# **Time Delayed Non-minimum Phase Slave Tele-Robotics**

S.F. Atashzar, H.A. Talebi, M.Shahbazi, F. Towhidkhah, R.V. Patel

Abstract— Challenges created by non-minimum phase slave robots in time delayed telerobotics systems is the main focus of this paper. This new category of telerobotics systems has many practical applications such as minimally invasive time delayed tele-surgeries. The non-minimum phase (NMP) telemanipulation creates many challenges especially in presence of communication time delay. In this article, a thorough analysis on the impacts of NMP slaves on the stability and transparency of the time delayed telerobotics is given. Additionally, a controlarchitecture is proposed for this category of teleoperation. The transparency and stability of the proposed controller are mathematically analyzed. This architecture called the Pseudo Three-Channel (PTC). Experimental results are performed to illustrate the effectiveness of the proposed control structure.

### I. INTRODUCTION

relerobotics have received a great deal of interest during L the last decade, since it creates remote presentation. Many control issues have been investigated in this area. Such as: the effects of time varying [1] or constant timedelays [2], perfect transparency [3], stability in hard/soft contact [4], multi master-multi slave structures [5], etc. In order to deal with the aforementioned challenges, many control architecture are established such as: Two, Three and Four-Channel architectures [6], passivity based controllers and wave variables [7], robust, adaptive and nonlinear controllers [8] and etc. As a brilliant result, telerobotics systems are currently utilized in many practical applications where the manipulation is extremely precise, hazardous or not-reachable for human [7]. One of the most attractive applications of such systems is "Telesurgery". In fact, utilizing tele-surgical robots, the performance and precision of surgery is considerably enhanced due to scaling ability, vibration alleviation feature, direct-contact free and sterilized manipulations [9],[10]. A fascinating application of tele-surgical robots is Minimally Invasive Surgery (MIS), where miniaturized manipulators are utilized as slave robots to minimized the invasion and maximize likelihood of success in surgeries [11],[12]. Towards this end, thin and cable driven surgical instruments are utilized in mechanical structure of the slave robots [13]. This miniaturization causes compliance, flexibility (in joint/link) and nonminimum phase behavior for slave manipulators [14], [17]. Some researchers have been performed to deal with the elasticity of miniaturized slave robots [15], [16], [19]. In [15] and [16],[25] the effects of structural miniaturization is modeled as quasi-static flexion and joint stiffness respectively. Consequently in the literature other degrading issues such as NMP behavior are disregarded.

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In this article first, the effects of NMP slave robots on telerobotics systems are studied afterward, a new architecture is developed to deal with the challenges of time delayed NMP tele-manipulation. The performance and stability utilizing the proposed controller for time delayed system, is analyzed. Flexible arms are well-known examples of NMP manipulators [17]. Flexibility and non-minimum phase behavior in the arm is caused by structural miniaturization [18],[16]. In this article, first, it is shown that, since the impedance tuning and cancelling is not applicable for NMP systems, the ideal transparency is not achievable even when the time-delay is not presented. Then, it is shown that, even utilizing ideal Four-Channel telerobotics architecture, some parts of NMP slave impedance is undesirably transmitted to the operator which degrades the performance, accuracy and transparency. Consequently, a new control strategy called "Pseudo-Three-Channel (PTC) architecture" is proposed to enhance the transparency and stability of the system. The stability of the proposed structure is analyzed utilizing Nyquist stability theorem. It is shown that, utilizing the proposed controller (PTC), the bandwidth requirement of one of the communication channels is reduced. Hence, the name "Pseudo Three-Channel" is selected. Afterward, the performance and stability of the proposed architecture (PTC) is investigated when the challenge of communication time delay exists. Finally, experimental validation is given.

#### II. GENERAL STANDARD MODELING

Non-minimum phase Slave Teleoperation (NMPST) is caused by miniaturization for slave robots. In this section, a general modeling for NMP slaves is proposed. The general model of NMP Slave is shown in Fig. (1) and Eq. s (1)-(3):

$$Z_s V_e = Z_2 F_{cs} - Z_1 F_e \tag{1}$$

$$Z_{2} = \prod_{i=1}^{n} \left( \frac{1}{w_{i}} s - 1 \right), \quad w_{i} = a_{i} + j. \, b_{i} \, , a_{i} > 0$$
(2)  
$$F_{e} = F_{e}^{*} + Z_{e} V_{e}$$
(3)

$$F_e^* + Z_e V_e$$



Where " $Z_s$ ", " $Z_1$ ", " $Z_2$ " are related to dynamics of slaveside impedances and " $Z_e$ " is related to the environmental Impedance as (3).

$$Z_s V_e = Z_2 F_{cs} - Z_1 F_e \tag{1}$$

$$Z_2 = \prod_{i=1}^{n} (\frac{1}{w_i} s - 1), \quad w_i = a_i + j. \, b_i \, , a_i > 0$$
(2)

$$F_e = F_e^* + Z_e V_e \tag{3}$$

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 $F_{cs}$  is the control input,  $F_e$  is the interaction force between the slave and environment,  $V_e$  is the velocity of the environment (and the slave) and  $F_e^*$  is the exogenous force of the environment. As it's clear in (2),  $Z_2$  includes RHP (Right Half Plane) roots. The mentioned roots have considerable effects on teleoperation and will be discussed in this paper. In (2), " $N_z$ " is the number of RHP roots of  $Z_2$ . Dynamics of the minimum phase rigid master is written in (4), (5) [6],[3]:

$$\begin{split} & Z_m V_h = F_{cm} + F_h \\ & F_h = F_h^* - Z_h V_h \end{split} \tag{4}$$

where  $F_{cm}$  is the control effort of the master robot,  $V_h$  and  $F_h$  are related to the hand velocity and interaction force between the hand and the master,  $Z_h$  is the impedance of operator hand and  $F_h^*$  is the hand exogenous force. Note that, in this paper the **linear** dynamics of the slave manipulator is utilized to exhibit the potential degrading effects of NMP slave robot for the first time.

#### III. IDEAL TRANSPARENCY IN NMP TELE-MANIPULATION

In this section, the feasibility of ideal transparent NMP telemanipulation is investigated. Note that, in order to highlight the effects of NMP characteristics of the slave robot, the challenge of time delay is not considered ( $\tau_d = 0$ ) in this section. Four-Channel (4C) architecture is well-known as the most general architecture in telerobotics [6],[7], [19] since all possible feedback and feed-forward signals are utilized. So, first of all, the performance of 4C architecture is evaluated for NMP-slave telerobotics. The general structure is shown in Fig.(2). Control efforts are drawn in (12), (13).

$$F_{cm} = -C_m V_h + C_6 F_h - C_4 V_e - C_2 F_e$$
(12)  
$$F_{cs} = -C_s V_e - C_5 F_e + C_1 V_h + C_3 F_h$$
(13)

In (12) and (13),  $C_m$  and  $C_s$  are local position controllers. Additionally  $C_5$ ,  $C_6$  are local force controllers of the slave and the master robots. Moreover  $C_1$ ,  $C_2$ ,  $C_3$ ,  $C_4$  are the remote feed forward compensators. Utilizing (1), (4), (12) and (13) the hybrid matrix of NMPST is calculated as (14)-(17).

$$h_{11} = \frac{Z_{cm}(Z_s + C_s Z_2) + C_1 C_4 Z_2}{(1 + C_6)(Z_s + C_s Z_2) - C_3 Z_2 C_4}$$
(14)

$$h_{12} = \frac{c_2(Z_s + C_s Z_2) - c_4(Z_1 + C_5 Z_2)}{(1 + C_6)(Z_s + C_s Z_2) - C_3 Z_2 C_4}$$
(15)

$$h_{21} = -\frac{C_3 Z_2 Z_{CM} + C_1 Z_2 (1 + C_6)}{(1 + C_6) (Z_s + C_s Z_2) - C_3 Z_2 C_4}$$
(16)  
$$h_{21} = -\frac{(Z_1 + C_5 Z_2) (1 + C_6) - C_2 C_3 Z_2}{(1 + C_6) - C_2 C_3 Z_2}$$
(17)

$$h_{22} = \frac{(1 + C_6)(Z_5 + C_5Z_2) - C_3Z_2C_4}{(1 + C_6)(Z_5 + C_5Z_2) - C_3Z_2C_4}$$
(17)  
Where  $\begin{bmatrix} F_h \\ U \end{bmatrix} = H \begin{bmatrix} V_h \\ F_b \end{bmatrix}$  where  $H = \begin{bmatrix} h_{11} & h_{12} \\ h_{12} & h_{13} \end{bmatrix}$ 

$$\begin{bmatrix} -V_e \end{bmatrix} \begin{bmatrix} F_e \end{bmatrix} \quad \begin{bmatrix} h_{21} & h_{22} \end{bmatrix}$$
  
So, the characteristics equation can be calculated as (18):  
$$\Delta = (Z_s + C_s Z_2 + (Z_1 + C_5 Z_2) Z_e)(Z_{cm} + (1 + C_6) Z_h) + \quad (18)$$
$$(C_1 Z_2 - C_2 Z_2 Z_h)(C_h + C_2 Z_e) = 0$$

In above equations  $Z_{cm} = Z_m + C_m$ . In telerobotics systems the ideal transparent and stable architecture has the *ideal hybrid matrix* denoted in (19) [7],[6],[2].

$$H_{ideal} = \begin{bmatrix} 0 & 1\\ -1 & 0 \end{bmatrix} \tag{19}$$

If the ideal hybrid matrix is satisfied, then:  $F_e = F_h$  and  $V_e = V_h$ . In the literature [6] it has been shown that, the idealizing conditions can be achieved using  $C_1-C_4$ , moreover  $C_5$ ,  $C_6$  are

two additive degrees of freedoms which have been used to enhance the performance and stability of Minimum Phase (MP) teleoperation. Additionally utilizing  $C_5$ ,  $C_6$  "3-Channel architecture" has been achieved which ensures the optimized communication structure and ideal transparency simultaneously for MP tele-manipulations [6]. Accordingly, conditions of ideal transparent *NMP teleoperation* can be achieved solving (14)-(17) & (19). The results are shown as:

$$\begin{cases} C_1 = (Z_s + C_s Z_2)/Z_2, \ C_2 = 1 + C_6 \\ C_3 = (Z_1 + C_5 Z_2)/Z_2, \ C_4 = -Z_{cm} \end{cases}$$
(20)

Considering  $C_1$  and  $C_3$  in (20), in addition to (2) and (18), it can be inferred that, in order to establish ideal transparent NMP teleoperation, the achieved closed loop system is not internally stable due to RHP pole/zero cancellation in hybrid matrix and the characteristic equation of the system ( $\Delta$ ). The result can be summarized in Corollary I.

**Corollary I:** If any of the slave robots in a telerobotics system exhibits non-minimum phase behavior then, the ideal transparency is not achievable.



#### IV. EFFECTS OF NON-MINIMUM PHASE SLAVE

In the previous section the impacts of NMP slave on ideal transparency is investigated. In this section, the effects of NMP slave on the performance of the *applicable* architecture are studied. As mentioned, ideal transparency is not achievable for NMPST. Consequently, as the first suggestion, performance of the applicable regular Four-Channel strategy, defined in [6], and denoted in (21), can be investigated for delay-free NMPST.

$$\begin{cases} C_1 = Z_s + C_s, C_2 = 1 + C_6 \\ C_3 = 1 + C_5, C_4 = -Z_{cm} \end{cases}$$
(21)

The hybrid matrix applying (21) is achieved as (22)-(25):  $Z_{1} = Z_{2}(1 - Z_{2})$ 

$$h_{11} = \frac{2c_{cm}Z_{S}(1-Z_{2})}{(1+C_{6})(Z_{s}+C_{s}Z_{2})+(1+C_{5})Z_{cm}Z_{2}}$$
(22)

$$\begin{array}{c} (1+C_6)(Z_s+C_sZ_2) + (1+C_5)Z_{cm}Z_2 \\ (Z_1+C_5Z_2)(1+C_6) - (1+C_6)(1+C_5)Z_2 \end{array}$$

$$h_{22} = \frac{(1+C_5)(Z_5+C_5)(Z_2+C_5))$$

Characteristic equation of the system is defined in (26):

 $\Delta' = (C_2(Z_s + C_sZ_2) + C_3Z_{cm}Z_2)(Z_h + Z_e) = 0$  (26) As it can be inferred by comparing (22)-(25) with ideal condition denoted in (19), transparency of the system is degraded. In order to minimize the destabilizing effects of  $Z_2$  (26), Three-Channel architecture can be applied as  $C_5 =$ -1. So the hybrid matrix can be re-written as (27)-(30):

$$h_{11} = \frac{Z_{cm}Z_s(1-Z_2)}{(1+C_6)(Z_{cs})}$$
(27)

$$h_{12} = \frac{(1 + C_6)(Z_{cs}) + Z_{cm}(Z_1 - Z_2)}{(1 + C_6)(Z_{cs})}$$
(28)

$$h_{21} = -((Z_s + C_s)Z_2)/Z_{cs}$$
<sup>(29)</sup>

$$h_{22} = (Z_1 - Z_2)/Z_{cs} \tag{30}$$

Note that, in (26)-(30),  $Z_{cs} = Z_s + C_s Z_2$  and  $C_s$  should be designed to keep the roots of  $\Delta'$  (which are equal to the roots of  $Z_{cs}$ , when  $C_3 = 0$ ) in Left Half Plane (LHP). In (27)-(30) if  $C_6 \gg 1$ , then  $h_{11} \rightarrow h_{11-id} = 0$  and  $h_{12} \rightarrow h_{12-id} = 1$ , where the index of "*id*" denotes the ideal value. However,  $C_6$  cannot improve the undesired reflected slave impedance in " $h_{21}$ " and " $h_{22}$ ", which considerably degrades the transparency. So the result can be summarized as Fact I. **Fact I: Utilizing conventional Four-Channel architecture for NMP phase slave telerobotics, parts of NMP slave impedance which are not eliminated, are reflected in Hybrid matrix and degrades the transparency of the system. Additionally it is** 

observed that, increasing  $C_6$ , conventional Three-channel has a better behavior in comparison with Four-channel structure but the performance of both structures cannot be idealized and suffers from unwanted reflected NMP slave impedances.

For MP slave, utilizing 3-Channel architecture,  $C_5$  or  $C_6$  can be applied to cancel dynamics of the environment-side or operator-side and eliminate one communication channel [6]. However, considering (14)-(18), for NMP slave telerobotics, it is not permitted to consider  $C_6 = -1$  and eliminate the environment force feed-forward channel. The reason is: assuming  $C_6 = -1$ , roots of " $Z_2$ " play the role of RHP poles for hybrid elements and the characteristic equation denoted in (18), so the internal stability is lost. In addition to above, since the multiplication of local force controller  $C_5$  and  $Z_2$ exists in hybrid elements (considering (15), (17)), complete cancelling of slave-side dynamics is not permitted, so eliminating hand force feed forward  $(C_3 = 1 + C_5 = 0)$ cannot considerably enhance the performance by impedance cancelling. The mentioned results are summarized as Fact II. Fact II: In the presence of non-minimum phase slave:

1- Elimination of environmental force feed forward is not permitted.

2- Elimination of hand force feed forward communication channel can be performed however  $C_5$  cannot improve the performance by cancelling the slave-side dynamics. As a conclusion non-minimum phase behavior degrades the fascinating features of common control architectures.

## V. THE NOVEL PSEUDO-THREE-CHANNEL ARCHITECTURE

It has been shown in previous section that, conventional Three and Four-Channel architectures fail to guarantee acceptable performance for NMP slave telerobotics. In this section, a novel architecture is proposed with mathematical proof of stability for NMP tele-manipulation which is called "Pseudo-Three-Channels" (PTC). The number of required communication channels of PTC is similar to Four-Channel architecture. However, the main control concept and communication issues are roughly inferred from Three-Channel architecture. The proposed control technique is introduced in (31). Note that, the performance and stability of the proposed technique in the presence of *time delay* is investigated in the next section. In this section the main concept of PTC is illustrated when  $\tau_d = 0$ . In the proposed structure utilizing frequency content of NMP part of the system ( $Z_2$ ), the proposed technique is achieved as:

$$\begin{cases} C_1 = (Z_s + C_s Z_2)/\hat{Z}_2 , C_2 = 1 + C_6 \\ C_3 = (Z_1 + C_5 Z_2)/\hat{Z}_2 , C_4 = -Z_{cm} \end{cases}$$
(31)

Where 
$$Z_2 = \prod_{i=1:N_Z} \left( \frac{1}{w_i} s + 1 \right)$$
 (32)

The hybrid matrix applying this strategy is achieved as (33)-(36):

$$h_{11} = \frac{Z_{cm}(Z_s + C_s Z_2) - Z_{cm}(Z_s + C_s Z_2) \frac{Z_2}{\hat{Z}_2}}{(1 + C_6)(Z_s + C_s Z_2) + Z_{cm}(Z_1 + C_5 Z_2) \frac{Z_2}{\hat{Z}_2}}$$
(33)

$$h_{12} = \frac{(1+C_6)(Z_s+C_sZ_2) + Z_{cm}(Z_1+C_5Z_2)}{(1+C_6)(Z_s+C_sZ_2) + Z_{cm}(Z_1+C_5Z_2)\frac{Z_2}{\hat{Z}_2}}$$
(34)

$$h_{21} = -\frac{(1+C_6)(Z_s+C_sZ_2)\frac{Z_2}{Z_2} + Z_{cm}(Z_1+C_5Z_2)\frac{Z_2}{Z_2}}{(1+C_6)(Z_s+C_sZ_2) + Z_{cm}(Z_1+C_5Z_2)\frac{Z_2}{Z_2}}$$
(35)  
$$h_{22} = \frac{(Z_1+C_5Z_2)(1+C_6) - (Z_1+C_5Z_2)(1+C_6)\frac{Z_2}{Z_2}}{(1+C_6)(Z_s+C_sZ_2) + Z_{cm}(Z_1+C_5Z_2)\frac{Z_2}{Z_2}}$$
(36)

Note that, the amplitude of  $(Z_2/\hat{Z}_2)$  is equal to unity. This fact is utilized to obtain the optimized teleoperation architecture for NMP slaves. Considering (33)-(36), in high frequencies since  $(Z_2/\hat{Z}_2) \rightarrow 1 < 0^\circ$  the ideal hybrid matrix is achieved. Utilizing PTC, since the multiplication of " $C_5$ " and  $Z_2$  exists in  $C_3$ , eliminating feed forward communication channel related to the hand force is not permitted  $C_3 \neq 0$ . Consequently to enhance the transparency,  $C_5$  is designed as (37) to *minimize* the degrading effects of the slave dynamics.  $C_5 = -(Z_1/\hat{Z}_2)$  (37)

Note that utilizing (37), in high frequencies  $C_3 \rightarrow 0$  consequently, the required band width of the feed forward communication channel related to the hand force is reduced. Therefore, this control structure called **Pseudo-Three-Channel** architecture. In order to recover the performance of the system in low frequencies  $C_6$  is employed as a sufficient degree of freedom. Considering (33)-(36) it can be inferred that if  $C_6 \gg 1$  the hybrid matrix can be re-written as (38).

$$\begin{pmatrix}
h_{11} = 0 & , h_{12} = 1, h_{21} = -(Z_2/Z_2) \\
h_{22} = \frac{Z_1 \left(1 - \frac{Z_2}{\hat{Z}_2}\right)^2}{(Z_s + C_s Z_2)}
\end{cases}$$
(38)

Considering (38),  $h_{11}$  and  $h_{22}$  are ideally achieved increasing  $C_6$ . Additionally " $h_{21}$ " illustrates a frequency based decreasing phase difference between  $V_e$  and  $V_h$ . " $h_{22}$ " includes the remaining-reflected impedance  $Z_r = (Z_1/(Z_s + C_s Z_2)))$  generated by NMP slave dynamics. " $Z_r$ " can potentially degrades the transparency however, utilizing the PTC, aforementioned impedance  $Z_r$  is multiplied by  $(1 - (Z_2/\hat{Z}_2))^2$  which alleviates the effects of reflected impedance especially in *high frequencies*. Note that, increasing local velocity feedback  $C_s$ , alleviates the effects of " $Z_r$ " even in low frequencies. However it enhances the destabilizing effects of  $Z_2$ . Consequently a tradeoff should be performed. The results are summarized in Corollary II.

Corollary II: Utilizing Pseudo Three-Channel architecture: 1- The ideal transparency is achieved in high frequencies

2- Increasing  $C_6$ , performance of the system is optimized since the effects of reflected slave side impedance is minimized.

3- The degrading effects of slave side dynamics are alleviated (not cancelled) by  $C_5$  especially in high frequencies.

4- The effects of remaining reflected slave side impedance are minimized by  $(1 - (Z_2/\hat{Z}_2))^2$  and can be alleviated by  $C_s$ .

5- The bandwidth of the required forth channel, related to hand force feed forward, is decreased.

In order to analyze the stability of the PTC, one alternative is Llewellyn's criterion [7]. Considering (33)-(36), analyzing the stability utilizing this criterion is so complicated. So in this paper similar to [6] Nyquist stability theorem is utilized. The system block diagram can be re-configured as Fig.(3). So the characteristics equation can be expressed as:

$$\Delta(s) = (Z_{cs} + (1 - Z_2/\hat{Z}_2)Z_1Z_e)(Z_{cm} + (1 + C_6)Z_h) - (Z_{cs} - (1 - Z_2/\hat{Z}_2)Z_1Z_h)(Z_2 (39) /\hat{Z}_2)(Z_{cm} - (1 + C_6)Z_e) = 0$$

The characteristic equation can be written as:

 $\hat{\Delta}(s) = 1 + \Delta_1 = 0$ 

$$\Delta_{1} = \frac{\left(1 - \frac{Z_{2}}{\hat{Z}_{2}}\right) Z_{cm} (Z_{1} Z_{e} + Z_{1} Z_{h} \frac{Z_{2}}{\hat{Z}_{2}} + Z_{cs})}{C_{2} Z_{cs} \left(Z_{h} + Z_{e} \frac{Z_{2}}{\hat{Z}_{2}}\right) + \left(1 - \frac{Z_{2}}{\hat{Z}_{2}}\right) \left(1 - \frac{Z_{2}}{\hat{Z}_{2}}\right) Z_{1} Z_{h} C_{2} Z_{e}}$$
(41)

Considering (40)-(41), in low frequencies where  $(Z_2/\hat{Z}_2)$  generates maximum phase difference, then the achieved hybrid matrix has maximum variation from ideal conditions. So degrading effects of reflected slave-side impedance are maximized. Consequently, the worst case situation appears in **critical** frequency when $(Z_2/\hat{Z}_2) \rightarrow -1$ . Considering (39), general stability analysis is so complicated consequently the stability is analyzed in worst case situation. It's clear that, if the system is stable in worst case, then the stability is preserved. The characteristic equation in worst case is:

$$\Delta_w(s) = C_2 Z_{cs} (Z_h - Z_e) + 4C_2 Z_1 Z_h Z_e + 2Z_{cm} (Z_1 Z_e - Z_1 Z_h + Z_{cs}) = 0$$
(42)
The characteristic equation can be written equ

The characteristic equation can be written as:

$$\hat{\Delta}_{w}(s) = 1 + \frac{2Z_{cm}(Z_{1}Z_{e} - Z_{1}Z_{h} + Z_{cs})}{C_{2}(Z_{cs}(Z_{h} - Z_{e}) + 4Z_{1}Z_{h}Z_{e})} = 0$$
(43)

Consequently, utilizing Nyquist theorem the stability condition is calculated as (44) and (45):

$$DE is Hurwitz \to DE = (Z_{cs}(Z_h - Z_e) + 4Z_1Z_hZ_e)$$
(44)  
$$C_2 > \left| \frac{2Z_{cm}(Z_1Z_e - Z_1Z_h + Z_{cs})}{(Z_{cc}(Z_h - Z_e) + 4Z_1Z_hZ_e)} \right|$$
(45)

Considering (43)-(45), increasing  $C_2$  ensures the stability. Note that, as a consequence of Corollary II increasing  $C_2$ enhances the performance as well. Therefore utilizing  $C_2$ , both the transparency and stability are preserved. However, increasing  $C_2$  magnifies the conservative behavior of masterside controller. This may cause some undesirable phenomena such as actuator saturation or noise magnification. Less conservative behavior can be achieved applying impedance tuning for  $Z_m$  in addition with proper choice of  $C_s$  and  $C_m$ . Considering (45) decreasing  $Z_m$  causes less conservative  $C_2$ . Additionally, considering the first condition in (44), the problematic term is  $(Z_h - Z_e)$ , which can potentially degrades the stability. In order to preserve the stability, the impedance of the operator hand should be larger than the impedance of the environment  $(||Z_h(jw)|| >$  $||Z_e(jw)||)$ . This condition is satisfied in many applications however the stability in hard contact is not guaranteed. It is roughly similar to the stability condition of 3-Channel architecture ( $C_3 = 0$ ) for MP tele-manipulation [6]. The mentioned results is summarized as Corollary III.



Fig. (3). Re-configured block diagram

Corollary III: The stability of the PTC architecture is preserved if  $C_2$  has sufficient large gain especially in low and critical frequencies. Proper tuning of  $Z_m$ ,  $C_s$  and  $C_m$  leads to less conservative  $C_2$ .

### VI. TIME DELAY ISSUE

In this section, stability and performance of the PTC is analyzed in the presence of unknown-constant time delay. System interconnection is drawn in Figs. (2), (3). The hybrid matrix of the system is shown in (46)-(49):

$$h_{11} = \frac{Z_{cm}(Z_s + C_s Z_2) + C_1 C_4 Z_2 e^{-2s\tau_d}}{(1 + C_6)(Z_s + C_s Z_2) - C_3 Z_2 C_4 e^{-2s\tau_d}}$$
(46)

$$h_{12} = \frac{C_2(Z_s + C_s Z_2)e^{-s_t d} - C_4(Z_1 + C_5 Z_2)e^{-s_t d}}{(1 + C_6)(Z_s + C_s Z_2) - C_3 Z_2 C_4 e^{-2s_t d}}$$
(47)

$$h_{21} = -\frac{C_3 Z_2 Z_{cm} e^{-S\tau_d} + C_1 Z_2 (1 + C_6) e^{-S\tau_d}}{(1 + C_6) (Z_s + C_s Z_2) - C_3 Z_2 C_4 e^{-2s\tau_d}}$$
(48)  
$$(Z_1 + C_5 Z_2) (1 + C_6) - C_2 C_3 Z_2 e^{-2s\tau_d}$$

$$h_{22} = \frac{(Z_1 + C_5 Z_2)(1 + C_6)}{(1 + C_6)(Z_5 + C_5 Z_2) - C_3 Z_2 C_4 e^{-2s\tau_d}}$$
(49)

Considering (46)-(49) and also  $C_6 \gg 1$  and  $C_5 = -Z_1/Z_2$  the hybrid matrix is achieved as:

$$\begin{cases} h_{11} = 0, h_{12} = e^{-s\tau_d}, h_{21} = -(Z_2/\hat{Z}_2) e^{-s\tau_d} \\ h_{22} = \hat{Z}_r \left( 1 - \frac{Z_2}{\hat{Z}_2} \right) \text{ and } \hat{Z}_r = \frac{Z_1 \left( 1 - \frac{Z_2}{\hat{Z}_2} e^{-2s\tau_d} \right)}{(Z_s + C_s Z_2)} \end{cases} (50)$$

Considering (50) in high frequencies when  $(Z_2/\hat{Z}_2) \rightarrow 1$ then  $h_{11} = 0$ ,  $h_{12} = e^{-s\tau_d}$ ,  $h_{21} = -e^{-s\tau_d}$ ,  $h_{22} = 0$ . It means that:  $V_e = e^{-s\tau_d}V_h$  and  $F_e = e^{s\tau_d}F_h$ . Considering  $h_{22}$  in (50), in near to critical frequencies, the effects of reflected slave side dynamics  $(\hat{Z}_r)$ , degrades the velocity tracking. Increasing frequency, this reflected impedance is alleviated by  $(1 - Z_2/\hat{Z}_2)$ . Moreover, considering (50), the effects of  $\hat{Z}_r$  in near to critical frequencies can be alleviated increasing  $C_s$ . However, magnification of " $C_s$ " increases the destabilizing effects of  $Z_2$ . So " $C_s$ " should satisfy the

(40)

stability condition which will be introduced. Additionally, if the effects of reflected impedance is alleviated by " $C_s$ " then, considering  $h_{21}$ , the slave velocity lags " $(Z_2/\hat{Z}_2)V_h$ ". So, " $Z_2/\hat{Z}_2$ " degrades the transparency near to critical frequencies. Characteristic equation in is denoted in (51):

$$\Delta_{del}(s) = (Z_s + C_s Z_2 + (Z_1 + C_5 Z_2) Z_e) (Z_{cm} + (1 + C_6) Z_h) + (C_4 + C_2 Z_e) (C_1 Z_2 - C_3 Z_2 Z_h) e^{-2sTd} = 0$$
Similar to (42) worst case stability should be investigated as:

$$\Delta_{del-W}(s) = (Z_{cs} + 2Z_1Z_e)(Z_{cm} + C_2Z_h) + (Z_{cm} - C_2Z_e)(Z_{cs} - 2Z_1Z_h)e^{-2sTd} = 0$$
(52)

Worst case characteristic equation can be rewritten as:

$$\hat{\Delta}_{del-W} = 1 + \frac{(Z_{cm} - C_2 Z_e)(Z_{cs} - 2Z_1 Z_h)}{(Z_{cs} + 2Z_1 Z_e)(Z_{cm} + C_2 Z_h)} e^{-2sTd} = 0$$
(53)

Utilizing impedance tuning,  $Z_{cm}$  can be designed such that  $Z_{cm} \ll C_2 Z_e$ ,  $Z_{cm} \ll C_2 Z_h$ . So, increasing  $C_2$  and decreasing  $Z_{cm}$ , (53) can be re-written as:

$$\hat{\hat{\Delta}}_{del-W}(s) = 1 + \frac{-Z_e(Z_{cs} - 2Z_1Z_h)}{Z_h(Z_{cs} + 2Z_1Z_e)}e^{-2sTd} = 0$$
(54)

Considering (54), the stability condition can be achieved as: (1)  $DE_{2}is$  Hurwitz where  $DE_{2} = Z_{cs} + 2Z_{1}Z_{e}$ 

$$2) \left| \frac{-Z_e(Z_{cs} - 2Z_1Z_h)}{Z_h(Z_{cs} + 2Z_1Z_e)} \right| < 1$$
(55)

$$\left( \rightarrow |Z_{int} + Z_h Z_{cs}| > |Z_{int} - Z_e Z_{cs}| ; Z_{int} = 2Z_1 Z_h Z_e \right)$$

Note that  $Z_{cs} = Z_s + C_s Z_2$ . Consequently to preserve the stability, " $C_s$ " should satisfies the first condition denoted in (55). The second condition is roughly similar to (45), where the stability condition results in  $||Z_h(jw)|| > ||Z_e(jw)||$ .

Corollary IV: PTC for time delayed NMP Teleoperation:

1) Time delay degrades the stability and performance of the system, especially in near to critical frequencies. However, utilizing PTC, the slave-side force leads the master side force.

2) Although increasing  $C_s$ , the performance of velocity tracking is improved, however, magnification of  $C_s$  increases the destabilizing effects of  $Z_2$ .

3) In near to critical frequencies, " $(Z_2/\hat{Z}_2)$ " degrades the performance of the velocity tracking.

4) To preserve the stability of the system " $C_s$ ", " $Z_h$ " and " $Z_e$ " should satisfy the stability conditions denoted in (55).

### VII. EXPERIMENTAL RESULTS

In this section, some experimental results are presented. The master robot is an Omni Phantom from SensAle and the slave is simulated in Matlab. The Master and Slave processors are connected via a LAN network. The experiments are performed in one Cartesian direction. In the first experiment the proposed PTC is evaluated, then in Experiment II conventional Three-Channel architecture is compared with PTC. Finally in Experiment III the performance is evaluated when the time delay exists.



## **Experiment I:**

Considering (38), (45) to have a less conservative  $C_6$  it's better to have a low magnitude  $Z_{cm}$ . In this experiment a PD controlled master is utilized and desired impedance " $Z_{cm}$ " is assigned. The parameters are shown in Table I.

Table	т
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$Z_1 = 3s + 1$	$C_{s} = 1, C_{m} = 30$	$B_m = 10 N.sec/m$
$Z_s = 10s + 40$	$C_{6} = 20$	$M_m = 2Kg$
$Z_2 = s - 1$	$B_e = 2 N.Sec/m$	$K_m = 100 N/m$
$M_m = 2Kg$	$M_e = 1Kg$	$K_e = 50N/m$

Fig.(6) & (7) represent the velocity an force tracking respectively. Considering (38), Figs. (6) & Fig.(7), it can be inferred that, utilizing PTC, force-tracking is almost idealized since  $h_{11} \rightarrow 0$  and  $h_{12} \rightarrow 1$ . And almost ideal velocity tracking in high frequencies is achieved, since  $h_{22} \rightarrow 0$  and  $h_{21} \rightarrow -1$ . However, the effects of reflected slave-side dynamics, degrades the velocity tracking in low frequencies (considering Fig. (6) when *time* > 4sec).

## **Experiment II:**

In this experiment the performance of conventional Three-Channel architecture is investigated. The results are shown in Figs (8) and (9). Fig.(8) & (9) represent the velocity an force tracking respectively. The effects of not eliminated slave-side impedances on transparency can be observed. According to Fact I, utilizing 3-channel architecture the force tracking is idealized (as Fig(9)). However increasing " $C_6$ " cannot idealize the velocity tracking, so the reflected slave-side impedance degrades the velocity tracking as Fig. (8) and (9). Considering Figs. (6)-(9), the superior behavior of the PTC can be observed. Utilizing PTC, the effects of NMP slave are minimized and the transparency is optimized.





Fig. (9). Force Tracking,

*Experiment III:* In this experiment time delay is assigned equal to 140 ms. The slave dynamics are similar to table (1) other parameters in this experiment are denoted in Table II:

I able. II			
$C_6 = 100$	$C_s = 3$	$B_m = 0.1 N. sec/m$	
$B_e = 10 N.Sec/m$	$K_m = 0.1 N/m$	$M_m = 0.1 Kg$	
$M_e = 1Kg$	$K_e = 25N/m$	$C_m = 1$	

The results are shown in Figs. (10) and (11).





In Figs. (10) , (11) high capabilities of the proposed technique in presence of unknown constant time delay are illustrated. Considering Fig. (11), the slave-side force  $(F_e)$  leads the master side force  $(F_h)$  despite the applied frequency. Additionally, considering Fig. (10), the slave-side velocity approximately lags the master-side velocity  $(V_h)$  in *high frequencies.* However the phase deference is not exactly equal to the force issue (as (50)). It should be noted

that, the performance of velocity tracking is degraded in low frequencies as it clear in Fig (10).



Fig. (11). Force tracking, utilizing PTC, communication delay=140 ms.

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