2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Stabilizing Composite Control for Systems Modeled by Singularly Perturbed Itô Differential Equations with Two Small Time Constants

Vasile Dragan and Hiroaki Mukaidani

Abstract— **In this paper, we discuss the stabilizing composite control design for a class of multiparameter singularly perturbed systems governed by Itô differential equations. The asymptotic stability in mean square (ASMS) of the closed-loop system is addressed. First, the asymptotic structure of solutions of suitable Lyapunov type equation via multimodeling analysis is established. It is shown that the dominant part of this solution can be obtained by solving a parameter-independent system of coupled algebraic linear equations which define a resolvent positive operator. Moreover, it is noteworthy that this is the first time conditions for the existence of the stabilizing feedback gain. These conditions are expressed in terms of solvability of a system of linear matrix inequalities. Finally, in order to demonstrate the effectiveness of the proposed design method, a numerical example is provided.**

I. INTRODUCTION

The deterministic and stochastic stability, control, filtering and dynamic games for a class of singularly perturbed systems (SPS) have been investigated extensively by several researchers (see e.g., [7], [8]). Afterward, various aspects of the problem of designing of a stabilizing feedback gain for systems modeled by singularly perturbed Itô differential equations with one small time constant have been well documented in many literatures (see e.g., [10]). However, such an approach is not adequate to the multiparameter singularly perturbed systems (MSPS) since in case that the parameters ε_j are not known exactly, they cannot be transformed to the SPS [13].

The problem of designing a feedback strategy for a multimodeling system has been subject of many papers during the past three decades (see e.g., [13]). Recent advance in theory of the stochastic approach has allowed a revisiting of the control problems for the MSPS [16], [17]. These literatures, however, the special structure for the fast subsystems are imposed. As a result, these results do not give more general framework because there is no interconnection for each fast subsystem.

In this paper, the stabilizing composite control design for a class of MSPS governed by Itô differential equation is considered. It is worth pointing out that although the optimal and H_∞ control problems for the stochastic MSPS has been investigated [17], the conservative restriction over the fast subsystems has been imposed. As compared with the deterministic case [13], the sufficient condition is established for the first time such that the asymptotic stability in mean square (ASMS) of the closed-loop system is attained. Moreover, necessary and sufficient conditions for the existence of feedback gains such that the system of reduced algebraic Lyapunov equations have positive definite solutions are expressed in terms of solvability of some suitable linear matrix inequalities. Finally, in order to demonstrate the efficiency of the proposed algorithm, a numerical example is given.

Notation: The superscript T denotes matrix or vector transpose. $\mathbf{E}[\cdot]$ denotes the expectation operator. I_n denotes the $n \times n$ identity matrix. **block diag** denotes the block diagonal matrix.

II. PROBLEM FORMULATION

Consider the controlled system modeled by the following system of singularly perturbed Itô differential equations with both state and control multiplicative white noise:

$$\begin{aligned} & dx_0(t) \\ &= [A_{00}x_0(t) + A_{0f}x_f(t) + B_0u(t)]dt \\ &\quad + \sum_{p=1}^{r}[A_{p00}x_0(t) + A_{p0f}x_f(t) + B_{p0}u(t)]dw_p(t), \quad \text{(1a)} \\ & \Pi_\varepsilon dx_f(t) \\ &= [A_{f0}x_0(t) + A_f x_f(t) + B_f u(t)]dt \\ &\quad + \Pi_\mu \sum_{p=1}^{r}[A_{pf0}x_0(t) + A_{pf}x_f(t) + B_{pf}u(t)]dw_p(t), \text{(1b)} \end{aligned}$$

where $x_0(t) \in \Re^{n_0}$ is the slow state variable. $x_f(t) = \left[\begin{array}{cc} x_1^T(t) & x_2^T(t) \end{array} \right]^T \in \Re^{n_1} \oplus \Re^{n_2}$ are the fast state variables. $u(t) \in \Re^m$ is the vector of control parameter. Let us define the following matrices.

$$\begin{aligned} & A_{0f} := \left[\begin{array}{cc} A_{01} & A_{02} \end{array} \right], \ A_{p0f} := \left[\begin{array}{cc} A_{p01} & A_{p02} \end{array} \right], \\ & A_{f0} := \left[\begin{array}{c} A_{10} \\ A_{20} \end{array} \right], \ A_{pf0} := \left[\begin{array}{c} A_{p10} \\ A_{p20} \end{array} \right], \\ & A_f := \left[\begin{array}{cc} A_{11} & A_{12} \\ A_{21} & A_{22} \end{array} \right], \ A_{pf} := \left[\begin{array}{cc} A_{p11} & A_{p12} \\ A_{p21} & A_{p22} \end{array} \right], \\ & B_f := \left[\begin{array}{c} B_1 \\ B_2 \end{array} \right], \ B_{pf} := \left[\begin{array}{c} B_{p1} \\ B_{p2} \end{array} \right], \\ & A_{ij}, \ A_{pij} \in \Re^{n_i \times n_j}, \ B_i, \ B_{pi} \in \Re^{n_i \times m}, \ i = 0, \ 1, \ 2. \end{aligned}$$

Moreover, $\Pi_\varepsilon :=$ **block diag** $\left(\begin{array}{cc} \varepsilon_1 I_{n_1} & \varepsilon_2 I_{n_2} \end{array} \right)$, $\Pi_\mu :=$ **block diag** $\left(\begin{array}{cc} \mu_1 I_{n_1} & \mu_2 I_{n_2} \end{array} \right)$, where $\varepsilon_i > 0$, $\mu_i > 0$, $i = 1, \ 2$ are small parameters. It may be noted that they are not exactly known.

In (1) $\{w(t)\}_{t \geq 0}$, $w(t) = \left[\begin{array}{ccc} w_1(t) & \cdots & w_r(t) \end{array} \right]^T$ is r-dimensional standard Wiener process on a given probability space $(\Omega, \ \mathcal{F}, \ \mathcal{P})$ [11], [12]. It should be noted that such

V. Dragan is with Institute of Mathematics of the Romanian Academy, P.O.Box 1-764, RO-014700, Romania. Vasile.Dragan@imar.ro

H. Mukaidani is with Graduate School of Education, Hiroshima University, 1-1-1 Kagamiyama, Higashi-Hiroshima, 739-8524 Japan. mukaida@hiroshima-u.ac.jp

systems (1) typically arise in the multi-area power systems [17].

Consider the control laws of the following form:

$$u(t) = F_0 x_0(t) + F_f x_f(t), \quad (2)$$

where $F_0 \in \Re^{m\times n_0}$, $F_f = \begin{bmatrix} F_1 & F_2 \end{bmatrix}$, $F_i \in \Re^{m\times n_i}$, $i = 1,\ 2$.

Our aim is to develop a methodology which allows us to design the gain matrices $\begin{bmatrix} F_0 & F_f \end{bmatrix}$ not depending upon the small parameters ε_i and μ_i such that the corresponding control law of the form (2) stabilizes the stochastic systems (1) for any $\varepsilon_i > 0$, $\mu_i > 0$ small enough, that is the trajectories of the closed-loop systems to satisfy

$$\lim_{t\to\infty} E[\|x_0(t)\|^2 + \|x_f(t)\|^2] = 0$$

for all initial conditions $x(0) = \begin{bmatrix} x_0^T(0) & x_f^T(0) \end{bmatrix}^T \in \Re^n$, $n = n_0 + n_f$, $n_f = n_1 + n_2$.

It is worth mentioning that any performance specification, other than stabilization of given system is not imposed to the designed control. It should be noted that for the case of systems modeled by singularly perturbed Itô differential equations with state and control multiplicative white noise, the problem to construct a stabilizing control in a state feedback form is more complicated than the case of deterministic singularly perturbed systems (see e.g., [7], [8]). At the end of this section, let us remark that in the general case $\varepsilon_1 = \varepsilon_2 = \varepsilon$, $\mu_1 = \mu_2 = \varepsilon^\delta$ with $\delta \geq 1/2$, the problem stated before reduces to that showed in [10].

III. MAIN RESULTS

A. Lyapunov type equations associated with the closed-loop systems

Consider the following closed-loop system.

$$\begin{aligned} d\boldsymbol{x}(t) &= [A(\varepsilon) + B(\varepsilon)F]\boldsymbol{x}(t)dt \\ &\quad + \sum_{p=1}^{r}[A_p(\varepsilon,\ \mu) + B_p(\varepsilon,\ \mu)F]\boldsymbol{x}(t)dw_p(t), \end{aligned} \quad (3)$$

where $F = \begin{bmatrix} F_0 & F_f \end{bmatrix}$ with

$$\boldsymbol{x}(t) = \begin{bmatrix} x_0(t) \\ x_f(t) \end{bmatrix},\ A(\varepsilon) := \begin{bmatrix} A_{00} & A_{0f} \\ \Pi_\varepsilon^{-1}A_{f0} & \Pi_\varepsilon^{-1}A_f \end{bmatrix},$$

$$A_p(\varepsilon,\ \mu) := \begin{bmatrix} A_{p00} & A_{p0f} \\ \Pi_\varepsilon^{-1}\Pi_\mu A_{pf0} & \Pi_\varepsilon^{-1}\Pi_\mu A_{pf} \end{bmatrix},$$

$$B(\varepsilon) := \begin{bmatrix} B_0 \\ \Pi_\varepsilon^{-1}B_f \end{bmatrix},\ B_p(\varepsilon,\ \mu) := \begin{bmatrix} B_{p0} \\ \Pi_\varepsilon^{-1}\Pi_\mu B_{pf} \end{bmatrix}.$$

Definition 1: We say that the closed-loop system (3) is:

(i) asymptotic stable in mean square (ASMS) if $\lim_{t\to\infty} E[\|\boldsymbol{x}(t)\|^2] = 0$ for any initial conditions $\boldsymbol{x}(0) = \boldsymbol{x}^0 \in \Re^n$.

(ii) exponentially stable in mean square (ESMS) if there exist $\beta \geq 1$, $\alpha > 0$ such that $E[\|\boldsymbol{x}(t)\|^2] \leq \beta \exp^{-\alpha t}\|\boldsymbol{x}^0\|^2$ for all $t \geq 0$, $\boldsymbol{x}^0 \in \Re^n$.

Throughout this paper, $\mathcal{S}_d \in \Re^{d\times d}$ stands for the linear subspace of the real symmetric matrix. Based on the coefficients of the stochastic systems (3), we construct the linear operator $\boldsymbol{\mathcal{L}} : \mathcal{S}_n \to \mathcal{S}_n$, by

$$\begin{aligned} \boldsymbol{\mathcal{L}}(X) &:= [A(\varepsilon) + B(\varepsilon)F]^T X + X[A(\varepsilon) + B(\varepsilon)F] \\ &\quad + \sum_{p=1}^{r}[A_p(\varepsilon,\ \mu) + B_p(\varepsilon,\ \mu)F]^T \\ &\quad \times X[A_p(\varepsilon,\ \mu) + B_p(\varepsilon,\ \mu)F] \end{aligned} \quad (4)$$

for all $X \in \mathcal{S}_n$. The following result shows the role of the linear operators $\boldsymbol{\mathcal{L}}$ in the characterization of the stability in mean square of the stochastic systems of the type (3).

Proposition 1: [5], [6] Suppose that the ratios of the small parameters $\varepsilon_i > 0$ have strict bounds. For the fixed values of the small parameters $\varepsilon_i > 0$, $\mu_i > 0$, the following are equivalent:

(i) the stochastic system (3) is ASMS,

(ii) the stochastic system (3) is ESMS,

(iii) the eigenvalues of the linear operator $\boldsymbol{\mathcal{L}}$ are located in the half plane $\mathbf{C}^- = \{z \in \mathbf{C} \mid \mathrm{Re}(z) < 0\}$,

(iv) the linear equation on $\mathcal{S}_n$;

$$\boldsymbol{\mathcal{L}}(X) + I_n = 0 \quad (5)$$

has a solution $X > 0$,

(v) there exist $Y \in \mathcal{S}_n$, $Y > 0$ satisfying

$$\boldsymbol{\mathcal{L}}(Y) < 0. \quad (6)$$

In the sequel, in order to obtain a control (2) stabilizing the stochastic systems (1) for $\varepsilon_i > 0$, $\mu_i > 0$ sufficiently small, we show how we can construct gain matrices F_0, F_f with appropriate size such that the linear equation (5) has a positive definite solution $X(\varepsilon,\ \mu)$. We look for solution of (5) of the following form.

$$X = \begin{bmatrix} X_{00} & \varepsilon_1 X_{01} & \varepsilon_2 X_{02} \\ \varepsilon_1 X_{01}^T & \varepsilon_1 X_{11} & \varepsilon_2 X_{12} \\ \varepsilon_2 X_{02}^T & \varepsilon_2 X_{12}^T & \varepsilon_2 X_{22} \end{bmatrix} = \begin{bmatrix} X_{00} & X_{0f}\Pi_\varepsilon \\ \Pi_\varepsilon X_{0f}^T & \lambda_\varepsilon(X_f)\Pi_\varepsilon \end{bmatrix}, \quad (7)$$

where

$$X_{ij} \in \Re^{n_i\times n_j},\ i,\ j = 0,\ 1,\ 2,\ i \leq j,$$
$$X_{ii} = X_{ii}^T,\ X_{0f} := \begin{bmatrix} X_{01} & X_{02} \end{bmatrix},$$
$$X_f := \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix},\ \lambda_\varepsilon(X_f) := \begin{bmatrix} X_{11} & X_{12} \\ \dfrac{\varepsilon_2}{\varepsilon_1}X_{12}^T & X_{22} \end{bmatrix}. \quad (8)$$

We remark that $X_f \to \lambda_\varepsilon(X_f) : \mathcal{S}_{n_f} \to \Re^{n_f\times n_f}$ is a linear operator. We have the following relation.

$$\lambda_\varepsilon(X_f)\Pi_\varepsilon = \Pi_\varepsilon \lambda_\varepsilon^T(X_f). \quad (9)$$

By using (7), we have

$$\Pi_\mu \Pi_\varepsilon^{-1}\lambda_\varepsilon(X_f)\Pi_\mu = \begin{bmatrix} \dfrac{\mu_1^2}{\varepsilon_1}X_{11} & \dfrac{\mu_1\mu_2}{\varepsilon_1}X_{12} \\ \dfrac{\mu_1\mu_2}{\varepsilon_1}X_{12}^T & \dfrac{\mu_2^2}{\varepsilon_2}X_{22} \end{bmatrix} \quad (10)$$

for all $X_f = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix} \in \mathcal{S}_{n_f}$. From (8) and (10) one sees that we need to know something about the behavior of the quantities $\varepsilon_2/\varepsilon_1$, μ_i^2/ε_i and $\mu_1\mu_2/\varepsilon_1$ when $\varepsilon \to +0$, $\mu \to +0$, $i = 1,\ 2$.

It must be mentioned that from the mathematical point of view we cannot know a priori what are the values of the limits $\lim_{\substack{\varepsilon_1 \to +0 \\ \varepsilon_2 \to +0}} \frac{\varepsilon_2}{\varepsilon_1}$, $\lim_{\substack{\varepsilon_i \to +0 \\ \mu_i \to +0}} \frac{\mu_i^2}{\varepsilon_i}$. So, to be able to do the asymptotic analysis of the stochastic system (5) when $\varepsilon_i \to +0$, $\mu_i \to +0$, we have to assume that the quantities $\varepsilon_2/\varepsilon_1$, μ_i^2/ε_i, 1, 2 are around of some nominal values $\rho > 0$ and $\rho_1 \geq 0$, $\rho_2 \geq 0$, respectively.

Remark 1: **(a)** In many papers (see for example [17]), one takes $\mu_i = \varepsilon_i^{\delta}$, $\delta > 1/2$. In that case $\rho_i = 0$ if $\delta > 1/2$ or $\rho_i = 1$ if $\delta = 1/2$. In the present paper, the small parameters μ_i are not necessary function of ε_i. The single available information is that $|\mu_i^2/\varepsilon_i - \rho_i|$, $i = 1,\ 2$ are sufficiently small, when $\rho_i \geq 0$ are nominal value determined in the process of modeling.
(b) Without loss of generality, we may assume that $\rho = 1$. Indeed, if $\varepsilon_2/\varepsilon_1 \to \rho > 0$ then $\varepsilon_2/(\rho\varepsilon_1) \to 1$. In this case, we may replace ε_1 by $\rho\varepsilon_1$ if $\rho < 1$ or ε_2 by ε_2/ρ if $\rho > 1$. In this way, we obtain a new singularly perturbed stochastic system for which $\rho = 1$.

In the sequel, we perform the asymptotic analysis of the solutions of the stochastic systems (5) with respect to the vector of parameters

$$\boldsymbol{\nu} = \begin{bmatrix} \varepsilon_1 & \varepsilon_2 & \mu_1 & \mu_2 & \frac{\varepsilon_2}{\varepsilon_1} & \frac{\mu_1^2}{\varepsilon_1} & \frac{\mu_2^2}{\varepsilon_2} \end{bmatrix}.$$

Letting $\boldsymbol{\nu} \to \tilde{\boldsymbol{\nu}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & \rho_1 & \rho_2 \end{bmatrix}$, we obtain:

$$\lim_{\boldsymbol{\nu} \to \tilde{\boldsymbol{\nu}}} \Pi_\mu \Pi_\varepsilon^{-1} \lambda_\varepsilon(X_f) \Pi_\mu = \Gamma(X_f) \tag{11}$$

when $\Gamma\ :\ \mathcal{S}_{n_f} \to \mathcal{S}_{n_f}$ is the linear operator defined by

$$\Gamma(X_f) = \begin{bmatrix} \rho_1 X_{11} & \rho_{12} X_{12} \\ \rho_{12} X_{12}^T & \rho_2 X_{22} \end{bmatrix} \tag{12}$$

for all

$$X_f = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix} \in \mathcal{S}_{n_f}, \tag{13}$$

where $\rho_{12} := \sqrt{\rho_1 \rho_2}$.

Taking $\boldsymbol{\nu} \to \tilde{\boldsymbol{\nu}}$ in (5), we obtain the following system:

$$\begin{aligned}
&(A_{00} + B_0 F_0)^T X_{00} + X_{00}(A_{00} + B_0 F_0) \\
&\quad + (A_{f0} + B_f F_0)^T X_{0f}^T + X_{0f}(A_{f0} + B_f F_0) \\
&\quad + \sum_{p=1}^{r} [(A_{p00} + B_{p0} F_0)^T X_{00} (A_{p00} + B_{p0} F_0) \\
&\quad + (A_{pf0} + B_{pf} F_0)^T \Gamma(X_f)(A_{pf0} + B_{pf} F_0)] \\
&\quad + I_{n_0} = 0, \qquad (14a)\\
&(A_{f0} + B_f F_0)^T X_f + X_{00}(A_{0f} + B_0 F_f) \\
&\quad + X_{0f}(A_f + B_f F_f) \\
&\quad + \sum_{p=1}^{r} [(A_{p00} + B_{p0} F_0)^T X_{00} (A_{p0f} + B_{p0} F_f) \\
&\quad + (A_{pf0} + B_{pf} F_0)^T \Gamma(X_f)(A_{pf} + B_{pf} F_f)] = 0, \qquad (14b)\\
&(A_f + B_f F_f)^T X_f + X_f(A_f + B_f F_f) \\
&\quad + \sum_{p=1}^{r} [(A_{p0f} + B_{p0} F_f)^T X_{00} (A_{p0f} + B_{p0} F_f) \\
&\quad + (A_{pf} + B_{pf} F_f)^T \Gamma(X_f)(A_{pf} + B_{pf} F_f)] \\
&\quad + I_{n_f} = 0. \qquad (14c)
\end{aligned}$$

If A_f and $A_f + B_f F_f$ are invertible matrices, we may introduce the following matrices:

$$\begin{aligned}
&A_{0s} := A_{00} - A_{0f} A_f^{-1} A_{f0},\ B_{0s} := B_0 - A_{0f} A_f^{-1} B_f, \\
&A_{p0s} := A_{p00} - A_{p0f} A_f^{-1} A_{f0},\ B_{p0s} := B_{p0} - A_{p0f} A_f^{-1} B_f, \\
&A_{pfs} := A_{pf0} - A_{pf} A_f^{-1} A_{f0},\ B_{pfs} := B_{pf} - A_{pf} A_f^{-1} B_f, \\
&Q_s = I_{n_0} + (A_{f0} + B_f F_0)^T (A_f + B_f F_f)^{-T} \\
&\qquad \times (A_f + B_f F_f)^{-1} (A_{f0} + B_f F_0). \qquad (15)
\end{aligned}$$

Proposition 2: If A_f is an invertible matrix, then the following statements are true:
(i) If $F_0 \in \Re^{m \times n_0}$ and $F_f \in \Re^{m \times n_f}$ are gain matrices such that $A_f + B_f F_f$ is invertible and if $(X_{00},\ X_{0f},\ X_f) \in \mathcal{S}_{n_0} \oplus \Re^{n_0 \times n_f} \oplus \mathcal{S}_{n_f}$ is a solution of the system (14), then $(X_{00},\ X_f)$ is a solution of the following system of linear equations:

$$\begin{aligned}
&(A_{0s} + B_{0s} F_s)^T X_{00} + X_{00}(A_{0s} + B_{0s} F_s) \\
&\quad + \sum_{p=1}^{r} [(A_{p0s} + B_{p0s} F_s)^T X_{00} (A_{p0s} + B_{p0s} F_s) \\
&\quad + (A_{pfs} + B_{pfs} F_s)^T \Gamma(X_f)(A_{pfs} + B_{pfs} F_s)] \\
&\quad + I_{n_0} = 0, \qquad (16a)\\
&(A_f + B_f F_f)^T X_f + X_f(A_f + B_f F_f) \\
&\quad + \sum_{p=1}^{r} [(A_{p0f} + B_{p0} F_f)^T X_{00} (A_{p0f} + B_{p0} F_f) \\
&\quad + (A_{pf} + B_{pf} F_f)^T \Gamma(X_f)(A_{pf} + B_{pf} F_f)] \\
&\quad + I_{n_f} = 0, \qquad (16b)
\end{aligned}$$

where $F_s = (I_m + F_f A_f^{-1} B_f)^{-1} (F_0 - F_f A_f^{-1} A_{f0})$.
(ii) If $F_s \in \Re^{m \times n_0}$, $F_f \in \Re^{m \times n_f}$ are gain matrices such that $A_f + B_f F_f$ is invertible and if $(X_{00},\ X_f) \in \mathcal{S}_{n_0} \oplus \mathcal{S}_{n_f}$ is a solution of the system (16), then $(X_{00},\ X_{0f},\ X_f)$ is a solution of the system (14) corresponding to

$$F_0 = (I_m + F_f A_f^{-1} B_f) F_s + F_f A_f^{-1} A_{f0} \tag{17}$$

and

$$\begin{aligned}
X_{0f} = -\Big[&(A_{f0} + B_f F_0)^T X_f + X_{00}(A_{0f} + B_0 F_f) \\
&+ \sum_{p=1}^{r} [(A_{p00} + B_{p0} F_0)^T X_{00} (A_{p0f} + B_{p0} F_f) \\
&+ (A_{pf0} + B_{pf} F_0)^T \Gamma(X_f)(A_{pf} + B_{pf} F_f)]\Big] \\
&\times (A_f + B_f F_f)^{-1}. \qquad (18)
\end{aligned}$$

The proof is done by direct calculations. The details are omitted.

For each pair $F = \begin{bmatrix} F_s & F_f \end{bmatrix}$ of gain matrices $F_s \in \Re^{m \times n_0}$, $F_f \in \Re^{m \times n_f}$, we define the linear operator:

$\boldsymbol{L}_F \ : \ \mathcal{S}_{n_0} \times \mathcal{S}_{n_f} \to \mathcal{S}_{n_0} \times \mathcal{S}_{n_f}$ by

$$\begin{aligned}
&\boldsymbol{L}_F(\boldsymbol{X}) = (\boldsymbol{L}_{1F}(\boldsymbol{X}),\ \boldsymbol{L}_{2F}(\boldsymbol{X})),\\
&\quad \boldsymbol{L}_{1F}(\boldsymbol{X})\\
&= (A_{0s} + B_{0s}F_s)^T X_0 + X_0(A_{0s} + B_{0s}F_s)\\
&\quad + \sum_{p=1}^{r} [(A_{p0s} + B_{p0s}F_s)^T X_0 (A_{p0s} + B_{p0s}F_s)\\
&\quad + (A_{pfs} + B_{pfs}F_s)^T \Gamma(X_f)(A_{pfs} + B_{pfs}F_s)], \quad (19a)\\
&\quad \boldsymbol{L}_{2F}(\boldsymbol{X})\\
&= (A_f + B_f F_f)^T X_f + X_f(A_f + B_f F_f)\\
&\quad + \sum_{p=1}^{r} [(A_{p0f} + B_{p0}F_f)^T X_0 (A_{p0f} + B_{p0}F_f)\\
&\quad + (A_{pf} + B_{pf}F_f)^T \Gamma(X_f)(A_{pf} + B_{pf}F_f)], \quad (19b)
\end{aligned}$$

for all $\boldsymbol{X} = (X_0,\ X_f) \in \mathcal{S}_{n_0} \times \mathcal{S}_{n_f}$.

With these notations, the system of type (16) corresponding to the pair $F = \begin{bmatrix} F_s & F_f \end{bmatrix}$ can be written in the compact form:

$$\boldsymbol{L}_F(\boldsymbol{X}) + \boldsymbol{Q} = 0, \quad (20)$$

where $\boldsymbol{Q} = (Q_0,\ I_{n_f}) \in \mathcal{S}_{n_0} \times \mathcal{S}_{n_f}$.

B. Properties of the operators of type $\boldsymbol{L}_F$

Let $\mathcal{X} = \mathcal{S}_{n_0} \times \mathcal{S}_{n_f}$. One sees that $\mathcal{X}$ has a structure of Hilbert space induced by the inner product

$$\langle \boldsymbol{X},\ \boldsymbol{Y} \rangle = \mathbf{Tr}[X_0 Y_0] + \mathbf{Tr}[X_f Y_f], \quad (21)$$

for all $\boldsymbol{X} = (X_0,\ X_f)$, $\boldsymbol{Y} = (Y_0,\ Y_f) \in \mathcal{X}$.

On $\mathcal{X}$, we introduce the order relation induced by the convex cone $\mathcal{X}^+ = \{\boldsymbol{X} = (X_0,\ X_f) \mid X_0 \geq 0,\ X_f \geq 0\}$. Here, $Z \geq 0$ means that Z is a symmetric positive semidefinite matrix.

Lemma 1: The linear operator $\Gamma(\cdot)$ is a positive operator. This mean that $\Gamma(X_f) \geq 0$ if $X_f \geq 0$.

Proof: Let us assume for the beginning that $\rho_2 > 0$. Let $X_f \in \mathcal{S}_{n_f}$ be a positive semidefinite matrix. Without loss of generality, we may assume that X_f has the structure

$$X_f = \begin{bmatrix} X_{11} & \hat{X}_{12} & 0 \\ \hat{X}_{12}^T & \hat{X}_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

with $\hat{X}_{22} > 0$. Using the Schur complement technique, we have that

$$X_f \geq 0 \text{ iff } X_{11} - \hat{X}_{12}\hat{X}_{22}^{-1}\hat{X}_{12}^T \geq 0, \quad (22)$$

$$\Gamma(X_f) \geq 0 \text{ iff } \rho_1 X_{11} - \frac{\rho_{12}^2}{\rho_2}\hat{X}_{12}\hat{X}_{22}^{-1}\hat{X}_{12}^T \geq 0. \quad (23)$$

Using (12) and (22), we obtain $\rho_1 X_{11} - \rho_{12}^2/\rho_2 \hat{X}_{12}\hat{X}_{22}^{-1}\hat{X}_{12}^T = \rho_1(X_{11} - \hat{X}_{12}\hat{X}_{22}^{-1}\hat{X}_{12}^T) \geq 0$. This shows that if (22) holds then (23) is also true.

To end the proof, let us remark that if $\rho_2 = 0$ then

$$\Gamma(X_f) = \begin{bmatrix} \rho_1 X_{11} & 0 \\ 0 & 0 \end{bmatrix} \geq 0$$

because $X_{11} \geq 0$ if $X_f \geq 0$.

Finally, if $\rho_1 = \rho_2 = 0$ then $\Gamma(X_f) = 0$ for all $X_f \in \mathcal{S}_{n_f}$. In this case, the assertion of the lemma is obvious, thus the proof is complete. ∎

Lemma 2: The operator $\Gamma \ : \ \mathcal{S}_{n_f} \to \mathcal{S}_{n_f}$ is a self-adjoint operator with respect to the usual inner product on $\mathcal{S}_{n_f}$:

$$\langle X_f,\ Y_f \rangle = \mathbf{Tr}[X_f Y_f],$$

for all $X_f,\ Y_f \in \mathcal{S}_{n_f}$.

Proof is done by direct calculations based on the definition of the adjoint operator.

We recall that a linear and bounded operator $\mathcal{L} \ : \ \boldsymbol{Y} \to \boldsymbol{Y}$ ($\boldsymbol{Y}$ being a real ordered Banach space) is called resolvent positive operator, if there exists $\lambda_0 \in \Re$ such that for all $\lambda \geq \lambda_0$, the operator $(\lambda I_{\boldsymbol{Y}} - \mathcal{L})^{-1}$ is a positive operator on $\boldsymbol{Y}$.

Useful properties of the resolvent positive operators as well as criteria which guarantee the fact that the spectrum of such operator is in the half plane $\mathbf{C}^-$ may be found in [1], [2].

Proposition 3: The operators of type (19) associated to a pair $F = \begin{bmatrix} F_s & F_f \end{bmatrix} \in \Re^{m \times n_0} \oplus \Re^{m \times n_f}$ have the properties:

(i) the adjoint operator $\mathcal{L}_f^*$ of $\mathcal{L}_f$ with respect to the inner product (21) is given by

$$\mathcal{L}_f^*(\boldsymbol{y}) = (\mathcal{L}_{1f}^*(\boldsymbol{y}),\ \mathcal{L}_{2f}^*(\boldsymbol{y})),$$

where $\mathcal{L}_{1f}^*(\boldsymbol{y}) = (A_{0s} + B_{0s}F_s)Y_{11} + Y_{11}(A_{0s} + B_{0s}F_s)^T + \sum_{p=1}^{r}[(A_{p0s} + B_{p0s}F_s)Y_{11}(A_{p0s} + B_{p0s}F_s)^T + (A_{p0f} + B_{p0f}F_f)Y_f(A_{p0f} + B_{p0f}F_f)^T]$, $\mathcal{L}_{2f}^*(\boldsymbol{y}) = (A_f + B_f F_f)Y_f + Y_f(A_f + B_f F_f)^T + \sum_{p=1}^{r}[\Gamma(A_{pfs} + B_{pfs}F_s)^T Y_{11}(A_{pfs} + B_{pfs}F_s) + \Gamma(A_{pf} + B_{pf}F_f)^T Y_f (A_{pf} + B_{pf}F_f)]$ for all $\boldsymbol{y} = (Y_{11},\ Y_f) \in \mathcal{X}$.

(ii) the operator $\mathcal{L}_f$ is resolvent positive.

Proof:

(i) May be proved by direct calculations starting from the definition of on adjoint operator.

(ii) Proceeding as in the proof of Lemma 8.1 in [4] one shows firstly that

$$\exp[\mathcal{L}_f t](\boldsymbol{X}) \geq 0,\ \forall t \geq 0 \text{ if } \boldsymbol{X} \geq 0.$$

A simple computation results in the fact that $\mathcal{L}_f$ is a resolvent positive operator (see e.g., Proposition 3.2 in [10]). ∎

C. A sufficient condition for the existence of a stabilizing composite control for stochastic systems

Using the notation and the concepts introduced in the previous subsections, we are in a position to state and prove the main results of this paper.

Theorem 1: Assume : a) A_f is an invertible matrix. b) the gain matrices $F_s \in \Re^{m \times n_s}$, $F_f \in \Re^{m \times n_f}$ are designed such that the eigenvalues of the corresponding operator $\mathcal{L}_f$ associated via (19) to the pair $F = \begin{bmatrix} F_s & F_f \end{bmatrix}$ are located in the half plane $\mathbf{C}^-$. We construct the gain matrix $F_0 \in \Re^{m \times n_0}$ as in (17). Under these conditions there exists $\sigma^* > 0$ with the property that the control

$$u(t) = F_0 x_0(t) + F_f x_f(t) \quad (24)$$

stabilizes the stochastic systems (1) for any values of the small parameters $\varepsilon_i,\ \mu_i$ which satisfy

$$\varepsilon_1^2+\varepsilon_2^2+\mu_1^2+\mu_2^2+\left(\frac{\varepsilon_2}{\varepsilon_1}-1\right)^2 + \left(\frac{\mu_1^2}{\varepsilon_1}-\rho_1\right)^2+\left(\frac{\mu_2^2}{\varepsilon_2}-\rho_2\right)^2 \le (\sigma^*)^2. \quad (25)$$

Proof: We show that under the considered assumptions the Lyapunov type equation (5) associated to the closed-loop stochastic systems has a positive definite solution for any $\varepsilon_i > 0,\ \mu_i > 0,\ i = 1,\ 2$, which satisfy a condition of type (25). First, let us remark that if the eigenvalues of the linear operator $\mathcal{L}_F$ are in the half plane $\mathbf{C}^-$, we deduce via Theorem 2.11 in [2] that there exists $\boldsymbol{Y} = (Y_s,\ Y_f) \in \mathcal{S}_{n_0}\times\mathcal{S}_{n_f},\ Y_s > 0,\ Y_f > 0$ which satisfy $\mathcal{L}_F < 0$. From here one obtains

$$(A_f + B_fF_f)^TY_f + Y_f(A_f + B_fF_f) < 0,\ Y_f > 0.$$

So, we may conclude that $A_f + B_fF_f$ is a Hurwitz matrix, which means that it is an invertible matrix. Hence, under the considered assumptions the matrix Q_f is well defined via (15). Further, applying for example Theorem 4.5 in [4], we obtain that the equation (22), or equivalently the system (16) has a unique solution $\tilde{\boldsymbol{X}} = (\tilde{X}_0,\ \tilde{X}_f) > 0$. Construct F_0 and $\tilde{X}_{0f}$ via (17)-(18) by using $\tilde{X}_0,\ \tilde{X}_f$ instead of $X_s,\ X_f$.

Applying Proposition 2 **(ii)**, we deduce that $(\tilde{X}_0,\ \tilde{X}_{0f},\ \tilde{X}_f)$ is a solution of the system (19). Moreover, applying Proposition 2 (i), we deduce that $(\tilde{X}_0,\ \tilde{X}_{0f},\ \tilde{X}_f)$ is the unique solution of the system (19). Taking into account that (19) is obtained from (16) for $\boldsymbol{\nu} = \tilde{\boldsymbol{\nu}}$, we deduce via implicit function theorem [9] that there exist $\boldsymbol{\nu}_1$ and the analytic function $\boldsymbol{\nu} \to (X_0(\boldsymbol{\nu}),\ X_{0f}(\boldsymbol{\nu}),\ X_f(\boldsymbol{\nu}))$ defined for $\boldsymbol{\nu}$ in the ball $\|\boldsymbol{\nu}-\tilde{\boldsymbol{\nu}}\| < \boldsymbol{\nu}_1$ which verify the system (10) and

$$\lim_{\boldsymbol{\nu}\to\tilde{\boldsymbol{\nu}}} X_0(\boldsymbol{\nu}) = \tilde{X}_0,\ \lim_{\boldsymbol{\nu}\to\tilde{\boldsymbol{\nu}}} X_{0f}(\boldsymbol{\nu}) = \tilde{X}_{0f},\quad \lim_{\boldsymbol{\nu}\to\tilde{\boldsymbol{\nu}}} X_f(\boldsymbol{\nu}) = \tilde{X}_f. \quad (26)$$

Set $(\tilde{X}_0(\varepsilon,\ \mu),\ \tilde{X}_{0f}(\varepsilon,\ \mu),\ \tilde{X}_f(\varepsilon,\ \mu))$, the restriction of $(X_0(\boldsymbol{\nu}),\ X_{0f}(\boldsymbol{\nu}),\ X_f(\boldsymbol{\nu}))$ for $\boldsymbol{\nu} = \left[\ \varepsilon_1\ \ \varepsilon_2\ \ \mu_1\ \ \mu_2\ \ \varepsilon_2/\varepsilon_1\ \ \mu_1^2/\varepsilon_1\ \ \mu_2^2/\varepsilon_2\ \right]$ which satisfy $\|\boldsymbol{\nu}-\tilde{\boldsymbol{\nu}}\| < \boldsymbol{\nu}_1$ and $\varepsilon_i > 0,\ \mu_i > 0,\ i = 1,\ 2$,

$$\tilde{X}(\varepsilon,\ \mu) = \begin{bmatrix} \tilde{X}_0(\varepsilon,\ \mu) & \tilde{X}_{0f}(\varepsilon,\ \mu)\Pi_\varepsilon \\ \Pi_\varepsilon\tilde{X}_{0f}^T(\varepsilon,\ \mu) & \lambda_\varepsilon\tilde{X}_f(\varepsilon,\ \mu)\Pi_\varepsilon \end{bmatrix}. \quad (27)$$

One verifies by direct calculations that $\tilde{X}(\varepsilon,\ \mu)$ is a solution of the equation (5). It remains to show that $\tilde{X}(\varepsilon,\ \mu) > 0$ for any $\varepsilon_i > 0,\ \mu_i > 0,\ i = 1,\ 2$ sufficiently small. To this and let us remark that (26) allows us to deduce that there exist $\sigma_1 < \mu_1$ such that

$$\bar{X}_0(\boldsymbol{\nu}) \ge \frac{1}{4}\lambda_{\min}(\tilde{X}_0),\ \bar{X}_f(\boldsymbol{\nu}) \ge \frac{1}{4}\lambda_{\min}(\tilde{X}_f), \quad (28)$$

for all which satisfy $\|\boldsymbol{\nu}-\tilde{\boldsymbol{\nu}}\| < \sigma_1$.

Finally, from (8), (27) and (28), we deduce that there exists $0 < \sigma^* \le \sigma_1$ such that

$$X_0(\boldsymbol{\nu}) - X_{0f}(\boldsymbol{\nu})[\lambda_\varepsilon X_f(\boldsymbol{\nu})\Pi_\varepsilon]^{-1}X_{0f}^T(\boldsymbol{\nu}) > 0 \quad (29)$$

for all $\varepsilon_i > 0,\ \mu_i > 0,\ i = 1,\ 2$ which (25). Combining (28) and (29) together with the Schur complement technique, we conclude that $\tilde{X}(\varepsilon,\ \mu) > 0$ for any $\varepsilon_i > 0,\ \mu_i > 0,\ i = 1,\ 2$, which satisfy (25). This complete the proof. ∎

The next result provides the procedure to construct a gain matrix $F = \left[\ F_s\ \ F_f\ \right]$ such that the assumption b) of the previous Theorem should be fulfilled.

Theorem 2: If A_f is an invertible matrix, then the following are equivalent:

(i) there exist the gain matrices $F = \left[\ F_s\ \ F_f\ \right] \in \Re^{m\times n_0}\oplus\Re^{m\times n_f}$ with the property that the corresponding operator $\mathcal{L}_f$ associated via (18) has the eigenvalues in the half plane $\mathbf{C}^-$.

(ii) there exist $Z_0 \in \mathcal{S}_{n_0},\ Z_f \in \mathcal{S}_{n_f},\ V_s \in \Re^{m\times n_s},\ V_f \in \Re^{m\times n_f}$ which solve the following system of linear matrix inequalities (LMIs):

$$\begin{bmatrix} \Psi_{0s}(Z_s,\ V_s) & \Psi_{1s}(\boldsymbol{Z},\ \boldsymbol{V}) & \Psi_{2s}(\boldsymbol{Z},\ \boldsymbol{V}) & \cdots & \Psi_{rs}(\boldsymbol{Z},\ \boldsymbol{V}) \\ \Psi_{1s}^T(\boldsymbol{Z},\ \boldsymbol{V}) & -\boldsymbol{Z} & 0 & \cdots & 0 \\ \Psi_{2s}^T(\boldsymbol{Z},\ \boldsymbol{V}) & 0 & -\boldsymbol{Z} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Psi_{rs}^T(\boldsymbol{Z},\ \boldsymbol{V}) & 0 & 0 & \cdots & -\boldsymbol{Z} \end{bmatrix} < 0, \quad (30)$$

$$\begin{bmatrix} \Psi_{0f}(Z_s,\ V_s) & \Psi_{1f}(\boldsymbol{Z},\ \boldsymbol{V}) & \Psi_{2f}(\boldsymbol{Z},\ \boldsymbol{V}) & \cdots & \Psi_{rf}(\boldsymbol{Z},\ \boldsymbol{V}) \\ \Psi_{1f}^T(\boldsymbol{Z},\ \boldsymbol{V}) & -\boldsymbol{Z} & 0 & \cdots & 0 \\ \Psi_{2f}^T(\boldsymbol{Z},\ \boldsymbol{V}) & 0 & -\boldsymbol{Z} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Psi_{rf}^T(\boldsymbol{Z},\ \boldsymbol{V}) & 0 & 0 & \cdots & -\boldsymbol{Z} \end{bmatrix} < 0, \quad (31)$$

where $\boldsymbol{Z} = \textbf{block diag}(Z_s,\ Z_f),\ \boldsymbol{V} = (V_s,\ V_f)$, $\Psi_{0s}(Z_s,\ V_s) = A_{0s}Z_s + Z_sA_{0s}^T + B_{0s}V_s + V_s^TB_{0s}^T$, $\Psi_{ps}(\boldsymbol{Z},\ \boldsymbol{V}) = \left[\ A_{p0s}Z_s + B_{p0s}V_s\ \ A_{p0f}Z_f + B_{p0}V_f\ \right]$, $\Psi_{0f}(Z_s,\ V_s) = A_fZ_f + Z_fA_f^T + B_fV_f + V_f^TB_f^T$, $\Psi_{pf}(\boldsymbol{Z},\ \boldsymbol{V}) = \textbf{block diag}(\sqrt{\rho_1}I_{n_1}\ \ \sqrt{\rho_2}I_{n_2})[A_{pfs}Z_s + B_{pfs}V_s\ \ A_{pf}Z_f + B_{pf}V_f]$, $\Psi_{ps}(\boldsymbol{Z},\ \boldsymbol{V}) \in \Re^{n_0\times n}$, $\Psi_{pf}(\boldsymbol{Z},\ \boldsymbol{V}) \in \Re^{n_f\times n},\ 1 \le p \le r$.

Moreover, if $(\tilde{Z}_s,\ \tilde{Z}_f,\ \tilde{V}_s,\ \tilde{V}_f)$ is a solution of the systems (30)-(31), then the operator $\mathcal{L}_{\tilde{F}}$ constructed via (19) for $\tilde{F} = \left[\ \tilde{F}_s\ \ \tilde{F}_f\ \right]$, when $\tilde{F}_s = \tilde{V}_s\tilde{Z}_s^{-1},\ \tilde{F}_f = \tilde{V}_f\tilde{Z}_f^{-1}$ has the eigenvalues in the half plane $\mathbf{C}^-$.

Proof: Since $\mathcal{L}_F$ is a resolvent positive operator, we deduce via Theorem 4.4 in [4] that its eigenvalues are in the half plane $\mathbf{C}^-$ iff there exists $\boldsymbol{Y} = (Y_0,\ Y_f) > 0$ such that

$$\mathcal{L}_F^*(\boldsymbol{Y}) < 0. \quad (32)$$

By using (19) together with the Schur complement technique, we deduce that (32) is equivalent with the systems (30)-(31) with $Z_s = Y_0,\ Z_f = Y_f,\ V_s = F_sY_0$ and $V_f = F_fY_f$. So the proof is complete. ∎

Remark 2: Consider the following special form of (1)

$$\begin{aligned} &dx_0(t) \\ &= [A_{00}x_0(t) + A_{01}x_1(t) + A_{02}x_2(t) \\ &\quad + B_{01}u_1(t) + B_{02}u_2(t)]dt \end{aligned}$$

$$+\sum_{p=1}^{r}[A_{p00}x_0(t)+A_{p01}x_1(t)+A_{p02}x_2(t)$$
$$+B_{p01}u_1(t)+B_{p02}u_2(t)]dw_p(t), \quad (33a)$$
$$\varepsilon_i dx_i(t)$$
$$=[A_{i0}x_0(t)+A_{ii}x_i(t)+B_{ii}u_i(t)]dt$$
$$+\mu_i\sum_{p=1}^{r}[A_{pi0}x_0(t)+A_{pii}x_i(t)+B_{pii}u_i(t)]dw_p(t), \quad (33b)$$

where $x_i(t)\in\Re^{n_i},\ i=0,\ 1,\ 2,\ u_i(t)\in\Re^{m_i},\ i=1,\ 2.$

The structure of the desired stabilizing control is of the form:

$$u_i(t)=F_{i0}x_0(t)+F_{ii}x_i(t),\quad i=1,2. \quad (34)$$

If we apply Theorem 1 to obtain a stabilizing control for (34) we shall see that if $\rho_1>0$, $\rho_2>0$, we cannot obtain Z_f with diagonal structure in order to have a feedback gain F_f also with diagonal structure. However, this happens if $\rho_1=0$ or/and $\rho_2=0$.

IV. NUMERICAL EXAMPLE

In order to demonstrate the efficiency of the proposed composite controller, we present results for a simple numerical example. The system matrices are given as follows.

$$\mu_i=\sqrt{\varepsilon_i},\ i=1,\ 2,\ r=1,\ A_{00}=\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix},$$
$$A_{0f}=\begin{bmatrix}0 & 1\\ 0 & 2\end{bmatrix},\ A_{f0}=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\ A_f=\begin{bmatrix}-1 & 1\\ 2 & -0.5\end{bmatrix},$$
$$A_{p00}=\begin{bmatrix}0.01 & 0\\ 0 & 0.02\end{bmatrix},\ A_{p0f}=\begin{bmatrix}0 & -0.01\\ 0 & 0\end{bmatrix},$$
$$A_{pf0}=\begin{bmatrix}0 & 0\\ 0 & -0.01\end{bmatrix},\ A_{pf}=\begin{bmatrix}0.01 & 0\\ 0 & 0.001\end{bmatrix},$$
$$B_0=\begin{bmatrix}0\\ 1\end{bmatrix},\ B_f=\begin{bmatrix}1\\ 1\end{bmatrix},\ B_{p0}=\begin{bmatrix}0\\ 0.01\end{bmatrix},\ B_{pf}=\begin{bmatrix}0.01\\ 0.02\end{bmatrix}.$$

By solving the LMIs (30) and (31), the gains of the composite parameter-independent controller are given as follows.

$$F_0=\begin{bmatrix}-2.7014e+02 & -5.1154e+01\end{bmatrix},$$
$$F_f=\begin{bmatrix}-1.2109e+02 & -1.3594e+02\end{bmatrix}.$$

In order to verify the ASMA of the closed-loop stochastic system, we solve the following stochastic algebraic Lyapunov equation (SALE).

$$PA_c(\varepsilon)+A_c(\varepsilon)P+A_{cp}^T(\varepsilon,\ \mu)PA_{cp}(\varepsilon,\ \mu)+I_4=0, \quad (35)$$

where $A_c(\varepsilon,\ \mu) := A(\varepsilon,\ \mu)+B(\varepsilon,\ \mu)F$ and $A_{cp}(\varepsilon,\ \mu):=A_p(\varepsilon,\ \mu)+B_p(\varepsilon,\ \mu)F$. The small parameters are chosen as $\varepsilon_1=0.001$ and $\varepsilon_1=0.002$. In this case, it is easy to observe that the SALE (35) has the positive definite solution. In fact, the eigenvalues of P is $\lambda(P)=\left\{1.0259,\quad 2.4071e-01,\quad 3.7248e-04,\quad 6.4445e-06\right\}$ Therefore, since the SALE (35) has the positive definite solution, the composite controller $u(t)=Fx(t)=\begin{bmatrix}F_0 & F_f\end{bmatrix}$ attains the ASMA.

V. CONCLUSION

In this paper, the composite stabilizing control problem for a class of stochastic controlled linear systems modeled by systems of multiparameter singularly perturbed Itô differential equations were considered. The asymptotic structure of the solution of the algebraic Lyapunov equations of stochastic control associated to this problem has been established for the first time under the assumption that the small perturbation parameters had same order magnitude. Necessary and sufficient conditions which guarantee the existence of stabilizing feedback gains not depending upon the small parameters ε_i and μ_i have been expressed in terms of solvability of some linear matrix inequalities (Theorem 2). Finally, the numerical example has shown the validity of the proposed method.

The interest in stability is essentially an interest in robustness of the model with respect to small changes. There is no physical process such as white noise, so one needs some sort of approach that could deal with a wide-band noise model that can be suitably approximated by an Itô equation. This issue will be addressed in the future work.

REFERENCES

[1] T. Damm, *Rational Matrix equations in Stochastic Control*, Berlin Heidelberg: Springer-Verlag, 2004.

[2] T. Damm and D. Hinrichsen, (2003), Newton's method for concave operators with resolvent positive derivatives in ordered Banach spaces, *Linear Algebra and its Applications*, 363, 43-64.

[3] V. Dragan and A. Halanay, *Stabilization of Linear Systems*, Boston: Birkhauser, 1999.

[4] V. Dragan, T. Damm, G. Freiling and T. Morozan, Differential Equations with Positive Evolutions and Some Applications, *Results in Mathematics*, vol.48, 2005, pp 206-236.

[5] V. Dragan, T. Morozan and A.M. Stoica, *Mathematical Methods in Robust Control of Linear Stochastic Systems*, New York: Springer-Verlag, 2006.

[6] R.Z. Khasminskii, *Stochastic Stability of Differential Equations*, Alphen aan den Rijn: Sijthoff and Noordhoff, 1980

[7] P.V. Kokotovic, H.K. Khalil, J. O'Reilly, *Singular Perturbations Methods in Control: Analysis and Design, (Classics in Applied Mathematics)*, Philadelphia: SIAM, 1999.

[8] D.S. Naidu , *Singular Perturbation Methodology in Control Systems*, Stevenage Herts: IEE and Peter Peregrinus, 1988.

[9] L. Schwartz, *Analyse Mathematique, Vol.1*, Paris: Hermann, 1967.

[10] V. Dragan, Stabilizing Composite Control for a Class of Linear Systems Modeled by Singularly Perturbed Itô Differential Equations, *Automatica*, vol.47, no.1, 2011, pp 122-26.

[11] A. Friedman, *Stochastic Differential Equations and Applications, Vol.1*, New York: Academic Press, 1975.

[12] B. Oxendal, *Stochastic Differential Equations*, Berlin Heidelberg: Springer-Verlag, 1998.

[13] H.K. Khalil and P.V. Kokotović, Control Strategies for Decision Makers Using Different Models of the Same System, *IEEE Trans. Automatic Control*, vol. 23, no. 2, 1979, pp 289-298.

[14] V. Dragan, T. Morozan and P. Shi, Asymptotic Properties of Input-Output Operators Norm Associated with Singularly Perturbed Systems with Multiplicative White Noise, *SIAM J. Control and Optimization*, vol. 41, no. 1, 2002, pp 141-163.

[15] V. Dragan, The Linear Quadratic Optimization Problem for a Class of Singularly Perturbed Stochastic Systems, *Int. J. Innovative Computing, Information and Control*, vol. 1, no. 1, 2005, pp 53-64.

[16] H. Mukaidani, Local Uniqueness for Nash Solutions of Multiparameter Singularly Perturbed Systems, *IEEE Trans. Circuits & Systems II*, vol. 53, no. 10, 2006, pp 1103-1107.

[17] M. Sagara, H. Mukaidani and V. Dragan, Near-Optimal Control for Multiparameter Singularly Perturbed Stochastic Systems, *Optimal Control, Applications and Methods*, vol.32, no.1, 2011, pp 113-125.