2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

A Two-Port Approach to Networked Feedback Stabilization

Guoxiang Gu§ and Li Qiu†

***Abstract*— Modeling and control of networked feedback systems are studied in this paper. Motivated by the two-port circuit theory, new network models are proposed which take inter-channel interference into consideration. Quadratic stability and stabilization are employed to study stability of the closed-loop system over the networked channel leading to the framework of $\mathcal{H}_\infty$ control. This paper will focus on formulation for networked stabilization in presence of the uncertainties induced by imperfect channels and quantization errors.**

I. Introduction

Networked feedback control (NFC) has received great attention recently, witnessed by the special issues in [1], [2] and ever-increasing volume of research papers in archival journals and professional conferences. While many research results exist, this paper is motivated by feedback control under limited information as pioneered in [8], [10], [20]. Most of the existing work in this problem area focuses on derivation of bounds [3], [4], [12], [13], [14], [19] on the minimal data rate required to stabilize an NFC system. See also [4], [11], [15], [16], [17], [18] and references therein.

An NFC system differs from the traditional feedback system in the presence of a network between the plant and controller, which serves as the communication media and can be wireless. Because of the bandwidth limit and because of the channel fading, information exchange between the plant and controller is not only limited but also distorted. How to mitigate the information loss and distortion has been a challenging issue in design of NFC systems. A simple solution is to increase the network resource including bandwidth and signal-to-noise ratio (SNR). However perfect information exchange requires infinite network resource that is unrealistic. A more sophisticated solution is to design the feedback control law capable of tolerating uncertainties induced by imperfect channels and quantization errors. The latter gives rise to the following two research problems. The first is how to model the network employed in NFC systems, and the second is the characterization of the minimum network resource for a given NFC system. We are particularly attracted to the resemblance of the communication network in the NFC system to the two-port network of electric circuits. Indeed both are used "over a distance" and admit the same external structure. While the two-port circuit network models the transport of energy, the communication network serves the information exchange. For this reason we propose to use two-port descriptions from the circuit theory [6], [7], [9] in modeling the communication network between the plant and controller. Specifically the limited and distorted information will be modeled by the ideal channel or network corrupted by nonlinear and time-varying perturbations. In fact four different network models will be proposed to characterize the uncertainties induced by imperfect channels and quantization errors, which are parallel to those in the two-port circuit network. In this paper we consider deterministic perturbation as the first attempt. By restricting perturbations to sector bounded uncertainty, we are able to develop an independent theory and provide solution methods based on $\mathcal{H}_\infty$ control for design of multi-input/multi-output (MIMO) NFC systems under output feedback control. Our work provides the characterization of the network resource in terms of the stability margin.

This paper will begin with the two-port circuit theory in Section 2 and introduce the notion of stability margin in connection with the network resource. In Section 3 new network models are proposed to classify the uncertainties induced by imperfect channels and quantization errors. The paper will be concluded in Section 4. The notation adopted in this paper is standard and will be introduced as we proceed.

*The research is supported in part by the US Air Force with grant number FA9550-08-1-0440, and by the Research Grants Council of Hong Kong with grant number 619209.

§ Department of Electrical and Computer Engineering, Louisiana State University, Baton Rouge, LA 70803-5901.

† Department of Electrical and Electronic Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong.

II. A Two-Port Approach

NFC systems are large scale distributed systems in which the information exchange between subsystems is via communication networks. There are many different formulations for networked control problems. One basic setup is shown on left of Fig. 1.

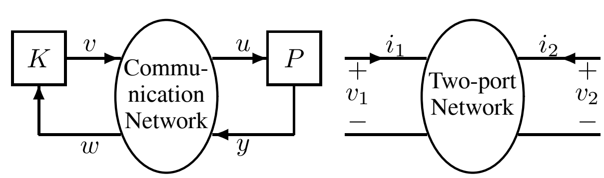

Fig. 1 NFC system (left) and two-port network (right)

The difference between this configuration and the standard feedback system lies in the presence of a network between the plant and the controller. The information exchange between the plant P and controller K is via the communication network. In general, the up-link transmission from y to w, and the down-link transmission from v to u may use the same network. Hence there exists interference between the up-link and down-link channels. The networked control aims at design of a stabilizing controller so that the feedback control system performs satisfactorily.

A natural question arises: how can we model the communication network connecting the two? This has been an unsettled issue for a long time. Since the external structure of the communication network has a remarkable resemblance to a two-port electrical circuit network, used for transport of energy, as shown on right of Fig. 1, we propose to use two-port descriptions in the circuit theory to model the communication network between the plant and controller. Another similarity of an electrical two-port and the communication network in an NFC system is that both are often used "over a distance": the former is often used to model a transmission line for distance energy transport while the latter is often used for remote information exchange, which provides a justification.

The study of electrical two-ports is now standard in circuit theory. See textbooks [6], [7], [9] and handbook [5]. An electrical two-port has four external variables, which are two port currents $I_1(s), I_2(s)$ and two port voltages $V_1(s), V_2(s)$, represented in the s-domain under Laplace transform. These variables are not completely independent but rather related, dependent on the internal structure of the circuit. Assume linear and time-invariant (LTI) circuits. Then the relationships of these variables may be represented in several different ways. We restrict to only those of the two-port applicable to modeling the communication network in this paper, although other representations can also be potentially useful.

The first is the *admittance description*. If we set the port voltages $V_1(s)$ and $V_2(s)$ as independent variables, the port currents $I_1(s)$ and $I_2(s)$ can be expressed as

$$\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} Y_{11}(s) & Y_{12}(s) \\ Y_{21}(s) & Y_{22}(s) \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix},$$

where $Y(s) = [\ Y_{ij}(s)\ ]_{i,j=1,1}^{2,2}$ is called the admittance matrix or the Y-matrix. The elements of $Y(s)$ are called admittance parameters or Y-parameters.

The second is the *impedance description*. If we set the port currents $I_1(s)$ and $I_2(s)$ as independent variables, the port voltages $V_1(s)$ and $V_2(s)$ can be expressed as

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} Z_{11}(s) & Z_{12}(s) \\ Z_{21}(s) & Z_{22}(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix},$$

where $Z(s) = [\ Z_{ij}(s)\ ]_{i,j=1,1}^{2,2}$ is called the impedance matrix or the Z-matrix. The elements of $Z(s)$ are called impedance parameters or Z-parameters. Apparently, there holds $Y(s)^{-1} = Z(s)$.

The third is the *Transmission A-matrix description*. If we consider port 1 and port 2 as the output port and the input port respectively, and set the input port variables $V_2(s)$ and $I_2(s)$ as independent variables, then the output port variables $V_1(s)$ and $I_1(s)$ can be expressed as

$$\begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix} = \begin{bmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{bmatrix} \begin{bmatrix} V_2(s) \\ I_2(s) \end{bmatrix},$$

where $T_A(s) = [\ A_{ij}(s)\ ]_{i,j=1,1}^{2,2}$ is called transmission A-matrix. The elements of $T_A(s)$ are called transmission A parameters.

The final one is the *Transmission B-matrix description*. If we reverse the position of the input port and output port, then $V_1(s)$ and $I_1(s)$ become independent variables, and $V_2(s)$ and $I_2(s)$ can be expressed as

$$\begin{bmatrix} V_2(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} B_{11}(s) & B_{12}(s) \\ B_{21}(s) & B_{22}(s) \end{bmatrix} \begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix},$$

where $T_B(s) = [\ B_{ij}(s)\ ]_{i,j=1,1}^{2,2}$ is called transmission B-matrix. The elements of $T_B(s)$ are called transmission B parameters. Apparently, there holds $T_B(s) = T_A(s)^{-1}$.

Coming back to the communication network in Fig. 1, we name the two ports in the network as the controller port and the plant port. Each port has an input variable and an output variable. Here we need to have several leaps of faith. First, we will deal with discrete-time systems instead of continuous-time circuits. So the frequency domain is now z-domain instead of s-domain. Secondly, since we will deal with nonlinear and stochastic parameters, it will be more convenient for us to work in the time domain. Thus the elements of the Y, Z, T_A, and T_B matrices become operators on discrete-time signal spaces. Thirdly, the port variables are vector-valued and no longer voltages and currents. We will analogously treat the input variables v and y as voltages and the output variables u and w as currents. After the translation, we obtain the descriptions of the communication network as

$$\begin{bmatrix} w \\ u \end{bmatrix} = Y \begin{bmatrix} v \\ y \end{bmatrix}, \qquad \begin{bmatrix} v \\ y \end{bmatrix} = Z \begin{bmatrix} w \\ u \end{bmatrix}, \tag{1}$$

$$\begin{bmatrix} v \\ w \end{bmatrix} = T_A \begin{bmatrix} y \\ u \end{bmatrix}, \quad \begin{bmatrix} y \\ u \end{bmatrix} = T_B \begin{bmatrix} v \\ w \end{bmatrix}. \tag{2}$$

In the ideal situation, the network transmits signals faithfully and instantaneously, yielding $u = v$ and $w = y$. Let m and p be dimension of u and y, respectively. This means that nominally, the Y-matrix, Z-matrix, A-matrix, B-matrix all take trivial values

$$Y_0 = T_{B_0} = \begin{bmatrix} 0 & I_p \\ I_m & 0 \end{bmatrix}, \quad T_{A_0} = Z_0 = \begin{bmatrix} 0 & I_m \\ I_p & 0 \end{bmatrix}$$

where I_k is the identity matrix of size k. In practice, transmission errors and interference occur. We deem the errors and interferences as generated by perturbations on the Y, Z, T_A, and T_B matrices, giving rise to

$$\begin{array}{ll} Y = Y_0 + \Delta, & Z = Z_0 + \Delta, \\ T_A = T_{A_0} + \Delta, & T_B = T_{B_0} + \Delta. \end{array} \tag{3}$$

By partitioning Δ compatibly, the above gives rise to the same form of channel representation:

$$J_\Delta = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} + \Delta = \begin{bmatrix} \Delta_{11} & I + \Delta_{12} \\ I + \Delta_{21} & \Delta_{22} \end{bmatrix} \tag{4}$$

where Δ_{12} and Δ_{21} are used to model the transmission distortion and Δ_{11} and Δ_{22} are used to model the inter-channel interference. Although we use the same symbol Δ to denote the perturbation of the four matrices, it has different physical meanings when it is used in association with the different two-port descriptions and it enters the networked system in different ways. We will develop a parallel theory for all these descriptions. The user will need to make a choice of which description to use as his/her model based on the understanding of the network under consideration. In this paper we focus on deterministic modeling of the network channel, and assume that the perturbation Δ as a double-input/double-output (DIDO) possibly nonlinear, time-varying, and dynamic system with norm bound

$$\begin{aligned} \|\Delta\|_\infty &:= \sup_{\|s\|_2 \neq 0} \frac{\|\Delta s\|_2}{\|s\|_2} \leq \delta, \\ \|s\|_2 &:= \left(\sum_{t=0}^{\infty} \|s(t)\|^2 \right)^{1/2}, \end{aligned} \tag{5}$$

for some $\delta > 0$ where $s(t)$ has dimension $m + p$ with time t integer valued, and $\|\cdot\|$ denotes the Euclidean norm.

The aforementioned four network models are important because they not only describe the network channel, but also characterize the loss and distortion of the information exchange between the plant and controller. Moreover they provide a rough measure to the network resource through δ^{-1} with δ the norm bound in (5). Roughly speaking, large δ^{-1} (i.e., small δ) implies more network resource at the expense of high SNR and large bandwidth in transmission and receiving of signals, while small δ^{-1} (i.e., large δ) implies less network resource consumed. Network control aims at design of a controller $K(z)$ that stabilizes the underlying networked feedback system in Fig. 1 for the network model in which Δ is bounded by $\delta > 0$ as in (5). Because of the scarce of the network resource, there is an incentive to maximize the norm bound δ subject to stability of the NFC system. The associated maximum, denoted by $\delta_{\max}$, is termed *stability margin*. The following small gain result is now well known [21].

Lemma 1 *Consider the feedback system in* Fig. 2 *where the transfer matrix $T(z)$ represents an* LTI *system and Δ represents a linear/nonlinear and time varying dynamic system. Denote $\overline{\sigma}(\cdot)$ the maximum singular value. Then the feedback system is stable for all possible Δ satisfying the norm bound in* (5)*, if and only if $\|T\|_{\mathcal{H}_\infty} := \sup_{|z|>1} \overline{\sigma}[T(z)] < \delta^{-1}$.*

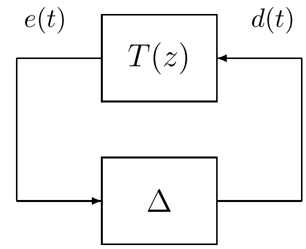

Fig. 2 Block diagram of the feedback system

III. Network Models and Their Analysis

Under the two-port approach, the NFC system admits the following block diagram:

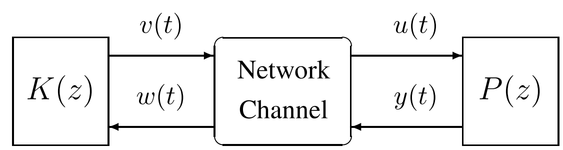

Fig. 3 NFC system as a two-port network

In the ideal case of no channel distortion or the channel having unlimited bandwidth,

$$\begin{bmatrix} w(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ y(t) \end{bmatrix} =: J_0 \begin{bmatrix} v(t) \\ y(t) \end{bmatrix}. \tag{6}$$

When distortion is present, the nominal channel gain J_0 is assumed to be perturbed to $\widehat{J}_\Delta(t)$ that is time-varying and represented by a lower linear fractional transform (LFT) $\widehat{J}_\Delta(t) = \mathcal{F}_\ell(J, \Delta_t)$, i.e.,

$$\widehat{J}_\Delta(t) = J_{11} + J_{12}\Delta_t(I - J_{22}\Delta_t)^{-1}J_{21} \tag{7}$$

where $\overline{\sigma}(\Delta_t) \leq \delta$ for some $\delta > 0$ at each time t. The nonlinearity and dynamics of Δ_t are removed to simplify the presentation, but our results in this paper hold for the general case. The unstructured form of Δ_t is attributed to the interference between input and output channels of the plant/controller. The matrix J is specified by

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}, \quad J_{11} = J_0. \tag{8}$$

This section aims to derive the expression of J for the LFT in (7) under four different two-port descriptions proposed in (1) and (2), and analyze their corresponding network models.

The distorted channel and the plant/controller form a closed-loop. Denote q as the unit advance operator. On one hand the forward path can be described by

$$\begin{aligned} \begin{bmatrix} v(t) \\ y(t) \end{bmatrix} &= H(q) \begin{bmatrix} w(t) \\ u(t) \end{bmatrix}, \\ H(z) &= \operatorname{diag}[K(z), P(z)]. \end{aligned} \tag{9}$$

On the other hand the feedback path is described by

$$\begin{bmatrix} w(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} \widehat{J}_{11}(\Delta) & \widehat{J}_{12}(\Delta) \\ \widehat{J}_{21}(\Delta) & \widehat{J}_{22}(\Delta) \end{bmatrix} \begin{bmatrix} v(t) \\ y(t) \end{bmatrix} \tag{10}$$

where $\widehat{J}_\Delta := \begin{bmatrix} \widehat{J}_{ij}(\Delta) \end{bmatrix}$ is partitioned compatibly with J_0. See the illustration in the next figure.

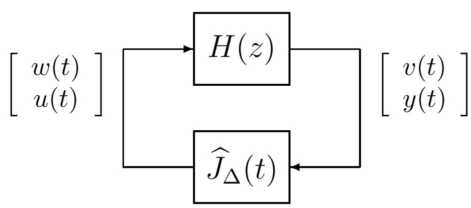

Fig. 4 Two port network as single feedback loop

It follows that feedback stability amounts to stability and invertibility of $(I - \widehat{J}_\Delta H)$ that is not easy to study, if $\widehat{J}_\Delta$ is nonlinear and time-varying. By recalling $\widehat{J}_\Delta(t)$ in (7), the above figure can be converted equivalently to the next block diagram:

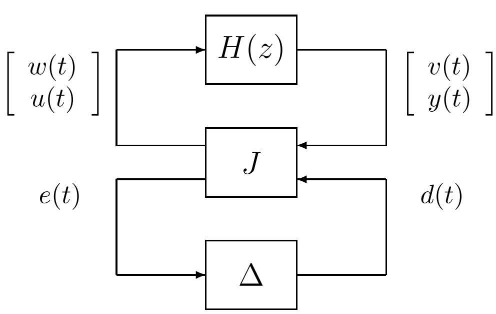

Fig. 5 Equivalent LFT feedback system

As a result Δ can be isolated by defining an upper LFT $T(z) = \mathcal{F}_u[J, H(z)]$, i.e.,

$$T(z) = J_{22} + J_{21}H(z)[I - J_{11}H(z)]^{-1}J_{12} \tag{11}$$

that is the transfer matrix from $d(t)$ to $e(t)$. As such the feedback loop in Fig. 4 is equivalent to the one in Fig. 2 of Section 2. Hence if $T(z)$ is stable, then the two port network is stabilized, if and only if $\|T\|_{\mathcal{H}_\infty} < \delta^{-1}$, in light of Lemma 1, in the case of nonlinear/dynamic Δ_t.

If $K(z)$ in Fig. 3 is kept intact in the feedback path, and $P(z)$ and Δ_t are lumped together to form a perturbed plant, then the uncertain plant for Fig. 3 is found to be

$$P_\Delta = \mathcal{F}_\ell(\widehat{J}_\Delta, P) = \widehat{J}_{11} + \widehat{J}_{12}P(I - \widehat{J}_{22}P)^{-1}\widehat{J}_{21} \tag{12}$$

where $\widehat{J}_\Delta$ as partitioned in (10) is compatible with that of J in (8), yielding the feedback loop (a) in Fig. 6.

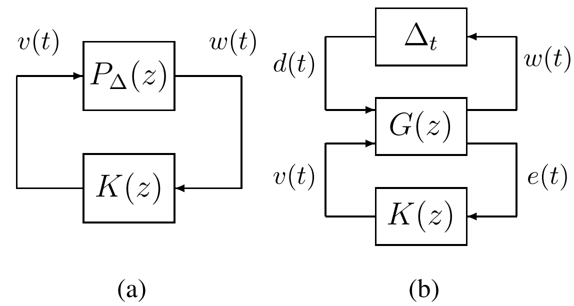

Fig. 6 Equivalent LFT in alternative forms

Because $\widehat{J}_\Delta$ is fractional of Δ_t, $P_\Delta(z)$ is fractional of Δ_t as well. That is, there exists $G(z) = [\ G_{ij}(z)\ ]_{i,j=1,1}^{2,2}$ that is a function of $P(z)$ such that

$$P_\Delta(q) = G_{22}(q) + G_{21}(q)\Delta_t[I - G_{11}(q)\Delta_t]^{-1}G_{12}(q),$$

or $P_\Delta(q) = \mathcal{F}_u[G(q), \Delta_t]$. A controller $K(z)$ can then be synthesized such that $T(z) = \mathcal{F}_\ell[G(z), K(z)]$, i.e.,

$$T = G_{11} + G_{12}K[I - G_{22}K]^{-1}G_{21} \tag{13}$$

is internally stable that in turn leads to the feedback system in Fig. 2. It should be clear that the above $T(z)$ is identical to the one in (11). Thus quadratic stability is again ensured by the condition $\|T\|_{\mathcal{H}_\infty} < \delta^{-1}$. The maximum of the channel distortion bound δ under stability constraint is the same as the stability margin $\delta_{\max}$ leading to $\mathcal{H}_\infty$ optimal control.

In the rest of this section we will derive the network or channel models corresponding to the four different two-port descriptions in (1) $\sim$ (4).

Case 1 Admittance description: The channels involve additive distortion by (1) and (3). Thus

$$\widehat{J}_\Delta = J_0 + \Delta = \begin{bmatrix} \Delta_{11} & I + \Delta_{12} \\ I + \Delta_{21} & \Delta_{22} \end{bmatrix}. \tag{14}$$

The feedback system over additively distorted channels is illustrated in Fig. 7 next.

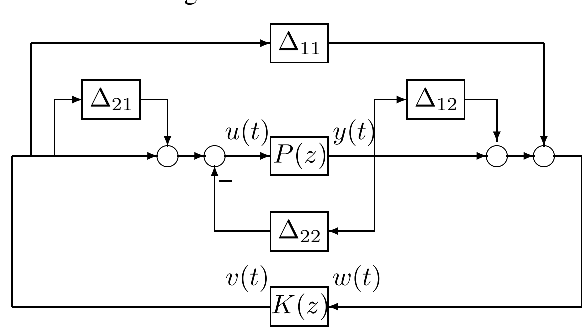

Fig. 7 NFC system under admittance description

The presence of $(\Delta_{11}, \Delta_{22})$ represents the interference between the input/output channels of the plant/controller or inter-channel interference. To write $\widehat{J}_\Delta$ in form of LFT in (7), we identify

$$J_{11} = J_0, \quad J_{12} = I, \quad J_{21} = I, \quad J_{22} = 0. \tag{15}$$

The channel distortion in Case 1 can be motivated by logarithmic quantization at both the input and output of the plant which induces multiplicative error. Specifically if interferences between input and output channels of the plant are ignored, then $\Delta_{11} = 0$, $\Delta_{22} = 0$.

To compute $T(z)$ in (11), we note that $J_{11} = J_0$. Direct calculation shows that

$$T = H \begin{bmatrix} (I - PK)^{-1} & P(I - KP)^{-1} \\ K(I - PK)^{-1} & (I - KP)^{-1} \end{bmatrix}. \tag{16}$$

Denoting $T(z)$ by $T_1(z)$ to signify Case 1 yields

$$T_1 = \begin{bmatrix} K(I - PK)^{-1} & KP(I - KP)^{-1} \\ PK(I - PK)^{-1} & P(I - KP)^{-1} \end{bmatrix}. \tag{17}$$

Networked stabilization requires

$$\overline{\sigma}(\Delta_t)\|T_1\|_{\mathcal{H}_\infty} \leq \delta \|T_1\|_{\mathcal{H}_\infty} < 1.$$

In Case 1, the equivalently perturbed plant $P_\Delta(z)$ is given by $P_\Delta(z) = \mathcal{F}_\ell(\widehat{J}_\Delta, P)$, leading to

$$P_\Delta = \Delta_{11} + (I + \Delta_{12})P(I - \Delta_{22}P)^{-1}(I + \Delta_{21})$$

It can be verified that $P_\Delta(z) = \mathcal{F}_u[G(z), \Delta]$ with

$$G(z) = \left[\begin{array}{c|c} G_{11} & G_{12} \\ \hline G_{21} & G_{22} \end{array}\right] = \left[\begin{array}{cc|c} 0 & 0 & I \\ 0 & P & P \\ \hline I & P & P \end{array}\right]. \tag{18}$$

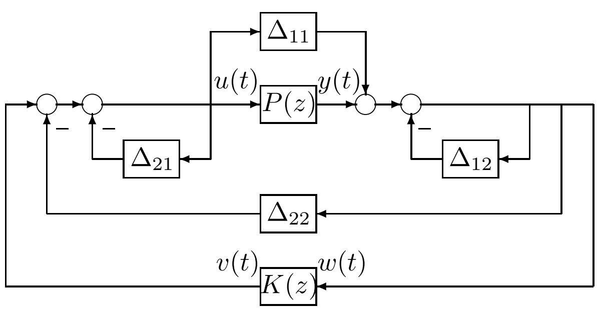

Fig. 8 NFC system under impedance description

Case 2 Impedance description: Channels involve feedback distortion. By (1) and (3),

$$\widehat{J}_\Delta = (J_0 + \Delta)^{-1} = J_0 - J_0\Delta(I + J_0\Delta)^{-1}J_0, \tag{19}$$

by $J_0^{-1} = J_0$. Clearly with

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} J_0 & -J_0 \\ J_0 & -J_0 \end{bmatrix},$$

the LFT form in (7) holds. The NFC system over channels with feedback distortion is illustrated in Fig. 8 where Δ_{11} and Δ_{22} are the inter-channel interference.

To compute the corresponding $T(z)$ in (11), denoted by $T_2(z)$, note that $J_0 = J_0^3$. Therefore $T_2(z) = -J_0^3 - J_0T_1(z)J_0 = -J_0(J_0 + T_1)J_0$, leading to

$$T_2 = -\begin{bmatrix} P(I - KP)^{-1} & (I - PK)^{-1} \\ (I - KP)^{-1} & K(I - PK)^{-1} \end{bmatrix}. \tag{20}$$

In upper LFT form, $P_\Delta(z) = \mathcal{F}_u[G(z), \Delta]$ with

$$G(z) = \left[\begin{array}{c|c} G_{11} & G_{12} \\ \hline G_{21} & G_{22} \end{array}\right] = \left[\begin{array}{cc|c} 0 & I & I \\ I & P & P \\ \hline I & P & P \end{array}\right]. \tag{21}$$

Case 3 Transmission A description: The relations in (2) and (3) can be brought to

$$\begin{bmatrix} w(t) \\ u(t) \end{bmatrix} = \widehat{J}_\Delta(\Delta) \begin{bmatrix} v(t) \\ y(t) \end{bmatrix}, \quad \widehat{J}_\Delta(\Delta) = \mathcal{F}_\ell(J, \Delta).$$

By straightforward calculation, $\widehat{J}_\Delta(\Delta)$ is obtained as

$$\begin{aligned} \widehat{J}_\Delta &= \begin{bmatrix} \Delta_{22} \\ I \end{bmatrix} (I + \Delta_{12})^{-1} \begin{bmatrix} I & -\Delta_{11} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & I + \Delta_{21} \\ 0 & 0 \end{bmatrix}. \end{aligned} \tag{22}$$

The above identifies the LFT representation $\widehat{J}_\Delta(\Delta) = \mathcal{F}_\ell(J, \Delta)$ by $J_{11} = J_{21} = J_0$ and

$$J_{12} = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}, \quad J_{22} = -\begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}.$$

See Fig. 9 next.

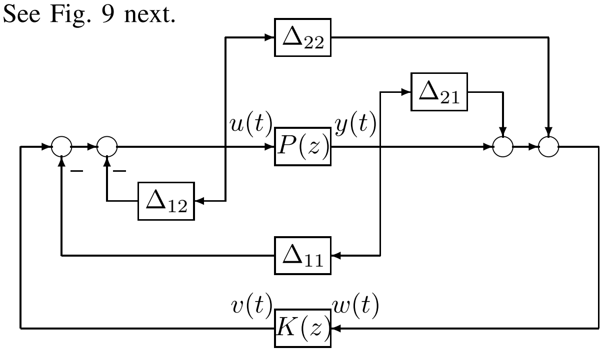

Fig. 9 NFC system under transmission A description

We use (17) to compute $T(z)$ in (11), denoted by $T_3(z)$:

$$\begin{aligned} T_3 &= -\begin{bmatrix} 0 & 0 \\ -I & 0 \end{bmatrix} + \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} T_1 \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \\ &= \begin{bmatrix} P \\ I \end{bmatrix} (I - KP)^{-1} \begin{bmatrix} -I & K \end{bmatrix}. \end{aligned} \tag{23}$$

Let $P = NM^{-1}$ be right coprime factorization. The equivalently perturbed plant in (12) can be written as

$$P_\Delta = \widehat{J}_{11} + \widehat{J}_{12}N(M - \widehat{J}_{22}N)^{-1}\widehat{J}_{21}.$$

By the expressions in (22) and after lengthy calculation, we obtain $P_\Delta = N_\Delta M_\Delta^{-1}$, specified by

$$P_\Delta = (\Delta_{11} + (I + \Delta_{12})P)(\Delta_{22}P + I + \Delta_{21})^{-1}.$$

In upper LFT form, $P_\Delta(z) = \mathcal{F}_u[G(z), \Delta]$ with

$$G(z) = \left[\begin{array}{c|c} G_{11} & G_{12} \\ \hline G_{21} & G_{22} \end{array}\right] = \left[\begin{array}{cc|c} -P & 0 & P \\ -I & 0 & I \\ \hline -P & I & P \end{array}\right]. \quad (24)$$

If the interference is ignored or $\Delta_{11} = 0$ and $\Delta_{22} = 0$, then $P_\Delta = (I + \Delta_{12})P(I + \Delta_{21})^{-1}$.

Case 4 Transmission B description: We have

$$\begin{bmatrix} w(t) \\ u(t) \end{bmatrix} = \widehat{J}_\Delta(\Delta) \begin{bmatrix} v(t) \\ y(t) \end{bmatrix}, \quad \widehat{J}_\Delta(\Delta) = \mathcal{F}_\ell(J, \Delta),$$

by (2) and (3) where $\widehat{J}_\Delta(\Delta)$ is obtained as

$$\begin{aligned} \widehat{J}_\Delta &= \begin{bmatrix} I \\ \Delta_{22} \end{bmatrix} (I + \Delta_{12})^{-1} \begin{bmatrix} -\Delta_{11} & I \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & 0 \\ I + \Delta_{21} & 0 \end{bmatrix}. \end{aligned} \quad (25)$$

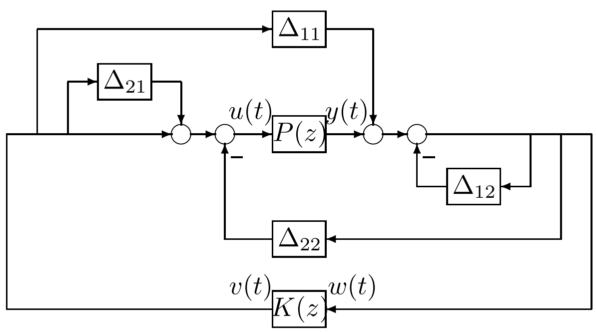

Fig. 10 NFC system in transmission B description

For the corresponding NFC system illustrated above, $\widehat{J}_\Delta = \mathcal{F}_\ell(J, \Delta)$ is specified by $J_{11} = J_0$, $J_{21} = I$, and

$$J_{12} = \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix}, \quad J_{22} = -\begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}.$$

It follows that $T_4(z)$, to signify Case 4, is given by

$$\begin{aligned} T_4 &= -\begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix} + T_1(z) \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix} && (26) \\ &= \begin{bmatrix} K \\ I \end{bmatrix} (I - PK)^{-1} \begin{bmatrix} -I & P \end{bmatrix}. && (27) \end{aligned}$$

To compute the equivalently perturbed plant, let $P = \tilde{M}^{-1}\tilde{N}$ be left coprime factorization. The equivalently perturbed plant as given in (12) can be written as

$$P_\Delta = \widehat{J}_{11} + \widehat{J}_{12}(\tilde{M} - \tilde{N}\widehat{J}_{22})^{-1}\tilde{N}\widehat{J}_{21} = \tilde{M}_\Delta^{-1}\tilde{N}_\Delta.$$

After length calculation and noting $P = \tilde{M}^{-1}\tilde{N}$ lead to

$$P_\Delta = (I + \Delta_{12} - P\Delta_{22})^{-1}\left(P(I + \Delta_{21}) - \Delta_{11}\right)$$

In upper LFT form, $P_\Delta = \mathcal{F}_u(G, \Delta)$ with

$$G = \left[\begin{array}{c|c} G_{11} & G_{12} \\ \hline G_{21} & G_{22} \end{array}\right] = \left[\begin{array}{cc|c} 0 & 0 & I \\ -I & P & P \\ \hline -I & P & P \end{array}\right]. \quad (28)$$

IV. Conclusion

NFC systems are investigated in this paper focusing on network or channel modeling, and problem formulation under $\mathcal{H}_\infty$ framework. Due to the page limit, the $\mathcal{H}_\infty$ solution to the networked stabilization problem as formulated in this paper will be presented elsewhere. Although only deterministic channel distortions are considered, our framework can be adapted easily to accommodate the stochastic ones such packet drop etc.

References

[1] Special Issue on Networked Control Systems, *IEEE Trans. Automat. Contr.*, vol. 49, September 2004.

[2] Special Issue on on Technology of Networked Control Systems, *Proc. of IEEE*, vol. 95. Jan. 2007.

[3] J. Baillieu, "Feedback coding for information based control: Operating near the data rate limit," in *Proc. 41st IEEE Conf. Decision Contr.*, Las Vegas, NV, 2002.

[4] J.H. Braslavsky, R.H. Middleton, and J.S. Freudenberg, "Feedback stabilization over signal-to-noise ratio constrained channels," *IEEE Trans. Automat. Contr.*, vol. 52:1391-1403, Aug. 2007.

[5] W.K. Chen, *The Circuits and Filters Handbook*, CRC Press, 1995.

[6] J. Choma, Jr., *Electrical Networks, theory and analysis*, John-Wiley & Sons, New York, 1985.

[7] L. O. Chua, C.A. Desoer, and E.S. Kuh, *Linear and Nonlinear Circuits*, McGraw Hill, New York, 1987.

[8] D.F. Delchamps, "Stabilizing a linear system with quantized state feedback," *IEEE Trans. Automat. Contr.*, vol. 35:916-924, 1990.

[9] C.A. Desoer and E.S. Kuh, *Basic Circuit Theory*, McGraw Hill, New York, 1969.

[10] N. Elia and S.K. Mitter, "Stabilization of linear systems with limited information," *IEEE Trans. Automat. Contr.*, vol. 46:1384-1400, Sept. 2001.

[11] M. Fu and L. Xie, "The sector bound approach to quantized feedback control," *IEEE Trans. Automat. Contr.*, vol. 50:1698-1711, Nov. 2005.

[12] J. Hespanha, A. Ortega, and L. Vasudevan, "Towards the control of linear systems with minimum bitrate," *Proc. Int. Symp. Math. Theory of Networks and Sys.*, Aug. 2002.

[13] K. Li and J. Baillieul, "Robust quantization for digital finite communication bandwidth (DFCB) control," *IEEE Trans. Automat. Contr.*, vol. 49:1573-1584, Sept. 2004.

[14] G.N. Nair and R.J. Evans, "Stabilization with data rate limited feedback: tightest attainable bounds," *Syst. Contr. Lett.*, vol. 41:49-56, 2000.

[15] G. Nair, R. Evans, I. Mareels, and W. Mohan, "Topological feedback entropy and nonlinear stabilization," *IEEE Trans. Automat. Contr.*, vol. 49:1585-1597, Sept. 2004.

[16] A.V. Savkin, "Analysis and synthesis of networked control systems: topological entropy, observability, robustness and optimal control," *Automatica*, vol. 42:51-62, Jan. 2006.

[17] A. Sahai and S. Mitter, "The necessity and sufficiency of anytime capacity for stabilization of a linear system over a noisy communication link, part 1: Scalar systems," *IEEE Trans. Info. Theory*, vol. 52:3369-3395, Aug. 2006.

[18] S. Tatikonda and S. Mitter, "Control over noisy channels," *IEEE Trans. Automat. Contr.*, vol. 49:1196-1201, July 2004.

[19] S. Tatikonda, A. Sahai, and S.K. Mitter, "Control of LQG systems under communication constraints," *Proc. 37th IEEE Conf. Decision Contr.*, Tampa, FL, Dec. 1998.

[20] W.S. Wong and R.W. Brockett, "Systems with finite communication bandwidth constraint," *IEEE Trans. Automat. Contr.*, vol. 42:1294-1299, 1997 and vol. 44:1049-1053, May 1999.

[21] G. Zames, "On the input-output stability of time-varying nonlinear feedback systems, Part I," *IEEE Trans. on Automat. Contr.*, vol. 11, pp. 228-238, April 1966.