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A Market-Based Mechanism for Providing Demand-Side Regulation Service Reserves*

Ioannis Ch. Paschalidis,† Binbin Li,‡ Michael C. Caramanis§

Abstract— **We develop a market-based mechanism that enables a building Smart Microgrid Operator (SMO) to offer regulation service reserves and meet the associated obligation of fast response to commands issued by the wholesale market Independent System Operator (ISO) who provides energy and purchases reserves. The proposed market-based mechanism allows the SMO to control the behavior of internal loads through price signals and to provide feedback to the ISO. A regulation service reserves quantity is transacted between the SMO and the ISO for a relatively long period of time (e.g., a one hour long time-scale). During this period the ISO repeatedly requests from the SMO to decrease or increase its consumption. We model the operational task of selecting an optimal short time-scale dynamic pricing policy as a stochastic dynamic program that maximizes average SMO and ISO utility. We then formulate an associated non-linear programming static problem that provides an upper bound on the optimal utility. We study an asymptotic regime in which this upper bound is tight and the static policy provides an efficient approximation of the dynamic pricing policy. We demonstrate, verify and validate the proposed approach through a series of Monte Carlo simulations of the controlled system time trajectories.**

Index Terms— **Electricity demand response, electricity regulation service, smart-grid, pricing, electricity markets, welfare maximization, dynamic programming.**

I. INTRODUCTION

We address advanced demand control in next generation intelligent buildings or neighborhoods that are (i) equipped with a sub-metering and actuation capable *smart-microgrid* accessible by occupants as well as by a *Smart Microgrid Operator (SMO)*, and (ii) connected to a cyber infrastructure enhanced *smart grid* that can support close-to-real-time power market transactions including participants connected at the distribution level. In particular, we consider demand control for offering capacity reserve ancillary services to the *Independent System Operator (ISO)* who clears short-term power markets. In this respect we note that five minute up and down capacity reserves, known as *Regulation Service (RS)* reserves, are important to meeting the required energy balance and preserving power system stability. As clean, but alas intermittent and volatile, renewable generation is increasingly integrated into the grid, RS reserve requirements increase as well [1]. Considering that today's RS reserves are procured simultaneously with energy, correspond to 1% of load, and command market clearing prices comparable to the price of energy, an increase in RS requirements without a commensurate increase in the supply of RS reserves may well be a show stopper for wind generation expansion. Since centralized generating units are today the only contributor of RS, enabling buildings to offer RS and compete in the power markets promises a major contribution in terms of affordable RS reserve cost and lower CO_2 emissions due to the associated adoption of clean generation.

Wholesale power markets were introduced in the US in the mid 1990's [2]. These markets clear simultaneously energy and several types of reserve requirements. For simplicity in exposition we consider here only RS reserves. Most markets have not yet allowed the demand side to participate in RS reserves. One of the ISO's, PJM, has allowed loads to participate in energy and reserve transactions since 2006 [3], while other ISO's are contemplating to follow suit. Of the existing short-term markets we point out briefly ([4], [5], [6], [7]) the: (i) *day ahead* markets that close at noon of the previous day and clear energy and reserve bids for each of the 24 hours of the next day, (ii) hour ahead *adjustment* markets that close an hour in advance and reveal energy and reserve prices, and (iii) 5-minute *real-time* economic dispatch markets that determine actual ex post variable marginal cost of energy at each bus or node of the transmission system. We assume that with the advent of the *smart grid* ([8], [9]) a *Load Aggregator (LA)* will be able to participate in power markets on a par basis with centralized generators. In particular we assume that a LA will be able to buy energy on an hourly basis at the corresponding clearing price and sell RS reserves for which it will be credited at the system RS clearing price. An ISO who procures R_h KW of RS is entitled to consider it as a stand by increment or decrement of consumption that it can utilize at will in total or in part. The ISO may send commands to the RS provider to request that it modulates its consumption either up or down by an amount that does not exceed R_h. These requests may arrive at inter-arrival times of 5 seconds or longer. To observe RS reserve contractual obligations, the RS provider must deliver the requested increase or decrease in its load with a ramp rate of $R_h/5$ KW per minute. The ISO typically re-dispatches the power system in 5 minute intervals. At each 5 minute system dispatch, the ISO schedules slower response tertiary reserves so as to reset the utilized RS reserves to their set points. As a result, although not guaranteed, the RS reserve provider's tracking of ISO commands is for all

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† Corresponding author. Dept. of Electrical & Computer Eng., and Division of Systems Eng., Boston University, 8 St. Mary's St., Boston, MA 02215, email: yannisp@bu.edu, url: http://ionia.bu.edu/.

‡ Division of Systems Eng., Boston University, e-mail: libinbin@bu.edu.

§ Dept. of Mechanical Eng., and Div. of Systems Eng., Boston University, 15 St. Mary's St., Brookline, MA 02446, e-mail: mcaraman@bu.edu.

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practical purposes energy neutral over the long time-scale of an hour and beyond. To meet the aforementioned contractual requirements, an SMO must be capable of controlling loads through the smart microgrid and a higher *decision support and communication* layer that interacts with users of energy in order to adapt their demand behaviors to ISO's requests. The lower SMO layer consists of sensing and actuation components that collect building state information and actuate so as to safely implement goals determined at the higher level and authorized by building occupants.

This paper focuses expressly on providing the higher decision support layer with a virtual market that operates on the building side of the meter for the purpose of eliciting a collaborative response of building occupants. Our objective is to derive an optimal SMO *pricing or incentive policy* towards building occupants so that they consent to the sale of RS reserves to the ISO and collaborate in meeting ISO's RS utilization requirements. To the best of our knowledge, little relevant work has been published, and we are the first to propose such a market based policy for demand control aiming at the provision of RS reserves. Methodologically, related techniques have been used in pricing Internet services [10], [11]. In Sec. II, we detail our internal market based model and formulate a related welfare maximization problem. In Sec. III we cast the problem into a *Dynamic Programming (DP)* framework to obtain the optimal dynamic policy. We then proceed to develop performance bounds and approximations. In Sec. IV we develop a static policy and in Sec. V we derive an easily computable upper bound on the optimal performance. Based on this bound, we establish in Sec. VI the asymptotic optimality of the static policy as the load class specific consumption level becomes smaller with a commensurate increase in the number of active loads. Further, we extend the asymptotic optimality results to account for constraints that model energy neutrality over the long time-scale and the upper limit in the RS delivery requested by the ISO . We present numerical results in Sec. VII, and conclude in Sec. VIII.

II. Problem Formulation

This section models the short time-scale interaction of the SMO with microgrid occupants/loads and the ISO in conjunction with RS reserves.

The SMO can sell R_h KW of regulation service for the duration of the long time-scale (e.g., one hour), provided that its microgrid's average consumption, R, exceeds R_h and its consumption capacity is at least $R + R_h$. We envision microgrid load classes that can be potentially active during the relevant long time period to include, among others, lights, HVAC zones, computers, electrical appliances and the like. We denote the event of a load unit becoming active as an *internal arrival* (i.e., internal to the building) and associate a class-specific electricity demand increment with each arrival. We similarly denote the event of a load unit becoming inactive as an *internal departure*. An actively consuming load unit derives a positive utility. With the sale of R_h KW of RS the SMO agrees to be on standby and respond to short time-scale (e.g., seconds to minutes) ISO requests for an increment or decrement of the building's consumption. We denote the event of an ISO request as an *external arrival* (i.e., external to the building). The termination of an ISO request is modeled as an *external departure*. Note that the cumulative ISO increment or decrement requests can not exceed R_h or $-R_h$ respectively. As mentioned, the SMO's response does not have to be instantaneous. It must adhere, however, to a response rate of roughly $R_h/5$ KW per minute. ISO requests that are met by the SMO result in positive utility. In addition, in its periodic 5 minute system re-dispatch, the ISO typically attempts to reset its cumulative increment or decrement requests to zero in order to enable RS providers to respond to new requests during future inter-dispatch 5 minute periods. This suggests that the long time-scale average deviation of building consumption from its R level equals zero. Hence, the sale of RS reserves has an energy neutral impact on long time-scale building consumption.

The primary objective is to maximize the sum of SMO and ISO welfare associated with internal and external arrivals. Hard and soft constraints are added to model adherence to the contractual requirements and long time-scale energy neutrality described above. To achieve these goals, the SMO controls the active internal loads and external requests by communicating external and internal-class-specific *prices* that may be interpreted as dynamic demand control and RS activation feedback signals as much as a monetary exchange.

We assume M classes of internal loads $i = 1, \dots, M$, that arrive according to a Poisson process and require r_i KW for an exponentially distributed period with rate μ_i. Let $\mu = (\mu_1, \dots, \mu_M)$. Each internal arrival of class i pays an SMO determined price u_i; we define $\mathbf{u} = (u_1, \dots, u_M)$. We assume that the arrival rate of class i loads is a known demand function $\lambda_i(u_i)$ which depends on u_i and satisfies Assumption A below. We denote the number of *active* class i internal loads at time t by $n_i(t)$, $i = 1, \dots, M$, and define $\mathbf{N}(t) = \big(n_1(t), \dots, n_M(t)\big)$.

Assumption A

For every i, there exists a price $u_{i,\max}$ beyond which the demand $\lambda_i(u_i)$ becomes zero. Furthermore, the function $\lambda_i(u_i)$ is continuous and strictly decreasing in the range $u_i \in [0, u_{i,\max}]$.

ISO requests for the dynamic activation of RS reserves are modeled as a special external class. External RS activation requests occur at a rate $a(y)$ where y is an SMO set price and $a(y)$ satisfies Assumption B below. While they are active, external arrivals require r_e KW each. They become inactive upon their departure which follows an exponential distribution with rate d. Denoting the number of active external class loads at time t by $m(t)$, we can express the request for increased or decreased building energy consumption as $R + R_h - m(t)r_e$. We impose the following two constraints:

$$\mathbf{N}(t)^{'}\mathbf{r} + m(t)r_e = \sum_{i=1}^{M} n_i(t)r_i + m(t)r_e \le R + R_h, \quad (1)$$

$$m(t)r_e \le 2R_h, \quad (2)$$

where prime denotes transpose. Inequality (1) ensures that at any time t the total capacity usage of all active loads does not exceed the maximal building consumption capacity. Inequality (2) ensures that the ISO can not request that the average building consumption R be increased beyond $R+R_h$ or decreased below $R-R_h$.

Assumption B

There exists a price $y_{\max}$ beyond which the demand $a(y)$ becomes zero. Furthermore, the function $a(y)$ is continuous and strictly decreasing in the range $y \in [0, y_{\max}]$.

To render the proposed constrained welfare maximization problem more meaningful, we first detail the arrival models and the underlying demand functions. An arrival of an internal load of class i generates utility U_i, where U_i is a non-negative random variable taking values in the range $[0, u_{i,\max}]$ with a continuous probability density function $f_i(u_i)$. Arrivals of internal class i loads are a fraction of *potential* class i arrivals generated according to a Poisson process with constant rate $\lambda_{i,\max}$. A potential arrival becomes a real arrival if and only if the random utility realization, U_i, exceeds the SMO set price u_i. This implies that internal class i arrivals occur according to a randomly modulated Poisson process with rate $\lambda_i(u_i(t)) = \lambda_{i,\max}\mathbb{P}[U_i \geq u_i(t)]$. Furthermore, the expected utility conditioned on the fact that a potential arrival has been accepted under a current price of u_i, is equal to $\mathbb{E}[U_i|U_i \geq u_i]$. We therefore conclude that the expected long-term average rate at which utility is generated by the arrival of internal loads is given by:

$$\lim_{T\to\infty} \frac{1}{T}\sum_{i=1}^{M} \mathbb{E}\left[\int_0^T \lambda_i(u_i(t))\mathbb{E}[U_i|U_i \geq u_i(t)]dt\right].$$

Following a similar argument, the welfare generated from external RS class arrivals can be expressed as:

$$\lim_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\int_0^T a(y(t))\mathbb{E}[Y|Y \geq y(t)]dt\right],$$

where Y stands for the welfare from the admission of a potential external RS arrival, and $a(y(t)) = a_{\max}\mathbb{P}[Y \geq y(t)]$ where $a_{\max}$ is the maximal arrival rate of the external RS class. An interesting interpretation of the long-term average utility generated by external RS class arrivals is that it represents the reservation reward level that the ISO might be willing to pay the SMO for standby RS reserves.

Finally, recall that building response to active ISO RS requests implies that the modified building load must equal $R + R_h - m(t)r_e$. This is to avoid compliance by energy dumping, and we impose the following penalty:

$$\lim_{T\to\infty}\frac{1}{T}\mathbb{E}\left[\int_0^T P\Big((R+R_h) - \Big(\sum_{i=1}^{M} n_i(t)r_i + m(t)r_e\Big)\Big)dt\right],$$

where $P(\cdot)$ denotes the penalty function. We make specific assumptions on $P(x)$ later.

The optimal pricing policy can now be described as the $\arg\max$ of:

$$\lim_{T\to\infty}\frac{1}{T}\mathbb{E}\Big[\sum_{i=1}^{M}\int_0^T \lambda_i(u_i(t))\mathbb{E}[U_i|U_i \geq u_i(t)]dt$$
$$+ \int_0^T a(y(t))\mathbb{E}[Y|Y \geq y(t)]dt$$
$$- \int_0^T P\Big((R+R_h) - \Big(\sum_{i=1}^{M} n_i(t)r_i + m(t)r_e\Big)\Big)dt\Big]. \quad (3)$$

Due to Assumption A and B, functions $\lambda_i(u_i)$ and $a(y)$ have inverse functions which we denote by $u_i(\lambda_i)$ and $y(a)$, respectively. The inverse functions are defined on $[0, \lambda_{i,\max}]$ and $[0, a_0]$, respectively, and are continuous and strictly decreasing. This allows us to use the arrival rates λ_i and a as the SMO's decision variables and write the instantaneous reward rates as $\lambda_i\mathbb{E}[U_i|U_i \geq u_i(\lambda_i)]$ and $a\mathbb{E}[Y|Y \geq y(a)]$.

III. Dynamic Programming Formulation

The problem introduced in Sec. II is in fact a finite-state, continuous-time, average reward DP problem. Note that the set $\{\mathscr{U}, \mathscr{Y}\} = \{(\mathbf{u}, y)|0 \leq u_i \leq u_{i,\max}, \forall i; y \leq y_{\max}\}$ is compact and that all states communicate assuring that there exists a (proper) policy that is associated with finite first passage time from any state $(\mathbf{N}, m)$ to any other state $(\mathbf{N}', m')$. Standard DP theory results assert that an optimal stationary policy exists [12].

Since the process $(\mathbf{N}(t), m(t))$ is a continuous-time Markov chain and the total transition rate out of any state is bounded by $\nu = \sum_{i=1}^{M}(\lambda_{i,\max} + \mu_i\lceil(R+R_h)/r_i\rceil) + (a_{\max} + d\lceil(R+2R_h)/r_e\rceil)$, we can uniformize this Markov chain and derive the following Bellman equation [12]:

$$J^* + h(\mathbf{N}, m) = \max_{\mathbf{u}\in\mathscr{U}}\Big[\sum_{i\in C(\mathbf{N},m)} \lambda_i(u_i)\mathbb{E}[U_i|U_i \geq u_i]$$
$$+\mathbf{1}_{D(N,m)}a(y)\mathbb{E}[Y|Y \geq y] - P\Big((R+R_h) - (\mathbf{N}'\mathbf{r} + mr_e)\Big)$$
$$+ \sum_{i\in C(\mathbf{N},\mathbf{M})} \frac{\lambda_i(u_i)}{\nu}h(\mathbf{N}+\mathbf{e}_i, m) + \sum_{i=1}^{M}\frac{n_i\mu_i}{\nu}h(\mathbf{N}-\mathbf{e}_i, m)$$
$$+ \mathbf{1}_{D(N,m)}\frac{a(y)}{\nu}h(\mathbf{N}, m+1) + \frac{md}{\nu}h(\mathbf{N}, m-1)$$
$$+ \Big(1 - \sum_{i\in C(\mathbf{N},\mathbf{M})}\frac{\lambda_i(u_i)}{\nu} - \sum_{i=1}^{M}\frac{n_i\mu_i}{\nu}$$
$$- \mathbf{1}_{D(N,m)}\frac{a(y)}{\nu} - \frac{md}{\nu}\Big)h(\mathbf{N}, m)\Big]. \quad (4)$$

Here, $C(\mathbf{N}, m) = \{i|(\mathbf{N}+\mathbf{e}_i)'\mathbf{r} + mr_e \leq R + R_h\}$ is the set of internal class arrivals that can be admitted in state $(\mathbf{N}, m)$, $D(\mathbf{N}, m) = \{(\mathbf{N}, m) \mid \mathbf{N}'\mathbf{r} + (m+1)r_e \leq R + R_h,\ (m+1)r_e \leq 2R_h\}$ describe the conditions under which external RS class arrivals can be admitted to the system, and $\mathbf{1}_{\mathscr{A}}$ denotes the indicator of some set $\mathscr{A}$. The above Bellman equation has a unique solution J^* and $h(\cdot)$ for an arbitrarily selected special state, say 0 at which we specify the value of the differential cost function, for example $h(\mathbf{0}) = 0$ [12]. The scalar J^* stands for the optimal expected social welfare per unit and $h(\mathbf{N}, m)$ denotes the relative reward in state $(\mathbf{N}, m)$. Solution of Bellman's equation yields an optimal policy that maps any state $(\mathbf{N}, m)$ to the optimal price vector $(\mathbf{u}, y)$ that maximizes the right-hand side of Equation (4). Unfortunately, the *curse of dimensionality* stipulates that Bellman's equation is only solvable for a small state space. We therefore seek a near optimal solution that is applicable to

SMO's managing relatively large buildings or neighborhoods with a large population of internal loads.

IV. Static Pricing Policy

We consider a *static* pricing policy, namely a fixed price vector $(\mathbf{u}, y)$ independent of the system state, for two reasons: (1) the computation effort of solving for optimal dynamic prices increases exponentially in the number of classes and active loads, and (2) good static prices can be constructed tractably and under reasonable conditions lead to reasonable behaved provision of RS. Indeed, under a static pricing policy $(\mathbf{u}, y)$, the system evolves as a continuous-time Markov chain with corresponding average welfare:

$$J\big((\mathbf{u},y)\big) = \sum_{i=1}^{M} \lambda_i(u_i)\mathbb{E}[U_i|U_i \geq u_i]\big(1 - \mathbb{P}^i_{\text{loss}}[(\mathbf{u},y)]\big) + a(y)\mathbb{E}[Y|Y \geq y]\big(1 - \mathbb{Q}_{\text{loss}}[(\mathbf{u},y)]\big) - \mathbb{E}\Bigg[P\Big((R+R_h) - \big(\sum_i n_i r_i + m r_e\big)\Big)\Bigg], \tag{5}$$

where $\mathbb{P}^i_{\text{loss}}[(\mathbf{u},y)]$ denotes the steady-state probability $\mathbb{P}[\mathbf{N}'\mathbf{r}+r_i+mr_e > R+R_h]$ that an internal class i arrival is rejected, and $\mathbb{Q}^i_{\text{loss}}[(\mathbf{u},y)]$ denotes the steady-state probability $\mathbb{P}[\mathbf{N}'\mathbf{r}+(m+1)r_e > R+R_h$ or $(m+1)r_e > 2R_h]$ that an external RS class arrival is rejected. Moreover, the expected penalty cost is also given by the steady-state probability associated with the same static policy $(\mathbf{u}, y)$.

The optimal static welfare is defined by

$$J_s = \max_{(\mathbf{u},y)\in\{\mathscr{U},\mathscr{Y}\}} J\big((\mathbf{u},y)\big), \tag{6}$$

and the following proposition holds.

Proposition IV.1 $J_s \leq J^*$.

V. Optimal Performance Upper Bound

In this section we develop an upper bound on J^* and use it to quantify the suboptimality of the static policy.

Using the inverse demand functions $u_i(\lambda_i)$, and internal class i arrival rate λ_i, the instantaneous reward rate is $F_i(\lambda_i) = \lambda_i\mathbb{E}[U_i|U_i \geq u_i(\lambda_i)]$. Similarly, $G(y) = a\mathbb{E}[Y|Y \geq y(a)]$. Assume that the functions F_i and G are *concave*. Let $J_{\mathbf{ub}}$ be the optimal value of the following *Non-Linear Programming (NLP)* problem:

$$\begin{aligned}
\max \quad & \sum_i F_i(\lambda_i) + G(a) \\
& - P\Big((R+R_h) - \big(\sum_i n_i r_i + m r_e\big)\Big) \\
\text{s.t.} \quad & \lambda_i = \mu_i n_i, \quad \forall i \\
& a = dm \\
& \sum_i n_i r_i + m r_e \leq R + R_h \\
& m r_e \leq 2R_h.
\end{aligned} \tag{7}$$

Remark: The non-negativity constraints $n_i \geq 0$ and $m \geq 0$ are ignored here. Notice that the departure rates μ_i and d are positive, and the arrival rates λ_i and a are also non negative by definition. Thus n_i and m are also non-negative under *well-defined* demand functions.

Proposition V.1 *If the functions $F_i(\lambda_i)$ and $G(a)$ are concave and $P(\cdot)$ is convex, then $J^* \leq J_{\mathbf{ub}}$.*

Proof: The proof is similar to a result in [10] and is omitted. ■

The optimal solution of NLP (7) provides an upper bound for the optimal social welfare. Moreover, if the objective function of (7) is concave, the NLP is very easy to solve.

VI. Asymptotic Behavior

In this section, we consider an asymptotic regime and discuss how to derive the optimal policy while satisfying additional system behavior requirements.

A. Many Small Loads

If R and R_h are large relative to the required power of a typical arrival, we expect that the laws of large numbers will dominate, attenuate statistical fluctuations, and allow us to carry out an essential deterministic analysis. To capture a situation of this nature, we start with a base system characterized by finite capacity R and R_h and finite demand functions $\lambda_i(u_i)$. We then scale the system through a proportional increase of capacity and demand.

More specifically, let $c \geq 1$ be a scaling factor. The scaled system has resources $R^c+R^c_h$, with $R^c+R^c_h = cR+cR_h$, and demand functions $\lambda^c_i(u_i)$, $a^c_j(y_j)$ given by $\lambda^c_i(u_i) = c\lambda_i(u_i)$ and $a^c(y) = ca(y)$. Note that the other parameters r_i, μ_i, and r_e, d are held fixed. We will use a superscript c to denote various quantities of interest in the scaled system.

In this case, consider the NLP problem (7). The upper bound J^c_{ub} is obtained by maximizing

$$\sum_i c\lambda_i(u_i)\mathbb{E}[U_i|U_i \geq u_i] + ca(y)\mathbb{E}[Y|Y \geq y] - P\Big(\big(cR+cR_h\big) - \big(\sum_i \frac{c\lambda_i(u_i)}{\mu_i} r_i + \frac{ca}{d} r_e\big)\Big),$$

subject to the constraints $\sum_i \frac{c\lambda_i(u_i)}{\mu_i} r_i + \frac{ca(y)}{d} r_e \leq cR+cR_h$ and $\frac{ca(y)}{d} r_e \leq 2cR_h$.

It can seen that, if the penalty function $P(\cdot)$ is linear, then the optimal solution for (7), denoted by $\mathbf{u}^*_{ub} = (u^*_{ub,1}, \ldots, u^*_{ub,M})$ and y^*_{ub}, is independent of c, and $J^c_{ub} = cJ^1_{ub}$.

We summarize in the following assumption the property of the penalty function.

Assumption C
$P(Kx) = Kx$ for some $K > 0$.

We summarize the above result as follows:

Proposition VI.1 *Under Assumption C, the optimal objective value of (7) in the scaled system increases linearly with c, i.e., $J^c_{ub} = cJ^1_{ub}$.*

We are interested in determining the gap between the two bounds derived in Sec. IV and Sec. V. We show that in the regime of many small users, the following result holds:

Theorem VI.2 *Assume that functions $F_i(\lambda_i)$ and $G(a)$ are concave, and Assumptions A, B, and C hold. Then,*

$$\lim_{c\to\infty}\frac{1}{c}J_s^c = \lim_{c\to\infty}\frac{1}{c}J^{*,c} = \lim_{c\to c}\frac{1}{c}J_{ub}^c. \quad (8)$$

Proof: The proof is omitted due to space limitations.

In the next two subsections, while staying in the regime of many small loads, we extend the asymptotic optimality results to accommodate additional system behavior requirements.

B. Energy Neutrality

We impose energy neutrality, i.e., require the long-term average cumulative active requests of the external RS class to equal R_h. We show that energy neutrality can be achieved if the SMO can appropriately influence the demand function of the RS class.

We assume linear demand $\lambda_i(u_i) = \lambda_{i,\max}(1 - \frac{u_i}{u_{i,\max}})$ and $a(y) = a_{\max}(1 - \frac{y}{y_{\max}})$. Suppose that the welfare U_i is uniformly distributed on $[0, u_{i,\max}]$ and Y is uniformly distributed on $[0, y_{\max}]$.Then, $F_i(\lambda_i) = u_{i,\max}(\lambda_i - \frac{\lambda_i^2}{2\lambda_{i,\max}})$ and $G(a) = y_{\max}(a - \frac{a^2}{2a_{\max}})$ are concave in λ_i and a, respectively.

The NLP (7) can now be written as:

$$\begin{aligned}\min \quad & -\sum_i u_{i,\max}(\lambda_i - \frac{\lambda_i^2}{2\lambda_{i,\max}}) - y_{\max}(a - \frac{a^2}{2a_{\max}}) \\ & \quad + K\Big((R+R_h) - (\sum_i \frac{\lambda_i}{\mu_i}r_i + \frac{a}{d}r_e)\Big) \\ \text{s.t.} \quad & \sum_i \frac{\lambda_i}{\mu_i}r_i + \frac{a}{d}r_e \le R + R_h, \\ & \frac{a}{d}r_e \le 2R_h. \end{aligned} \quad (9)$$

For ease of exposition but without loss of generality, we consider next a system involving 2 internal and 1 external RS class.

Note that the NLP problem (9) can be re-formulated into the following *Quadratic Programming (QP)* problem:

$$\begin{aligned}\min \quad & \frac{1}{2}[\lambda_1 \ \lambda_2 \ a] \begin{bmatrix} \frac{u_{1,\max}}{\lambda_{1,\max}} & & \\ & \frac{u_{2,\max}}{\lambda_{2,\max}} & \\ & & \frac{y_{\max}}{a_{\max}} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ a \end{bmatrix} \\ & + \begin{bmatrix} -K\frac{r_1}{u_1} - u_{1,\max} \\ -K\frac{r_2}{u_2} - u_{2,\max} \\ -K\frac{r_e}{d} - a_{\max} \end{bmatrix}^T \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ a \end{bmatrix} \\ \text{s.t.} \quad & \begin{bmatrix} \frac{r_1}{\mu_1} & \frac{r_2}{\mu_2} & \frac{r_e}{d} \\ & & \frac{r_e}{d} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ a \end{bmatrix} \le \begin{bmatrix} R + R_h \\ 2R_h \end{bmatrix}. \end{aligned} \quad (10)$$

The dual of (10) is also a QP problem. We denote the optimal solution of the primal QP (10) by $(\lambda_1^*, \lambda_2^*, a^*)$, and the optimal solution of the dual QP by (q_1^*, q_2^*).

Under energy neutrality, the long-term average of active external RS class requests is R_h, i.e., $r_e a^*/d = R_h$. By complementary slackness, we have the following optimality conditions:

$$y_{\max}(1 - \frac{1}{a_{\max}} \cdot \frac{dR_h}{r_e}) = \frac{\lambda_{1,\max}\frac{r_1}{\mu_1} + \lambda_{2,\max}\frac{r_2}{\mu_2} - R}{\frac{\lambda_{1,\max}}{u_{1,\max}} \cdot \frac{r_1^2}{\mu_1^2} + \frac{\lambda_{2,\max}}{u_{2,\max}} \cdot \frac{r_2^2}{\mu_2^2}} \cdot \frac{r_e}{d},$$

$$\begin{aligned} R + R_h &\le \lambda_{1,\max}\frac{r_1}{\mu_1} + \lambda_{2,\max}\frac{r_2}{\mu_2} + a_{\max}\frac{r_e}{d}, \\ R &\le \lambda_{1,\max}\frac{r_1}{\mu_1} + \lambda_{2,\max}\frac{r_2}{\mu_2}. \end{aligned} \quad (11)$$

Under conditions (11), the optimal social welfare is:

$$\begin{aligned} & -\frac{1}{2}\frac{\Big(\lambda_{1,\max}\frac{r_1}{\mu_1} + \lambda_{2,\max}\frac{r_2}{\mu_2} + a_{\max}\frac{r_e}{d} - (R + R_h)\Big)^2}{\frac{\lambda_{1,\max}}{u_{1,\max}} \cdot \frac{r_1^2}{\mu_1^2} + \frac{\lambda_{2,\max}}{u_{2,\max}} \cdot \frac{r_2^2}{\mu_2^2} + \frac{a_{\max}}{y_{\max}} \cdot \frac{r_e^2}{d^2}} \\ & + \frac{1}{2}\lambda_{1,\max}u_{1,\max} + \frac{1}{2}\lambda_{2,\max}u_{2,\max} + \frac{1}{2}a_{\max}y_{\max}. \end{aligned} \quad (12)$$

We summarize the above result as follows:

Proposition VI.3 *Given (11), in the regime of many small loads, the long-term average of active requests of the external RS class is R_h, and the optimal performance is given by (12).*

VII. Numerical Experiments

In this section, we report numerical experiments that verify and validate our results.

Assume that the SMO can support a maximal consumption of 1200 KW with $R = 1000$ KW and $R_h = 200$ KW. This consumption is consistent with the Boston University (BU) Photonics building housing the office of the first two co-authors. Consider two internal classes characterized by (all arrival rates are in arrivals/minute and departure rates in departures/minute): $\lambda_1(u_1) = 1600 - 80u_1$, $\lambda_2(u_2) = 800 - 80u_2$, $u_{1,\max} = 20$, $u_{2,\max} = 10$, $\lambda_{1,\max} = 1600$, $\lambda_{2,\max} = 800$, $r_1 = 2$ KW, $r_2 = 1$ KW, $\mu_1 = 1$, $\mu_2 = 2$. The RS class arrival rate is: $a(y) = 1000(1 - y/y_{\max})$ with $y_{\max}$ to be determined, $a_{\max} = 1000$, $r_e = 1$ KW, $d = 2$. The penalty function has a slope of $K = 1000$. Assume that the social welfare U_i is uniformly distributed on $[0, u_{i,\max}]$ and Y is uniformly distributed on $[0, y_{i,\max}]$. With these values we can solve the NLP problem (9) and obtain asymptotically optimal static prices.

Consider a typical regulation service cycle consisting of three 5-minute periods. Each cycle starts with a full RS standby state, namely, with all RS active loads totalling R_h. This is the result of the ISO 5 minute dispatch which we model by tuning the value of $y_{\max}$. In the following two periods within the cycle, ISO requests are modeled as random samples from a uniform distribution over $[0, 2R_h]$ which are instantiated by setting the corresponding value of $y_{\max}$. This random cycle is statistically neutral over the long time-scale. In this experiment, $y_{\max}$ changes every 5 minutes and the SMO must control internal class loads to meet ISO requests within the 5 minute requirement of RS reserves. By formulating and solving the NLP problem (9) at the beginning of every period, the SMO is able to appropriately set the prices that result in the required arrivals of internal classes. We simulate the system for the long time-scale of one hour consisting of 12 periods of 5 minutes each and report the results below.

The steady-state arrival rates for the two internal classes and the RS class in these periods are shown in Tab. I. The evolution of the total consumption due to internal loads and the total load of the RS class are shown in Fig. 1. Note

TABLE I

THE ARRIVAL RATES OF INTERNAL CLASSES AND THE RS CLASS.

Period	Internal class 1	Internal class 2	RS class
1	376	494	400
2	409	502	258
3	346	486	527
4	376	494	400
5	309	477	683
6	409	502	257
7	376	494	400
8	322	480	630
9	445	511	106
10	376	494	400
11	403	500	286
12	321	480	635

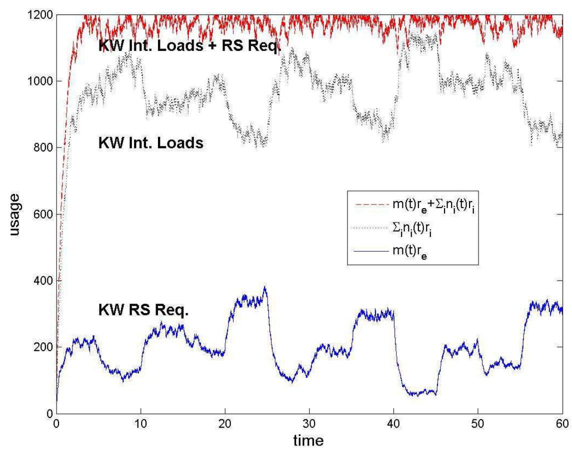

Fig. 1. Energy consumption by internal classes and active RS requests.

that by applying static pricing policies that are piece-wise constant over each 5-minute period, internal loads converge to the ISO request. Recalling that RS reserves are required to respond with a ramp of $R_h/5$ KW per minute, the response of internal class loads conforms well to requirements. Indeed, since $R_h = 200$ KW in this example, the rate at which $n_1(t)r_1 + n_2(t)r_2 + m(t)r_e$ move away from and then approach the 1200 KW level should be close to 40 KW per minute. Figure 1 demonstrates this to be the case. The SMO's decision to offer 200 KW of RS is consistent with its capability to perform according to the associated contractual requirements. In Figure 2, where we plot the number of internal loads and RS requests, we note that there are on average 350 active loads of class 1 with a 2 KW consumption rate – these might be HVAC heating zone loads – and 250 active loads of class 2 with a 1 KW consumption rate. These quantities are consistent with the BU Photonics building which features several hundred heating zones.

VIII. CONCLUSIONS

The prospect of a paradigm shift in the capabilities of the electric power grid as well as building side of the meter microgrids through cyber-physical system (CPS) infrastructure development is within sight. Such CPS infrastructure will certainly enable loads to participate in power markets on a par basis with generating units, not only in the provision

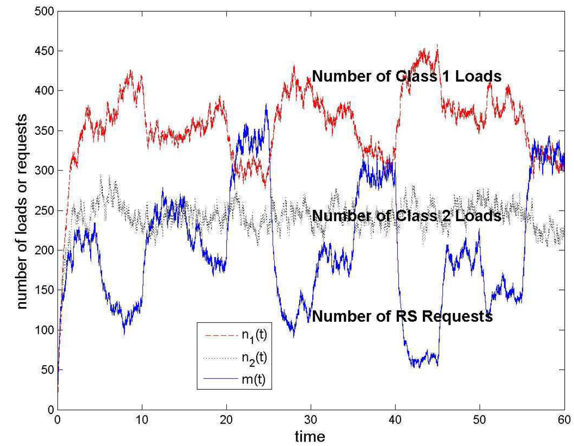

Fig. 2. Number of active internal loads and active RS requests.

of electric energy, but also in the provision of fast reserves. In this paper we develop and test a market based approach for a Smart Microgrid Operator (SMO) to control numerous and diverse loads and provide such services. We start by formulating a detailed dynamic optimal control problem and then derive an associated tractable and yet near optimal non-linear optimization model that is capable of determining both short term (at the minutes time-scale) operational decision support to the SMO as well as longer time-scale transaction quantities (at the hourly time-scale). Our model is elaborated and validated by numerical simulation results.

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