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A nonlinear active noise control scheme with on-line model structure selection

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Abstract—**In nonlinear active noise control (ANC) applications the on-line tuning of the parameters of the nonlinear control filter is not sufficient to guarantee the required model accuracy, and a suitable model structure adaptation scheme must be included. However, the ANC setting configures an indirect model identification problem which does not give direct access to the target output signal for the filter model. As such, the filter adaptation problem cannot be solved with the linear regression tools usually employed for model selection. A modified ANC scheme is here proposed, where the controller adaptation loop is reconfigured as a direct identification problem, to allow for model selection, and an auxiliary adaptation loop is introduced to compensate for the error resulting from the scheme modification. Some simulation examples are reported to show the algorithm effectiveness.**

I. Introduction

ACTIVE Noise Control (ANC) methods address the problem of acoustic noise reduction through the generation of secondary acoustic signals designed to interfere negatively with the noise, [1]–[2]. Typically, linear adaptive filters are employed to control the secondary sources, the parameters of which are tuned on-line with a Least Mean Squares (LMS) type algorithm. A reference signal (acoustic or not, depending on the application), highly correlated with the noise, is used as input to the control filter, and the cancellation performance is evaluated by measuring through a microphone the sound resulting from the combined application of the noise and the secondary signals.

Several extensions to the nonlinear case have been proposed in the literature to deal with nonlinearities related to the characteristics of both the noise [3] and the involved acoustic paths [4]. For example, devices such as microphones, amplifiers, loudspeakers and converters commonly suffer from distortion and saturation problems. The related ANC methods and schemes are collectively denoted NANC (Nonlinear ANC).

Various classes of nonlinear models have been employed in NANC methods, such as truncated Volterra expansions [5]–[7], radial basis functions [5], multi–layer neural networks [8]–[9], functional link artificial neural networks with trigonometric functional expansions, [7], [10]–[11], or piecewise linear functional expansions [12], adaptive bilinear filters [13], general function expansion nonlinear filters [14], polynomial nonlinear autoregressive models with exogenous variables (NARX) [15].

Independently of the adopted model class, the size of the model is of great concern for ANC applications, since the computational complexity of the filter adaptation algorithm scales with the number of model parameters. This discourages the use of non-recursive models of the NFIR type (Nonlinear Finite Impulse Response), such as Volterra expansions, or functional link artificial neural networks, that typically require a large number of parameters to achieve the necessary model accuracy.

Besides, it is well known from the model identification literature that all sorts of undesired problems may arise when a nonlinear system is identified using an incorrect model structure, such as overfitting, parameter fluctuation, poor model generalization capabilities and even model instability (see, *e.g.*, [16]–[17]). The general class of recursive NARX models has been here employed, since it guarantees sufficient flexibility (it encompasses both Volterra and bilinear filters), and – which is more important – comes also equipped with model structure selection techniques [18], which can be employed to carefully tune the model structure on-line.

These techniques are essentially based on efficient orthogonalization techniques that require a linear regression formulation of the identification problem. Unfortunately, the standard ANC setting configures an indirect identification problem that does not generally yield a linear regression in the filter parameters even if the control filter has a linear-in-the-parameters structure. In view of this, an alternative NANC scheme is here introduced where the control filter adaptation is reconfigured in a direct identification fashion. This scheme rearrangement is essentially equivalent to commuting the control filter block with a suitably modified secondary acoustic path system. The latter system is adapted through an auxiliary adaptation loop, seeking to minimize the commutation error, hence the name Dual Filtering LMS (DFLMS). The novel NANC scheme allows for simultaneous on-line parameter estimation and model structure tuning. Some simulations are reported to demonstrate the algorithm effectiveness in harshly time-varying conditions.

II. Active noise control

The basic feedforward ANC setting is represented in Fig. 1. The offending noise signal $d(k)$ is modeled as a filtered version of the (measurable) reference signal $x(k)$ through an un-

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known primary acoustic path P. The same reference signal feeds the control filter C to produce the driving signal $y(k)$ for the secondary acoustic source, ideally designed so that the resulting signal $y'(k)$ at the error measurement location cancels the primary noise. Signal $y'(k)$ is obtained by filtering $y(k)$ through the secondary path S, which accounts for the measurement and control chain, as well as the acoustic path from the secondary source to the error microphone.

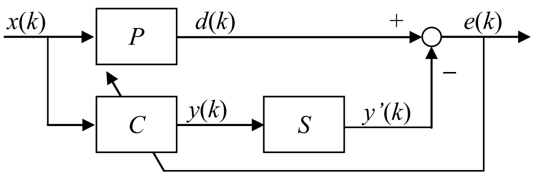

Fig. 1. Block diagram of an adaptive feedforward ANC system.

The controller is implemented as an adaptive parametric filter, whose coefficients (or weights) are tuned by means of an adaptive algorithm of the LMS family to minimize the error $e(k) = d(k) - y'(k)$.

The ANC scheme of Fig. 1 configures an *indirect* identification problem, where one is interested in the estimation of C, but direct access to the target outputs of C is not available, since S is generally non-invertible. In the linear framework, this issue is inessential, since blocks C and S can be exchanged (at least in the slow adaptation hypothesis, where C can be assumed almost time invariant). This results in the *direct* adaptation scheme of Fig. 2, where direct access to the target output of C is possible, through the error measurement.

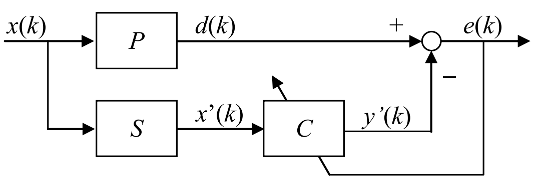

Fig. 2. ANC scheme with S and C commuted.

As a result, C can be adapted, *e.g.* with a LMS weight update rule, *as if* it had the filtered reference signal $x'(k)$ (obtained by filtering $x(k)$ through an estimated version of the secondary path dynamics, $\hat{S}$) as input and $y'(k)$ as output. This is the rationale behind the well known Filtered-x LMS (FXLMS) and Filtered-u LMS (FULMS) algorithms, as depicted in Fig. 3. In the latter case, since an Infinite Impulse Response (IIR) structure is assumed for the control filter, the filtering applies not only to the reference signal but also to the control variable $y(k)$.

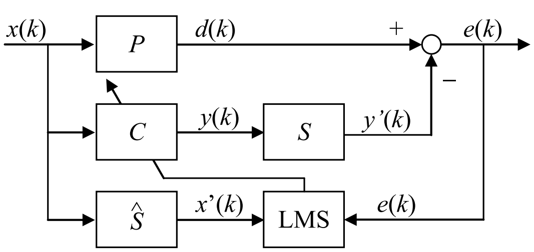

Fig. 3. The FXLMS paradigm.

In the nonlinear case, block commutation is not possible, and the FXLMS/FULMS paradigm can be applied only in very specific structural assumptions on both C and S. For example, in the Volterra FXLMS (VFXLMS) [6], the Filtered-S LMS (FSLMS) [10]–[11] and the Bilinear FXLMS (BFXLMS) [13] schemes S is assumed linear and C has either an NFIR (non-recursive) or output affine structure. The more general case where S is nonlinear is examined in [14] and [15], where the indirect identification problem is solved by accounting for the gradient of S in the weight update mechanism, as in the Nonlinear Filtered Gradient LMS (NFGLMS) developed in [15] for NARX models.

III. Model Structure Selection Methods

A. Polynomial NARX models

The class of NARX models is frequently used in black-box non-linear model identification applications in view of its representation capabilities and the flexibility of the model structure [19]. Under mild assumptions, a discrete-time nonlinear system can be represented with a recursive input–output model of the type:

$$y(k) = f(y(k-1), ..., y(k-L), x(k), ..., x(k-L)), \quad (1)$$

where $f(\cdot)$ is a generic nonlinear function, $y(k)$ and $x(k)$ are the output and input signals, respectively, and L is the maximum lag (assumed equal for the input and output signals, for simplicity). Model (1) is the deterministic version of the NARX model. In Equation (1), a polynomial expansion is commonly adopted to represent the nonlinearity $f(\cdot)$, resulting in a linear regression:

$$y(k) = \boldsymbol{f}(\boldsymbol{u}(k))^T \boldsymbol{w}, \quad (2)$$

where $\boldsymbol{f}(\boldsymbol{u}(k)) = [f_1(\boldsymbol{u}(k)) \ f_2(\boldsymbol{u}(k)) \ ... \ f_n(\boldsymbol{u}(k))]^T$ is the regressor vector, $f_j(\boldsymbol{u}(k))$ being a monomial in the arguments $\boldsymbol{u}(k) = [y(k-1) \ ... \ y(k-L) \ x(k) \ ... \ x(k-L)]$, and $\boldsymbol{w} = [w_1 \ w_2 \ ... \ w_n]^T$ is the parameter vector (vector quantities are indicated with boldface lower case letters).

The full polynomial expansion is seldom employed, since it typically yields largely over-parameterized models and a model structure selection procedure is generally adopted to find the appropriate model regressors.

B. On-line model structure selection

The identification of a polynomial NARX model is a complex problem combining parameter estimation with model structure selection. The model building process typically alternates forward and backward regression steps, usually in combination with Orthogonal Least Squares (OLS) techniques to decouple the estimation of the parameters associated with different regressors [20]. Both batch and on-line algorithms have been developed. Notable examples of the first category are the Forward-Regression Orthogonal Estimator (FROE) [21] and the Simulation Error Minimization with Pruning (SEMP) approach [17].

The Givens rotation with Forward selection and Exponential windowing (GFEX) [22], and Givens rotation with Forward selection and SLiding windowing (GFSL) [23] algorithms extend the OLS regression method to the adaptive case. Both algorithms employ an efficient data updating mechanism based on a recursive implementation of the *QR* decomposition of the regressor matrix (with different windowing and forgetting factor options). Model selection is operated by swapping regressor positions in the orthogonalized regressor matrix. This requires the use of several matrix operations to preserve the orthogonality of the regressor matrix when the model structure is changed. An enhanced version of the GFSL has been recently introduced in [18], with a more articulate selection policy, including both forward and backward regression operations. The resulting algorithm, denoted Recursive Forward Regression with Pruning (RFRP) is here employed.

Briefly, when a new datum arrives the current model is re-estimated and evaluated. If the accuracy has fallen below a given threshold, a model structure modification is triggered. This consists in one or more model addition/pruning iterations until the desired accuracy level is recovered. At each of these iterations the RFRP evaluates for possible inclusion all the regressors left out of the current model, based on the Error Reduction Ratio (ERR) criterion [21], which basically measures the marginal model improvement obtained by adding individually each regressor. The regressor with the highest ERR is then added to the model. Subsequently, it prunes redundant terms, as long as the combined addition/ elimination of terms improves the model accuracy. The main tuning knobs of the RFRP are the dimension W of the data window and the accuracy thresholds for the overall model (J_{thres}) and the model increment (ERR_{thres}), respectively.

From an algorithmic point of view, the key points of the GFEX, the GFSL and RFRP are the recursive updating of the *QR* decomposition of the regressor matrix operated when new data are available, and the retriangularization of the R matrix every time the model structure is changed. The interested reader is addressed to [18] for details on the implementation of the FPRP algorithm.

IV. DUAL ADAPTATION ANC SCHEME

The mentioned model selection techniques are all designed for a direct identification problem, formulated as a linear regression. As already discussed, the standard ANC scheme with nonlinear secondary path is instead an indirect identification problem, and does not result in a linear regression in the control filter parameters. In the following, an alternative ANC scheme is illustrated that addresses the control filter adaptation in a direct identification mode, at the cost of introducing a second adaptation loop.

For ease of notation denote as

$$y(k) = [C](x(k)), \tag{3}$$

$$y'(k) = [S](y(k)), \tag{4}$$

the nonlinear control filter and secondary path. Now, in the standard ANC scheme, one obtains:

$$y'(k) = [S](y(k)) = [S]([C](x(k))) = [S\circ C](x(k)), \tag{5}$$

where '∘' denotes the nonlinear series block composition operator. Clearly, in general

$$y'(k) = [S\circ C](x(k)) \neq [C\circ S](x(k)),$$

even if both nonlinear systems are assumed time-invariant.

However, there may exist a nonlinear system $\widetilde{S}$, generally different from S, such that:

$$y'(k) = [C\circ \widetilde{S}](x(k)). \tag{6}$$

In that case, the resulting ANC scheme would be similar to that of Fig. 2, with $\widetilde{S}$ in place of the true secondary path S, and a direct identification scheme could be applied for the identification of C, using the filtered reference signal

$$\widetilde{x}(k) = [\widetilde{S}](x(k)) \tag{7}$$

as input to C and the measured signal $e(k)$ as modeling error.

More precisely, using the stochastic gradient approach [1], the parameter update equation is derived as follows:

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) - \frac{\mu}{2}\frac{\partial e(k)^2}{\partial \boldsymbol{w}(k)}^T = \boldsymbol{w}(k) - \mu \frac{\partial e(k)}{\partial \boldsymbol{w}(k)}^T e(k) = $$
$$= \boldsymbol{w}(k) + \mu \frac{\partial y'(k)}{\partial \boldsymbol{w}(k)}^T e(k), \tag{8}$$

where $\boldsymbol{w}(k)$ is the parameter vector of linear regression (2) estimated at the kth iteration. Now, $y'(k)$ can be approximately expressed as follows:

$$y'(k) \cong \widetilde{y}(k) = [C\circ\widetilde{S}](x(k)) = [C](\widetilde{x}(k)) = \boldsymbol{w}(k)^T \boldsymbol{f}(\widetilde{\boldsymbol{u}}(k))$$

with $\boldsymbol{f}(\widetilde{\boldsymbol{u}}(k)) = [f_1(\widetilde{\boldsymbol{u}}(k))\ f_2(\widetilde{\boldsymbol{u}}(k))\ \ldots\ f_n(\widetilde{\boldsymbol{u}}(k))]^T$, $f_j(\widetilde{\boldsymbol{u}}(k))$ being a monomial in the arguments $\widetilde{\boldsymbol{u}}(k) = [y'(k-1)\ \ldots\ y'(k-L)\ \widetilde{x}(k)\ \ldots\ \widetilde{x}(k-L)]$. The resulting weight update law is then simply:

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) + \mu\, \boldsymbol{f}(\widetilde{\boldsymbol{u}}(k))e(k), \tag{9}$$

where μ is the step size. Notice that Equation (9) is computationally equivalent to the FXLMS/FULMS rule.

The existence of a system $\widetilde{S}$ such that the swapping relation (6) holds exactly is conditioned to the exact invertibility of C and is therefore not guaranteed in general. It is however arguable that an exact representation of $\widetilde{S}$ is actually necessary. In the linear framework the system $\hat{S}$ (see Fig. 3) used to pre-filter the reference signal in the FXLMS is only required to be within a ±90° phase range from S at every frequency, for algorithm stability. Although a similar condition has not been proven in the nonlinear domain, the simulation examples shown in the sequel prove that the cancellation performance can be excellent even with a non neglectable commutation error, defined as

$$\widetilde{e}(k) = [S\circ C](x(k)) - [C\circ\widetilde{S}](x(k)). \tag{10}$$

Accordingly, an estimation procedure has been set up for the estimation of $\widetilde{S}$, with the aim of minimizing the commutation error $\widetilde{e}(k)$. Observe that the estimation of $\widetilde{S}$

configures an indirect identification problem where the (nonlinear) control filter C has the role of the secondary path, but, given the much less stringent accuracy requirements on $\widetilde{S}$ compared to C, a fixed structure can be assumed for $\widetilde{S}$, and an indirect algorithm such as the NFGLMS [15] can be employed for its estimation, given the current controller C.

More precisely, representing $\widetilde{S}$ also as a NARX model:

$$\begin{aligned}\tilde{x}(k) &= g(\tilde{x}(k-1), \ldots, \tilde{x}(k-M), x(k), \ldots, x(k-M)) = \\ &= \boldsymbol{g}(\boldsymbol{z}(k))^T \boldsymbol{v}(k) \\ &= [g_1(\boldsymbol{z}(k)) \;\; g_2(\boldsymbol{z}(k)) \ldots g_m(\boldsymbol{z}(k))]^T \boldsymbol{v}(k), \qquad (11)\end{aligned}$$

where $\boldsymbol{z}(k) = [\tilde{x}(k-1) \ldots \tilde{x}(k-M)\; x(k) \ldots x(k-M)]$, $\boldsymbol{v}(k)$ is a vector of coefficients, and $g_j(\boldsymbol{z}(k))$ is a monomial in the arguments $\boldsymbol{z}(k)$, the NFGLMS weight update rule is given by:

$$\boldsymbol{v}(k+1) = \boldsymbol{v}(k) + \eta \frac{\partial \tilde{y}(k)}{\partial \boldsymbol{v}(k)}^T \tilde{e}(k), \qquad (12)$$

where η is the step size. The derivative in Eq. (12) is computed using the chain rule as:

$$\frac{\partial \tilde{y}(k)}{\partial \boldsymbol{v}(k)} = \boldsymbol{w}(k)^T \frac{\partial \boldsymbol{f}(\tilde{\boldsymbol{u}}(k))}{\partial \tilde{\boldsymbol{u}}(k)} \frac{\partial \tilde{\boldsymbol{u}}(k)}{\partial \boldsymbol{v}(k)}, \qquad (13)$$

using the gradient of C, where $\tilde{\boldsymbol{u}}(k) = [\tilde{y}(k-1) \ldots \tilde{y}(k-L)\; \tilde{x}(k) \ldots \tilde{x}(k-L)]$. Efficient rules for computing the recursive expression (13) are explained in [15].

Note that both weight update equations (9) and (12) are computed on the basis of pre-filtered signals. In (9) the filtering function is $\widetilde{S}$, whereas the filtered gradient of C is used in (12). Accordingly, the overall algorithm is named Dual Filtering LMS (DFLMS).

The proposed strategy is schematically depicted in Fig. 4, where the dual adaptation loops are highlighted in color.

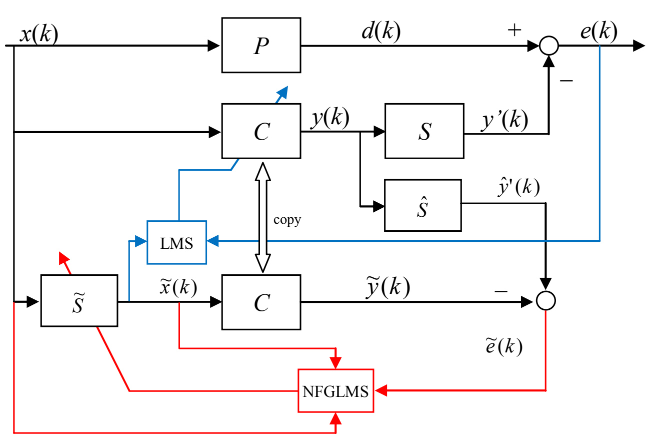

Fig. 4. DFLMS scheme for NANC using NARX filters: dual adaptation of $\widetilde{S}$ (red) and C (blue).

Notice that a model of the secondary path is required for the computation of the auxiliary error $\tilde{e}(k)$, while a copy of the current control filter is used in the auxiliary adaptation loop. In summary, the algorithm consists of the following main steps performed at each sample time:

1) Computation of the controller output $y(k)$;
2) Updating of the weights of the auxiliary model $\widetilde{S}$;
3) Filtering of the reference signal $x(k)$ through the new $\widetilde{S}$;
4) Updating of the weights of the control filter C.

In order to reduce the impact of the auxiliary adaptation loop on the overall scheme in terms of computational load, the related weight updating process can be suitably downsampled, thus distributing the related computations over several consecutive samples. Notice also that the two adaptation laws can be parallelized for increased efficiency.

The main achievement of the DFLMS scheme consists in having reformulated the adaptation problem of the control filter C as a direct identification problem, using the approximate block commutation scheme by means of the auxiliary system $\widetilde{S}$. In this scheme, the identification of C amounts to a linear regression problem (the NARX model is linear-in-the-parameters), where the input to the model is $\tilde{x}(k)$ and the model error is directly represented by the measured signal $e(k)$. This allows the direct implementation of an on-line model structure selection algorithm, as *e.g.* the mentioned RFRP, alongside the parameter updating rules. The resulting algorithm is denoted RFRP-DFLMS. Notice that the model selection rule can also be downsampled with respect to the control filter's parameter adaptation loop, for containment of the computational load.

V. Simulation Experiments

A. Example 1

The first example aims at a performance evaluation of the DFLMS scheme against the NFGLMS and the FXLMS in terms of disturbance rejection and speed of convergence. A simple time-invariant setting has been studied, where the primary path is modeled as a 2nd order NARX model:

$$d(k) = 0.5d(k-1) - 0.3x(k-2) + 0.25x(k-1)^2 x(k-2), \qquad (14)$$

and the secondary path is modeled as a non-minimum phase 4th order FIR system, as in [6]:

$$y'(k) = y(k-2) + 1.5y(k-3) - y(k-4), \qquad (15)$$

A 500 Hz sinusoidal wave (sampled at 8 kHz) has been used as reference signal:

$$x(k) = \sqrt{2}\,\sin\!\left(\frac{2\pi\cdot 500\cdot k}{8000}\right) + v(k), \qquad (16)$$

where an additive white noise process $v(k)$ with Gaussian distribution has also been assumed. The variance of $v(k)$ is chosen such that the power signal-to-noise ratio (SNR) is equal to 40 dB.

A fixed NARX structure has been assumed for the control filter, *i.e.* a full 3rd order polynomial expansion with a maximum lag of 3, for a total of 119 parameters. The DFLMS scheme has been analyzed first with no adaptation of $\widetilde{S}$ (algorithm denoted NFXLMS), and then with the auxiliary adaptation on, using two different structures for $\widetilde{S}$, *i.e.* an 8-

tap FIR filter and a 2nd order nonlinear FIR (NFIR) model with maximum lag 8, respectively. In all cases, the auxiliary model $\tilde{S}$ is initialized to S.

The NANC schemes are also compared to the standard linear FXLMS, where the order of the FIR controller has been chosen equal to the number of parameters of the NARX controller, for fairness of comparison. For all the schemes the adaptation gains have been maximized, so as to guarantee the best convergence speed without compromising the stability of the algorithms (see Table I). Moreover, all the control filter weights are initially set to zero.

TABLE I
PARAMETERS OF THE NANC ALGORITHMS FOR SETTING 1

Method	Controller adaption gain	Auxiliary model adaption gain
FXLMS	$1\cdot10^{-3}$	-
NFGLMS	$5\cdot10^{-4}$	-
NFXLMS	$5\cdot10^{-5}$	0
DFLMS FIR	$1\cdot10^{-4}$	$1\cdot10^{-2}$
DFLMS NFIR	$1\cdot10^{-4}$	$1\cdot10^{-2}$

Fig. 5 displays the algorithms' performance in terms of the Normalized Mean Square Error (NMSE), expressed as $NMSE = 10\log_{10}(E[e(k)^2]/\sigma_d^2)$, where σ_d^2 represents the power of the disturbance noise [6]. The NMSE has been computed on a window of 1000 data. The FXLMS achieves a mere −3.8 dB noise attenuation, while the NFGLMS obtains the best results (–27.2 dB). The DFLMS variants provide an intermediate performance level, that approaches the steady-state attenuation of the NFGLMS when the on-line adaptation of $\tilde{S}$ is active.

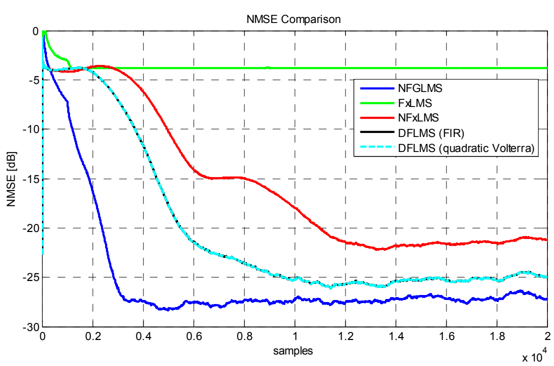

Fig. 5. Comparison of different NANC strategies for setting 1: NFGLMS (blue), FXLMS (green), NFXLMS (red), DFLMS with a FIR auxiliary model (black), DFLMS with a quadratic Volterra filter as auxiliary model (cyan dashed line).

Figure 6 illustrates the performance of the auxiliary adaptation scheme, designed to reduce the commutation error (compare the DFLMS variants with the NFXLMS), significantly improving both the speed of convergence and the canceling performance (–25 dB).

B. Example 2

A time-varying ANC setting has been examined next, to test the ability of the RFRP-DFLMS to track system variations. In particular, the secondary path is initially described by Eq. (15) but at sample $\bar{k}$ = 10000 changes to:

$$y'(k) = 0.7756y(k) + 0.5171y(k-1) - 0.362y(k-2), \quad (17)$$

The primary noise is generated by the following 3rd order polynomial model [6]:

$$d(k) = w(k-2) + 0.02w(k-2)^2 - 0.04w(k-2)^3, \quad (18)$$

$$w(k) = x(k-3) - 0.3\,x(k-4) + 0.2\,x(k-5), \quad (19)$$

where the reference noise $x(k)$ is a band-limited white noise in the (normalized) frequency interval [0.01, 0.1], with unitary variance.

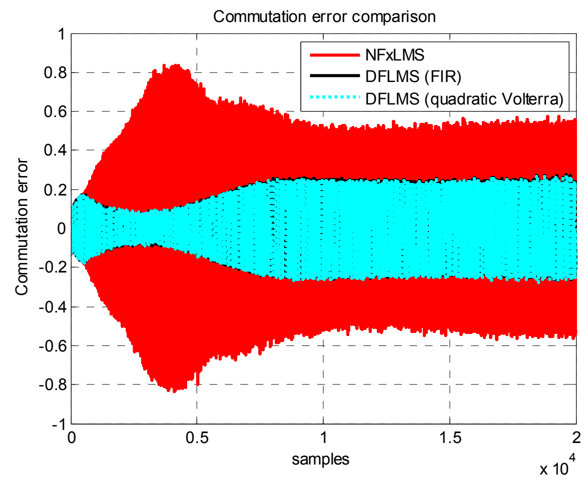

Fig. 6. Time-history of the commutation error in setting 1: NFXLMS (red), DFLMS with a FIR auxiliary model as $\tilde{S}$ (black), DFLMS with a quadratic Volterra filter as auxiliary model (cyan dashed line).

The RFRP-DFLMS has been compared to the NFGLMS and the plain DFLMS. A 2nd order polynomial NARX model with a maximum lag of 10 has been assumed for the latter two algorithms. The RFRP-DFLMS operates the model selection over the corresponding set of candidate regressors, starting from a FIR filter, for simplicity. The initial control filter parameters are set to zero and the adaptation gains are selected to guarantee the best trade-off between performance and stability (see Table II).

TABLE II
PARAMETERS OF THE NANC ALGORITHMS FOR SETTING 2

Method	Controller adaption gain	Filtering model adaption gain	Model selection parameters
NFGLMS	$5\cdot10^{-4}$	-	-
DFLMS	$1\cdot10^{-4}$	$1\cdot10^{-2}$	-
RFRP-DFLMS	$1\cdot10^{-3}$	$1\cdot10^{-1}$	$W = 600$ $J_{thres} = 1\cdot10^{-6}$ $ERR_{thres} = 5\cdot10^{-5}$

The performance results of the considered algorithms in terms of the NMSE are plotted in Fig. 7. After the structural modification of the secondary path, the RFRP-DFLMS reacts much more rapidly and efficiently than both the other algorithms, achieving in only 100 samples the level of rejection performance reached by the NFGLMS after 20000 iterations (the plain DFLMS is even slower).

Notice that the RFRP-DFLMS selects models of less than 10 terms over a set of 252 candidate regressors (see Fig. 8), which corresponds to a reduction of the computational time related to the controller weights updating by a factor of 25.

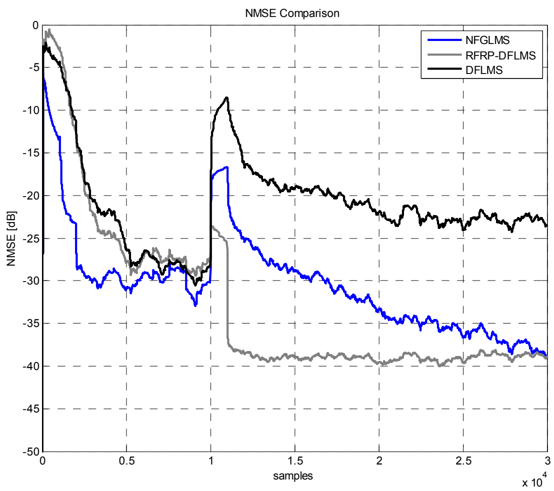

Fig. 7. NMSE results for setting 2: NFGLMS (blue), DFLMS (black) and RFRP-DFLMS (grey)

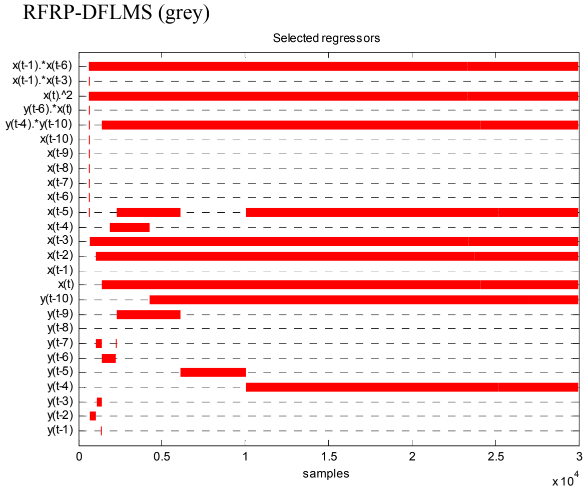

Fig. 8. Regressors selected by the RFRP algorithm in experiment 2. Only regressors that have been selected at least once are shown.

VI. Conclusions

A novel NANC scheme for NARX models has been proposed that reformulates the control filter adaptation as a direct identification problem, thanks to an auxiliary adaptation loop that estimates the correct filtering system. Both the main and the auxiliary adaptation loops operate with weight update rules based on the use of suitable pre-filtered signals. An on-line model structure selection procedure, the RFRP, is then combined with the control filter parameter tuning mechanism for enhanced accuracy in time-varying conditions.

The proposed method has been tested on both time invariant and time-varying nonlinear settings. The slight performance loss of the proposed DFLMS scheme in terms of steady state noise reduction compared to the NFGLMS algorithm is more than compensated by the performance gains achievable using the RFRP in time-varying conditions. The RFRP-DFLMS scheme is also very efficient in reducing the model size, thereby cutting to a minimum the computational load of the primary – time critical – adaptation loop.

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