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Robust Pointwise Min-Norm Control of Distributed Systems with Fluid Flow

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Abstract— **This paper reports a Pointwise Min-Norm control (PWMN) general result for a class of distributed systems that include transport phenomena associated with fluid flow in pipes and open pool canals. The main goal is to find a numerical control scheme that ultimately can be embedded in a more general Nonlinear Model Predictive Control formulation as an alternative to ensure closed-loop stability, for moderate values of the receding horizon without increasing dramatically the computational effort. In fact the PWMN control can be viewed as the NMPC limit stabilizing solution when the predictive horizon value goes to zero. A tubular system with finite escape traveling time is used to illustrate the control performance. An application to a canal pool modeled by Saint-Venant's equations is also given. Canals are formed by a sequence of pools separated by gates. Water distribution canals provide interesting examples of distributed parameter plants for nonlinear control application.**

I. INTRODUCTION

This paper describes a Robust Pointwise Min-Norm (RPWMN) control general result for a class of distributed systems with fluid flow, heat transfer and (bio)reactions processes modeled by a set of hyperbolic and/or parabolic PDE equations including convective and dispersive phenomena. This broad class of models arises from physical conservation principles by balances of mass, energy and linear momentum and can describe the temperature and (bio)chemical species distribution on moving fluids through pipes [2], [24] and also fluids hydraulics in open pool channels at atmospheric pressure, predicting velocity and mass, space and time behavior [6].

Pointwise Control for nonlinear finite dimensional systems can be found on several seminal nonlinear control text books [30], [9]. For the control of infinite dimensional see [4] and [5]. The first introduces linear control theory for infinite dimensional systems using semigroups in a mathematical background. In the second robust control methodologies to deal with hyperbolic and parabolic distributed systems are developed and a prototype system closely related with the present class of systems is introduced.

In the last decade the use of a PWMN stability condition for predictive control was studied in [26] and [12] using a slightly different approach. Similar Control Lyapunov Function (CLF) techniques for adaptive control of hyperbolic systems are described in [16].

Nonlinear Model Predictive Control (NMPC) for lumped systems is a well understood control methodology and its main results can be found in [10] and [27]. For a survey see [28]. Predictive control of hyperbolic PDE systems, namely transport-reaction processes, was studied in [8] and [29] for SISO cases. In the former the controller is based on a predictive model developed using the method of characteristics and does not consider constraints. In the latter finite differences for space discretization and a space distributed actuator were used with good results.

Adaptive predictive control was obtained via Orthogonal Collocation reduced modeling, for SISO hyperbolic tubular transport-reaction processes, can be found in [13] and [14]. A successful application, for a distributed uncertain solar power plant, where NMPC was combined with Feedback Linearizing is presented in [18] and with Lyapunov Adaptation in [15]. A wide bibliography on water distribution in open canals control is available. For a selection of the controller structure combined with robust design methods in order to achieve a compromise between water resources management and disturbance rejection see [23]. Predictive control with adaptation is considered in [11] and [21] and also in [17] for a multivariate NMPC application to a water canal pool.

In this paper the main goal is to establish a numerical PWMN control scheme that ultimately can be combined in a more general NMPC design as a way to ensure closed-loop stability, for moderate values of the prediction horizon, and without increasing dramatically the computational effort or massive off-line computation. In fact, the PWMN control can be viewed as the NMPC limit stabilizing solution when the horizon value goes to zero. The rest of the paper is organized as follows: Section 2 introduces the class of systems under study. Section 3 states the main result for RPWMN control and discuss its use as a stabilizing limit solution for continuous NMPC formulation. Section 4 presents a detailed example for a tubular system with finite escape traveling time and section 5 is dedicated to a canal pool modeled by Saint-Venant equations application to illustrate by simulation the proposed control scheme. Section 6 draws conclusions.

II. MODELS

Consider the following class of PDE models:

$$\begin{aligned}\frac{\partial x(z,t)}{\partial t} + \mathscr{L}(x(z,t),u(t);\theta) &= s(x(z,t),u(t);\theta) \quad (1)\\ \mathscr{M}(x(z,t),u;\theta) &= 0\end{aligned}$$

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$$y_k(t) = \int_0^1 b_k(z)\, h_k(x(z,t))\, dz \quad (k = 1,...,p) \tag{2}$$

where $(z,t) \in [0,1] \times \mathbb{R}_{\geq 0}$, the state trajectories $x(.,t) \in X$,a n-dimensional-like vector of smooth functions defined on the space interval $[0,1]$, the manipulated input $u(t) \in U \subset \mathbb{R}^m$ and is bounded and where the output $y(t) \in Y \subset \mathbb{R}^p$ and is also bounded. The operator $\mathscr{L}(.,u;\theta)$ is a quasi-linear matrix space operator. Boundary conditions are given by the nonlinear space operator $\mathscr{M}(.,u;\theta)$. Vector functions $s(x,d)$, $h(x)$ are smooth vectors of nonlinear functions, $s(x,d) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, $h(x) : \mathbb{R}^n \to \mathbb{R}$. The space weight $b_k(z) > 0 : [0,1] \to \mathbb{R}^+$ satisfies $\int_0^1 b_k(z)\, dz = 1$.

The column vector $\theta \in \Theta \subset \mathbb{R}^q$ denotes the uncertain parameters or additive disturbances that lies within a known open convex set $\Theta = \{\theta : \underline{\theta} < \theta < \overline{\theta}\}$.

The $\mathscr{L}$ operator must include convection terms $v\frac{\partial(.)}{\partial z}$ and will depend on the manipulated variable(s) when $u \equiv v$. If the manipulated variable is space weighted additive in the production term then:

$$s(x,u;\theta) = s_x(x;\theta) + s_x(x;\theta)\, w(z)\, u \tag{3}$$

In both cases the manipulated variable(s) is explicit on equation (1) and it will be implicit when it only appears in the boundary condition.

These prototype configurations allow the study of a wide variety of processes with transport phenomena, see for instance [2], including tubular reactors [20], bio-reactors [7], heat exchangers, solar fields [3], and fluid flow in water distribution canals. This class of distributed nonlinear systems may exhibit complex dynamical behavior, with strong space dependency, such as unstable dynamics [1], unstable with traveling finite escape time, non minimum-phase behavior [25], *hot spots* characteristics [20] and fluid flow traveling waves and oscillation [24].

III. POINTWISE MIN-NORM CONTROL

An implicit stabilizing controller may be design obtained using the following optimization statement: *if* $V(e) > \phi$ *then*

$$\begin{aligned} &\min_{u \in U} \quad u^T u \\ &s.t.\ (1)\ \textit{and}\ \max_{\theta \in \Theta}\{\dot{V}(e;\theta)\} + \alpha V(e) < 0 \end{aligned} \tag{4}$$

where $V(e) : X \to \mathbb{R}_{\geq 0}$ is a continuously differentiable, positive definite and radially unbounded function in respect to the L_2 norm of e, see [5], and $e(z,t) = x(z,t) - x_r(z;\theta_0)$ is the difference between the actual state and a steady state profile along space length, where $x_r \in X$, obtained for parameter nominal value θ_0. In other words, $V(e)$ is simply a robust CLF candidate whose derivative maximum can be made less than $-\alpha V(e)$ pointwise by the choice of control values outside a small region around the origin. This region can be made arbitrarily small by picking ϕ sufficiently small. Small control property, see [30], and numerical issues can be avoided in this way. The parameter α relax convergence.

The most obvious choice for the CLF candidate is:

$$V(e) = \frac{1}{2}\int_0^1 e^T q(z) e\, dz \tag{5}$$

where $\int_0^1 q(z)\, dz = 1$, with $q(z)$ positive definite.

Using Lyapunov stability arguments [22]: any $e(z,t)$ solution, originating in a bounded region, will asymptotically tend to the included invariant region parameterized by ϕ as $t \to \infty$, if $\dot{V} < 0$ $(\forall\, e \neq 0)$.

In this case, the time derivative of (5) yields:

$$\dot{V} = \frac{1}{2}\int_0^1 \frac{\partial}{\partial t}\left(e^T(z,t)\, q(z)\, e(z,t)\right)\, dz \tag{6}$$

using (1):

$$\dot{V} = \int_0^1 (s(x,u;\theta) - \mathscr{L}(x,u;\theta)^T q(z) e(z,t)\, dz \tag{7}$$

and the robust optimization condition, for the class of systems under study, is given by:

$$\max_{\theta}\left\{\int_0^1 (s(x,u;\theta) - \mathscr{L}(x,u;\theta))^T q(z) e(z,t)\, dz\right\} + \frac{\alpha}{2}\int_1^0 e^T q(z) e\, dz < 0 \tag{8}$$

Remark that one of the following conditions must hold *a priori* in relation to the manner how the inputs appear in (1). If the corrected condition does not hold then the candidate V is not a RCLF and a different candidate or method must be used. Condition $\int_0^1 (\frac{\partial x}{\partial z})^T A_v\, q(z) e\, dz \neq 0$, where A_v is a diagonal matrix with one or zero in the main diagonal, must hold if the corresponding state is related with the manipulated velocity. Or $\int_0^1 (s_x(x;\theta)\, w(z)\, u)^T q(z) e\, dz \neq 0$ *iff* $u \neq 0$ if u is related with the production term. Finally $\mathscr{M}(x,u,\theta) \neq 0$ *iff* $u \neq 0$ if u is implicit through boundary conditions. If these conditions do not hold then controllability from u to the 'output' $\equiv V$ is lost. Remark also that these conditions depend on $q(z)$ for finding V.

The above results can be summarized in the following propositions:

Proposition 1: Consider the class of distributed systems $\Sigma_\Delta = (\mathscr{L}, \mathscr{M}, s, U, \Theta, Y, R)$ with solutions $x(z,t)$ defined by (1), then the function V given by (5) is a RCLF for Σ_Δ if and only if exists scalars ϕ, $\alpha \in \mathbb{R}^+$ such that:

$$\min_{u \in U}\max_{\theta \in \Theta}\{\dot{V}(e;\theta)\} + \alpha V(e) < 0$$

whenever $V(e) > \phi$.

Proposition 2: The $u \in U$ referred in proposition 1 can be obtained by the optimization statement (4) when feasible, meaning that the set $U \supset u$ is large enough.

Feasibility in the last proposition rises only from the fact that in general unstable systems cannot be stabilized globally when input constrains are present. Without feasibility there is no guarantee for global stability.

An important PWMN control feature is evident from the remark that it can be viewed as the NMPC limit stabilizing solution when the horizon value goes to zero. Consider the following NMPC formulation:

$$\begin{aligned} &\min_{u \in U} \quad \int_t^{t+\mathscr{T}} (V(e(z,\tau)) + \rho u^T u) d\tau \\ &s.t.\ \ (1)\ \textit{and}\ \ \max_{\theta \in \Theta}\{\dot{V}(e;\theta)\} + \alpha V(e) < 0 \end{aligned} \tag{9}$$

Consider what happens as the horizon $\mathscr{T}$ tends to zero:

$$\min_{u} \lim_{\mathscr{T}\to 0} \frac{1}{\mathscr{T}} \int_{t}^{t+\mathscr{T}} (V(e(z,\tau)) + \rho\, u^T u) d\tau$$
$$= \min_{u} \{V(e(z,\tau)) + \rho\, u^T\, u\} \Leftrightarrow \min_{u} u^T u \quad (10)$$

Remark that dividing (9) by $\mathscr{T}$ has no effect on the optimization problem and also that when $\mathscr{T}$ goes to zero there is no need to include the term $V(e)$ because it is not affected by u and so forth constant. Hence this simple observation, stated in [26], indicates that as $\mathscr{T}$ goes to zero receding horizon controllers loses the ability to maintain acceptable performance just by minimizing input energy. Performance degradation can lead in general to closed-loop instability if the RPWMN condition or some other equivalent mechanism is not included to assure stability. Remark also that the constraint requires V to be a RCLF for any receding horizon value and by that closed-loop stability is guaranteed.

IV. FINITE ESCAPE TRAVELING TIME DISTRIBUTED SYSTEM

Consider a possibly uncertain distributed hyperbolic system with finite traveling escape time, is given by:

$$\frac{\partial x}{\partial t} + \frac{u(t)}{L} \frac{\partial x}{\partial z} = \theta x^2 \quad (11)$$

with parameter $\overline{\theta} > \theta > \underline{\theta} > 0$, solving for $x(0,t) = 0$ it yields:

$$x(z,t) = \frac{1}{\varphi(z,0) - \theta t} \quad (12)$$

where $\varphi(z,0) = x^{-1}(z,0)$. Clearly for constant velocity $u > \theta L$ to stabilize the system around $x(z,t) = 0$ if $x(z,0) = 1$, see Fig. 1.

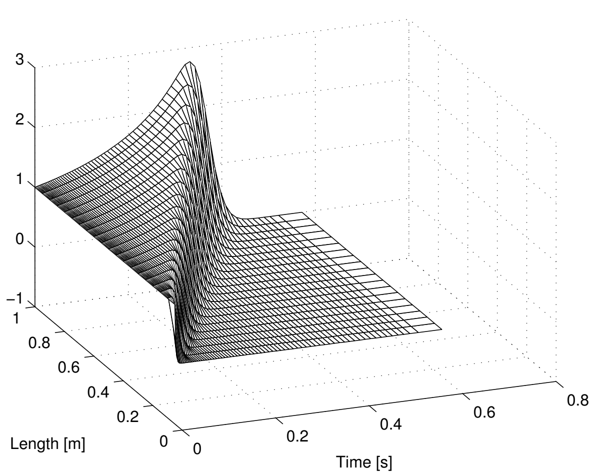

Fig. 1. Numerical solution, by 200 finite differences, for u=2.5 m/s, $\theta = 2$ and $x(z,0) = 1$.

The space stationary profile, defined by $\frac{\partial x}{\partial t} \equiv 0$, can be obtained from:

$$\frac{dx(z)}{dz} = \gamma x^2 \quad (13)$$

where $\gamma = \frac{\theta L}{u}$. Denoting $x(0,t) = x_{in}$ for the boundary condition, then the solution for the stationary state profile is:

$$x_r(z) = \frac{x_{in}}{1 - \gamma_r x_{in} z} \quad (14)$$

With the output set-point $x_r(1,t) = r$, and using (14) it follows:

$$r = \frac{x_{in}}{1 - \gamma_r x_{in}} \quad \text{and} \quad u_r = \frac{\theta\, r x_{in}}{r - x_{in}}, \quad (15)$$

with $u_r > 0$ which implies $r > x_{in}$. Substituting (15) in (14) it yields:

$$x_r = \frac{r x_{in}}{r - rz + x_{in} z} \quad (16)$$

for $x_r(0) = x_{in}$ and $x_r(1) = r$.

Stabilizing around the stationary profile, using the candidate Lyapunov function:

$$V = \frac{1}{2} \int_0^1 e^2 dz \quad (17)$$

with $e = x - x_r$. Differentiating (17) with respect to time:

$$\dot{V} = -\frac{u}{L} \int_0^1 e\, \frac{\partial x}{\partial z} dz + \int_0^1 e\, \theta x^2 dz \quad (18)$$

The feasible optimization problem can be written as:

$$\min_{u>0} \quad u^2 \quad (19)$$
$$s.t. \quad \frac{\partial x}{\partial t} + \frac{u}{L}\frac{\partial x}{\partial z} = \theta x^2$$
$$-\frac{u}{L}\int_0^1 e\,\frac{\partial x}{\partial z} dz + \max_{\theta}\left\{\theta \int_0^1 e\, x^2 dz\right\} + \alpha \frac{1}{2}\int_0^1 e^2 dz < 0$$

yielding the following dynamical bound $\max\{\dot{V}\} + \alpha V < 0$. In this case, an analytical solution can be found for (19):

$$u = \frac{\check{\Theta} \int_0^1 e\, x^2 dz + \alpha\, V}{\int_0^1 e\, \frac{\partial x}{\partial z} dz} L \quad (20)$$

where $\check{\Theta}$ switches from $\underline{\theta}$ to $\overline{\theta}$ with $sign(\int_0^1 e\, x^2 dz)$. Remark that u is well-defined because $\int_0^1 e\, \frac{\partial x}{\partial z} dz \neq 0$ outside the curve level $V(e) = \phi$ and the space operators converge exponentially to zero as $e(z,t) \to 0$. In fact this can be shown observing that the PDE solution $x(z,t) = (x_{in}^{-1} - \theta z L/u(t))^{-1}$, along characteristic curve $\dot{z} = u(t)/L$, implies $\frac{\partial x}{\partial t} > 0$ and $e(z,t) \neq 0$ for any $z \neq 0$, outside the invariant region. Remark also that equation (20) coincides with the control law obtained through feedback linearization with $y \equiv V$ with a characteristic index equal to one outside the invariant region [4].

Figs. 2 and 3 show $x(z,t)$ and $u(t)$ when the set-point suddenly changes from 3 to 2 at $t = 5\ s$. This simulation results were obtained solving (19) for 200 space finite differences. Figs. 4 and 5 show $x(z,t)$ and $u(t)$ when the set-point changes from 3 to 2 at $t = 5\ s$, and from 2 to 3 again at $t = 10\ s$ for bounded uncertain θ. Remark that in both cases u_r is unknown.

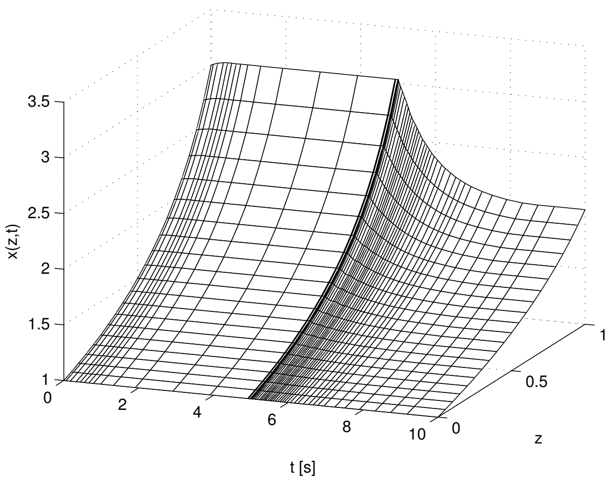

Fig. 2. State $x(z,t)$ transition, for $\theta = 2$, $\alpha = 2$ and $\phi = 0.0001$.

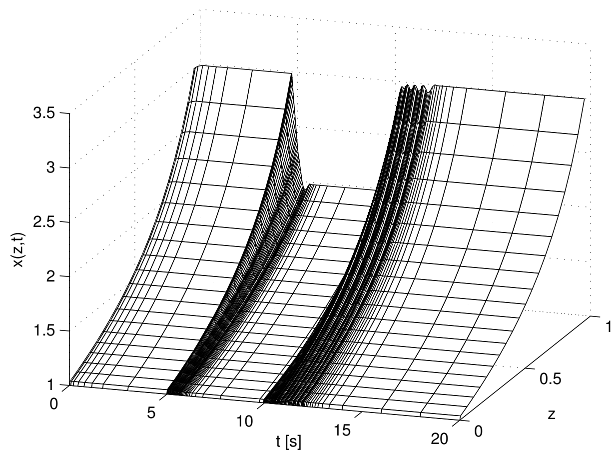

Fig. 4. Robust state $x(z,t)$ transition, for $\theta \in [1.8\ 2.2]$, $\alpha = 2$ and $\phi = 0.0001$.

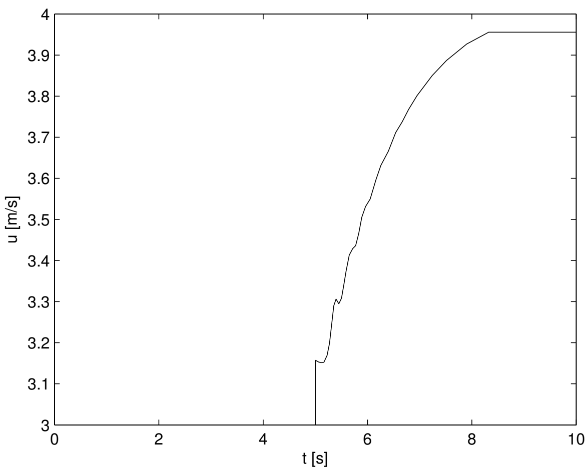

Fig. 3. Manipulated velocity u [m/s].

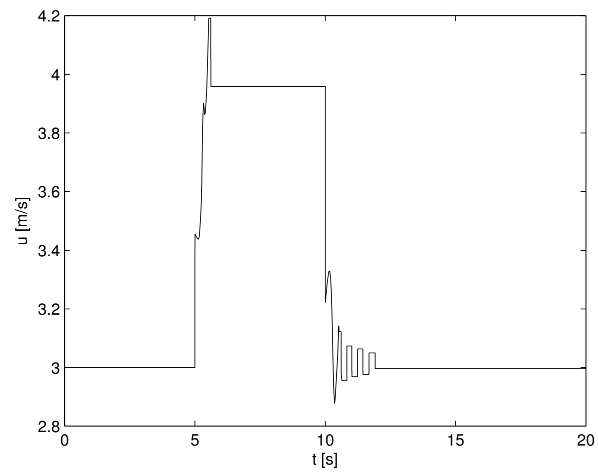

Fig. 5. Manipulated velocity u [m/s].

V. WATER DISTRIBUTION CANAL POOL

Water distribution canals provide interesting examples of distributed parameter plants for which nonlinear control may be applied. Canals are formed by a sequence of pools separated by gates. Output variables are the pool level at certain points, manipulated variables are the position of the gates and disturbances are the outlet water flows for agricultural use. The operation of this system is subject to a number of constraints. These are the minimum and maximum positions of the gates, gate slew-rate and the minimum and maximum water level. The objective considered in this application is to drive the canal pool level to track a reference in the presence of disturbances. The pool level is a function of both time and space that satisfies the Saint-Venant equations. These are a set of hyperbolic partial differential equations that embody mass and momentum conservation.

Saint-Venant equations for a single pool model without infiltration are given by:

$$\begin{aligned} \frac{\partial h}{\partial t} + \frac{v}{L}\frac{\partial h}{\partial z} + \left(\frac{da}{dh}\right)^{-1}\frac{a(h)}{L}\frac{\partial v}{\partial z} &= 0 \\ \frac{\partial v}{\partial t} + \frac{v}{L}\frac{\partial h}{\partial z} + \frac{g}{L}\frac{\partial v}{\partial z} + g(\mathscr{I}(h,v) - \mathscr{J}) &= 0 \end{aligned} \tag{21}$$

where $h(z,t)$ and $v(z,t)$ are respectively level and water velocity distributions along space ($z \in [0,1]$) and time, wet surface $a(h)$ and friction $\mathscr{I}(h,v)$ are nonlinear functions, g is the gravity acceleration, J is the constant canal slope and L is the pool length. Boundary conditions are given by flow at upstream and downstream gates:

$$v(0,t) = c_d A_d(u)\sqrt{(2g(H_u(t) - h(0,t)))}/a(h(0,t)) \tag{22}$$

TABLE I

POOL PHYSICAL PARAMETERS.

Parameter		Value	Units
Gravitational constant	g	9.8	ms^{-1}
Manning coefficient	n	1.0	$m^{-1}s^{-3}$
Discharge coefficient	c_d	0.6	$-$
Discharge area	$A_d(u)$	$0.49\,u$	m^2
Bottom width	b	0.15	m
Trapezoid slope	d	0.15	$-$
Canal slope	$\mathscr{J}$	2×10^{-3}	$-$
Upstream elevation	H_u	2.0	m
Downstream elevation	H_d	1.0	m

$$v(1,t) = c_d A_d(u)\sqrt{(2g(h(1,t)-H_d(t)))}/a(h(1,t)) \quad (23)$$

In this paper a single trapezoidal reach with two pools, two moving gates (upstream ends) and a fixed gate at the downstream end is considered. The water elevation immediately before and after the reach, $H_u(t)$ and $H_d(t)$ respectively, are assumed known. See table I for physical parameters values, the canal pool is depicted in Fig. 6.

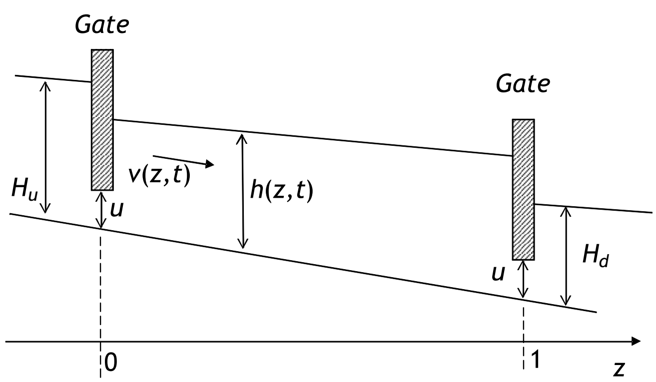

Fig. 6. Canal pool schematic.

Using PWMN optimization statement (20) a water elevation control was developed using the upstream gate position as control signal. The error signal was obtained by considering a water level measure and the corresponding reference at some fixed distance from the gate, velocity distribution was not included in the optimiztion process to keep low computational effort.

Figs. 7 and 8 show respectively water elevation and gate manoeuver for RPWMN level control at 1.7 m, upstream, when downstream gate opens from 0 to 0.1 m at $t = 0\,s$. The pool was initially at rest with 1.5 m at $z = 0.5$. Numerical results were obtained with 200 space finite differences, more details about model space reduction can be found in [17]. Controller parameters values: $\alpha = 0.5$ and $\phi = 1\times10^{-5}$.

Figs. 9 and 10 show the same experiment when downstream gate opens from 0 to 0.1 m at $t = 10\,s$. Remark that in the first experiment the wave back propagation effect on gate opening is very small causing only a small overshoot. In

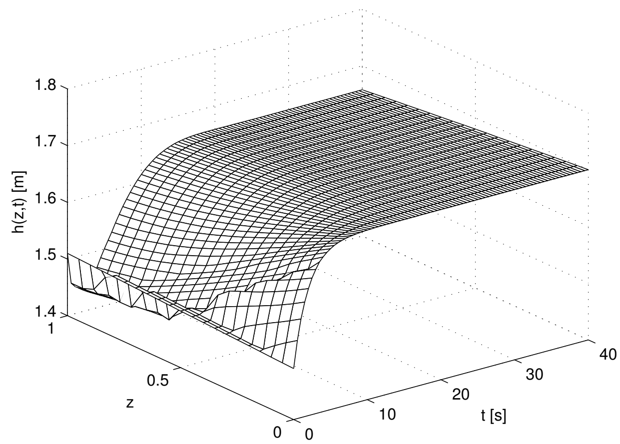

Fig. 7. Water elevation $h(z,t)$ $[m]$.

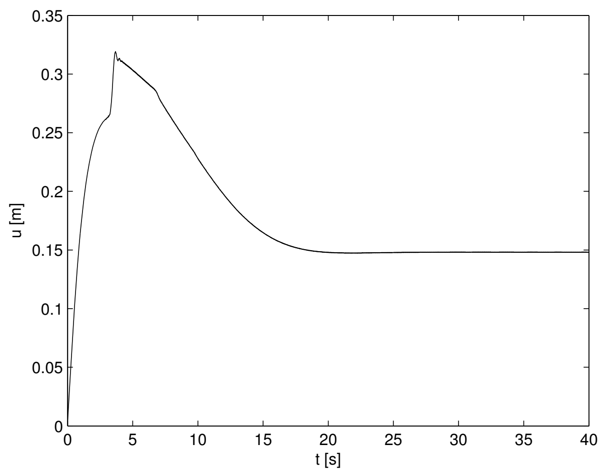

Fig. 8. Upstream gate opening u [m].

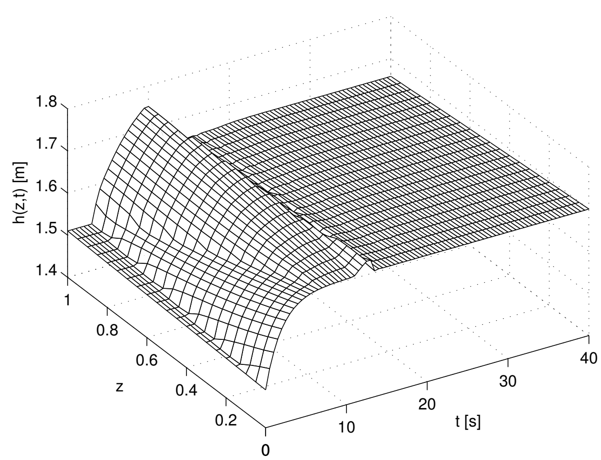

Fig. 9. Water elevation $h(z,t)$ $[m]$.

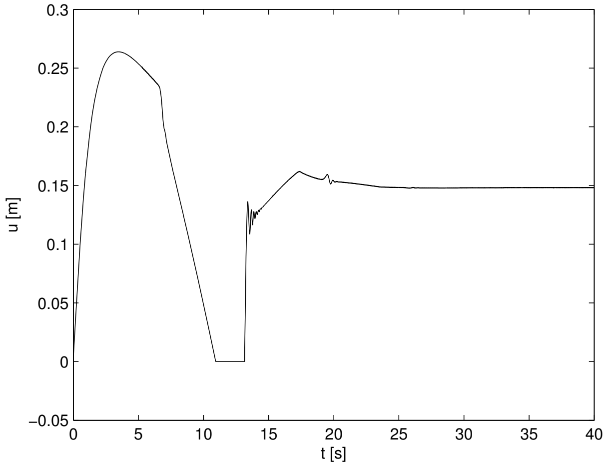

Fig. 10. Upstream gate opening u [m].

the last case there are two combined observable facts. Until $t = 10\,s$ the pool level is increasing and the downstream gate is closed creating a large water rise near it. After $t = 10\,s$ the downstream gate opens letting the water down flow. During this period the upstream gate opening must actuate accordingly causing the constraint $u \geq 0$ to be active for a considerable period of time. This last experiment illustrates how this type of control can handle hard constraints.

VI. CONCLUSIONS

A general result for distributed fluid flow systems stabilization around a stationary space profile was derived using RPWMN methodologies. The derived optimization statement can be interpreted as a stabilizing limit solution, when included in a predictive receding horizon control formulation, ensuring by that way closed-loop stability for any horizon value. The viability of this alternative technique was shown along with the advantage of small increase on the computational effort, observed in the numerical example and application. The results obtained for water distribution canal pool RPWMN control form a basis of more complex studies on canal engineered architectures that by combining robust and predictive design methods can achieve a fair compromise between water resources management and disturbance rejection.

REFERENCES

[1] Bošković, D. and M. Krstić. *Backstepping control of chemical tubular reactors*, Computers & Chemical Engineering, Vol. 26, 15 August 2002, pages 1077-1085.

[2] Bird, R. B., W. E. Steward and E. N. Ligthfoot. *Transport Phenomena*, John Willey, 1960.

[3] Camacho, E. F., M. Berenguel and F. Rubio. *Advanced Control of Solar Plants*, Springer-Verlag, 1997.

[4] Christofides, P. D. *Nonlinear and Robust Control of PDE Systems*, Birhäuser, 2001.

[5] Curtain, R. F. and H. J. Zwart. *An Introduction to Infinite-Dimensional Linear Systems Theory*, Texts in Applied Mathematics 21, Springer-Verlag, 1995.

[6] Debnath, L. *Nonlinear Partial Differencial Equations for Scientists and Engineers*, Birkhäuaer, 1997.

[7] Dochain, D., J. P. Babary and N. Tali-Maamar. *Modelling and Adaptive Control of a Nonlinear Distributed Parameter Bioreactors via Orthogonal Collocation*, Automatica, Vol. 28, N° 5, pages 873-883, 1992.

[8] Dubljevic, S., P. Mhaskar, N. H. El-Farra and P. Cristofides. Predictive control of transport-reaction processes. *Computer and Chemical Engineering*, Vol. 29, pages 2335-2345, 2005.

[9] Freeman, R. A. and P. V. Kokotović. *Robust Nonlinear Control Design*, Birkhauser Boston, 2008.

[10] Findeisen, R. and F. Allgöwer. *An Introduction to Nonlinear Model Predictive Control*, 21st Benelux Meeting on Systems and Control, Veidhoven, 2002.

[11] Gomez, M., J. Rodellar and J. Mantecon. Predictive control method for decentralised operation of irrigation canals. *Applied Mathematical Modelling*, 26:1039-1056, 2002.

[12] He, Y. and J. Han. *Nonlinear Model Predictive Control Enhanced by Generalized Pointwise Min-Norm Scheme*, Proceedings of the 46th IEEE Conference on Decision, 2007.

[13] Igreja, J. M., J. M. Lemos and R. N. Silva. *Adaptive Receding Horizon Control of a Distributed Collector Solar Field*, Proceedings of the 44th IEEE CDC-ECC, 2005.

[14] Igreja, J. M., J. M. Lemos and R. N. Silva. *Adaptive Receding Horizon Control of Tubular Bioreactors*, Proceedings of the 44th IEEE CDC-ECC, 2005.

[15] Igreja, J. M., J. M. Lemos and R. N. Silva. *Nonlinear Predictive Control of a Solar Plant Based on Reduced Complexity Models*, Proceedings of the IFAC 2005, Czech Republic, 2005.

[16] Igreja, J. M., J. M. Lemos and R. N. Silva. *Adaptive Control of Hyperbolic Systems: A CLF approach*, Proceedings of the European Control Conference, Kos, 2007.

[17] Igreja, J. M. and J. M. Lemos. *Nonlinear Model Predictive Control of a Water Distribution Canal Pool*, *Model Predictive Control: Towards New Challenging Applications*, In L. Magni, D. M. Raimondo and F. Algöwer, editors, pages 521-525, Springer, 2009.

[18] Isidori, A. *Nonlinear Control Systems*, Springer, 1995.

[19] Krstić, H., I. Kanellakopoulos and P. Kokotović. *Nonlinear & Adaptive Control Design*, John Willey & Sons, 1995.

[20] Lefèvre, L., D. Dochain, S. Feyo de Azevedo and A. Magnus. *Optimal selection of orthogonal polynomials applied to the integration of chemical reactor equations by collocation methods*, Computers & Chemical Engineering, Vol. 24, pages 2571-2588, 2000.

[21] Lemos, J. M., L. M. Rato, F. Machado, N. Nogueira, P. Salgueiro, R. N. Silva and M. Rijo. Predictive adaptive control of water level in canal pools. *Proc. 16th Int. Conf. Systems Science*, Wroclaw, Poland, pages 139-148, 2007.

[22] La Salle, J. and S. Lefschetz. *Stability by Liapunov's Direct Method*, Academic Press, 1961.

[23] Litrico, X., and V. Fromion. H_∞ Control of an Irrigation Canal Pool With a Mixed Control Politics. IEEE Trans. Control Syst. Tech., 14(1):99-111, 2006.

[24] Marquardt, W. *Traveling waves in chemical processes*, International Chemical Engineering, Vol. 30, N 4, 1990.

[25] Nihtila, M. T., J. P. Tervo, J. P. Kaipio and J. P. Babary. *Controller design issues and algorithms for a nonlinear distributed-parameter process*, IEEE International Conference on Computational Cybernetics and Simulation 1997, Systems, Man, and Cybernetics, Vol. 3, pages 2414-2419, 1997.

[26] Primbs, J. A., V. Nevistić and J. C. Doyle. *A Receding Horizon Generalization of Pointwise Min-Norm Controllers*, IEEE Transactions on Automatic Control, Vol. 45, N 5, 2000.

[27] Rawlings, J. B. and D. Q. Mayne. *Model Predictive Control: Theory and Design*, NobeHill Publishing, 2009.

[28] Rawlings, J. B., D. Q. Mayne, C. V. Rao and P. O. M. Scokaert. *Constrained model predicitve control: Stability and optimality*, Automatica 36 (2000) 789-814.

[29] Shang, H., J. F. Forbes and M. Guay. Model preditive control for quasilinear hyperbolic distributed parameters systems. *Industrial and Enginnering Chemical Research*, Vol. 43, pages 2140-2149, 2004.

[30] Sontag, E. D. Mathematical Control Theory: Deterministic Finite Dimensional Systems, Springer, 1998.