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Online Selection of H_∞ Controllers for a Faulty Linear System

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***Abstract*—This paper investigates optimal selections of controllers online for a linear system with actuator faults in the framework of H_∞ control. Actuator faults are modeled into three states: normal, loss of effectiveness and outage, and the development of the actuator faults is predicted by using a Markov chain model. We present five methods to construct a smaller bank of controllers. Four selection schemes are proposed to select an optimal controller from a bank of controllers for different demands. Another selection scheme and one pre-selection scheme are presented when there may be false fault detection and isolation (FDI). Considered together, they provide a powerful solution for all kinds of demands. Finally, we utilize an example to explain and verify the proposed methods.**

I. Introduction

Fault tolerant control systems (FTCS) can maintain stability and acceptable performances after system component faults occur [1]. Generally, the design techniques for FTCS can be classified as passive approach (PFTCS) and active approach (AFTCS) [2]. PFTCS is easy to implement, but it always results in limited recoverable faults and a low overall performance level. AFTCS is always implemented either by synthesizing a new control law online in real-time or selecting a pre-computed control law online. In the approach of selecting a pre-computed control law online, a pre-computed control bank is established for the fault modes, and then a suitable controller is selected for the fault mode determined by a designed mechanism of FDI [1], [2]. The advantage of selecting a control law online is that the constraints of real-time computation are relaxed compared with the redesign of the control law [3]. This paper mainly falls in the category of online selection of controllers for actuator faults.

There are many investigations about the online selection approach, such as gain scheduling [4], multiple model adaptive method (MM) [5]–[8], selection based on the separation of set [9], dissipativity-based switching [10] and general management of controllers for selection [3]. All of these methods have made progress with the online selection FTCS. Generally, one fault mode may be recovered by many controllers and one controller may also recover many fault modes in a fault tolerant system. However, the optimal selection and management of the controllers has not received enough attention. The basic selection schemes based on optimizing performance index and minimizing switching times have been used in the literature [3], [7]. We offer six optimal selections in the framework of H_∞ control to accommodate all kinds of applications, such as making a trade-off between the performances and switching times and enforcing reliability when there may be false FDI.

The designs of H_∞ fault tolerant controllers have been studied in many works [11]–[14] due to its robustness to model uncertainty and external disturbances. In this paper, an essential relationship is proposed between the set of H_∞ controllers and the successional fault modes in Theorem 1. By modeling the actuators into three states: normal, loss of effectiveness and outage, the proposed H_∞ control law reduces some degree of the conservatism compared with the existing methods [12]. The occurrence of actuator faults is modeled by a time-homogeneous Markov chain [15], which is used to predict the development of actuator faults, and thus helps to design a selection scheme.

The set of all controllers is always too large to select a proper controller. Thus five methods are presented to construct a small set of controllers according to different demands. Then four selection schemes are proposed to determine how to select a proper controller from a given set of controllers. The designs are motivated by optimizing H_∞ gain, minimizing switching times, making a trade-off between optimizing H_∞ gain and minimum-switching times, and minimizing the expedition-like cost of H_∞ gain, respectively. What is more, another selection based on the probability of false FDI is proposed for the case when there may be false FDI. A general pre-selection scheme is also put forward to make the selections more robust for the case of possible false FDI, and this pre-selection approach can be combined with any other selection schemes. Finally, the fault tolerant longitudinal control of a F-18 aircraft is designed to explain and illuminate the superiorities of the proposed H_∞

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control law and selections.

This article is organized as follows. Section II describes system and fault models. Section III develops the related H_∞ control theories under actuator fault modes. Section IV offers five methods to build a small set of controllers. Four selections are proposed in Section V. Section VI addresses tow selection schemes when there may be false FDI. Section VII shows an illustrative example to explain and verify the proposed methods. A conclusion is made in Section VIII

II. System and Fault Description

Consider a linear system with external disturbance

$$\begin{cases} \dot{x}(t) = Ax(t) + B_f u(t) + B_w w(t) \\ z(t) = Cx(t) + D_f u(t) + D_w w(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the control input, $w(t) \in L_2$ is the exogenous disturbance and $z(t)$ is the penalized output. A, B_f, B_w, C, D_f and D_w are constant matrices with appropriate dimensions.

The actuator faults are modeled by $B_f = B(\Lambda + \Delta)$, where $\Lambda = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_m]$ is a known matrix to classify different fault modes and $\Delta = \text{diag}[\delta_1, \delta_2, \ldots, \delta_m]$ is an unknown matrix to compensate errors. The other types of fault such as actuator bias or blocked are considered as exogenous disturbance. Correspondingly, let $D_f = D(\Lambda + \Delta)$. B and D are coefficient matrices for the normal system.

Moreover, assume that FDI can ensure accurate detection for the k^{th} actuator in normal case, i.e., when $\lambda_k = 1$, $\delta_k = 0$. When the k^{th} actuator is loss of effectiveness, $0 < \lambda_k + \delta_k < 1$. Assume $0 < \sigma_k < 1$ is the known lower bound of effectiveness of the k^{th} actuator. Then let $\lambda_k = \sigma_k$ in the case of loss of effectiveness. When the k^{th} actuator is outage, $\lambda_k = 0$ and $0 \le \delta_k < 1$. Hence we use $\{\lambda_k^1, \lambda_k^{\sigma_k}, \lambda_k^0\}$ to describe the k^{th} actuator in the state of normal, loss of effectiveness and outage, respectively.

Define a set $\mathcal{F} = \{\Lambda \mid \lambda_k \in \{\lambda_k^1, \lambda_k^{\sigma_k}, \lambda_k^0\}$. The number of elements of $\mathcal{F}$ is $|\mathcal{F}| = 3^m$. So by using the index set $\mathcal{I} = \{0, 1, 2, \ldots, 3^m - 1\}$, either $\Lambda_i \in \mathcal{F}$ or $i \in \mathcal{I}$ denotes the i^{th} fault mode. Generally, $i = 0$ is associated with the normal system. For two fault modes $\Lambda_i = \text{diag}[\lambda_{i_1}, \ldots, \lambda_{i_m}]$ and $\Lambda_j = \text{diag}[\lambda_{j_1}, \ldots, \lambda_{j_m}]$, if $\lambda_{i_k} \ge \lambda_{j_k}$ holds for all $k = 1, \ldots, m$, then a partial ordering " $\succeq$ " on the set $\mathcal{I}$ can be defined by $\Lambda_i \succeq \Lambda_j$.

Based on above partial ordering " $\succeq$ ", the predecessors of Λ_i are defined by $\mathcal{P}(\Lambda_i) = \{\Lambda_j \mid \Lambda_j \succeq \Lambda_i\}$, and its successors are defined by $\mathcal{S}(\Lambda_i) = \{\Lambda_j \mid \Lambda_i \succeq \Lambda_j\}$.

Actuator failures are a random process and we assume it is a time-homogeneous Markov chain. Suppose the fault mode is $\Lambda(t_0)$ at initial time t_0. Then the probability of the fault mode Λ_i occurring at time $t_n = t_0 + nT$ is $Pr(\Lambda_i(nT)) = Pr(\lambda_{i_1}(nT) = \lambda_1^{u_1}, \ldots, \lambda_{i_m}(nT) = \lambda_m^{u_m})$, where $u_k \in \{1, \sigma_k, 0\}$ for $k = 1, \ldots, m$. Define the distribution $p^n \triangleq (Pr(\Lambda_0(nT)), Pr(\Lambda_1(nT)), \ldots, Pr(\Lambda_{3^m-1}(nT)))^T$, which is a $3^m \times 1$ vector. Due to the known fault mode $\Lambda(t_0)$ at time t_0, the distribution p^0 is known. According to reliable historic data, the transition matrix W from p^{n-1} to p^n can be estimated. Then $p^n = Wp^{n-1} = W^n p^0$.

III. H_∞ Fault Tolerant Control

Assume a state feedback control law of system (1) is $u = K_i x$ under the fault mode Λ_i. Then the transfer function matrix T_{wz}^i from w to z is given by

$$T_{wz}^i(s) = (C + D_{fi}K_i)(sI - (A + B_{fi}K_i))^{-1}B_w + D_w$$

where $B_{fi} = B(\Lambda_i + \Delta_i)$ and $D_{fi} = D(\Lambda_i + \Delta_i)$.

The H_∞ design specifications are to find a state feedback matrix K_i such that $(A + B_{fi}K_i)$ is Hurwitz and $\|T_{wz}^i(s)\|_\infty < \gamma$ for a given constant $\gamma > 0$.

The following definitions are used to classify fault modes and make the problems clearer and more accurate.

Definition 1: A feedback control $u = Kx$ is an admissible control law if the H_∞ specifications are achieved.

Definition 2: A fault mode $\Lambda_i, i \in \mathcal{I}$ is structure recoverable (SR) if there exists an admissible control $u = K_i x$.

So the fault set $\mathcal{I}$ can be divided as $\mathcal{I} = \mathcal{I}^+ \bigcup \mathcal{I}^-$, where $\mathcal{I}^+$ is a SR set and $\mathcal{I}^-$ is a non-SR set.

Definition 3: A fault tolerant strategy (FTS) is a surjective mapping from the SR set $\mathcal{I}^+$ on a control law set $\mathcal{K}$ and $|\mathcal{K}| \le |\mathcal{I}^+|$.

Definition 4: Extended fault tolerant strategy (EFTS) is a mapping from $\mathcal{I}^+$ on $2^{\mathcal{K}}$ (the power set of $\mathcal{K}$).

Definition 5: A fault mode $i \in \mathcal{I}^+$ is practical recoverable (PR) by a given FTS if this FTS gives a mapping from i to K_i and $u = K_i x$ is an admissible controller. If all SR are PR, then this FTS is the complete FTS. A complete EFTS is the extension of a complete FTS.

The H_∞ design specifications need to be converted to linear matrix inequalities (LMI) for further studies.

Lemma 1: If there exist a feedback matrix K_i and a symmetric positive definite matrix P_i such that the following inequality holds,

$$\begin{pmatrix} M1_{11} & P_i B_w + (C + D_{fi}K_i)^T D_w \\ (*) & D_w^T D_w - \gamma^2 I \end{pmatrix} < 0 \tag{2}$$

where $M1_{11} = (A + B_{fi}K_i)^T P_i + P_i(A + B_{fi}K_i) + (C + D_{fi}K_i)^T(C + D_{fi}K_i)$, and $(*)$ denotes a corresponding symmetric term. Then there is a control law $u = K_i x$ such that the H_∞ specifications are achieved for Λ_i.

Inequality (2) cannot be solved directly because of the unknown matrices B_{fi} and D_{fi}. Lemma 2 gives the solvable LMI to obtain an admissible controller for each SR fault mode.

Lemma 2: If there exists a symmetric positive definite matrix P_i and a positive semidefinite matrix Y_i such that the following inequality holds,

$$\begin{pmatrix} M2_{11} & P_i B_w + (C - D\Lambda_i^2 B^T Y_i P_i)^T D_w \\ (*) & 2D_w^T D_w - \gamma^2 I \end{pmatrix} < 0 \tag{3}$$

where $M2_{11} = P_i A + A^T P_i - P_i B \Lambda_i^2 B^T Y_i P_i - P_i Y_i^T B \Lambda_i^2 B^T P_i + (C - D\Lambda_i B^T Y_i P_i)^T (C - D\Lambda_i B^T Y_i P_i) + P_i Y_i^T B \Lambda_i (I - \Lambda_i) D^T D (I - \Lambda_i) \Lambda_i B^T Y_i P_i$. Then $u =$

$-\Lambda_i B^T Y_i P_i x$ such that the H_∞ specifications are achieved for Λ_i.

Thus we can get an admissible controller for a certain fault mode. A meaningful question is whether this controller is an admissible controller or not for other fault modes. Lemma 3 is used to answer this question.

Lemma 3: Suppose there exists a symmetric positive definite matrix P and a positive semidefinite matrix Y, and let $K = -\Lambda B^T Y P$ for a given fault mode Λ. If the following inequality holds for the current fault mode Λ_i,

$$\begin{pmatrix} M3_{11} & PB_w + (C + D\Lambda_i K)^T D_w \\ (*) & 2D_w^T D_w - \gamma^2 I \end{pmatrix} < 0 \qquad (4)$$

where $M3_{11} = PA + A^T P + PB\Lambda_i K + K^T \Lambda_i B^T P + (C + DK)^T (C + DK) + K^T (I - \Lambda_i) D^T D (I - \Lambda_i) K$, then $u = Kx$ such that the H_∞ specifications hold for Λ_i.

Theorem 1 proposes the compatible relationship of H_∞ controllers between successional fault modes on the basis of Lemmas 1, 2 and 3. This relationship is a fundamental theory for constructing EFTSs and selection schemes.

Theorem 1: Suppose $\Lambda_i \succeq \Lambda_j$ for $i, j \in \mathcal{I}^+$, if there exists solutions P_j and Y_j of inequality (3) for the fault mode Λ_j, and let $K_j = -\Lambda_j B^T Y_j P_j$, then $u = K_j x$ can also achieve the H_∞ specifications under the fault mode Λ_i.

Note 1: It is omitted to prove Lemmas 1, 2 3 and Theorem 1 because of the limitation of length of a Conference Paper.

IV. Design of EFTS

For the given H_∞ specifications, we can test inequality (3) to divide $\mathcal{I}$ into $\mathcal{I}^+$ and $\mathcal{I}^-$. EFTS shows a comprehensive relationship between $\mathcal{I}^+$ and the controller set $\mathcal{K}$ for further selecting a control law. Here five methods are addressed to construct EFTS due to different demands. Based on the relationship between H_∞ controllers and fault modes proposed by Theorem 1, EFTS can be constructed from aspects of fault modes (Method 1), H_∞ controllers (Methods 2, 3) or combining them together (Methods 4 and 5).

Method 1: Let $\mathcal{M}^+$ be the set of minimal elements of $\mathcal{I}^+$, and $u = K_j x$ be an admissible control for $j \in \mathcal{M}^+$. Let $\mathcal{M}_i^+ = \mathcal{S}(\Lambda_i) \bigcap \mathcal{M}^+$, a complete EFTS can be constructed by

$$i \in \mathcal{I}^+ \Longrightarrow u = Kx, K \in \{K_j \mid j \in \mathcal{M}_i^+\}.$$

For a given feedback matrix K, let $\mathcal{I}^+(K)$ be the subset of $\mathcal{I}^+$ such that $u = Kx$ is an admissible control for the fault modes in $\mathcal{I}^+(K)$. If some controllers must be involved in EFTS, the following method can achieve this goal by choosing the related fault modes.

Method 2: Let $\mathcal{I}^+(K_\mathcal{J}) \triangleq \bigcup_{i \in \mathcal{J}} \mathcal{I}^+(K_i)$ for a given set $\mathcal{J} \in \mathcal{I}^+$. If $\mathcal{I}^+(K_\mathcal{J}) = \mathcal{I}^+$, then a complete EFTS can be constructed by

$$i \in \mathcal{I}^+ \Longrightarrow u = Kx, K \in \{K_j \mid i \in \mathcal{I}^+(K_j), j \in \mathcal{J}\}.$$

For $\forall i \in \mathcal{I}^+$, define $\mathcal{K}_i = \{K_{i_1}, \ldots, K_{i_l}\}$ as the control law set such that $u = K_{i_j} x$ is an admissible control for the given fault mode $i \in \mathcal{I}^+$.

Method 3: Let $\mathcal{H}$ be a hitting set of the collection $\{\mathcal{K}_i, i \in \mathcal{I}^+\}$, and let $\mathcal{H}_i \triangleq \mathcal{H} \bigcap \mathcal{K}_i$, then a complete EFTS can be constructed by

$$i \in \mathcal{I}^+ \Longrightarrow u = Kx, K \in \mathcal{H}_i.$$

The following method can find the smallest EFTS.

Method 4: Let $\mathcal{H}^*$ be the minimal hitting set of the collection $\{\mathcal{K}_i, i \in \mathcal{M}^+\}$, and let $\mathcal{H}_i^* \triangleq \mathcal{H}^* \bigcap \mathcal{K}_i$, then the smallest complete EFTS can be constructed by

$$i \in \mathcal{I}^+ \Longrightarrow u = Kx, K \in \mathcal{H}_i^*.$$

The following method contains controllers to maximize system performances most of the time.

Method 5: Let m_0 be the greatest element of $\mathcal{I}^+$, and m_k is the maximal element of $\{\mathcal{I}^+ \setminus \{\bigcup_{0 \le j \le k-1} \mathcal{I}^+(K_{m_j})\}\}$. When $\{\bigcup_{0 \le j \le k} \mathcal{I}^+(K_{m_j})\} = \mathcal{I}^+$, a complete EFTS can be constructed by

$$i \in \mathcal{I}^+ \Longrightarrow u = Kx, K \in \{K_{m_j} \mid i \in \mathcal{I}^+(K_{m_j})\}.$$

V. The Optimal Selection of EFTS

An EFTS may contain many admissible control laws for each fault mode $i \in \mathcal{I}^+$. This section will give four selection policies to obtain the optimal FTS.

Optimizing performance index and minimizing switching times of controllers are the common desired performances for most of control systems, and have been used to select a controller in the existing literature [3], [7]. It is well known that the selection based on optimizing performance index always chooses the corresponding controller for a given fault mode (Selection 1). But from Theorem 1, we know that the selection based on minimizing switching times of controllers always chooses the controller designed for the fault mode with more faults (Selection 2). However these kinds of control laws always lead to a high H_∞ gain. Sometimes we need a trade-off between them to meet different requirements (Selections 3 and 4).

Selection 1 emphasizes to optimize H_∞ gain, which is fairly simple and straight-forward. Selection 2 focuses on minimizing the probability of switching of controllers. When a low H_∞ gain and a low-switching are required simultaneously, Selection 3 makes a trade-off between them. Another intractable case is that when a control law is the optimal selection for the fault mode Λ_i, and it is still an admissible control for other fault modes but has very bad performances. Selection 4 based on minimizing the expedition-like cost of H_∞ gain is proposed to handle this kind of problem. Considered together, these selections provide a powerful solution for all kinds of demands.

Selection 1: For a given EFTS, let $\mathcal{K}_i = \{K_{i_1}, \ldots, K_{i_n}\}$ be the corresponding control law set of the fault mode $\Lambda_i, i \in \mathcal{I}^+$. Suppose $\gamma_{i_j}^*$ is the optimal H_∞ gain by solving inequality (2) for each control law $K_{i_j} \in \mathcal{K}_i$ for $1 \le j \le n$. Then the best control law to be selected for Λ_i is K_{i_j} with the minimal H_∞ gain $\gamma_{i_j}^*$.

Selection 2: For the fault mode $\Lambda_i, i \in \mathcal{I}^+$ and the corresponding control law set $\mathcal{K}_i$, let $\mathcal{SK}_{ij} = \mathcal{S}(\Lambda_i)\bigcap \mathcal{I}^+(K_j), \forall K_j \in \mathcal{K}_i$. If the control law for the current fault mode Λ_i is K_j, then K_j is also the admissible control law for the rest fault modes in $\mathcal{SK}_{ij}$. So the switching of controllers is not required from Λ_i to the fault modes in $\mathcal{SK}_{ij}$. Let $\Phi(i, K_j, t_0, t_n)$ be the probability of non-switching for Λ_i and K_j from time t_0 to the expected ending time t_n, i.e., $\Phi(i, K_j, t_0, t_n) = \sum_{j\in\mathcal{SK}_{ij}} Pr(\Lambda_j(t_n) \mid \Lambda_i(t_0))$, where $Pr(\Lambda_j(t_n) \mid \Lambda_i(t_0))$ is conditional probability that the fault mode is Λ_j at t_n when the fault mode is Λ_i at t_0. Then the best selection is the control law K_j^* with maximal $\Phi(i, K_j^*, t_0, t_n)$.

Selection 3: Define a cost function

$$J(\Lambda_i, K_j) = \alpha\frac{\gamma^*_{i_j}}{\gamma} + \beta(1 - \Phi(i, K_j, t_0, t_n))$$

where $\alpha \geq 0$ and $\beta \geq 0$ are constant numbers. For a expected running time interval $[t_0, t_n]$ and the initial fault mode Λ_i, the best selection is the control law K_j^* with the minimal $J(K_j^*)$.

Selection 4: For the fault mode Λ_i, the associated control law set $\mathcal{K}_i$, the non-switching set $\mathcal{SK}_{ij}$, the initial time t_0 and the expected ending time t_n, the expedition-like cost of each control law $K_j \in \mathcal{K}_i$ is defined by

$$E_i(K_j, t_0, t_n) = \frac{\sum_{m\in\mathcal{SK}_{ij}} \gamma^*_{m_j} Pr(\Lambda_m(t_n) \mid \Lambda_i(t_0))}{\sum_{k\in\mathcal{S}(\Lambda_i)} Pr(\Lambda_k(t_n) \mid \Lambda_i(t_0))} + \frac{\kappa \sum_{l\in\{\mathcal{S}(\Lambda_i)\setminus\mathcal{SK}_{ij}\}} \overline{\gamma} Pr(\Lambda_l(t_n) \mid \Lambda_i(t_0))}{\sum_{k\in\mathcal{S}(\Lambda_i)} Pr(\Lambda_k(t_n) \mid \Lambda_i(t_0))}$$

where κ is a weighted factor of minimizing switching times and $\overline{\gamma}$ is an upper bound of γ. Then the best selection is the control law K_j^* with the minimal expedition-like cost $E_i(K_j^*, t_0, t_n)$.

VI. Optimal FTS with Possible False FDI

The above four selections are used to guarantee system reliability and performances when actuator faults occur. But all of them are based on the correct FDI. However, false FDI exists widely and is inevitable. The following selection is proposed to enforce system reliability when there may be false FDI.

Selection 5: Suppose the fault mode is $\Lambda_i, i \in \mathcal{I}^+$ at time t, FDI sends an alert f_{i_j} at time $t_r = t + T$. f_{i_j} means that the fault mode $\Lambda_j \in \mathcal{S}(\Lambda_i)$ is detected at time t_r. Let $\mathbf{K}_i = \bigcup_{\Lambda_j\in\mathcal{S}(\Lambda_i)} \mathcal{K}_j$ and $\Psi(K_j, f_{i_j}) = \sum_{k\in\mathcal{SK}_{ij}} Pr(\Lambda_k(t_r) \mid f_{i_j}), \forall K_j \in \mathbf{K}_i$, where $Pr(\Lambda_k(t_r) \mid f_{i_j})$ is the known conditional probability according to reliable historic data. $\Psi(K_n, f_{i_j})$ is the probability of K_j being an admissible control law when the fault report is f_{i_j}. So the best selection is the control law K_j^* such that $\Psi(K_j^*, f_{i_j})$ is the maximum.

TABLE I
The set of $\mathcal{I}^+(K_i)$ and $\gamma^*_{i_j}$

	ab	$a\tilde{b}$	a	$\tilde{a}b$	$\tilde{a}\tilde{b}$	b
K_{ab}	0.4450	-	-	-	-	-
$K_{a\tilde{b}}$	0.4909	0.4992	0.5025	-	-	-
K_a	0.5006	0.5006	0.5006	-	-	-
$K_{\tilde{a}b}$	1.7406	1.8531	1.9425	1.7351	-	-
$K_{\tilde{a}\tilde{b}}$	2.4812	2.4820	2.4822	2.4820	2.4820	-
K_b	2.1645	-	-	2.1645	-	2.1645

Generally, FDI is correct most of the time. The following pre-selection scheme offers a test to evaluate FDI, and then it can be mixed with any other selections to make the system more reliable when there may be false FDI.

Selection 6: Suppose the fault mode is Λ_i at t_0, and the fault report is f_{i_j} at $t_r = t_0 + nT$. Assume $Pr(\Lambda_k(t_r))$ is the maximum for $\forall k \in \mathcal{S}^+(\Lambda_i)$. We define a function $D(f_{i_j}, t_r) \triangleq \frac{Pr(\Lambda_j(t_r))}{Pr(\Lambda_k(t_r))}$. Given a threshold ε, if $D(f_{i_j}, t_r) \geq \varepsilon$, then select a control law in the set $\mathcal{K}_j$, otherwise select a control law in the set $\mathcal{K}_k$.

On the basis of the above pre-selection, further selection can be constructed by mixing with other schemes. Here a simple selection is presented. When $\mathcal{K}_j$ is the basic set to select a control law from, define $H(K_j) \triangleq \sum_{q\in\mathcal{SK}_{ij}} Pr(\Lambda_q(t_r))$, then the best selection is the control law K_j^* such that $H(K_j^*)$ is the maximum. When $\mathcal{K}_k$ is the basic set to select a control law from, define $G(K_k) \triangleq \sum_{q\in\mathcal{SK}_{ik}} Pr(\Lambda_q(t_r)) + NPr(\Lambda_j(t_r))$, where N is a given number large enough, then the best selection is the control law K_k^* such that $G(K_k^*)$ is the maximum.

VII. Illustrative Example

We take the longitudinal control of F-18 aircraft [12] as an example to explain the proposed methods,

$$A = \begin{pmatrix} -1.175 & 0.9871 \\ -8.458 & -0.8776 \end{pmatrix}, B = \begin{pmatrix} -0.194 & -0.03593 \\ -19.29 & -3.803 \end{pmatrix},$$

$$B_w = \begin{pmatrix} 1 \\ 4 \end{pmatrix}, C = \begin{pmatrix} 0 & 4 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, D = \begin{pmatrix} 0 & 0 \\ 2 & 0 \\ 0 & 2 \end{pmatrix} D_w = \begin{pmatrix} 0.09 \\ -0.105 \\ 0.15 \end{pmatrix}$$

Normal states of actuators are denoted by a, b respectively, and loss of effectiveness are denoted by $\tilde{a}, \tilde{b}$ respectively. When actuator is outage, the corresponding symbol is omitted. Assume the ranges of loss of effectiveness are $0.2 \leq \tilde{a} \leq 1$ and $0.2 \leq \tilde{b} \leq 1$. So $\lambda_a \in \{0, 0.2, 1\}$, $\lambda_b \in \{0, 0.2, 1\}$.

Let the maximal allowable H_∞ gain $\gamma < 2.5$, then $\mathcal{I}$ can be divided by $\mathcal{I}^+$ and $\mathcal{I}^-$, where $\mathcal{I}^+ = \{ab, a\tilde{b}, a, \tilde{a}b, \tilde{a}\tilde{b}, b\}$. We can present Table I to show the associated $\mathcal{I}^+(K_i)$ and $\gamma^*_{i_j}$ for optimal control laws obtained from LMI 3.

Assume the failures of a and b are dependent and the

TABLE II
THE EFTSs BASED ON METHODS 1-5

	ab	$a\tilde{b}$	a	$\tilde{a}b$	$\tilde{a}\tilde{b}$	b
M1	K_a	K_a	K_a	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$	K_b
M3	$K_{\tilde{a}\tilde{b}}$ K_b	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$	K_b		
M2	K_{ab}	$K_{\tilde{a}b}$	$K_{\tilde{a}b}$	$K_{\tilde{a}b}$	$K_{\tilde{a}\tilde{b}}$	K_b
M5	$K_{\tilde{a}b}$ $K_{\tilde{a}\tilde{b}}$ K_b	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$ K_b		
M4	$K_{\tilde{a}\tilde{b}}$ K_b	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$ K_b	$K_{\tilde{a}\tilde{b}}$	K_b

TABLE III
THE FTSs FOR SELECTIONS 1-4

FDI	ab	$a\tilde{b}$	a	$\tilde{a}b$	$\tilde{a}\tilde{b}$	b
Slt1	K_{ab}	$K_{\tilde{a}b}$	$K_{\tilde{a}b}$	$K_{\tilde{a}b}$	$K_{\tilde{a}\tilde{b}}$	K_b
Slt2	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$	K_b
Slt3	$K_{\tilde{a}b}$	$K_{\tilde{a}b}$	$K_{\tilde{a}b}$	$K_{\tilde{a}b}$	$K_{\tilde{a}\tilde{b}}$	K_b
Slt4	$K_{\tilde{a}b}$	$K_{\tilde{a}b}$	$K_{\tilde{a}b}$	$K_{\tilde{a}b}$	$K_{\tilde{a}\tilde{b}}$	K_b

transition matrix W_{ab} in time interval T is

$$W_{ab} = \begin{pmatrix} 0.955 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.01 & 0.95 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.005 & 0.025 & 0.94 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.025 & 0 & 0 & 0.92 & 0 & 0 & 0 & 0 & 0 \\ 0.0008 & 0.018 & 0 & 0.0238 & 0.9 & 0 & 0 & 0 & 0 \\ 0.0001 & 0.0027 & 0.05 & 0.004 & 0.05 & 0.92 & 0 & 0 & 0 \\ 0.004 & 0 & 0 & 0.05 & 0 & 0 & 0.93 & 0 & 0 \\ 0.00009 & 0.004 & 0 & 0.002 & 0.04 & 0 & 0.06 & 0.89 & 0 \\ 0.00001 & 0.0003 & 0.01 & 0.0002 & 0.01 & 0.08 & 0.01 & 0.11 & 1 \end{pmatrix}$$

Then we can construct the EFTSs as shown in Table II by using Methods 1-5.

Note 2: $M1$ is short for Method 1 and others are the same; $\mathcal{J} = \{ab, \tilde{a}b, \tilde{a}\tilde{b}, b\}$ for $M2$; the hitting set $\mathcal{H} = \{K_a, K_{\tilde{a}\tilde{b}}, K_b\}$ for $M3$; $\mathcal{H}^* = \{\tilde{a}\tilde{b}, b\}$ for $M4$; $\{m_0 = ab, m_1 = \tilde{a}b, m_2 = \tilde{a}\tilde{b}, m_3 = b\}$ for $M5$; $M1 = M3$ and $M2 = M5$ in this simple example.

It is shown that there may be many admissible control laws for some fault modes. So it is necessary to select the optimal one according to different demands. Then four FTSs are constructed in Table III according to Selections 1 - 4 based on EFTS M5.

Note 3: Slt1 is short for Selection 1 and others are the same; assume all actuators are normal at $t_0 = 0$ and give the expected ending time $t_n = 10T$ for Slt2, Slt3 and Slt4; the cost function for Slt3 is $J(ab, K_j) = \frac{\gamma^*_{0_j}}{2.5} + 3(1 - \Psi(ab, K_j, t_0, t_n))$; the weighted factor is $\kappa = 10$ for Slt4.

Now we design a simulation to verify the proposed fault tolerant control approaches. Suppose the system plans to run from $t_0 = 0T$ to $t_f = 10T$ and the actuators are normal at t_0. Then the first actuator is loss of effectiveness $a = 0.3$ at $t = 2T$, and it is outage $a = 0$ at $t = 5T$. The disturbance is chosen as

$$w(t) = \begin{cases} 1 + 0.2r(t), & 0.5 \leq t < 3.5 \\ -1 + 0.2r(t), & 3.5 \leq t < 7 \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

where $r(t)$ produces a random number in [0,1].

Selections 1-4 are used to construct fault-tolerant control strategies. The result is compared with that of Adaptive Reliable H_∞ controller (short for ARH in Fig. 1 and Table IV) in [12]. The FTSs from Slt1 and Slt4 are the same in this simple example. Fig. 1 shows the response curve of pitch rate $q(t)$ for different control methods in this faulty case. The actual H_∞ gain $\frac{\int_{t_0}^{t_n} z^T z dt}{\int_{t_0}^{t_n} w^T w dt}$ is shown in Table IV. It is observed that our methods have better performance to reject exogenous disturbance and accommodate actuator faults. The design requirement that H_∞ gain $\gamma < 2.5$ is fully satisfied by our methods but is violated by Adaptive Reliable H_∞ Controller. The H_∞ gains of Slt3 and Slt4 are a little better than that of Slt2. Selections 2, 3 and 4 incorporating Markov model require only two times of switching, but Selection 1 without considering Markov model requires three times of switching. Basically these results reflect the design motivations.

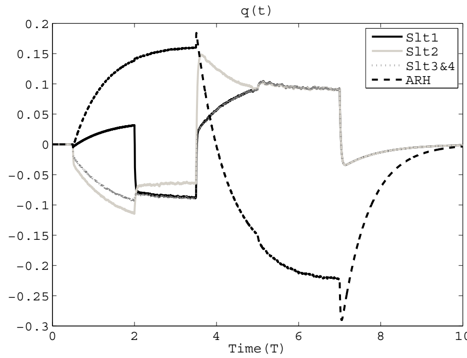

Fig. 1. Response Curve of $q(t)$ for Controllers of Selections 1-4 and Adaptive Reliable H_∞ Controller

TABLE IV
ACTUAL H_∞ GAINS OF SELECTIONS 1-4 AND ARH

	Slt1	Slt2	Slt3	Slt4	ARH
Gains	1.6973	1.7220	1.7207	1.7207	3.1985

TABLE V
THE CONDITION PROBABILITY OF EACH FDI FOR NORMAL MODE

	ab	$a\tilde{b}$	a	$\tilde{a}b$	$\tilde{a}\tilde{b}$	b	others
f_{0_0}	1	0	0	0	0	0	0
f_{0_1}	0.02	0.95	0.01	0	0.02	0	0
f_{0_2}	0.01	0.04	0.94	0	0	0	0.01
f_{0_3}	0.01	0.02	0	0.93	0.01	0.03	0
f_{0_4}	0	0.04	0	0.04	0.90	0	0.02
f_{0_5}	0.01	0	0	0.05	0	0.91	0.03

Suppose there may be false FDI to make a wrong detection for fault modes. If the fault mode is normal ab at time t_0, then suppose Table V shows the probability of each detection at time $t_r = t_0 + T$. The total fault modes are marked by $\{0, 1, 2, 3, 4, 5\}$, as shown in Table V.

Two FTSs are constructed in Table VI according to Selections 5 and 6 when there may be false FDI.

Note 4: Slt5 and Slt6 are based on EFTS M5; assume all actuators are normal at $t_0 = 0$ and the expected ending time is $t_n = 10T$; the FDI report time is $t_r = T$ for Slt5; let $\varepsilon = 0.10$ and $N = 10$ for Slt6.

In order to test the fault tolerant controllers incorporating

TABLE VI
THE FTSS FOR SELECTIONS 5 AND 6

FDI	$a\tilde{b}$	a	$\tilde{a}b$	$\tilde{a}\tilde{b}$	b
Slt5	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$	K_b	$K_{\tilde{a}\tilde{b}}$	K_b
Slt6	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$	$K_{\tilde{a}\tilde{b}}$	K_b

TABLE VII
TIMES OF FTCS SATISFYING $\gamma < 2.5$

	Slt1	Slt2	Slt3	Slt4	Slt5	Slt6
Success	8978	9186	9000	9000	9347	9186

with false FDI, assume actuator faults occur randomly during the running time 0 to $10T$. The accuracy of FDI is shown in Table V. We make an assumption that at most one fault mode is activated during one running. When the system runs 10000 times, the number of FTCS successfully satisfying H_∞ gain $\gamma < 2.5$ is shown in Table VII. It shows that the selections incorporating with false FDI are more successful to cope with potential false detection.

VIII. CONCLUSION

This paper presents a H_∞ control design for a linear system with actuator faults by selecting a pre-computed control law online. The actuator faults are modeled by three states and Markov chain. Five methods to construct a small set of controllers from the set of all controllers are given for different requirements. Four selection schemes are designed to get a proper controller from a bank of admissible controllers. Another two selection schemes are motivated by coping with the case when there may be false FDI. These six selections combining with the five EFTSs offer a powerful solution for all kinds of applications in the framework of H_∞ control. Finally, a numerical example is employed to illustrate the proposed methods and shows encouraging results.

However, the switching of controllers from one fault mode to another can induce transients. This kind of transient is harmful and easily makes the system unstable due to saturating the actuators and FDI delay. It is a critical problem for applications of the online selection approach of FTCS and will be the subject of future researches.

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