2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Explicit coordination for MPC-based distributed control with application to Hydro-Power Valleys

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***Abstract*— This paper discusses a decomposition-coordination control approach for large-scale systems, based on distributed MPC controllers and a specific coordination, with an application to the control of a so-called Hydro-Power Valley. The coordination strategy here explored can be characterized as an explicit interaction-prediction method, in the sense that the coordinator distributes predicted interactions to each subsystem on the basis of the information collected from those subsystems on the one hand, and takes advantage of explicit solutions for linear MPC control to globally update those predictions on the other hand. It is emphasized in the paper how this makes the approach suitable for real-time implementation, constraint handling, and communication limitations. In particular promising simulation results are provided for an industrial based Hydro-Power Valley case-study, chosen for the purpose of illustration, but using real data from French main electricity provider EDF.**

I. Introduction

Real-time control of large-scale systems is still an open problem since such a control a priori has to manage a large amount of information and to solve large-scale computational problems in very short periods of time [17], [15], [4]. Relying on faster computers with larger memory may even not be enough. In addition, such systems are subject to several unpredictable events which demand a more robust control solution in order to respect the system constraints and/or specifications. Several methodologies have been elaborated to cope with the complexity of such systems, most of them belonging to one of the following groups: decentralization [1], decomposition [3] or just model simplification.

The most cited decomposition-coordination techniques have been introduced more than 30 years ago for solving large-scale optimization problems. Methods such as *Model coordination*, *Goal coordination* or *Interaction prediction* have been originated by [14] and also considered by various other researchers who made significant contributions to those ideas [5], [22]. The present work is part of a renewed interest on such methods [8], [16], [25], [26]. Those techniques have not been extensively or successfully used in control applications for several reasons: one reason is the need for separability properties of the optimization problem; another reason is the fact that most of the decomposition-coordination algorithms must solve an optimization problem that fails to converge for nonconvex optimization problems; a third reason is the fact that these approaches require a large amount of data exchanges (which can take a long time and be subject to data losses) as well as adequate communication media. These aspects will be taken into account in the present work, in particular focusing on a specific *interaction-prediction coordination* method.

This work is motivated by the problem of finding an optimal control for a so-called Hydro-Power Valley (HPV) system [11], [12], namely a set of hydroelectric production plants, depending on interconnected water resources (typically along a river). Such systems have dynamics similar to those of irrigation networks, for which some decentralized control designs are discussed for instance in [13], [9] and [6].

On the other hand, distributed *Model Predictive Control* (MPC) schemes have already been proposed for interconnected systems, as in [20], [24], [19], [7] to cite a few examples.

Here, we explore the application of *Explicit* MPC in a decomposition-coordination scheme in order to solve for *real-time constrained* optimization problems in a more efficient computation time by reducing the computational complexity. This can indeed be possible by using explicit solutions of the MPC problem. Explicit solution of MPC problems have been discussed in [10], [21], [23] and [2]. In addition, we emphasize how to take advantage of such explicit solutions to propose some improved interaction-prediction coordination, by including global prediction updates at the coordinator level. This allows to reduce communication exchanges w.r.t. the two main types of coordination strategies more classically proposed (either price-driven or quantity-driven coordinations [5]) and relying on iterative schemes for convergence guarantees. The use of explicit solutions is extended to the case when constraints are also to be taken into account, and the method is applied to an example of HPV control, for which such decomposition-coordination approaches appear to be quite well suited.

The operation of a Hydro-power valley can indeed be described as follows: A daily power-generation program is proposed to each power-plant (or subsystem) and has to be respected as much as possible. However, in practice there are many unpredictable events such as, for instance, plant failures and meteorological changes which make the regulation problem more difficult. This is mainly because, in a power plant, the reservoir levels can be regulated by

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means of local controllers which drive the inlet valves of the turbines (turbine flow control) that determine at the same time the generated power (i.e. the generated power is proportional to the turbine flow). Thus, trying to regulate the reservoir levels in the presence of large unpredictable disturbances can lead to variations and modifications of the power generation program. Those systems are already well developed but a qualitative improvement is still possible by using an optimal real-time management of the resources that will maximize the efficiency of the hydro-power plants while respecting the environmental constraints (e.g. irrigation needs and navigation capabilities).

This paper is organized as follows: firstly, the problem formulation is discussed in section II, including recalls on decomposition-coordination methods. Then the proposed interaction prediction coordination method is introduced in section III, with emphasis on the coordinator task and local tasks. Real-data based case-study and simulation results are finally proposed in section IV, before concluding in section V.

II. Decomposition-Coordination

As recalled in the introduction, one attractive approach to solve for large-scale control problems is that based on a system "decomposition" into subsystems of lower dimension. This sounds of particular interest in the case of a Hydro-Power Valley, since the full system naturally results from the interconnection of several subsystems, each of them having its own controller. This also raises the problem of coordination between those subsystems, in particular in the context of power production. Both aspects are presented hereafter.

A. System decomposition

The configuration of the considered Hydro-Power Valley is depicted in figure 1. The whole system can be modelled, in a discrete-time state space representation, as follows:

$$x_{k+1} = Ax_k + Bu_k + Ed_k \quad (1)$$
$$y_k = Cx_k \quad (2)$$

where the current system state vector $x_k \in \mathbb{R}^n$ corresponds to the reservoir levels, the vector $x_{k+1} \in \mathbb{R}^n$ denotes the one-step ahead value of the system state. The control input vector $u_k \in \mathbb{R}^m$ gathers the turbine flows, and $d_k \in \mathbb{R}^n$ is the disturbance input vector. In a general decomposition approach, the model for each subsystem i ($i = 1, \cdots, n$; with n: the number of subsystems) is obtained by considering a relevant partition of the centralized model (1), and combining the effect of local variables with that of interconnection variables, that is:

$$x_{(i)k+1} = A_{ii}x_{(i)k} + B_{ii}u_{(i)k} + v_{(i)k} + E_i d_{(i)k} \quad (3)$$

where the *interactions* terms are given by

$$v_{(i)k} = \sum_{j \neq i}(A_{ij}x_{(j)k} + B_{ij}u_{(j)k}) \quad (4)$$

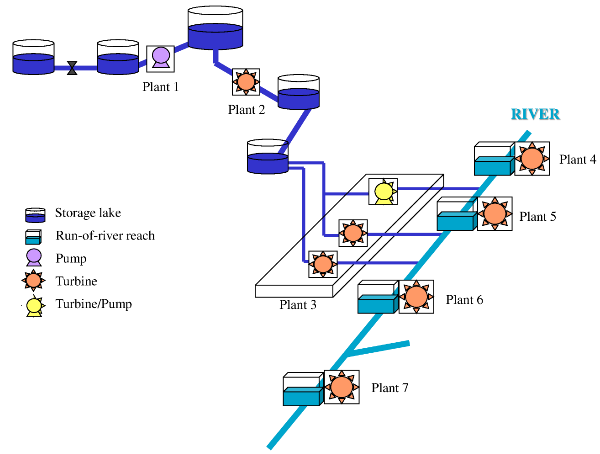

Fig. 1. The Hydro-power valley configuration.

In order to simplify the notations, we can rewrite $v_{(i)k}$ as follows:

$$v_{(i)k} = v_A x_k + v_B u_k \quad (5)$$

with
v_A = Matrix A with the main diagonal are all zero.
v_B = Matrix B with the main diagonal are all zero.

In the case of a HPV as in figure 1, the decomposition can naturally rely upon the pre-existence of separate plants, which can be chosen as subsystem.

B. Problem formulation and chosen approach

Aiming at following a daily power-generation program for the whole HPV, the coordination issue should also be considered.

A purely centralized control solution, when computable, could be seen as the "strongest" coordination since the control takes into account the full information at any computation time and from this provides the appropriate law to each actuator. An alternative decentralized approach can simply rely on direct interconnection information exchanges between subsystems without any coordination entity [18], still requiring a lot of communication. In effective coordination schemes, either driven by prices or by quantities (see e.g. [5]), the dynamical implementation classically proposed also means intensive information exchanges between the coordinator and the subsystems.

The purpose of the paper is thus to take advantage of such decomposition-coordination methods for the considered problem of HPV control, but at the same time to take into account the problem of communication constraints for such a system (limited amount of exchanged information, communication speed, or synchronization for instance). To that end, are combined the idea of interaction exchanges with that of coordination in the spirit of the interaction-prediction coordination described in [14], but with an additional stage of centralized updates by the coordinator at some communication times, taking also advantage of MPC and available explicit solutions for systems of the form

(1) - (2). This indeed relies on a decomposition approach, including a coordination which allows to keep the global performance under control, while also tuning the amount of communication exchanges.

In the next section the proposed interaction - prediction coordination method is introduced as well as the explicit model predictive control (MPC) which is used both at the coordinator and subsystems levels.

III. EXPLICIT INTERACTION-PREDICTION COORDINATION AND LOCAL CONTROLS

The interaction-prediction coordination here is from a practical point of view in the same spirit as the one described in [14], in the sense that the information computed and sent by the coordinator is the interactions between subsystems (this giving the name to the method). It also relies on the general framework of MPC, and is adapted from the original algorithm as follows: the coordinator receives at each communication time the state values from each subsystem. This information is then used to implement a global optimization which allows to compute an explicit optimal prediction of the interactions (i.e. including just information concerning the interaction variables). The coordinator sends these pieces of information to each subsystem. Each subsystem in turn solves a local optimization problem taking into account the interaction vector as a predicted disturbance of the subsystem.

Taking advantage of the fact that, in the proposed framework, explicit solutions are available [21], the interaction-prediction coordination can be applied in a very fast way, but our idea is to take also-into account the limitations in the communications, that is, to coordinate subsystems in communication times which can be chosen larger that local control times. Full formal details are given hereafter.

A. Coordinator

- *MPC formulation*

The coordinator is assumed to solve for a classical MPC problem from data collected from the subsystems at each communication time T_{com}, while each subsystem solves itself a similar problem with sampling time T_s. Classically, given steady state values x_s, u_s for both the state and the control, corresponding to the desired control purpose[1] under a steady state disturbance d_s, the finite horizon optimization problem to be solved by the coordinator consists in minimizing the control tracking error $\tilde{u}_k = u_k - u_s$ (power generation error) and the current state tracking error $\tilde{x}_k = x_k - x_s$ (reservoir levels regulation) over a finite horizon, together with the final state tracking error $\tilde{x}_{N_g} = x_{N_g} - x_s$ (reservoir levels regulation at final time N_g), for a system evolving according to (1) under the effect of disturbance d_k. Formally, this means finding the control sequence $\{u_0, \cdots, u_{N_g-1}\}$ which minimizes:

$$J = \frac{1}{2}\Big(\tilde{x}_{N_g}^{\mathsf{T}} P \tilde{x}_{N_g} + \sum_{k=0}^{N_g-1} \tilde{x}_k^{\mathsf{T}} Q \tilde{x}_k + \sum_{k=0}^{N_g-1} \tilde{u}_k^{\mathsf{T}} R \tilde{u}_k\Big) \quad (6)$$

where $Q \geq 0$, $R > 0$ are state and control weighting matrices and $\tilde{x}_{N_g}^{\mathsf{T}} P \tilde{x}_{N_g}$, $P > 0$ is the terminal cost function. Here N_g is the *coordinator* prediction horizon corresponding to a prediction time T_{pred}^{coord}. Assuming that T_{pred}^{loc} denotes the prediction time for the subsystems (or at least the maximum of subsystems prediction times), and that T_{com} stands for the communication time, then for the purpose of providing enough information by the coordinator to the subsystems for their local optimization, we need:

$$T_{pred}^{coord} \geq T_{pred}^{loc} + T_{com}$$

If N denotes the prediction horizon for the local controllers, and if the same sampling time T_s is the same for all controllers (which is not even strictly necessary), this means that one can choose:

$$N_g = \frac{T_{com}}{T_s} + N \quad (7)$$

when T_{com} is an integer multiple of T_s.

Now it can be pointed out that the fixed horizon optimization problem described in equation (6) for system (1) can be transformed into a *Quadratic Program* (QP). This is possible by defining the following sequences:

$$\begin{aligned} &\mathbf{u} = [u_0, \cdots, u_{(N_g-1)}]^T, \ \mathbf{u}_s = [u_{s0}, \cdots, u_{s(N_g-1)}]^T, \\ &\mathbf{d} = [d_0, \cdots, d_{(N_g-1)}]^T, \ \mathbf{d}_s = [d_{s0}, \cdots, d_{s(N_g-1)}]^T \end{aligned} \quad (8)$$

and matrices:

$$\Gamma = \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N_g-1}B & A^{N_g-2}B & \cdots & B \end{bmatrix}, \quad \Omega = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^{N_g} \end{bmatrix}$$

$$\Theta = \begin{bmatrix} E & 0 & \cdots & 0 \\ AE & E & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N_g-1}E & A^{N_g-2}E & \cdots & E \end{bmatrix} \quad (9)$$

$$\mathbf{Q} = \mathrm{diag}\{Q, \cdots, Q, P\}, \quad \mathbf{R} = \mathrm{diag}\{R, \cdots, R\}$$

The cost function can be expressed in terms of the control sequence $\mathbf{u}$:

$$J = \bar{V} + \frac{1}{2}\mathbf{u}^{\mathsf{T}}\mathbf{H}\mathbf{u} + \mathbf{u}^{\mathsf{T}}[\mathbf{K}_1 x_0 + \mathbf{c}] \quad (10)$$

where

$$\begin{aligned} &\bar{V} = \text{terms independent of } \mathbf{u}, \\ &\mathbf{c} = -\mathbf{K}_1 x_s + \mathbf{K}_2(\mathbf{d} - \mathbf{d}_s) - \mathbf{H}\mathbf{u}_s, \\ &\mathbf{H} = \Gamma^{\mathsf{T}}\mathbf{Q}\Gamma + \mathbf{R}, \ \mathbf{K}_1 = \Gamma^{\mathsf{T}}\mathbf{Q}\Omega, \ \mathbf{K}_2 = \Gamma^{\mathsf{T}}\mathbf{Q}\Theta \end{aligned} \quad (11)$$

The global state vector x_0 is composed by the current state information coming from the subsystems. The above Quadratic Problem without constraints can have the following explicit solution:

$$\mathbf{u}_{uc}^{opt} = -\mathbf{H}^{-1}[\mathbf{K}_1 x_0 + \mathbf{c}] \quad (12)$$

[1]We assume that signals u_s, x_s come from a supervisor system that establishes the desired energy-power generation and the desired water levels as a result, for example, of a pure off-line economical optimization.

- *Constraints*

In addition to the tracking purpose, input and state linear constraints are also taken into account in the control. To that end, those constraints must be expressed as a linear function of the action variable $\mathbf{u}$ of the form:

$$\mathbf{L}\mathbf{u} \leq \mathbf{W} \tag{13}$$

with

$$\mathbf{L} = \begin{bmatrix} \Phi \\ -\Phi \end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix} \bar{\Delta} \\ \underline{\Delta} \end{bmatrix} + \begin{bmatrix} -\Lambda \\ \Lambda \end{bmatrix} x_0 \tag{14}$$

where

$$\Phi = \begin{bmatrix} I_{Nm} \\ \Gamma \end{bmatrix} \quad \bar{\Delta} = \begin{bmatrix} \mathbf{u_{max}} \\ \mathbf{x_{max}} - \Theta \mathbf{d_{max}} \end{bmatrix}$$
$$\Lambda = \begin{bmatrix} \Omega \\ 0 \end{bmatrix} \quad \underline{\Delta} = \begin{bmatrix} \mathbf{u_{min}} \\ \mathbf{x_{min}} + \Theta \mathbf{d_{min}} \end{bmatrix} \tag{15}$$

Hence, the problem is to find the control sequence which minimizes (10), that is in fact a Quadratic Program (QP) problem subject to (1) and constraints (13). QP problems can be solved by using any standard QP solver, however we explore here the applicability of explicit solutions to obtain more computational efficient coordination.

- *Geometric characterization of QP*

The method of QP characterization proposed by [21] and [10] uses a coordinate- transformation (via the square root of the Hessian, that is, $\tilde{\mathbf{u}} = H^{\frac{1}{2}}\mathbf{u}$) and then based on the constraint polyhedron (13) dividing the full state space.

An active set "l" includes the indices of the active constraints, and by means of this one, it can be possible to identify the active face of the constraint polyhedron Φ_l and the associated constrained region.

This method then proposes that the optimal solution is given by the point on the corresponding active face at the closest Euclidean distance to the unconstrained optimal solution $\mathbf{u}_{uc}^{opt}$, leading to an explicit control $\mathbf{u}^{opt}$ as follows:

$$\mathbf{u}^{opt} = H^{-\frac{1}{2}}\tilde{\Phi}_l^{\mathsf{T}}[\tilde{\Phi}_l\tilde{\Phi}_l^{\mathsf{T}}]^{-1}(\Delta_l - \Lambda_l x_0) -$$
$$H^{-\frac{1}{2}}[I - \tilde{\Phi}_l^{\mathsf{T}}[\tilde{\Phi}_l\tilde{\Phi}_l^{\mathsf{T}}]^{-1}\tilde{\Phi}_l]H^{-\frac{1}{2}}(\mathbf{K}_1 x_0 + \mathbf{c}) \tag{16}$$

with matrices defined in (12), (15); $\tilde{\Phi} = \Phi\mathbf{H}^{-\frac{1}{2}}$. $\tilde{\Phi}_l$ and Λ_l corresponds to the "l" rows of the matrices $\tilde{\Phi}$ and Λ; Δ_l is given by:

$$\Delta_l = \bar{\Delta}_l \qquad if \quad \tilde{\Phi}_l\tilde{\mathbf{u}} = \bar{\Delta}_l - \Lambda_l x_0$$
$$\Delta_l = -\underline{\Delta}_l \qquad if \quad \tilde{\Phi}_l\tilde{\mathbf{u}} = -\underline{\Delta}_l - \Lambda_l x_0$$

In order to reduce the computational complexity, with the purpose of a real time implementation, the above described method is adapted in the present paper so as to focus on the identification of the region where the current state x_0 is located, without the need to characterize the geometry of the full state space. Then, if an active face Φ_l is known, the explicit optimal constrained control is given by (16). In the case when many adjacent faces are active, we choose the optimal control $\mathbf{u}^{opt}$ as the solution inside the polyhedron defined by (13) which verifies the minimal Euclidean distance to the unconstrained optimal solution $\mathbf{u}_{uc}^{opt}$.

It is clear that the coordinator has to solve a centralized optimization problem, but once again, its solution can be expressed explicitly. In this work we have:

$$\mathbf{v}_i = \mathbf{v}_i^{opt} = \text{diag}\{v_A, \cdots, v_A\}\mathbf{x} + \text{diag}\{v_B, \cdots, v_B\}\mathbf{u}^{opt}$$
$$\mathbf{v}_{si} = \text{diag}\{v_A, \cdots, v_A\}\mathbf{x}_s + \text{diag}\{v_B, \cdots, v_B\}\mathbf{u}_s \tag{17}$$

where $\mathbf{u}^{opt}$ is computed from (16), the sequence $\mathbf{u}_s$ is defined in (8) and v_A, v_B in (5).

To summarize, in the proposed framework of coordination based on MPC, the fixed horizon minimization problem (10) subject to (1) and constraints (13) is solved at each communication time T_{com} for the current state coming from the subsystems and disturbance values, and the resulting control sequence $\mathbf{u}^{opt}$ is used as the current control for the computation of the interaction-prediction vector $\mathbf{v}_i$ (17), while the procedure is repeated at the next communication time step.

B. Local control (Subsystems)

The local optimization problems can be written as QP problems too, as follows:

$$J_i = \bar{V} + \frac{1}{2}\mathbf{u}_i^{\mathsf{T}}\mathbf{H}_i\mathbf{u}_i + \mathbf{u}_i^{\mathsf{T}}[\mathbf{K}_{1i}x_{(i)_0} + \mathbf{c}_i] \tag{18}$$

subject to (3) and $\mathbf{L}_i\mathbf{u}_i \leq \mathbf{W}_i$,
where:

$$\mathbf{c}_i = -\mathbf{K}_{1_i}x_{s_i} + \mathbf{K}_{2_i}(\mathbf{d}_i - \mathbf{d}_{s_i}) + \mathbf{K}_{3_i}(\mathbf{v}_i - \mathbf{v}_{s_i}) - \mathbf{H}_i\mathbf{u}_{s_i} \tag{19}$$

and

$$\bar{V} = \text{terms independent of } \mathbf{u}_i,$$
$$\mathbf{H}_i = \Gamma_i^T\mathbf{Q}_i\Gamma_i + \mathbf{R}_i, \ \mathbf{K}_{1_i} = \Gamma_i^T\mathbf{Q}_i\Omega_i, \tag{20}$$
$$\mathbf{K}_{2_i} = \Gamma_i^T\mathbf{Q}_i\Theta_i, \ \mathbf{K}_{3_i} = \Gamma_i^T\mathbf{Q}_i\Psi_i$$

with matrices Γ_i, Ω_i, Θ_i as in (10) for subsystem i, and

$$\Psi_i = \begin{bmatrix} I & 0 & \cdots & 0 \\ A_{ii} & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{ii}^{N-1} & A_{ii}^{N-2} & \cdots & I \end{bmatrix}, \tag{21}$$
$$\mathbf{Q}_i = \text{diag}\{Q_i, \cdots, Q_i, P_i\}, \ \mathbf{R}_i = \text{diag}\{R_i, \cdots, R_i\}$$

Remark that, in (18) the interactions information $\mathbf{v}_i$ is not available at the local level. This information comes from the coordination algorithm. The explicit local control is given by:

$$\mathbf{u}_i^{opt} = H_i^{-\frac{1}{2}}\tilde{\Phi}_{(i)l}^{\mathsf{T}}[\tilde{\Phi}_{(i)l}\tilde{\Phi}_{(i)l}^{\mathsf{T}}]^{-1}(\Delta_{(i)l} - \Lambda_{(i)l}x_{(i)_0}) -$$
$$H_i^{-\frac{1}{2}}[I - \tilde{\Phi}_{(i)l}^{\mathsf{T}}[\tilde{\Phi}_{(i)l}\tilde{\Phi}_{(i)l}^{\mathsf{T}}]^{-1}\tilde{\Phi}_{(i)l}]H_i^{-\frac{1}{2}}(\mathbf{K}_{1_i}x_{(i)_0} + \mathbf{c}_i) \tag{22}$$

with matrices defined in (20)-(21).

Remark that, the local control law requires the information about the disturbances, in particular the interactions $\mathbf{v}_i$ in (19) in order to obtain a good compensation. Absence of this information means non-compensation of such disturbances (or interactions), producing then, a degradation of the system performance.

The first element of the resulting control sequence $\mathbf{u}_i^{opt}$ is then applied to the local system at the k instant. The subsystem improves the local optimization during $N_g - N + 1$

steps, that is, the time necessary to the next communication period T_{com} to receive the *interaction prediction vector*

$$\mathbf{v}_i = \begin{bmatrix} v_{(i)0} & v_{(i)1} & \dots & v_{(i)N_g-1} \end{bmatrix}^{\mathsf{T}} \tag{23}$$

from the coordinator to implement the next local optimizations.

Remark that this solution seems to be suitable for real-time applications, because only the interaction-prediction is imposed while the local controls are solutions of the local optimization problems. It means, that the subsystems have some independency, unlike what happens in a totally centralized control case.

In the next section, we will explore this interaction-prediction decomposition-coordination technique in a simulation with real-data, in an idealized context (i.e. well-known model, disturbances and without communication faults).

IV. Real-data based simulation

In this section, a case study model of a Hydro-Power Valley is considered for simulation, with parameters and input data taken from a real HPV system producing around 550 [MW], managed by the french group EDF. The HPV is modeled as reservoirs open to the atmosphere, perturbed by input disturbance flows. The water levels are controlled by output flows determined by the required power generation.
We assume the daily availability of the reservoir level measurements, and that the disturbances are well known. The dynamics of each reservoir can be modeled, in continuous time, as follows:

$$S\frac{dh}{dt} = q_{in} - q_{out} \tag{24}$$

where S denotes the area of the reservoir, h the reservoir level and q_{in}, q_{out} the input and output water flows. For simplicity, the matrices of the discrete-time state space representation (1) of the case study are the following: A and C are identity matrices of dimension 3, while matrices B and E depend on the reservoir surfaces S and the chosen sampling-time T_s,

$$B = \begin{bmatrix} \frac{-T_s}{S_1} & \frac{-T_s}{S_1} & 0 & 0 \\ 0 & \frac{T_s}{S_2} & \frac{-T_s}{S_2} & 0 \\ \frac{T_s}{S_3} & 0 & \frac{T_s}{S_3} & \frac{-T_s}{S_3} \end{bmatrix}, E = \begin{bmatrix} \frac{T_s}{S_1} & 0 \\ 0 & \frac{T_s}{S_2} \\ 0 & 0 \end{bmatrix} \tag{25}$$

The system has been decomposed into three subsystems according to section II-A.

Simulations are performed by using the following settings: The local prediction horizon is $N = 6$; Weighting matrices P, Q and R are identity matrices; The sampling time is $T_s = 5$ minutes; The communication time between the coordinator and the subsystems is $T_{com} = 30$ minutes; The coordinator horizon prediction then is $N_g = 12$ (according with (7)).
The level $[m]$ and flow $[\frac{m^3}{s}]$ constraints are defined as follows:

$$\mathbf{u_{max}} = [60 \;\; 0 \;\; 60 \;\; 60]^T, \; \mathbf{x_{max}} = [729.5 \;\; 287.5 \;\; 268.5]^T,$$
$$\mathbf{u_{min}} = [0 \;\; -6.6 \;\; 0 \;\; 0]^T, \; \mathbf{x_{min}} = [716.2 \;\; 282 \;\; 259.5]^T$$

The reference data of a day is known the night before. The variables are daily initialized with the first values of this reference data. The input disturbances are presented in figure 2. They correspond to an actual recording over 120 hours, that is, 5 days. All simulations will be provided for the same period. Figure 3 presents the controlled flows (blue)

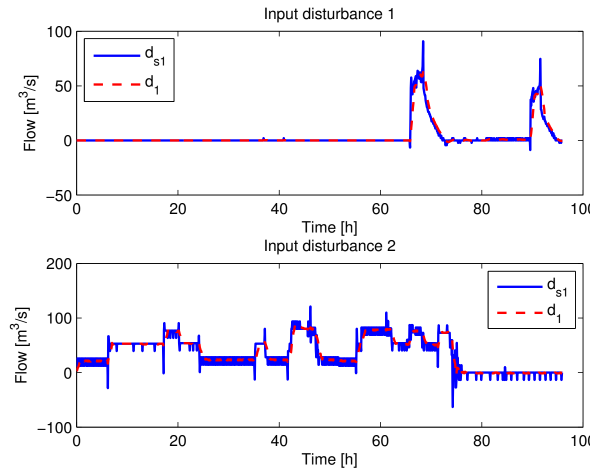

Fig. 2. Estimated input disturbances $\mathbf{d}_i$.

with respect to the imposed constraints (green) and tracking the demanded setpoints (magenta). The local controls have been calculated with equation (22). Figure 4 shows such the

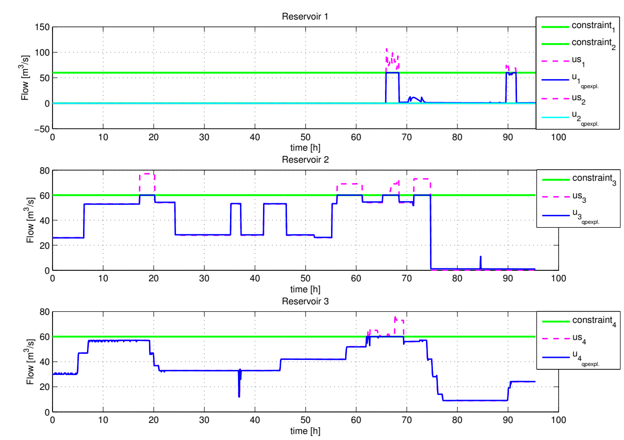

Fig. 3. Turbine flows (Control inputs $\mathbf{u}_i^{opt}$ and references $\mathbf{u}_{s_i}$).

reservoir level (blue) with respect to the imposed constraints (green) and their reference values to be tracked (magenta). The daily updating of the variables is clear at the beginning of each day. Remark that in an idealist case without communication constraints (communication time $T_{com} = T_s$) and enough computational power, the centralized control provides the best performance (see table I). But considering that communication limitations impose a communication time larger than T_s, simulations were also successfully performed for various values of T_{com}.

A performance comparison with the centralized solution in terms of achieved cost function is presented in Table I. It shows how the performance is affected when the communication time increases. From this, it would be interesting to add into the total cost function a *communication cost* term

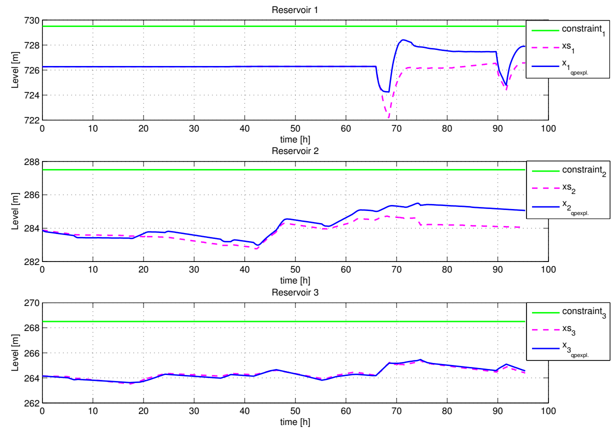

Fig. 4. Level of the reservoirs (system states $\mathbf{x}_i$ and references $\mathbf{x}_{s_i}$).

T_{com} [min]	5	30	90	180	∞
Performance $\left(J_{T_{com}}/J_{5_{min}}\right)$	1.0000	1.0018	1.1550	2.4204	26.793

TABLE I
PERFORMANCE OF THE EXPLICIT INTERACTION-PREDICTION METHOD FOR VARIOUS VALUES OF T_{com}

limiting the minimal possible value of T_{com}, and then to compute and/or choose the optimal value of the coordination time T_{com} in a more suitable way. In fact, the proposed scheme allows also to include possible adaptation of the T_{com} according to the desired performance, solver load, communications faults and/or an uncertainty level. These aspects will be explored in a future work.

V. CONCLUSIONS AND FUTURE WORKS

Classical coordination approaches are attractive for large-scale problems, but require a lot of communication exchanges between the coordinator and subsystems in order to converge to a suitable optimal value. This is very expensive in terms of communication requirements and computation-time.

In this paper we have discussed an explicit interaction prediction decomposition-coordination approach, with the purpose of real-time application to the control of Hydro-Power Valleys. In particular, it has been emphasized how to take advantage of explicit solutions to the considered optimization problem to reduce computational complexity, as well as take into account the communication constraints between the coordinator and subsystems.

These aspects are very useful for real-time applications and illustrative simulation results have provided for a real-data based example. Providing a finer stability and performance analysis in the presence of unknown disturbances, model uncertainties or network failures for instance will be part of future developments.

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