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Robust Repetitive Controller Design and Its Application on the Track-Following Control System in Optical Disk Drives

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Abstract— **A repetitive controller has been applied owing to its prominent capability for attenuating periodic disturbances and/or tracking periodic reference commands. A repetitive controller is generally added on the existing feedback control system to improve the tracking performance. However, the repetitive controller has been designed without utilizing the effective information such as the performance weighting function used in the design of the feedback controller. In this paper, we deal with the problem of a robust repetitive controller design for an uncertain feedback control system using its explicit performance information. We first show that a robust stability condition of repetitive control systems is closely related with the well-known robust performance condition of general feedback control systems. It is also shown that the steady-state tracking error of the repetitive control system is described in a simple form without time-delay element. From this result, we explain how different loop properties of the repetitive control system are from those of the feedback control system. Moreover, sufficient conditions are provided, which ensure that the power of the steady-state tracking error generated by the repetitive control system is less than or equal to that only by the feedback control system. Based on the obtained results, we present repetitive controller design criteria. Finally, to show the validity of the proposed method, application studies on the track-following control system of optical disk drives are performed.**

I. Introduction

Given a periodic reference signal or disturbance, repetitive control is a special control scheme to reduce a tracking error effectively. Its highly accurate tracking property originates from a periodic signal generator implemented in the repetitive controller. However, the positive feedback loop and the time-delay term to generate the periodic signal decreases the stability margin. Therefore, the tradeoff between stability and tracking performance has been considered as an important factor in the repetitive control system. Hara *et al.* [1] derived sufficient conditions for the stability of a repetitive control system and a modified repetitive control system which sacrifices tracking performance at high frequencies for system stability. Güvenç [2] applied the structured singular value to repetitive control systems in order to determine their stability and performance robustness in the presence of structured parametric modeling error in the plant. Li and Tsao [3] addressed analysis and synthesis of robust stability and robust performance repetitive control systems. Doh and Chung [4] proposed a method to design a repetitive control system ensuring robust stability for linear systems with time-varying uncertainties. M.-C. Tsai and W.-S. Yao derived upper and lower bounds of the repetitive controller parameters ensuring the stability and the desired performance [5] and an upper bound for square integral of the tracking error over one time period of periodic input signals based on Fourier analysis [6], respectively. Steinbuch *et al.* presented a design method of high-order repetitive controllers which is obtained by solving a convex optimization problem [7]. Doh and Ryoo presented a robust stability condition for the repetitive control system from the robust performance condition by selecting the performance weighting function as q filter [8].

However, there are two explicit problems to be solved to use a repetitive controller effectively. First, although a repetitive controller is added on the existing feedback control system, we have solved a totally separate problem to design a repetitive controller irrespective of a feedback controller design problem. Moreover, the cutoff frequency of the q filter in the repetitive controller should be found by many trials and errors. Therefore, we should consider positively finding a method to reduce design efforts. Second, not only the delay element in the repetitive controller decreases the phase margin but also plant uncertainty threatens overall system stability. Hence, the added repetitive controller should guarantee robust stability.

In this paper, a systematic design method of the add-on type repetitive control system is presented for the feedback control system with plant modeling perturbation. The real plant is represented as a multiplicative uncertainty model. We assume that a feedback controller is given, which is designed based on the performance weighting function describing the reference servo and ensures robust performance. We first propose a robust stability condition of repetitive control systems closely related with the well-known robust performance condition of general feedback control systems. Based on the robust stability condition, a q filter to satisfy the robust stability condition is obtained from the performance weighting function. It is also shown that if the robust stability condition is satisfied, the steady-state tracking error can be described in a simple form without the time-delay element. Through the analysis on the steady-state tracking errors of the repetitive control system and the feedback control system, we show the change of the loop gain and propose conditions under which it can be ensured that the steady-state tracking error of the repetitive control system is less than that of

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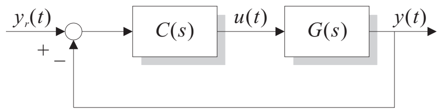

Fig. 1. Feedback Control System.

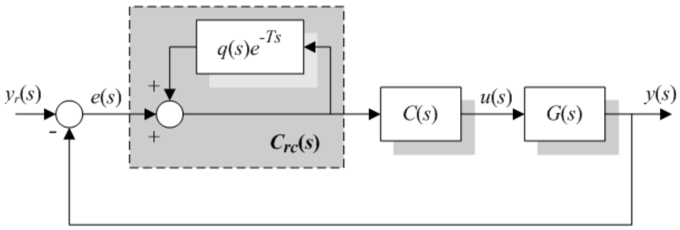

Fig. 2. Repetitive control system.

the feedback control system in the sense of power. From the obtained results, several design criteria of q filter are proposed to improve the tracking performance and satisfy the robust stability condition. Application studies on the track-following control system of optical disk drives are performed to show the validity of the proposed method.

II. Robust Stability Condition of Repetitive Control Systems

Consider the feedback control system in Fig. 1. In this figure, $y_r(t)$ is the reference trajectory and is assumed to be periodic and bounded within the period T, $y(t)$ is the plant output, and $u(t)$ is the feedback control input. $C(s)$ is the feedback controller that stabilizes the feedback control system and ensures robust performance. The plant $G(s)$ is described in the following multiplicative uncertain form:

$$G(s) = (1 + \Delta(s)W_u(s))G_n(s) \tag{1}$$

where $G_n(s)$ is the nominal plant model, $W_u(s)$ is a known stable uncertainty weighting function, and $\Delta(s)$ is an unknown stable function satisfying $\|\Delta\|_\infty \le 1$.

The following lemma, which is widely known as the robust performance condition in the robust control theory, will be used to derive our results.

Lemma 1: [9] Consider the feedback control system in Fig. 1 with the plant $G(s)$ described in (1). Then a necessary and sufficient condition for robust performance is

$$\|W_uT_n\|_\infty < 1 \text{ and } \left\| \frac{W_pS_n}{1+\Delta W_uT_n} \right\|_\infty < 1$$

which is equivalent to

$$\| |W_pS_n| + |W_uT_n| \|_\infty < 1 \tag{2}$$

where $W_p(s)$ is assumed to be a known stable performance weighting function, $S_n(s) = 1/(1 + G_n(s)C(s))$ is the nominal sensitivity function, and $T_n(s) = 1 - S_n(s)$ is the nominal complementary sensitivity function.

In order to effectively track the periodic reference signal, the repetitive controller $C_{rc}(s)$ is added to the existing feedback control system as an add-on module shown in Fig. 2 where $q(s)$ is a low-pass filter to ensure system stability. Note that $C_{rc}(s)$ is equivalent to the modified repetitive controller with $a(s) = 1$ as proposed by Hara *et al.* [1].

Theorem 1: Consider the repetitive control system in Fig. 2. Then the repetitive control system is robustly stable if there exists a $q(s)$ such that

$$\| |qS_n| + |W_uT_n| \|_\infty < 1 \tag{3}$$

is satisfied.

Corollary 1: Consider the repetitive control system in Fig. 2. Then the repetitive control system in Fig. 2 is robustly stable if the robust performance condition (2) of a general feedback control system is guaranteed.

According to Corollary 1, the feedback controller satisfying the robust performance condition can directly guarantee the robust stability of the repetitive control system. Therefore, there is no need to design a $q(s)$ in the repetitive controller ensuring robust stability in comparison with other methods [2]–[6]. The reason why this result can be obtained is that the performance weighting function $W_p(s)$, which is used to design the feedback control system satisfying the robust performance condition, plays a role of the q filter to ensure the robust stability of the repetitive control system. This result is equivalent to that of [8]. However, since $\|W_p\|_\infty$ is generally much larger than 1 to reduce the tracking error in the design of the feedback controller, Corollary 1 should be modified to solve practical problems.

Corollary 2: Consider the repetitive control system in Fig. 2. Assume that there exists a $C(s)$ such that the robust performance condition (2) is satisfied. Then the repetitive control system is robustly stable if there exists a $q(s)$ ensuring

$$\|q(s)/W_p(s)\|_\infty \le 1 \tag{4}$$

is satisfied.

Corollary 2 provides a design guideline of $q(s)$ to robustly stabilize the repetitive control system when the repetitive controller is added on to the feedback control system ensuring robust performance. In general, $W_p(s)$ has the information about the controlled system such as the control bandwidth, the amount of the tracking error, and so on. Therefore, it is a proper approach to determine the bandwidth of $q(s)$ using $W_p(s)$.

(Criterion 1) (Robust Stability) According to (4), the relative degree of $q(s)$ is more than or equal to that of $W_p(s)$. The cutoff frequency of $q(s)$ is selected in order for the magnitude envelope of $q(s)$ to exist inside that of $W_p(s)$.

The condition (4) is equivalent to the inequality

$$\| |qS_n| + |W_uT_n| \|_\infty < 1. \tag{5}$$

The inequality $\|W_uT_n\|_\infty < 1$ in (5) is the robust stability condition of the feedback control system as shown in Fig. 1. (5) can be rewritten as

$$\begin{aligned} & \| |qS_n| + |W_uG_nCS_n| \|_\infty \\ = \ & \| |S_n|(|q| + |W_uG_nC|) \|_\infty = \rho \end{aligned} \tag{6}$$

where ρ is defined as robustness measure of the repetitive control system. In other words, if a small ρ can be achieved by designing an adequate q filter, then the repetitive control system can be stabilized in spite of the large plant uncertainties. Since $|W_u G_n C|$ and $|S_n|$ are already fixed in (6) when $C(s)$ is designed, ρ can be determined according to the properties of the q filter. ρ increases as the magnitude of the q filter approaches to 1 irrespective of other terms. Since a low pass filter is selected as the q filter, the bandwidth and the relative degree of the q filter have significant effects on the robustness. The narrower bandwidth of the q filter is, the smaller ρ can be obtained. Moreover, a large relative degree of the q filter makes ρ small since the magnitude of the q filter decrease abruptly as the frequency increases.

(Criterion 2) (Robustness of Repetitive Control System) The robustness of the repetitive control system can be better as the bandwidth of the q filter is narrow. Also, for the case of the same bandwidth, the increase of relative degree of the q filter makes the robustness better.

III. Analysis on the Steady-State Tracking Error

The following theorem shows that the steady-state tracking error of the repetitive control system in Fig. 2 can be obtained irrespective of the time-delay element if the robust stability condition of the repetitive control system is satisfied.

Theorem 2: Consider the repetitive control system in Fig. 2. The tracking error $e(t)$ approaches to

$$e_{ss}(t) = \lim_{t\to\infty} \mathcal{L}^{-1}\left\{\frac{S_n(1-q)}{1+\Delta W_u T_n - qS_n} Y_r(s)\right\} \tag{7}$$

as $t \to \infty$ if the repetitive control system satisfies the condition (3).

From this result, we analyze the loop gain closely related with the steady-state tracking error. Let the transfer function form $y_r(t)$ to $e_{ss}(t)$ of the repetitive control system be defined as

$$S_{rc}(s) = \frac{1-q}{1-q+CG_n(1+\Delta W_u)} \tag{8}$$

which means the sensitivity function of the repetitive control system in the steady state. Similarly, that of the feedback control system is equivalent to the sensitivity function and can be rewritten as

$$S_{fb}(s) = \frac{1}{1+CG_n(1+\Delta W_u)} = \frac{1}{1+L_{fb}} \tag{9}$$

where $L_{fb}(s) = C(s)G_n(s)(1+\Delta(s)W_u(s))$ is the loop transfer function of the feedback control system. (8) can be written as a similar form with (9):

$$\begin{aligned} S_{rc}(s) &= \frac{1}{1+(CG_n(1+\Delta W_u))/(1-q)} \\ &= \frac{1}{1+L_{fb}/(1-q)} = \frac{1}{1+L_{rc}} \end{aligned} \tag{10}$$

where $L_{rc}(s) = L_{fb}(s)/(1-q(s))$.

Corollary 3: Consider the feedback control system in Fig. 1 and the repetitive control system in Fig. 2. $|L_{rc}(s)|$ is greater than or equal to $|L_{fb}(s)|$ if

$$\|1-q(s)\|_\infty \leq 1. \tag{11}$$

Corollary 3 means that if the condition (11) is satisfied, the repetitive control system has a much higher loop gain in the steady state than the feedback control system and then the steady-state tracking error of the repetitive control system is reduced much less than that of the feedback control system. (10) explains that the steady-state error of the repetitive control system reduces to zero as q is close to 1. Although it is the best choice that $q(s)$ is 1, the repetitive control system in Fig. 2 with a strictly proper plant cannot be stable [1]. As a result, a low pass filter with magnitude of 1 is generally used as q filter. The bandwidth of the q filter is selected sufficiently wider than that of $y_r(t)$ to reduce the tracking error. For the case of the same bandwidth, a lower relative degree of the q filter makes a better tracking performance since the error generated by the harmonics of $y_r(t)$ is reduced.

(Criterion 3) (Steady-State Tracking Error) The tracking performance can be improved as the relative degree of the q filter decreases and the bandwidth of the q filter becomes wider.

Although the loop gain of the repetitive control system is higher than that of the feedback control system, it does not always ensure that the steady-state tracking error of the repetitive control system is less than that of the feedback control system because $|S_{fb}(j\omega)|$ may be equal to or less than $|S_{rc}(j\omega)|$ in some frequency regions. In other words, even if the condition (11) is satisfied, there are frequency regions where $|S_{fb}(j\omega)|$ may be equal to or less than $|S_{rc}(j\omega)|$. Now, using $S_{fb}(s)$ and $S_{rc}(s)$ directly related with the steady-state tracking error, we determine conditions under which it can be ensured that the steady-state tracking error of the repetitive control system is less than that of the feedback control system in the sense of power. Before analyzing the steady-state tracking error in a view of power, we first define the powers of steady-state tracking errors of the repetitive control system and the feedback control system

$$P_{rc} = \frac{1}{T}\int_T |s_{rc}(t) * y_r(t)|^2 dt, \tag{12}$$

$$P_{fb} = \frac{1}{T}\int_T |s_{fb}(t) * y_r(t)|^2 dt, \tag{13}$$

where $s_{rc}(t) = \mathcal{L}^{-1}\{S_{rc}(s)\}$ and $s_{fb}(t) = \mathcal{L}^{-1}\{S_{fb}(s)\}$, respectively. The complex Fourier series representation of $y_r(t)$ is given by

$$y_r(t) = \sum_{k=-n}^{n} c_k e^{jk\omega_0 t} \tag{14}$$

where c_k is the kth Fourier coefficient and $\omega_0 = \frac{2\pi}{T}$. Let us select $q(s)$ as a low-pass filter with its magnitude of 1 according to the suggested design criteria. In the low-frequency region, $|S_{rc}(j\omega)|$ approaches to 0 and is much less than $|S_{fb}(j\omega)|$ as $q(j\omega) \approx 1$ and in the high-frequency region, $|S_{rc}(j\omega)| \approx |S_{fb}(j\omega)|$ as $q(j\omega) \approx 0$. However, there are some frequency regions between in the low frequency and in the high frequency where $|S_{rc}(j\omega)| - |S_{fb}(j\omega)|$ is larger than zero. By solving the equation

$$|S_{rc}(j\omega)| = |S_{fb}(j\omega)|, \tag{15}$$

we can find the frequencies where the sign of $|S_{rc}(j\omega)| - |S_{fb}(j\omega)|$ is changed. Let $q(j\omega)$ and $S_{fb}(j\omega)$ be defined as $|q(j\omega)|e^{j\phi_q(j\omega)}$ and $|S_{fb}(j\omega)|e^{j\phi_s(j\omega)}$, respectively, where $\phi_q(j\omega) = \angle q(j\omega)$ and $\phi_s(j\omega) = \angle S_{fb}(j\omega)$. Then, since the both sides have a common term $|S_{fb}(j\omega)|e^{j\phi_s(j\omega)}$, (15) can be written as

$$\left|\frac{1 - |q|(\cos(\phi_q) + j\sin(\phi_q))}{1 - |q||S_{fb}|\{\cos(\phi_q + \phi_s) + j\sin(\phi_q + \phi_s)\}}\right| = 1. \quad (16)$$

After some mathematical manipulation, we can get

$$(1 - |S_{fb}|^2)|q| = 2\cos(\phi_q) - 2|S_{fb}|\cos(\phi_q + \phi_s) \quad (17)$$

which is a simple form of (15). The following theorem gives analysis results on the steady-state tracking error in the sense of power.

Theorem 3: Consider the repetitive control system shown in Fig. 2. Assume that $q(s)$ has the frequency characteristics:

a) $q(j\omega) = 1$, $|\omega| \leq \omega_{q0}$
b) $|q(j\omega)| < 1$, $|\omega| > \omega_{q0}$

and $\omega_{q1} \geq \omega_{q0}$ is the least frequency satisfying (17).

i) Let $y_r(t)$ be a band-limited signal represented as (14) and $n\omega_0 \leq \omega_{q0}$. Then P_{rc} is zero.
ii) Let $y_r(t)$ have the same properties as i) except $n\omega_0 \leq \omega_{q1}$. Then $P_{rc} \leq P_{fb}$.
iii) Let $y_r(t)$ have the same properties as i) except $n\omega_0 > \omega_{q1}$. Then $P_{rc} \leq P_{fb}$ if

$$\begin{aligned}&\sum_{k=-m}^{m} |c_k|^2 \left(|S_{rc}(jk\omega_0)|^2 - |S_{fb}(jk\omega_0)|^2\right) \\ \leq\ & -2\sum_{k=m+1}^{n} |c_k|^2 \left(|S_{rc}(jk\omega_0)|^2 - |S_{fb}(jk\omega_0)|^2\right)\end{aligned} \quad (18)$$

where m is the integer satisfying $m\omega_0 \leq \omega_{q1} < (m+1)\omega_0$.

iv) Let $y_r(t)$ be a band-unlimited signal represented as $y_r(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$. Then $P_{rc} \leq P_{fb}$ if

$$\begin{aligned}&\sum_{k=-m}^{m} |c_k|^2 \left(|S_{rc}(jk\omega_0)|^2 - |S_{fb}(jk\omega_0)|^2\right) \\ \leq\ & -2\sum_{k=m+1}^{\infty} |c_k|^2 \left(|S_{rc}(jk\omega_0)|^2 - |S_{fb}(jk\omega_0)|^2\right).\end{aligned} \quad (19)$$

Theorem 3 offers useful information in which case the repetitive control system is effective through the Fourier series analysis. In other words, the repetitive control system has effects on reducing the steady-state tracking error rather than the feedback control system if the conditions proposed in Theorem 3 are satisfied.

IV. Application Studies

To verify the feasibility of the proposed method, we perform simulation studies on the track-following control system in DVD drives. A repetitive track-following control system is shown in Fig. 3 where $d(t)$ is a periodic disturbance with unknown magnitude, $e_{pd}(t)$ is an amplified signal of the tracking error $e(t)$ by the photo detector gain K_{PD}.

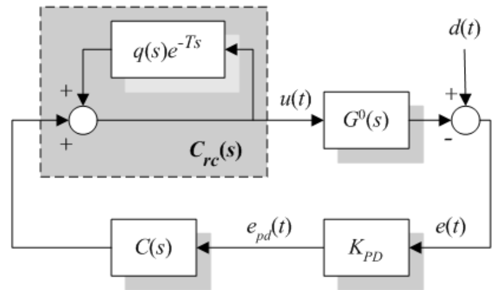

Fig. 3. Repetitive track-following control system.

TABLE I
Parameters of the Plant.

1st Resonance Frequency (ω_n)	62 Hz	ζ	0.08
2nd Resonance Frequency	20 kHz	K	5.031×10^{-4}
2nd Resonance Magnitude	10 dB	K_{PD}	5.4×10^6 (V/m)

A nominal tracking actuator is modeled as

$$G_n^0(s) = \frac{K \cdot \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \quad (20)$$

Although (20) is nearly exact in the low-frequency region, $G_n^0(s)$ is different from the actual tracking actuator $G^0(s)$ in the high-frequency region. In addition to the unmodeled component, resonances at high frequencies are barely considered in the plant model. To take into account the effect of the unmodeled dynamics, $G^0(s)$ should be modeled with consideration of the uncertainty. For the purpose, a multiplicative uncertainty model is adequate [9], [10] and $G^0(s)$ is given as

$$G^0(s) = (1 + \Delta(s)W_u(s))G_n^0(s) \quad (21)$$

where $W_u(s)$ is a known uncertainty weighting function given by $2s^2/(s+4000\pi)^2$ and $\Delta(s)$ is an unknown stable function satisfying $\|\Delta\|_\infty \leq 1$. K_{PD} is the conversion ratio of the position sensor from the distance between the track center and the laser spot to the electrical error signal. The track pitch of DVD disk is 0.74μm and the measurable range is $\pm0.37\mu$m, which corresponds to the tracking error signal amplitude of ±2.0V. Therefore, K_{PD} is 5.4×10^6 V/m. The plant $G(s)$ is $K_{PD}G^0(s)$ and has the characteristics given by Table I.

A feedback compensator satisfying robust performance is designed based on the reference servo for track-following recommended by the DVD standard [11]. Let the allowable disturbance, the maximum tracking error, and the expected maximum radial acceleration be $\pm50\mu$m, $\pm0.022\mu$m, and $1.1\mathrm{m/s}^2$, respectively, as explained in the DVD standard. Then, a minimal open-loop gain $|H_{\min}(j\omega)|$ (dash-dot) of the reference servo for track-following is depicted in Fig. 4. For an open-loop transfer function $H(s)$ of the reference servo, $|1 + H(j\omega)|$ is limited as schematically shown by the shaded surface of Fig. 4. A performance weighting

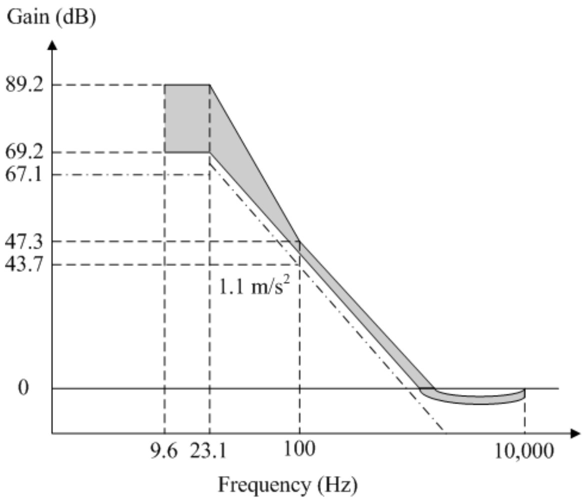

Fig. 4. Reference servo for track-following: $|1 + H(j\omega)|$ (the shaded surface), $|H_{\min}(j\omega)|$ (dash-dot).

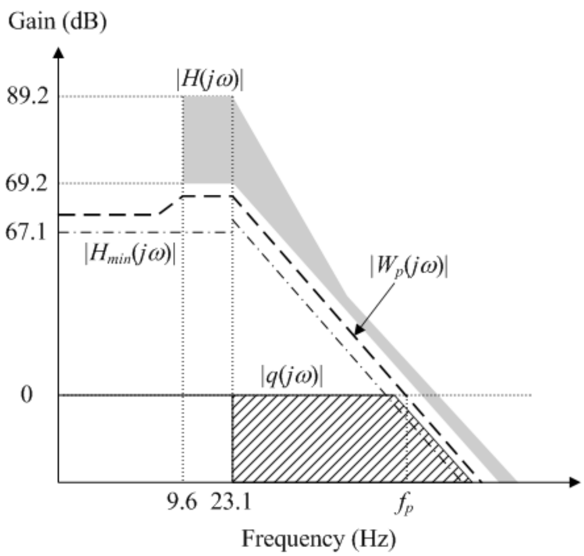

Fig. 5. Frequency properties of the designed $q(s)$ based on the reference servo for track-following.

function is selected from the relationship between the robust performance condition and the reference servo.

The performance weighting function is selected as

$$W_p(s) = \frac{6.0755 \times 10^7}{s^2 + 145.14s + 21066}. \tag{22}$$

to satisfy

$$|W_p(j\omega)| < |H(j\omega)|, \ \forall\omega. \tag{23}$$

A feedback controller is selected as

$$C(s) = \frac{19.5(s + 17592)(s + 35186)}{(s + 87965)(s + 105560)} \tag{24}$$

to consider the minimum stability margin for stable track-following pull-in [12].

Fig. 5 depicts frequency properties of the designed $q(s)$ based on the reference servo for track-following. $q(s)$ should

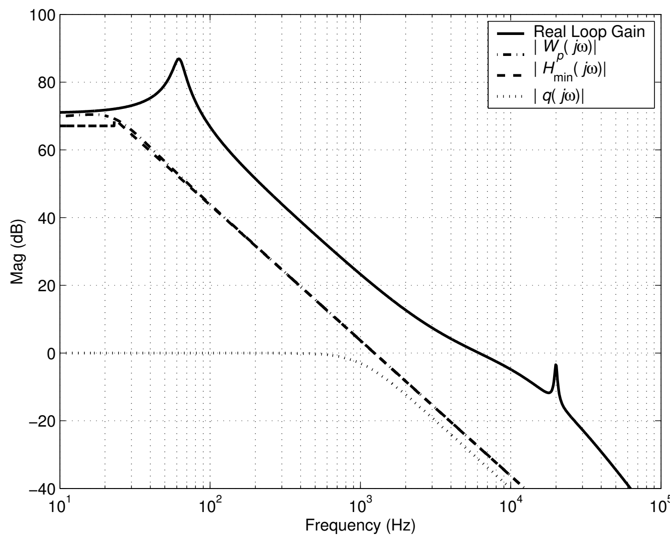

Fig. 6. Magnitude plots of the open loop gain with the real plant(solid), $|W_p(j\omega)|$(dash-dot), $|H_{\min}(j\omega)|$(dash), and $|q(j\omega)|$(dot).

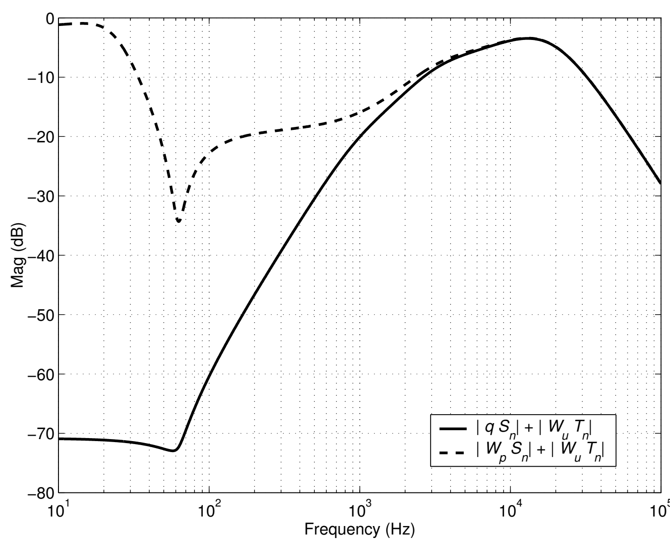

Fig. 7. Magnitude plots of $|q(j\omega)S_n(j\omega)|+|W_u(j\omega)T_n(j\omega)|$(solid) and $|W_p(j\omega)S_n(j\omega)| + |W_u(j\omega)T_n(j\omega)|$(dash).

have a gain of 1 between the maximum rotational frequency (23.1Hz) and the 0dB cross-over frequency of $W_p(s)$, f_p by the proposed criterions. To satisfy Criterion 1, the relative degree of $q(s)$ should be equal to or higher than that of $W_p(s)$. Moreover, to satisfy Criterion 2 and Criterion 3 simultaneously, we select a value between the maximum rotational frequency and f_p as the cutoff frequency of $q(s)$. Considering the design specifications, we select the following 2nd order filter with the cutoff frequency of 1kHz and the DC gain of 1

$$q(s) = \frac{3.948 \times 10^7}{s^2 + 8885.8s + 3.948 \times 10^7} \tag{25}$$

as $q(s)$. Fig. 6 shows the magnitude plots of the open loop with the real plant, $W_p(s)$, $H_{\min}(s)$, and $q(s)$ obtained from

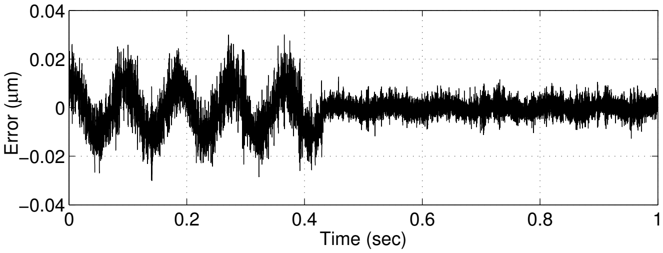

Fig. 8. Tracking error $e(t)$.

(22), (24), and (25). This result leads to

$$\||qS_n| + |W_uT_n|\|_\infty = 0.671,$$

$$\||W_pS_n| + |W_uT_n|\|_\infty = 0.897,$$

respectively, as shown in Fig. 7. Therefore, the robust stability of the repetitive control system is ensured with preserving the robust performance of the feedback control system.

The track-following control system including the repetitive controller was digitally implemented on a 32-bit floating point DSP, TMS320C6727. The program for the track-following control was executed at a sampling rate of 200kHz, which is an extremely high sampling rate for control applications but common in commercial DVD drives. The controller designed in continuous-time domain was transformed to a discrete-time controller by the pole-zero-mapping method.

In the experiment, the repetitive controller was turned on at 0.43sec. Fig. 8 shows the tracking error before and after the application of the repetitive controller. Although the results are affected by the measurement noise, the effect of a repetitive controller is evident in the results. The external disturbance of the disk rotational frequency 12Hz is almost completely attenuated by the repetitive controller. The repetitive controller enabled the track-following control system to reduce the tracking error to a value($\pm 0.1\mu$m) below the maximum allowable boundary($\pm 0.022\mu$m), resulting in more reliable reading/writing of data from/to the optical disk. The improved performance is clearly illustrated by the fast Fourier transform (FFT) results shown in Fig. 9. The repetitive controller reduced the tracking error remarkably at 12Hz, the frequency of disk rotation, which leads to the improved tracking accuracy. However, because the bandwidth of $q(s)$ is restricted to 1kHz, high-order harmonics of tracking error is hardly decreased.

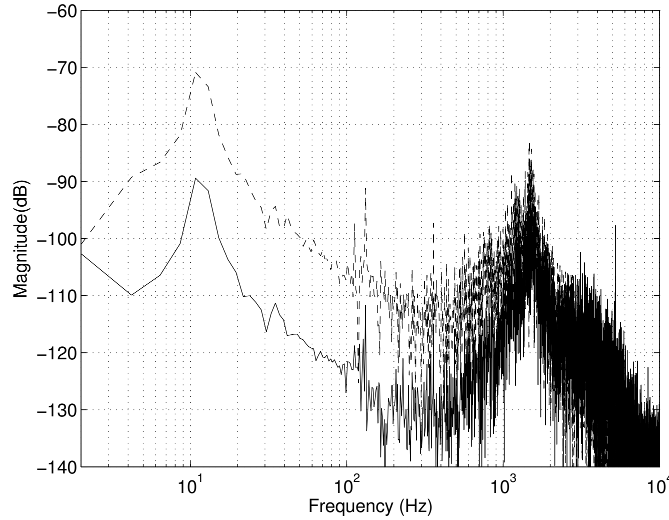

Fig. 9. FFT results of tracking errors without(dash line)/with(solid line) repetitive controller.

V. Conclusions

This paper considered the problem of repetitive controller design for an uncertain feedback system. The robust stability condition of the repetitive control system was obtained using the robust performance condition of the feedback control system. Through the analysis on the steady-state tracking error, the loop and the power of the steady-state tracking error in the repetitive control system was compared with those of the feedback control system. Based on the obtained results, several design criteria proposed to design a repetitive controller. Finally, the application study on the track-following control in optical disk drives were performed and the experimental results were presented to validate the effectiveness of the proposed method.

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