\}$, by defining $X = P^{-1}$, $Y = \bar{K}P^{-1}$, $\tilde{Q} = P^{-1}QP^{-1}$, and noting that $-Q^{-1} = -P^{-1}\tilde{Q}^{-1}P^{-1} \le \tilde{Q} - 2P^{-1}$, the corresponding matrices are transformed into $\tilde{\digamma}_1 = \bar{A}X + X\bar{A}^T - h^{-1}\tilde{Q}$, $\tilde{\digamma}_2 = [\bar{B}MY + h^{-1}\tilde{Q},\ \bar{B},\ 0,\ 0]$, $\tilde{\digamma}_3 = [1,\ 0,\ 0,\ 0]^T Y^T N[0,\ 0,\ 1,\ 0]$ and $\tilde{\digamma}_4 = [1,\ 0,\ 0,\ 0]^T Y^T N[0,\ 0,\ 0,\ 1]$, and (21) can be guaranteed by

$$\begin{bmatrix} -\varepsilon_2 I & \tilde{\Xi} \\ * & \tilde{\Omega} \end{bmatrix} < 0, \tag{22}$$

where, with the definition $\tilde{\Delta} = diag\{-h^{-1}\tilde{Q},\ -\varepsilon_1^{-1}I,\ -\varepsilon_1 I,\ -(h\varepsilon_2)^{-1}I\}$, $\tilde{\Xi}$ and $\tilde{\Omega}$ are given by $\tilde{\Xi} = [\bar{B}^T,\ 0,\ 0,\ 0,\ 0,\ 0]$,

$$\tilde{\Omega} = \begin{bmatrix} \tilde{Q} - 2X & \tilde{\Omega}_{12} \\ * & \tilde{\Omega}_{22} \end{bmatrix},\ \ \tilde{\Omega}_{22} = \begin{bmatrix} \tilde{\digamma}_1 & \tilde{\digamma}_2 \\ * & \tilde{\Delta} + \tilde{\digamma}_3 + \tilde{\digamma}_4 \end{bmatrix},$$
$$\tilde{\Omega}_{12} = \sqrt{h}\times[\bar{A}X,\ \bar{B}MY,\ 0,\ 0,\ 0].$$

Obviously, if the sampling interval bound h, the thrust sector nonlinearity domain bounds σ_{li} and σ_{hi} are given, (22) is a linear matrix inequality. Thus, (22) can be regarded as a solvable condition of the existence of $\bar{K}$.

On the other hand, by Schur complements, the inequality condition in (20) can be ensured by

$$\begin{bmatrix} -I & \sqrt{\rho} R_i \bar{K} \\ * & -u_{i,\max}^2 P \end{bmatrix} < 0, \tag{23}$$

for $i = x,\ y,\ z$. Pre- and post-multiplying (23) by $diag\{I, P^{-1}\}$ and by the definitions of $X = P^{-1}$ and $Y = \bar{K}P^{-1}$, the inequality in (23) equals to

$$\begin{bmatrix} -I & \sqrt{\rho} R_i Y \\ * & -u_{i,\max}^2 X \end{bmatrix} < 0. \tag{24}$$

Thus, the thrust constraints can be ensured by (24), which are linear matrix inequalities with the given positive scalar ρ and the maximum thrust bounds along x-, y- and z-axis.

Summarizing the above analyses about the stabilization and the thrust constraint, we give the following theorem as the main result of this paper.

Theorem 1: Consider the relative position holding problem between two adjacent spacecraft, whose relative motion can be described as (6). The thrust produced by the thruster with sector nonlinearity is described by (5); the sampling interval is not greater than h and the maximal thrust along three axes are given by $u_{i,\max}, (i = x,\ y,\ z)$. If there exist $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, positive symmetric matrices P and Q simultaneously satisfying (22) and (24), then, a sampled-data state-feedback control law in the form of (9) exists, such that the chaser holding at a given relative position y_r and the needed thrust satisfies the thrust constraint in (4). the proper state-feedback gain matrix $\bar{K}$ can be obtained by $\bar{K} = YX^{-1}$.

IV. Simulation

Assume that the mass of the chaser spacecraft is 200kg, the target is moving in a geosynchronous orbit of radius $r = 42241$km with an orbital period of 24 hours, the angle velocity $n = 1.117 \times 10^{-3}$ rad/s, and the maximum thrust along each axis is 100N. In the coordinate based on target frame, assume the initial state is $[50,\ 0,\ -30,\ 0,\ 0,\ 0]$, the desired holding position is $(0,\ -20,\ 0)$. In order to evaluate the control input thrust, we introduce $T_{\max}(t_k)$ to denote the maximum thrust at t_k, that is $T_{\max}(t_k) = \max\left\{u_x(t_k),\ u_y(t_k),\ u_z(t_k)\right\}$.

As discussed in above section, the maximum sampling interval can be adjusted by changing h, and the thrust sector nonlinearity can be denoted by the sector bounds σ_{li} and σ_{hi}. Here, we introduce a scalar δ and assume the thrust satisfies the following function

$$S(\mathbf{u}(t_k)) = \mathbf{u}(t_k) + \delta \mathbf{u}(t_k) \sin\left[\mathbf{u}(t_k)\right].$$

Then, for $i = x,\ y,\ z$, the scalars σ_{li} and σ_{hi} can be given by $\sigma_{li} = 1 - \delta$ and $\sigma_{hi} = 1 + \delta$. Thus, the sector nonlinearity level can be adjusted by changing δ. Assume $h = 0.5$s and $\delta = 0.1$. By Theorem 1, we obtain a sampled-data controller $\bar{K}_{sam}$ which is shown at the bottom of next page.

With the obtained controllers $\bar{K}_{sam}$, the positional outputs along three axes are depicted in Fig. 2. The thrust along the axes are depicted in Fig. 3, and Fig. 4 shows the maximum needed thrust $T_{\max}(t_k)$ during the orbital transfer process. The actual and desired thrust along z-axis are shown in Fig. 5. The situations along x- and y-axis are omitted here because of their similarity with Fig. 5.

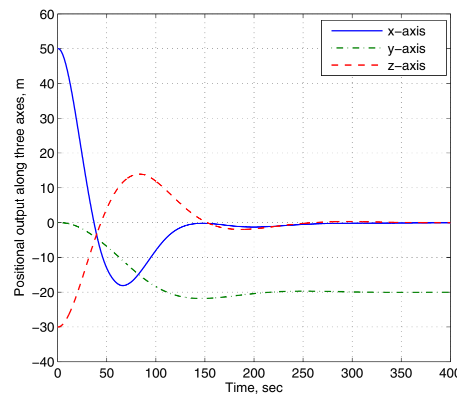

Fig. 2. Positional outputs along the axes.

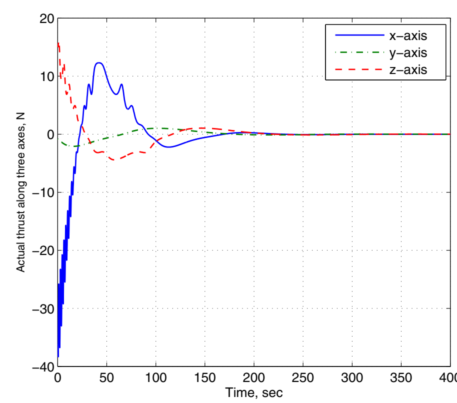

Fig. 3. Actual thrust along three axes.

$$\begin{aligned} \bar{K}_{sam} &= [K_{sam,\mathbf{x}} \mid K_{sam,\mathbf{e}}] \\ &= \left[\begin{array}{c|c} \text{-0.5915, 0.0243, -0.0017, -16.4717, -0.0049, -0.2346} & \text{-0.0120, 0.0004, -0.0002} \\ \text{-0.0248, -0.5928, -0.0000, 0.0019, -16.8264, 0.0007} & \text{-0.0004, -0.0123, 0.0000} \\ \text{-0.0017, 0.0003, -0.5921, -0.2351, 0.0007, -16.6949} & \text{-0.0002, 0.0000, -0.0122} \end{array}\right] \end{aligned}$$

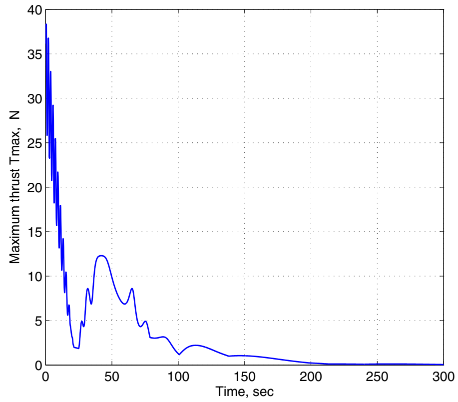

Fig. 4. Maximum thrust $T_{\max}(t_k)$.

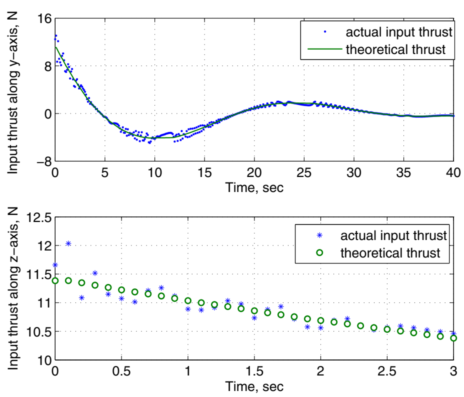

Fig. 5. Actual and desired thrust along z-axis.

We can see that the chaser can track to the desired position in spit of the thrust nonlinearity, and the maximum needed thrust during the transfer process is 38.6349N, which satisfies the thrust constraint we proposed.

It should be note that, the feasibility of the LMI conditions depends much on the given h and δ. Table I lists the maximum allowed $\delta_{\max}$ for different h. Table II lists the maximum allowed $h_{\max}$ for different δ. Fig. 6 shows the maximum needed thrust for different h and δ. We can see that longer sampling interval or greater sector nonlinearity would cause larger needed thrust.

TABLE I

MAXIMUM ALLOWED NONLINEARITY LEVEL $\delta_{\max}$ FOR DIFFERENT h

h	0.1s	0.2s	0.5s	1s	1.5s
$\delta_{\max}$ of K_{sam}	0.9388	0.8930	0.7450	0.6807	0.6030

V. CONCLUSIONS

The spacecraft relative position holding control problem has been investigated in this paper. The position holding problem has been transformed into a sampled-data output tracking problem. By considering the limited thrust constraint and the input sector nonlinearity, a sampled-data controller design method has been proposed. Simulations have shown that the designed controller is effective to make the chaser hold at the desired position in spite of the thruster nonlinearity and the thrust constraint.

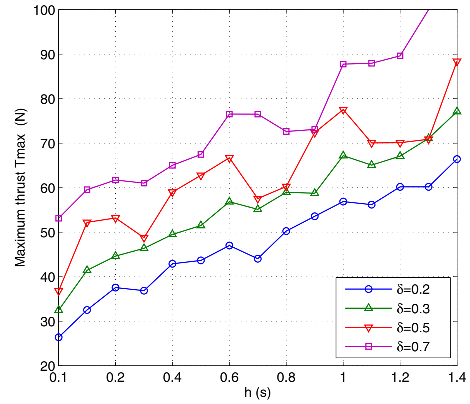

Fig. 6. Maximum needed thrust for different h and δ.

TABLE II

MAXIMUM ALLOWED SAMPLING INTERVAL $h_{\max}$ FOR DIFFERENT δ

δ	0.1	0.2	0.5	0.8	0.9
$h_{\max}$ of K_{sam}	27.8950s	16.0018s	2.5089s	0.3951s	0.1348s

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