2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Optimum Generation Units Dispatch for Fuel Consumption Minimization

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***Abstract*—The United States Marine Corps (USMC) utilizes Forward Operating Bases (FOBs) which employ multiple generation units, primarily powered by JP-8 (similar to diesel fuel). It is often true that logistical support to deliver fuel is both expensive and dangerous. On the other hand, the generation units deployed by USMC range from 2 KW to over 200 KW, with very different input-output characteristics. In order to minimize fuel usage, a more sophisticated dispatch approach is needed. This paper applies the Karush-Kuhn-Tucker (KKT) conditions for optimality (in the sense of minimizing the fuel consumption) to develop an approach to economically dispatch generators. Simulation results based on the KKT method are compared with several existing dispatch methods, showing that our approach reduces the fuel usage compared to current standard methods.**

***Index Terms*—fuel economy, Lagrangian functions, optimization, power generation dispatch, power systems.**

I. Introduction

The United States Marine Corps (USMC) utilizes generation units covering a broad range, from 2 KW to over 200 KW. These generation units can have very different fuel consumption curves. The highest efficiency normally occurs when the generators are loaded at near rated capacity, with larger generators being typically more efficient than smaller units. Considering fuel consumption as the primary cost, each generation unit has a different cost function.

For this study, a classic two-tier power generator configuration is assumed. Suppose several of these generators, with the same or different rated output power, are interconnected to provide sufficient power to meet the overall demand. At the same time, it is desired to minimize the fuel consumption. How should these generation units be dispatched? There are several existing dispatch methods, which include: all uniformly, descend uniformly, and maximum load uniformly. These approaches are easy to apply and widely used in the power industry [1] [2]. However, these methods are not sufficient for USMC applications, in that they do not guarantee to minimize fuel usage. Note further that optimal economic generator dispatch for a microgrid would find useful application in many other (smart grid) areas [3].

In this paper, the Karush-Kuhn-Tucker (KKT) conditions are used to optimally dispatch generation units for minimum fuel consumption [4]. All the feasible solutions are found, but only the optimum solution is actually used for generator dispatch. Following the introduction in section 1, the rest of the paper is organized as follows. In section 2 the general problem is set up, and the KKT condition are briefly reviewed. The KKT-based conditions for generator dispatch are established in section 3, so as to minimize the fuel consumption. A simulation tool is developed in Matlab$^{\mathrm{TM}}$, and the simulation results are presented in section 4, along with comparisons to existing methods. Finally, in section 5, we present some concluding remarks.

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II. Problem setup and the KKT Conditions

In this section, the general problem is set up. We wish to find an optimal approach, in the sense of minimizing fuel consumption, for dispatching generation units. This is an extremum problem with inequality constraints, and hence the KKT optimality conditions are briefly presented as well.

A. Generator Dispatch Problem Setup

Suppose there are n generation units ($i = 1, 2, 3 \ldots, n$), with each generation unit having its own input-output characteristic curve. Further suppose this curve can be expressed by a quadratic function [5] as:

$$\Phi_i = \alpha_2^i \beta_i^2 + \alpha_1^i \beta_i + \alpha_0^i \tag{1}$$

where α_2^i, α_1^i, and α_0^i are given parameters of the generation unit (typically obtained via identification experiments or manufacturer specifications). A typical 60 KW generation unit input-output characteristic curve is shown in Fig. 1. Let P_i be the output power of generation unit i, ranging from a minimum P_{min}^i to a maximum P_{max}^i [3]. We define β_i as: $\beta_i = P_i / P_{max}^i$ (i.e., output power as a fraction of maximum rated power). Suppose F_i is the fuel consumption of generation unit i (whilst producing corresponding output power P_i), and it varies from a minimum F_{min}^i to a maximum F_{max}^i. We define Φ_i as: $\Phi_i = F_i / F_{max}^i$ (i.e., fuel usage as a fraction of maximum fuel usage). Note of course that P_i and F_i are both physical quantities, which only assume non-negative values. Note further that β_i varies between 0 and 1 (i.e., zero power to maximum power), but when the generation unit is idling at zero output power, it still consumes some fuel. Hence, Φ_i varies from a small positive value to 1 (i.e., idling to maximum fuel usage). With these definitions, the total output power can be expressed as: $\sum_{i=1}^{n} P_{max}^i \beta_i$ and the total fuel consumption can be expressed as: $\sum_{i=1}^{n} F_{max}^i \Phi_i$. If storage of electrical energy is not considered, the generation units total output power always equals the total power demand, with power produced at essentially the same time as it is consumed [4]. In this case, the demanded power P_T, and the total fuel consumption F_T, of n generation units are given respectively as follows:

$$P_T = \sum_{i=1}^{n} P_{max}^i \beta_i \tag{2}$$

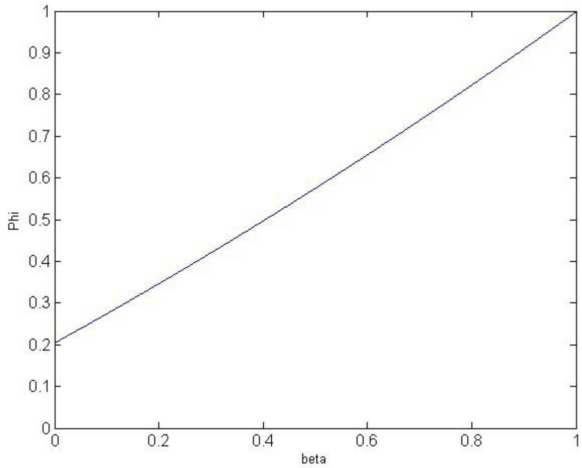

Fig. 1: A typical generation unit input-output characteristic curve

$$F_T = \sum_{i=1}^{n} F^i_{max}\Phi_i \tag{3}$$

The economic generator dispatch problem is to minimize the fuel usage, while at the same time ensuring the total generator output power meets the total power demand. This problem can now be formulated as [6]:

$$\text{minimize: } F_T = \sum_{i=1}^{n} F^i_{max}\Phi_i$$
$$\text{subject to: } P_T = \sum_{i=1}^{n} P^i_{max}\beta_i$$
$$0 \le \beta_i \le 1$$

The inequality constraint $0 \le \beta_i \le 1$ is trivially equivalent to $-\beta_i \le 0$ and $\beta_i - 1 \le 0$. It can be seen that this optimization problem contains both equality and inequality constraints. This kind of extremum problem can be solved by deploying the KKT optimality conditions, and hence before proceeding further we briefly review the general KKT conditions.

B. Karush-Kuhn-Tucker (KKT) Optimality Conditions

Considering the following problem:

$$\text{minimize } f(x)$$
$$\text{subject to } h(x) = 0,$$
$$g(x) \le 0$$

where f: $\mathbb{R}^n \to \mathbb{R}$, $h : \mathbb{R}^n \to \mathbb{R}^m$, $m \le n$, and $g : \mathbb{R}^n \to \mathbb{R}^p$ [7]. The KKT optimality conditions for the above problem comprise five components, which are shown below:

1) $\mu^* \ge 0$
2) $Df(x^*) + \lambda^{*T} Dh(x^*) + \mu^{*T} Dg(x^*) = 0^T$
3) $\mu^{*T} g(x^*) = 0$
4) $h(x^*) = 0$
5) $g(x^*) \le 0$

where x^* is the feasible solution, and also a local minimum. $\lambda^* \in \mathbb{R}^m$ is regarded as the Lagrange multiplier vector, and $\mu^* \in \mathbb{R}^p$ is taken as the Karush-Kuhn-Tucker (KKT) multiplier vector. Their components are referred to as the Lagrange multipliers and Karush-Kuhn-Tucker (KKT) multipliers, respectively. Df is defined as: $Df \equiv \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n}\right]$, with Dh and Dg defined similarly [8]. Accordingly, the original optimal generator dispatch problem is mapped into the following problem, in a form suitable for applying the KKT conditions:

$$\text{minimize: } f(\beta_i) = F_T = \sum_{i=1}^{n} F^i_{max}\Phi_i$$
$$\text{subject to: } h(\beta_i) = \sum_{i=1}^{n} P^i_{max}\beta_i - P_T = 0$$
$$g(\beta_i) = \begin{cases} -\beta_1 & \le 0 \\ \beta_1 - 1 & \le 0 \\ -\beta_2 & \le 0 \\ \beta_2 - 1 & \le 0 \\ \vdots & \\ -\beta_n & \le 0 \\ \beta_n - 1 & \le 0 \end{cases}$$

In the next section, the mapped optimal generator dispatch problem is solved via application of the appropriate KKT conditions.

III. KKT CONDITIONS FOR OPTIMAL DISPATCH

For general case, suppose there are n generation units interconnected to serve the load. Then the KKT conditions are given as:

1) $\mu = \begin{bmatrix}\mu^1_1 & \mu^1_2 & \mu^2_1 & \mu^2_2 & \dots & \mu^n_1 & \mu^n_2\end{bmatrix}^T \ge 0$
2) $$\begin{aligned} &F^1_{max}(2\alpha^1_2\beta_1 + \alpha^1_1) + \lambda P^1_{max} - \mu^1_1 + \mu^1_2 = 0 \\ &F^2_{max}(2\alpha^2_2\beta_2 + \alpha^2_1) + \lambda P^2_{max} - \mu^2_1 + \mu^2_2 = 0 \\ &\vdots \\ &F^n_{max}(2\alpha^n_2\beta_n + \alpha^n_1) + \lambda P^n_{max} - \mu^n_1 + \mu^n_2 = 0 \end{aligned}$$
3) $$\begin{bmatrix}\mu^1_1 & \mu^1_2 & \mu^2_1 & \mu^2_2 & \dots & \mu^n_1 & \mu^n_2\end{bmatrix} \begin{bmatrix} -\beta_1 \\ \beta_1 - 1 \\ -\beta_2 \\ \beta_2 - 1 \\ \vdots \\ -\beta_n \\ \beta_n - 1 \end{bmatrix} = 0$$
4) $h(\beta_i) = \sum_{i=1}^{n} P^i_{max}\beta_i - P_T = 0$
5) $$g(\beta_i) = \begin{bmatrix} -\beta_1 \\ \beta_1 - 1 \\ -\beta_2 \\ \beta_2 - 1 \\ \vdots \\ -\beta_n \\ \beta_n - 1 \end{bmatrix} \le 0$$

where the superscript i in μ^i_j denotes (that it is a KKT multiplier for) generation unit i. Note that each generation unit has two KKT multipliers, μ^i_1 and μ^i_2, which correspond to the lower and upper limits of each β^i. Now condition 3) is equivalent to:

$$-\mu_1^1\beta_1 + \mu_2^1(\beta_1 - 1) - \mu_1^2\beta_2 + \mu_2^2(\beta_2 - 1) + \dots \\ -\mu_1^n\beta_n + \mu_2^n(\beta_n - 1) = 0 \quad (4)$$

Note from condition 1), all μ_j^i's are nonnegative, and from condition 5), all elements of $g(\beta_i)$ are nonpositive. Equation (4) implies that all elements of $h(\beta_i)$ equal 0. Taking this into account, and for ease of presentation considering only the first generation unit, the first two terms of equation (4) are:

$$-\mu_1^1\beta_1 = 0 \quad (5)$$

and

$$\mu_2^1(\beta_1 - 1) = 0 \quad (6)$$

There are three possibilities for β_i, namely $\beta_i = 0$, $\beta_i = 1$, and $0 < \beta_i < 1$. Note that when $\beta_1 = 0$, from equation (6) immediately it can be observed that $\mu_2^1 = 0$, so that the only unknown is μ_1^1; When $\beta_1 = 1$, from equation (5), it is easy to see that $\mu_1^1 = 0$, and μ_2^1 is the only unknown. Similarly, when $0 < \beta_1 < 1$, from equation (5), it can be seen that $\mu_1^1 = 0$, and from equation (6) it follows that $\mu_2^1 = 0$, so that β_1 is the only unknown.

These three situations cover all the possible combinations. Notice that among the variables β_1, μ_1^1, and μ_2^1, two variables are always known, leaving us with only one unknown variable. The other generation units may be solved by the same approach. For the entire system, there is one more unknown, the Lagrange multiplier λ associated with the equality constraint in condition 2). It has been ignored in the discussion till now, since it does not appear in condition 3). Thus, for n generation units interconnected as a system, there are 3^n possible combinations, and for each combination we always have $2n$ variables known, with $n+1$ variables unknown, to be solved for from the resulting $n+1$ remaining equations [1].

Referring to the five components of the KKT conditions, note that conditions 1), 3), and 5) are constraints, with conditions 2) and 4) used to compute the unknown variables. By putting conditions 2) and 4) into matrix form, equation (7) is obtained as:

$$Ax = b \quad (7)$$

where $A \in \mathbb{R}^{(n+1)\times(3n+1)}$, $x \in \mathbb{R}^{3n+1}$ and $b \in \mathbb{R}^{n+1}$

$$x = \begin{bmatrix}\beta_1 & \beta_2 & \cdots & \beta_n & \lambda & \mu_1^1 & \mu_2^1 & \cdots & \mu_1^n & \mu_2^n\end{bmatrix}^T$$

$$b = \begin{bmatrix}-F_{max}^1\alpha_1^1 & -F_{max}^2\alpha_1^2 & \cdots & -F_{max}^n\alpha_1^n & P_T\end{bmatrix}^T$$

Note that of the $3n+1$ elements in x, only $n+1$ of them are unknown. In order to find all potential solutions, the 3^n possible combinations need to be solved via:

$$x_{ukn} = A_{ukn}^{-1}b \quad (8)$$

where $x_{ukn} \in \mathbb{R}^{n+1}$ represents the unknown elements in x. $A_{ukn} \in \mathbb{R}^{(n+1)\times(n+1)}$ only consists of the column elements of A that correspond to unknown (row) elements in x. Equation (8) computes all the unknown variables. By substituting the solution in x_{ukn} back into x, all the elements of x are now known. Thus, by using the objective function $f(\beta_i)$, the fuel consumptions of all the potential solutions may be computed. The minimum fuel consumption is now readily found, and the corresponding β_i's constitute the (global) optimal generator dispatch solution.

IV. Examples and Discussion

As an example, we consider a microgrid with three generation units. For the KKT-based dispatch method, the solution algorithm is implemented in Matlab$^{\text{TM}}$, with the corresponding flowchart shown in Fig. 2. In this section, the simulation results of the KKT-based dispatch method and several existing standard dispatch methods are compared. The existing methods used for comparison include: all uniformly dispatch (AUD), descend uniformly dispatch (DUD), and maximum load uniformly dispatch (MLUD).

Suppose there are n generation units available. The AUD method specifies that all n generation units are running, and they are all loaded to the same power ratio, i.e., $\frac{P_T}{P_R}$, where P_T is the total power demand, and P_R is the total rated output power of the (running) generation units. This explains the term 'uniformly' dispatch. The other approaches pre-select a group of generators, and then uniformly dispatch within that group.

In the DUD method, the available generation units are arranged in descending order, in the sense of their rated output power. So long as the total output rated power of the first m (where $m < n$) generation units is greater than or equal to the total power demand, only the first m generation units are running. The remaining $n-m$ generation units are either shut off or idling. If the total power demand is greater than the total generation capability of the first $n-1$ generation units, all generation units need to be running. At this point, the DUD method utilizes the same power ratio as the AUD method. The MLUD method involves first finding all possible combinations of the given n generation units that can meet the total power demand. Then, the algorithm chooses the particular combination P_C, that gives the minimum non-negative value of $P_C - P_T$. Generation units which form the particular combination P_C are running, with the remaining generation units shut off or idling [9].

In all simulations, the total demand power P_T sweeps from 1 KW to the total rated output power. Five different generation units are used, and their parameters are given in Table 1. We consider separately the 'idling' and 'shut-off' cases. In the idling case, as discussed earlier, even generation units not generating power still consume fuel to remain idling (see Fig. 1 and note that $\Phi_i \neq 0$ for $\beta_i = 0$). In the shut-off case, it is assumed that generators not contributing to the load are turned off and hence do not consume any fuel. Note that this involves a modified (KKT) solution, since the generator input-output curve is essentially discontinuous at $\beta_i = 0$, but for reasons of brevity we do not go into the details of this here.

TABLE I: Generation Units Parameters

No.	P_{max} (KW)	α_2	α_1	α_0	F_{max} (gal/hr)
1	20	0.071428571	0.753571429	0.183928571	1.6
2	30	0.064285714	0.748214286	0.193035714	2.9
3	40	0.057142857	0.742857143	0.202142857	4.0
4	60	0.103512881	0.689929742	0.203881733	4.8
5	150	0.167758847	0.676277851	0.160419397	10.9

$$A = \begin{bmatrix} 2F_{max}^1\alpha_2^1 & 0 & \cdots & 0 & P_{max}^1 & -1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 2F_{max}^2\alpha_2^2 & \cdots & 0 & P_{max}^2 & 0 & 0 & -1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & & & & \ddots & & \vdots \\ 0 & 0 & \cdots & 2F_{max}^n\alpha_2^n & P_{max}^n & 0 & 0 & 0 & 0 & \cdots & -1 & 1 \\ P_{max}^1 & P_{max}^2 & \cdots & P_{max}^n & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

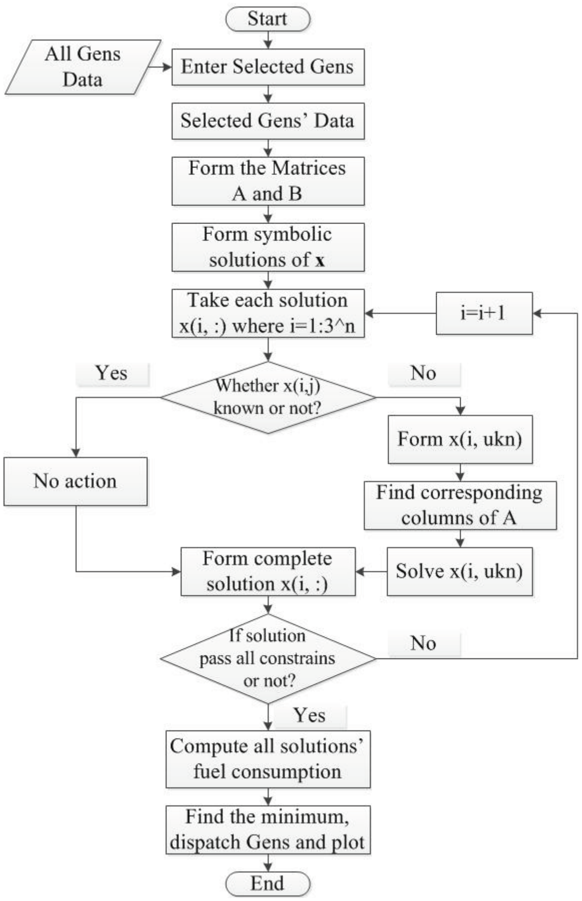

Fig. 2: Flowchart of the Matlab$^{\text{TM}}$ script file

Three different configurations are compared. We first consider generation units 1, 2, and 3, which represents a microgrid that consists of similar (but not the same) small rated output power generators. The second microgrid is constructed using three of the No. 4 generators, hence representing a microgrid with identical generators. The third microgrid consists of generators 1, 3, and 5, representing a microgrid containing very different generators. Simulation results for the shut-off case are shown in Fig. 3, Fig. 4, Fig. 5, and Fig. 6, and for the idling case in Fig. 7, Fig. 8, Fig. 9, and Fig. 10.

Fig. 3 illustrates the fuel consumption curves for each method tested on configuration 1 in the shut-off case. Clearly, it can be seen that the KKT-based approach consumes the least fuel. The AUD method, which keeps all generators on, consumes the most fuel. The DUD method initially has the 40 KW generator turned on. This is the one of the most inefficient generators at low loads. For the MLUD method, with total power demand between 30 KW and 40 KW, the 40 KW generator is turned on. By contrast, the KKT-based method combines more efficient 20KW and 30 KW generators to supply the load.

Fig. 4 shows how the generators are actually dispatched for the above test using the KKT-based method. At the beginning, the most efficient generator, namely 20 KW, is turned on. Once the total load demand exceeds 20 KW, the method turns off the 20 KW generator and starts the 30 KW generator, because it is more efficient than having two generators running. However, the combined 20 KW and 30 KW generators are more efficient than the 40 KW one. Hence, once the load exceeds 30 KW, the combined 20 KW and 30 KW generation units are used. When load exceeds 50 KW, the 20 KW and 40 KW generators are used, up to $P_T = 60$ KW, at which point all three generators are required serve the load.

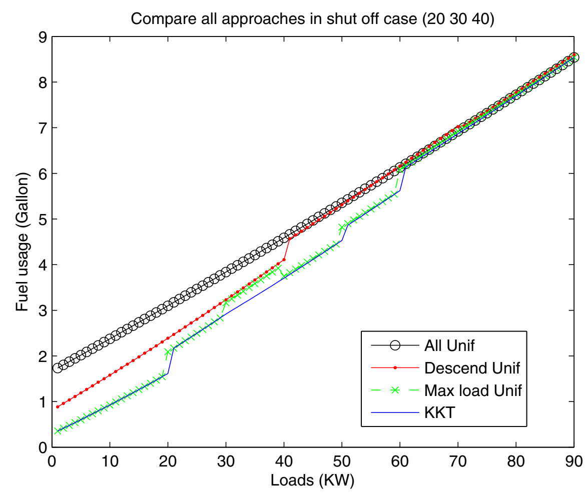

Fig. 3: Fuel consumption of 1^{st} configuration in shut-off case

Three identical generators form configuration 2, whose fuel consumption curves are shown Fig. 5. As one would expect, there is no preference in how to dispatch between them, so that the DUD method, MLUD method, and KKT-based method all consume the same amount of fuel, indicating that uniform dispatch is the optimal solution for this case. Note, however, that the above approaches utilize a sub-group of generators (big enough to serve the load). The AUD method has all generators running all the time, and so it consumes more fuel than the other methods.

In the shut-off case, when there are considerably big differences between the generators in the system, the AUD and DUD methods perform poorly. This is because both of these methods always have the biggest generators on, but big generators are inefficient when the load is too low. The MLUD method is considerably more efficient, since it is more flexible

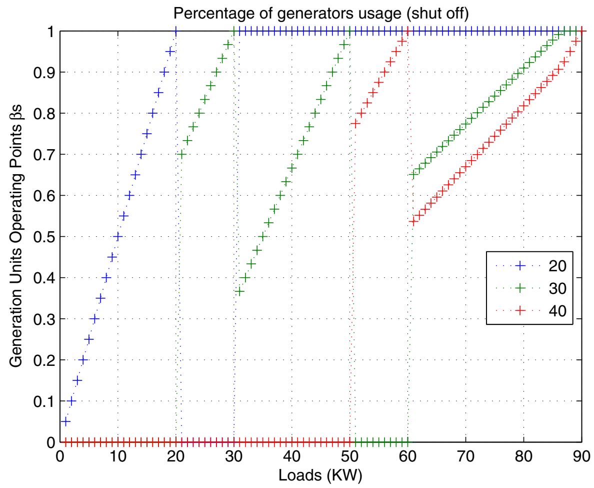

Fig. 4: 1^{st} configuration generators dispatch in shut-off case

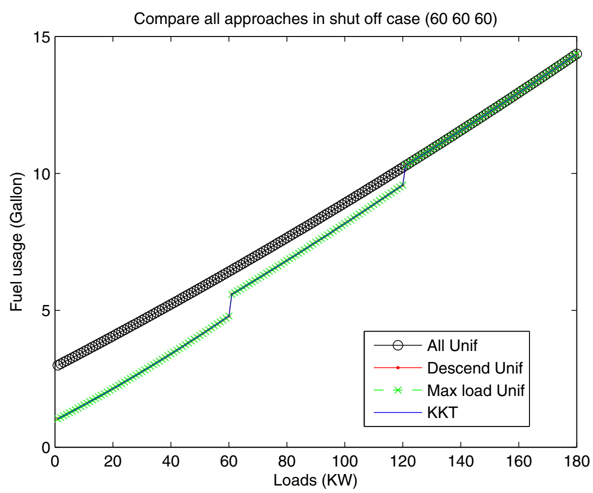

Fig. 5: Fuel consumption of 2^{nd} configuration in shut-off case

compared to the previous two methods. However, note that the objective function is to $\min P_C - P_T$, subject to: $P_C - P_T \geq 0$, which is not directly minimizing fuel consumption. Hence, it is still slightly out-performed by the KKT-based method, which finds the true optimal dispatch for least fuel consumption, as can be seen in Fig. 6, which shows the fuel consumption curves for configuration 3.

We now consider the idling case, and the fuel consumption curves for the 1^{st} configuration are shown in Fig. 7. Although the KKT-based method still consumes least fuel, its superiority over the other approaches is now less apparent. In the idling case all the generators are on all the time for all the methods. Hence, the space of fuel performance curves is compressed, with small differences between the approaches.

Fig. 8 shows how the KKT-based method dispatches the generation units for the above test. Note that this is much smoother than the dispatch schedule shown earlier in Fig. 4 for the shut-off case, because there is no longer any advantage to switching machines in and out. There are some similarities between Fig. 4 and Fig. 8, since we can see that both of them utilize the 20 KW generator first. However, note that in Fig. 8,

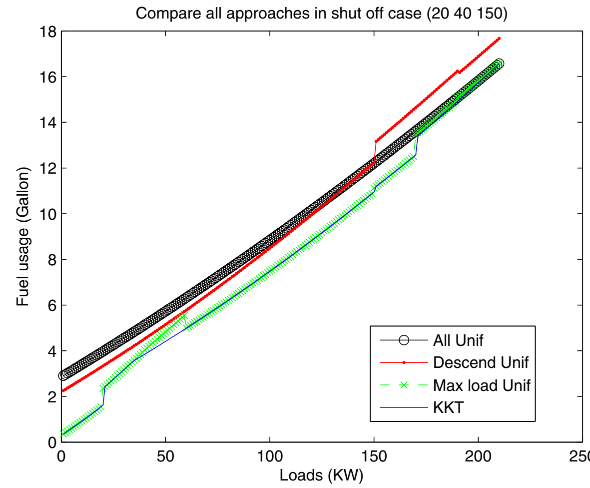

Fig. 6: Fuel consumption of 3^{rd} configuration in shut-off case

once the load exceeds 25 KW, all three generators supply the load (versus the shut-off case in Fig. 4). This happens since all the generators are idling anyway, so it is better to have them contribute to the load, rather than just idling and wasting fuel.

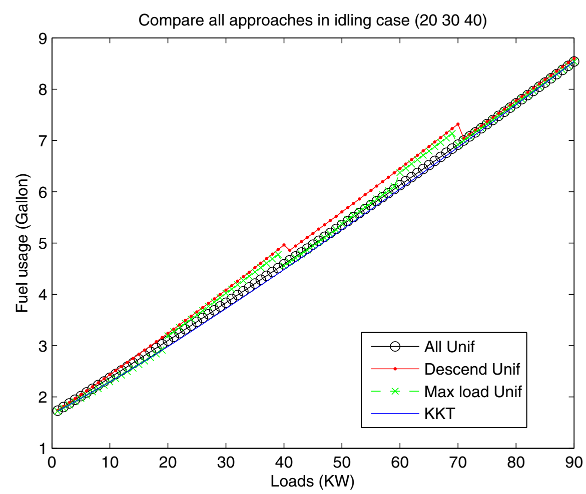

Fig. 7: Fuel consumption of 1^{st} configuration in idling case

For the second configuration, shown in Fig. 9, there is little difference between the approaches. In fact the AUD and KKT-based methods consume identical amounts of fuel, as do the DUD and MLUD methods. This makes perfect sense, since all the generators are identical, and all are kept idling, there is little room for optimal dispatch to make a big difference.

The third system is operated in the idling case to generate the curves in Fig. 10. It can be seen that the KKT-based method shows great advantages over the MLUD and DUD methods. The reasons are similar to the earlier discussion regarding Fig. 6, where uneconomical generators are turned on at low load. Conversely, the AUD method shows more economical behavior, and is largely similar to the KKT-based method, but is less efficient for mid-range total loads. This is because the AUD method dispatches all generators identically, but the individual generators themselves are inefficient at low

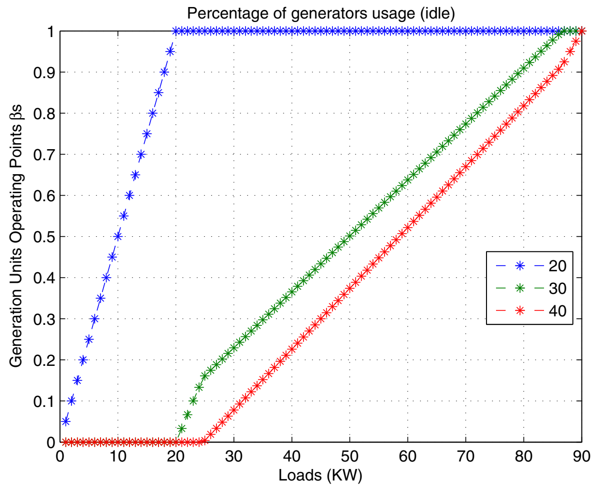

Fig. 8: 1st configuration generators dispatch in idling case

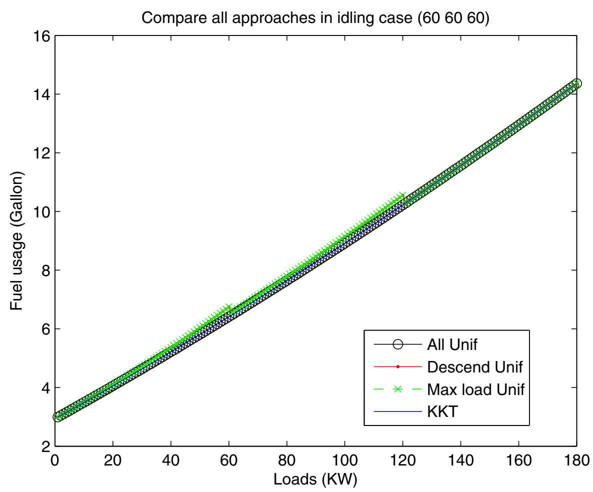

Fig. 9: Fuel consumption of 2nd configuration in idling case

and high loads. Hence, we see once again that the KKT-based dispatch method is the best in terms of fuel economy.

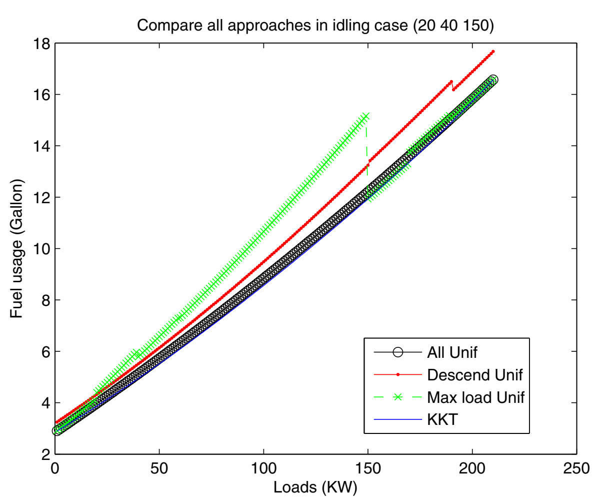

Fig. 10: Fuel consumption of 3rd system in idling case

V. Conclusion

This paper presents a KKT-based approach for optimal generator dispatch in a microgrid. The primary objective is to minimize fuel consumption, which has significant practical meaning for USMC operations and smart grid systems [10]. Simulation testing showed that the KKT-based dispatch method is the most economical, regardless of the system structure and operation situation. Standard existing dispatch methods can have similar performance in the right circumstances, but they all also exhibited poor performance under other circumstances, and the KKT-based approach was the only one that always dispatched the generators in the most economical manner.

Note that in this paper some factors are neglected, including the generator start-up fuel consumption. Also, for the shut-off case, the generators may be switched on and off as necessary when the load ramps up, which would not be acceptable in a practical application. These issues constitute a more complicated problem, which will require a more sophisticated control algorithm, and this is a subject of current research.

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