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PEM Fuel Cell/ DC-DC Boost Power Converter System Control Via Traditional and Higher Order Sliding Modes

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Abstract— **Proton Exchange Membrane fuel cell (PEMFC) is considered as a primary source of electrical energy for the DC-DC boost power converter. System's PEMFC/DC-DC boost power converter zero dynamics are analyzed and appeared to be stable. Relative degree approach is applied for direct control of the output load voltage and the fuel cell current in the presence of the model uncertainties. The sliding mode observer is employed for identification of the load resistance, which estimated value is used for generating the fuel cell current command profile. The decoupled traditional SMC and 2-SMC super-twisting sliding mode controller are designed to control the output voltage the PEMFC current respectfully. The efficacy and robustness of the proposed SMC and 2-SMC controllers are confirmed via computer simulations.**

I. Introduction

FUEL Cells [1, 2] are very important because they offer an alternative environmentally friendly fuel for transportation, which causes a major amount of the damage to or earth and the growing problem of global warming. The most common use of a fuel cell is to power automotive applications. Fuel cells, unlike batteries, use an external and continuous source of fuel and produce power continuously, as long as the fuel supply is maintained. Within such perspective fuel cells have been appeared as an ideal alternative because of their high generation efficiency, high generation power density, no-noise, zero pollution, module type structure, high reliability and durability. By converting an on-board fuel to electricity it could be effectively used to power an electric vehicle [2]. The fuel cell transforms hydrogen into DC power. In most stationary and mobile applications, fuel cells are used in conjunction with other power conditioning converters [3, 4] and a circuit model [12] would be beneficial, especially for power electronics engineers who in many cases have the task of designing converters associated with the fuel cell for various load applications.

Control of fuel cells using sliding mode control (SMC) technique [5-8] that is robust to the fuel cell model uncertainties and the external disturbances is studied in [9]. Sliding mode control design for power systems, which consist of fuel cell and boosts DC-DC power converter, is presented in [10, 11].

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In this paper we consider control of a power system that comprises Proton Exchange Membrane fuel cell (PEMFC) and a boost DC-DC power converter using sliding mode control techniques. The contribution of this paper is as follows

(1) Two types of sliding mode control (traditional SMC and 2-SMC, continuous super-twisting control) are used in the same time for controlling PEMFC/ boost DC-DC power converter system.
(2) It is observed that a two-loop SMC feedback control structure (2-SMC current and traditional SMC voltage loops) allows avoiding nonminimum phase property of boost DC-DC power converter, which output voltage is controlled directly, that simplifies the controller design.
(3) Use of sliding mode observer for on-line identification of the load resistance improves the robustness of the designed control system.

The structure of the paper is as follows. The physical description and schematic of PEMFC is given in Section II. Mathematical model of system PEMFC/boost DC-DC power converter is presented in Section III. Sections IV and V are dedicated to the inductance current and the output voltage tracking controller design. In Section VI the inductance current command generator is proposed. Identification of the load resistance via SMC observer is presented in Section VII. Section VIII presents the simulation study, and the conclusions can be found in Section IX.

II. Physical Description of PEMFC

There are different technologies of fuel cell. They are commonly classified according to temperature or the type of electrolyte. Among others, low-temperature fuel cell includes Proton Exchange Membrane (PEM). Proton exchange membrane fuel cells (PEMFC) are promising new power sources for vehicles and portable devices. The fuel cell transforms hydrogen into DC power while producing electrons, protons, heat and water. The fuel cell schematic [1, 2] is shown in Fig. 1. PEM fuel cells use an extremely thin solid polymer layer as a membrane (electrolyte). This membrane is sandwiched between two electrodes; the hydrogen electrode (anode) and the oxygen electrode (cathode). A very thin layer of catalyst is bonded to either side of the membrane or to the electrodes. This membrane electrode assembly (MEA) is sandwiched between separators to compose one cell. Two bipolar plates are positioned against the electrodes, one on each side of the MEA. The bipolar plates have two primary functions:

transmission of electric current through the elementary cells and release of heat to the external environment.

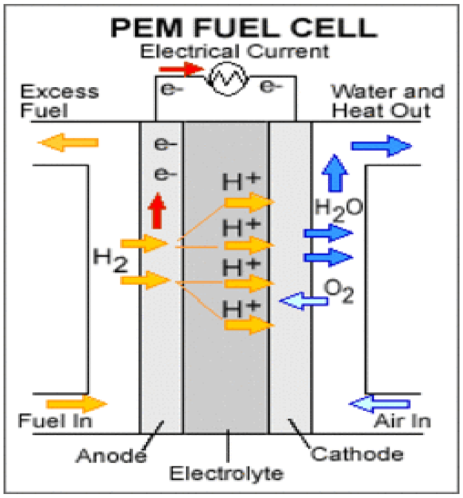

Fig. 1 Schematic of a Fuel cell

The reaction in a fuel cell produces only about 0.7 volts, so several fuel cells are connected in a series to attain a useful output. Fuel cells connected together are called a fuel cell stack. To obtain a fuel cell stack, multiple fuel cells and bipolar plates are sequentially assembled in series in a modular configuration. The input fuel passes over the anode (and oxygen over the cathode) where it splits into ions and electrons. The electrons pass through an external circuit to serve an electric load while the ions move through the electrolyte toward the oppositely charged electrode. At the electrode, ions combine to create by-products, primarily water and carbon dioxide. PEMFCs are capable of operation at pressures from 0.10 to 0 MPa (10 to 100 psig) and with suitable current collectors and supporting structure, these fuel cells may be capable of operating at pressures as high as 3000 psi. The chemical reactions that take place in PEMFC is illustrated in Fig. 2 [1, 2].

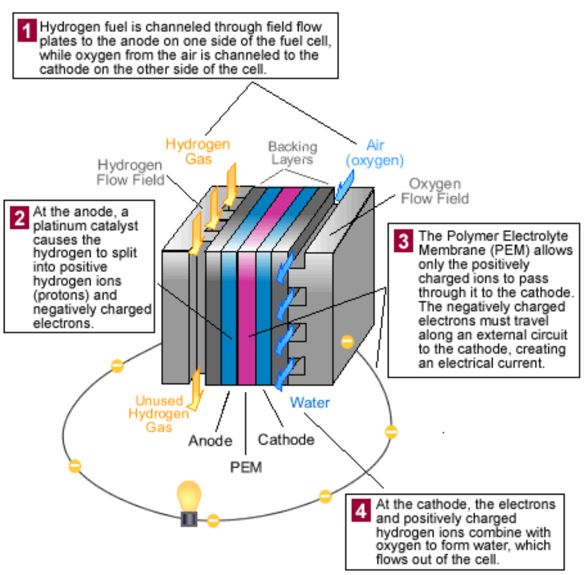

Fig. 2 Detailed Schematic of the PEMFC operation

The chemical reactions in PEMFC can be described by the following equations [1, 2]

$$\begin{cases} H_2 \rightarrow 2H^+ + 2e^+ \\ 2H^+ + \dfrac{1}{2}O_2 + 2e^- \rightarrow H_2O \\ H_2 + \dfrac{1}{2}O_2 \rightarrow H_2O \end{cases} \tag{1}$$

III. MATHEMATICAL MODEL OF PEMFC/BOOST DC-DC POWER CONVERTER

A stack of N considered PEMFCs are used as a primary source of electrical energy for the boost DC-DC power converter. The equivalent circuit of such system [1, 2] is presented in Fig. 3, where

$i_{fc} \rightarrow$ is the current of the fuel cell,

$R_L \rightarrow$ is the load,

$v_{cf} \rightarrow$ is the voltage across the capacitor,

$V_{tfc} \rightarrow$ is the voltage across the fuel cell,

$V_L \rightarrow$ is the output voltage of the DC/DC convertor,

$R_{var} \rightarrow$ is the variable internal resistor of the fuel cell.

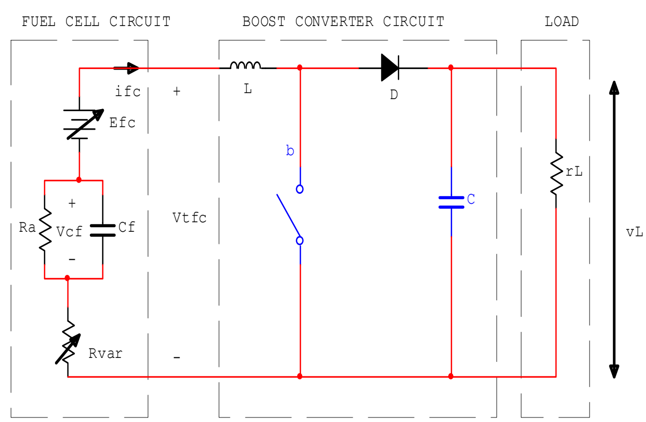

Fig. 3 Circuit Diagram of Fuel Cell and DC-DC converter

We have to acknowledge that a big variety of the PEMFC equivalent circuits are available [1, 2, 9-12]. The PEMFC equivalent circuit [11] considered in this work and presented in Fig. 3 is of a reasonable complexity and reflects the dynamics of the main electrical processes in the fuel cell.

A. *Mathematical Model of PEMFC*

Equations that describe the dynamics of the considered PEMFC output voltage v_{tfc} are obtained based on analysis of the circuit presented on Fig. 3. They are

$$\begin{cases} \dfrac{dv_{cf}}{dt} = -\dfrac{1}{C_f}\left(i_{fc} + \dfrac{v_{cf}}{R_a} \right) \\ v_{tfc} = E_{fc} + v_{cf} - i_{fc} R_{var} \end{cases} \tag{2}$$

The output voltage V_{tfc} of the stack of N PEMFCs connected in series is easily computed

$$V_{tfc} = N v_{tfc} = N E_{fc} + N v_{cf} - N i_{fc} R_{var} \tag{3a}$$

The dynamics of a stack of N PEMFCs can be controlled by pressure of H_2 and O_2 (see eqs. (1)). This pressure is directly proportional to E_{fc}. It is assumed that there exists some initial pressure of H_2 and O_2 that results in an initial value of $E_{fc0} = E_{fc}(0)$, and

$$E_{fc} = E_{fc0} + \tilde{E}_{fc} \tag{3b}$$

where $\tilde{E}_{fc}$ is considered to be a PEMFC control function.

$$u_1 = \tilde{E}_{fc} \tag{4}$$

B. Mathematical Model of Boost DC-DC power converter

The dynamics of the boost DC-DC power converter are well-studied, see for instance [3, 4], and are governed by the following system of bilinear differential equations

$$\frac{di_L}{dt} = -(1-u_2)\frac{1}{L}V_L + \frac{V_{tfc}}{L} \tag{5}$$

$$\frac{dV_L}{dt} = (1-u_2)\frac{1}{C}i_L - \frac{V_L}{R_L}\frac{1}{C} \tag{6}$$

where $u_2 \in [0,1]$ is a switch control function. In particular, u_2=1 when the switch is closed and u_2=0 when it is opened. In order to achieve the control symmetry, the control function is transformed as

$$u_2 = v_2 + 0.5 \tag{7}$$

Therefore, a new control function $v_2 \in [-0.5, 0.5]$ will replace u_2 in equations (5), (6). Combining equations (2)-(6), bearing in mind that $i_{fc} = i_L$, we obtain following equations of PMEFC/boost DC-DC converter

$$\begin{cases} \dfrac{di_L}{dt} = -\dfrac{1}{L}\left(\dfrac{1}{2}V_L - N\left(E_{fc0} + v_{cf} - i_L R_{var} \right) \right) + \dfrac{1}{L}\left(N u_1 + V_L v_2 \right) \\ \dfrac{dV_L}{dt} = \dfrac{1}{C}\left(\dfrac{1}{2}i_L - \dfrac{V_L}{R_L} \right) - \dfrac{1}{C}i_L v_2 \\ \dfrac{dv_{cf}}{dt} = -\dfrac{1}{C_f}\left(i_L + \dfrac{v_{cf}}{R_a} \right) \end{cases} \tag{7}$$

C. Problem formulation. Relative Degree Approach

The goal of this paper is to design the sliding mode controller in terms of controls u_1, v_2 that make the output voltage V_L of the boost DC-DC converter to follow a given reference profile $V_{Lc}(t)$ in the presence of unknown bounded parameters of PMEFC R_{var}, C_f, R_a and the load resistor R_L. In fact, direct control of the output voltage in the boost DC-DC power converters is a nonminimum phase problem [13] that adds complexity to the controller design [3, 4]. In the case of system (7) there exists a possibility to control some PMRFC variable using the control function u_1. Therefore, in addition to controlling the output voltage V_L we will drive the fuel cell current i_{fc}, which is equal to inductance current i_L (see Fig. 3), to a command profile $i_{Lc}(t)$ in finite time.

Denoting the controlled and measured outputs as

$$y_1 = i_L, \;\; y_2 = V_L \tag{8}$$

and using the relative degree approach [13], the *input-output* dynamics of system (7) are presented as

$$\begin{cases} \dfrac{di_L}{dt} = -\dfrac{1}{L}\left(\dfrac{1}{2}V_L - N\left(E_{fc0} + v_{cf} - i_L R_{var} \right) \right) + \dfrac{1}{L}\left(N u_1 + V_L v_2 \right) \\ \dfrac{dV_L}{dt} = \dfrac{1}{C}\left(\dfrac{1}{2}i_L - \dfrac{V_L}{R_L} \right) - \dfrac{1}{C}i_L v_2 \end{cases} \tag{9}$$

and the *internal* dynamics are

$$\frac{dv_{cf}}{dt} = -\frac{1}{C_f}\left(i_L + \frac{v_{cf}}{R_a} \right) \tag{10}$$

where v_{cf} is considered as an *internal* variable.

Assuming that the current i_L has been converged already to its reference profile $i_{Lc}(t)$, the respective forced *zero* dynamics [13]

$$\frac{dv_{cf}}{dt} + \frac{v_{cf}}{C_f R_a} = -\frac{1}{C_f} i_{Lc}(t) \tag{11}$$

are apparently stable.

Stability of internal dynamics makes system (7) of a minimum phase and simplifies the controller design. Now, the control functions u_1 and v_2 can be designed using only the input-output dynamics in eq. (9).

Assuming that the inductance current $i_L \neq 0$, and denoting

$$\omega_1 = N u_1 + V_L v_2 \tag{12}$$

eq. (9) is rewritten in an input-output *de-coupled* format

$$\frac{di_L}{dt} = \varphi_1(i_L, V_{L,} v_{cf}) + \frac{1}{L}\omega_1 \tag{13}$$

$$\frac{dV_L}{dt} = \varphi_2(i_L, V_{L,} R_L) - \frac{1}{C} i_L v_2 \tag{14}$$

where

$$\varphi_1(i_L, V_{L,} v_{cf}) = -\frac{1}{L}\left(\frac{1}{2}V_L - N\left(E_{fc0} + v_{cf} - i_L R_{var} \right) \right),$$

$$\varphi_2(i_L, V_{L,} R_L) = \frac{1}{C}\left(\frac{1}{2}i_L - \frac{V_L}{R_L} \right).$$

The disturbance $\varphi_1(i_L, V_{L,} v_{cf})$ and its derivative are assumed to be bounded, i.e.

$\left|\varphi_1(i_L, V_{L,} v_{cf})\right| \le q\Omega,\ 0 < q < 1,\ \Omega > 0$,

$\left|\dot{\varphi}_1(i_L, V_{L,} v_{cf})\right| \le M_1$; and the disturbance $\varphi_2(i_L, V_{L,} R_L)$ is also assumed to be bounded, i.e. $\left|\varphi_2(i_L, V_{L,} R_L)\right| \le M_2$

Now, the problem can formulated as designing the control functions ω_1, ω_2 so that $e_1, e_2 \to 0$ in finite time, where $e_1(t) = y_{1c}(t) - y_1(t),\ \ e_2(t) = y_{2c}(t) - y_2(t)$.

IV. Inductance current Super-twisting controller design

The inductance current controller is supposed to be continues, while driving $e_1 \to 0$ in finite time in accordance with eq. (13). The viable candidate for this controller is super-twisting control that belongs to a class of second order sliding mode controllers (2-SMC), since it also drives $\dot{e}_1 \to 0$ in finite time. The 2-SMC super-twisting controller is [6]:

$$\begin{aligned} &\omega_1 = L\left[\lambda \,|\, e_1 \,|^{1/2} \operatorname{sign}(e_1) + \varpi_1\right], \\ &\varpi_1 = \begin{cases} \omega_1, & |\,\omega_1\,| > \Omega, \\ \alpha \operatorname{sign}(e_1), & |\,\omega_1\,| \le \Omega. \end{cases} \end{aligned} \tag{15}$$

where the gains are to satisfy the following inequalities:

$$\begin{cases} \alpha > M_1, \\ \lambda > \sqrt{\dfrac{2}{(\alpha - M_1)}} \dfrac{(\alpha + M_1)(1+q)}{(1-q)} \end{cases} \tag{16}$$

The other recommended set of the controller gains is [6,7]:

$$\alpha = 1.1 M_1, \quad \lambda = 1.5\sqrt{M_1} \tag{17}$$

V. The SMC controller design for the output voltage of the boost DC-DC converter

The dynamics of the output voltage tracking error are derived

$$\dot{e}_2 = \underbrace{\dot{y}_{2c} - \varphi_2(i_L, V_{L,} R_L)}_{\psi(i_L, V_{L,} R_L)} + \frac{1}{C} i_L v_2 \tag{18}$$

where the combined disturbance $\psi(i_L, V_{L,} R_L)$ is assumed bounded, i.e. $\left|\psi(i_L, V_{L,} R_L)\right| \le \bar{M}_2$

The output voltage controller is supposed to be discontinuous, since it represents a switching function $v_2 \in [-0.5, 0.5]$. The viable candidate for this controller is a traditional sliding mode controller (SMC)

$$v_2 = -0.5 sign(e_2) \tag{19}$$

The sliding mode existence condition [5]:

$$e_2 \dot{e}_2 \le -\rho |e_2|,\ \rho > 0 \tag{20}$$

where ρ is to be selected to provide for a given reaching time $t_r \le \dfrac{e_2(0)}{\rho}$, are to be verified for the controller (19). Substituting eq. (19) into eq. (20) we obtain

$$\begin{aligned} &e_2 \dot{e}_2 = e_2\left(\psi - \frac{i_L}{2C} sign(e_2)\right) \le \\ &|e_2|\left(\bar{M}_2 - \frac{i_L}{2C}\right) \le -\rho |e_2| \end{aligned} \tag{21}$$

The existence condition (21) will be satisfied if

$$i_L \ge 2C(\bar{M}_2 + \rho) \tag{22}$$

Therefore, $y_{1c}(t) = i_{Lc}(t)$ must be large enough in order to fulfill inequality (22).

VI. The inductance current command generator

The 2-SMC (15) is supposed to drive $i_L(t)$ to the profile $y_{1c}(t) = i_{Lc}(t)$ given on line. This command is computed based on the power balance $P_{\mathrm{PMEFC}} = P_{Load}$ that is achieved if the current command is generated as

$$y_{1c}(t) = i_{Lc}(t) = \frac{\left(y_{2c}(t)\right)^2}{V_{tfc}^* R_L} \tag{23}$$

where V_{tfc}^* is the commanded value of the voltage across the fuel cell stack. The fuel cell current command (23) must satisfy eq. (22) by means of a corresponding choice of V_{tfc}^*.

VII. Identification of the load resistance

The unknown load resistance R_L that is supposed to be used in computing $y_{1c}(t)$ in eq. (23) is identified via the sliding mode observer using eq. (14)

$$\frac{dV_L}{dt} = \frac{1}{C}\left(\frac{1}{2} i_L - \frac{V_L}{R_L}\right) - \frac{1}{C} i_L v_2$$

The observer's equation is

$$\frac{d\hat{V}_L}{dt} = \frac{1}{C}\left(\frac{1}{2} i_L - \frac{V_L}{R_{L0}}\right) - \frac{1}{C} i_L v_2 + \beta \tag{24}$$

where i_L is measured, v_2 is known, R_{L0} is a nominal value of R_L, and β is an injection term. The dynamics of the output voltage estimation error $e_V = V_L - \hat{V}_L$ are derived

$$\frac{de_V}{dt} = \frac{V_L}{C}\left(\frac{1}{R_L} - \frac{1}{R_{L0}}\right) - \beta \tag{25}$$

The injection term that is designed in a SMC format

$$\beta = \bar{\rho} sign(e_V),\quad \bar{\rho} > \frac{V_L |R_L - R_{L0}|}{C R_{L0} R_L},\quad V_L > 0 \tag{26}$$

will drive $e_V \to 0$ in finite time. In the sliding mode $e_V = 0$ we obtain

$$\frac{V_L(R_L - R_{L0})}{C R_{L0} R_L} - \beta_{eq} = 0 \ \Rightarrow\ \hat{R}_L = R_{L0} \frac{V_L}{V_L - C R_{L0} \beta_{eq}} \tag{27}$$

were β_{eq} is an *equivalent* injection term [5] that can be obtained via low pass filtering of the injection term (26) in the sliding mode [5], for instance

$$\tau_1 \dot{\beta}_{eq} + \beta_{eq} = \beta, \quad \tau_1 > 0 \tag{28}$$

In order to avoid a possible singularity in eq. (27), it can be regularized

$$\hat{R}_L \approx R_{L0} \frac{V_L\left(V_L - CR_{L0}\beta_{eq}\right)}{\left(V_L - CR_{L0}\beta_{eq}\right)^2 + \varepsilon}, \quad \varepsilon > 0 \tag{29}$$

The estimated load resistance in eq. (29) is to be used in eq. (23) while computing the command profile for the inductance current.

Finally, the control law $u = [u_1, u_2]^T$ is designed

$$u_1 = \begin{cases} \frac{1}{N}\left(\omega_1 - 0.5V_L + 0.5V_L sign(e_2)\right), & if \quad t \le t_r \\ \frac{1}{N}\left(\omega_1 - V_L v_{2eq}\right), & if \quad t > t_r \end{cases} \tag{30}$$

$$u_2 = 0.5\left(1 - sign\left(e_2\right)\right)$$

where ω_1 is defined by eqs. (15), (17), t_r is a finite reaching time in the load voltage control loop, and v_{2eq} is equivalent control of v_2 that can be identified via v_2 low pass filtering, for instance, as

$$\tau_2 \dot{v}_{2eq} + v_{2eq} = v_2, \quad \tau_2 > 0 \tag{31}$$

VIII. Simulation Study

The system of electrical energy supply that consists of the boost DC-DC power converter, which receives a primary electrical energy from a stack of N PEMFCs and is controlled by SMC and 2-SMC (super-twisting control), is simulated using the mathematical model in eqs. (2)-(6) and controls given by eq. (30).

The following parameters were selected during the simulations:

$L = 7 \cdot 10^{-6} H, \quad C = C_f = 1.1 \cdot 10^{-3} F, \quad E_{fc}(0) = 24V, \quad R_{L0} = 30\Omega,$
$i_L(0) = 5A, \quad V_L(0) = 120V, \quad v_{fc}(0) = -0.7V$

The following load voltage command profiles were selected in accordance with [9]-[12]

$$V_{Lc}(t) = \begin{cases} 450V, & 0 \le t \le 0.73s \\ 400V, & 0.73s < t \le 1.05s \\ 430V, & t > 1.05s \end{cases} \tag{32}$$

The load resistor was changed at $t = 0.83s$:

$$R_L = \begin{cases} 20\Omega, & 0 \le t \le 0.83s \\ 50\Omega, & t > 0.83s \end{cases} \tag{33}$$

The fuel cell varying resistor changed its value at $t = 1.05s$

$$R_{\text{var}} = \begin{cases} 0.5\Omega, & 0 \le t \le 1.05s \\ 0.75\Omega, & t > 1.05s \end{cases} \tag{34}$$

The simulation plots are presented in Figs. 4-10. The plots on Fig. 4 illustrate high accuracy direct tracking of the load voltage command profile via classical sliding mode control (Fig. 8) in the presence of unknown changing load resistor

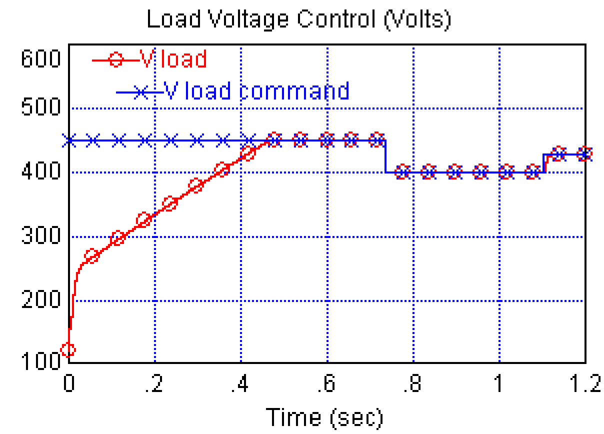

Fig. 4 Load voltage control

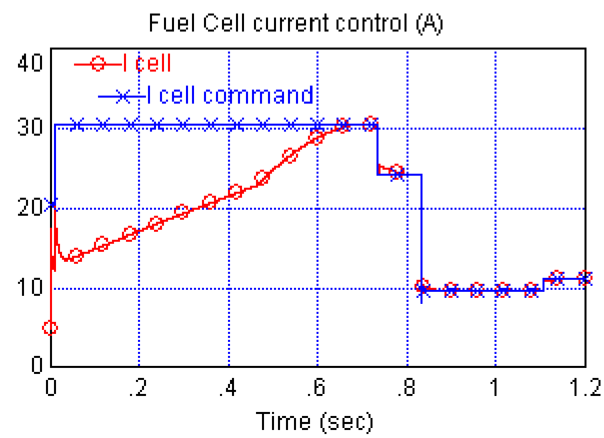

Fig. 5 Fuel cell current control

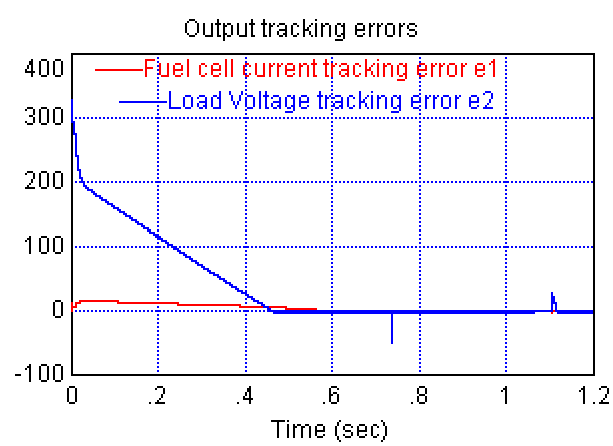

Fig. 6 Output tracking errors

(Fig. 10) that is accurately reconstructed by the sliding mode resistor observer in eqs. (24-(29), while the voltage command profile is changing instantaneously twice during simulation time. High accuracy tracking of the fuel cell current command profile that is generated on line using eq. (22) is confirmed via plots in Fig. 5. The output tracking error dynamics are shown in Fig. 6. Finite time convergence of tracking errors to zero in the presence of the unknown parameters, including R_L, R_{var} changing on line, is demonstrated. Fuel cell continuous control is shown in Fig 7, and time history of the voltage across the fuel cell internal capacitor is shown in Fig. 9.

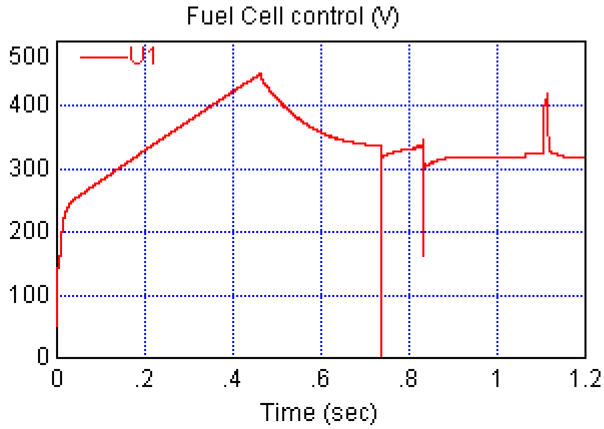

Fig. 7 Fuel cell current control

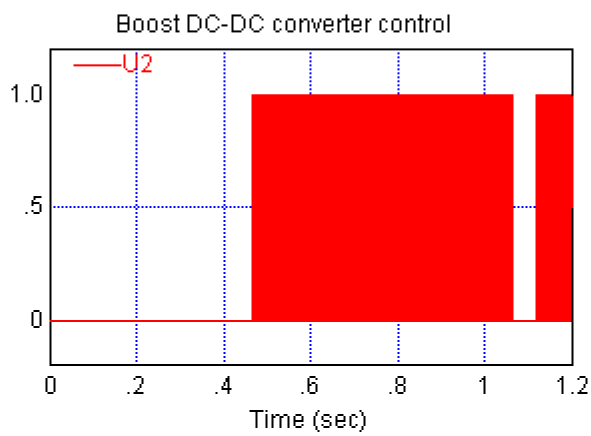

Fig. 8 Boost DC-DC power converter control

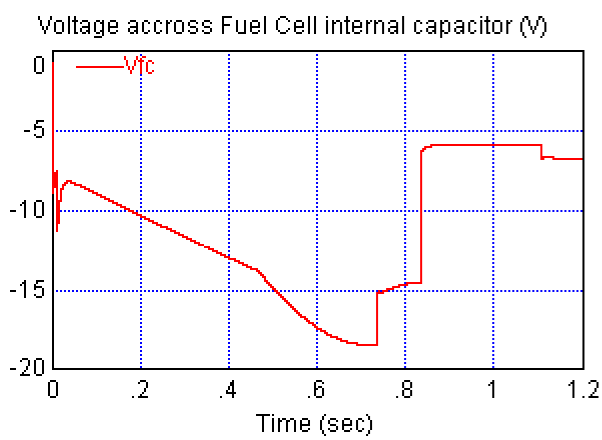

Fig. 9 Voltage across the fuel cell internal capacitor

IX. Conclusions

Control of a power system that comprises Proton Exchange Membrane fuel cell (PEMFC) and a boost DC-DC power converter is studied using sliding mode control techniques. Traditional SMC and 2-SMC, continuous super-twisting control, are used for controlling PEMFC/boost DC-DC power converter system. It is observed that a two-loop SMC feedback control structure allows avoiding nonminimum phase property of boost DC-DC power converter, which output voltage is controlled directly, that simplifies the controller design.

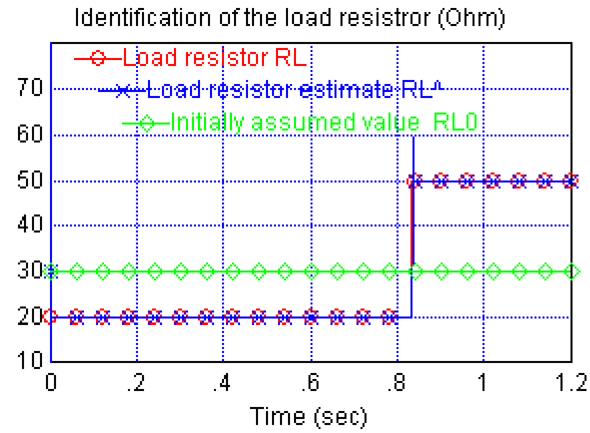

Fig. 10 Identification of load resistor

Use of sliding mode observer for on-line identification of the load resistance improves the robustness of the designed control system. The efficacy of the proposed control system is confirmed via computer simulations.

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