

Optimal and Robust Saturated Control for a Clock Generator

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Abstract—Fine-grain Dynamic Voltage and Frequency Scaling (DVFS) is becoming a requirement for Globally-Asynchronous Locally-Synchronous (GALS) architectures. However, the area overhead of adding voltage and frequency control engines in each voltage/frequency island must be taken into account to optimize the circuit. This paper focuses on the control for the frequency actuator. An optimal and robust saturated control law, with a minimum hardware implementation area is proposed for a Clock Generator, taking into account the delay introduced by the sensor. This controller is designed with Lyapunov-Krasovskii theory that ensures asymptotic stability, disturbance rejection as well as system robustness with respect to delay presence and parameter uncertainties. The closed-loop system presents a regional stabilization due to the actuator saturation. An estimation of a maximum attraction domain is provided. The performance achieved with this controller are shown in simulation.

I. INTRODUCTION

The continuous increase in clock frequency together with technology scaling has made the distribution of a single global clock over a digital chip tremendously difficult. Globally Asynchronous Locally Synchronous (GALS) design alleviates the problem of clock distribution by having multiple clocks, each one being distributed on a small area of the chip. An architecture with different clock frequency domains appears as a natural enabler for fine-grain power-aware architectures. Actually, power consumption is a limiting factor in Very Large Scale Integration (VLSI), especially for mobile applications. Voltage-Frequency Islands (VFI) [1] have proven to be highly effective to reduce the power consumption of the chip while meeting the performance requirements [2]. The key idea behind local DVFS is to control at fine grain the supply voltage and the frequency of an island at runtime to minimize its power consumption of the island considered while satisfying the computation/throughput constraints [3].

DVFS techniques mainly rely on two ‘actuators’. These actuators need to be dynamically controlled in order to reduce the power consumption while maintaining the required performance. More precisely, the control policy

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must be carefully designed in order to achieve high power efficiency at low area cost. The voltage actuator fixes the supply voltage of the VFI. It can be a classical buck converter [4] or a discrete Vdd-hopping converter [5], [6]. The frequency actuator is supposed to be a Clock Generator.

An important issue of technology scaling is the in-die and die-to-die process variability (P-variability). As a consequence, each VFI performance and optimal functional frequency will strongly differ from place to place within the chip. These changes are also induced by Voltage and Temperature variations (VT-variability) over time. Therefore, the optimum functional and energetic point of the whole circuit can be found if VFI number i has its functioning frequency in the range $[F_{min,i}, F_{max,i}]$ [7]. If the clock is generated for the whole circuit, and distributed in each VFI, the maximum acceptable frequency (i.e. that will ensure no timing fault for any VFI) will be $F_{max,i} = \min\{F_{max,i}, \forall i\}$, leading to a suboptimal circuit functioning. Therefore, the clock is locally generated and controlled by a Clock Generator according to Process, Voltage and Temperature (PVT) variations.

The main objective of this paper is to design a control law for the Clock Generator (see Fig.1) taking into account the next objectives:

- robustness with respect to PVT variations;
- low cost implementation;
- saturation;
- delay consideration and
- exogenous disturbance rejection.

Therefore, the designed controller must not only guarantee the set-point stabilization, but also other criterions.

An optimal and robust saturated control law for the Clock Generator is proposed in discrete time. The system is rewritten into a suitable state-space representation to formulate a robust H_∞ control problem that can be solved using Lyapunov-Krasovskii theory [8]. In this process, the saturation in the actuator input is considered [9]. The designed controller guarantees asymptotic stability, disturbance rejection as well as robustness of the system with respect to delays and uncertain parameters. The problem is expressed in terms of Linear Matrix Inequalities (LMIs). Likewise, an attraction domain is estimated in such a way that a regional stabilization for the saturated control is guaranteed. The robustness properties of the closed-loop system are confirmed by simulations.

The rest of this paper is organized as follows: in Section II, the error equation of the circuit model is derived as well as its properties. The robustness problem statements are presented in Section III. Likewise, in Section IV, the stabilization of the time-delay system is performed. This result is extended for a robust control design in Section V. An optimization procedure is proposed in Section VI. The control gains are computed in Section VII, being confirmed by simulations in Section VIII. The paper closes with a section of conclusion.

Notation. For a given $x \in \mathcal{R}$,

$$\text{sat}_m^M(x) \triangleq \begin{cases} M & \text{if } x > M \\ x & \text{if } m \leq x \leq M \\ m & \text{if } x < m. \end{cases}$$

and $\text{round}(x)$ is the nearest integer to x . For a given set \mathcal{S} , $\mathcal{Co}(\mathcal{S})$ denotes the convex hull of set \mathcal{S} . $\Delta\zeta \triangleq \zeta^+ - \zeta^-$, where ζ^+ and ζ^- are respectively $\zeta(k+1)$ and $\zeta(k)$, i.e., the value of ζ in two consecutive sampling times. Finally, \mathcal{L}_2 is the space of $\{x_k\}$ with $\|x_k\|_2^2 \triangleq \sum_{k=0}^{\infty} x_k^T x_k < \infty$.

II. ERROR EQUATION OF THE CLOCK GENERATOR

The main blocks of the Clock Generator considered in this paper are a digital to analog converter (DAC), a Voltage Controlled Oscillator (VCO) that provides at its output a frequency $f(t)$, a sensor to measure this frequency and a controller that contains a frequency comparator between the targeted frequency, f_r , and the output frequency and some ‘intelligent controller’. Note that this structure is similar to the one of a Digital Frequency-Locked Loop. Figure 1 shows a sketch of the Clock Generator.

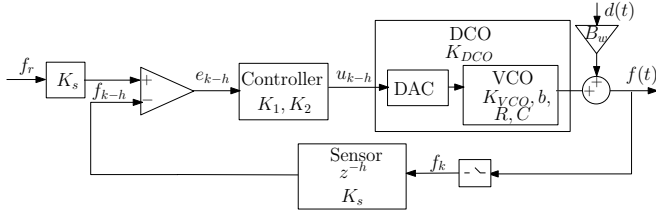


Fig. 1: Clock Generator block diagram.

The main blocks of this Clock Generator are modeled through design considerations and accurate simulations. The VCO can be modeled as a RC circuit [10]. Indeed, it can be approximately modeled as

$$\dot{f}(t) = -\frac{f(t)}{RC} + K_{VCO}u(t-\tau) + b + B_w d(t), \quad \tau = h(t_{k+1} - t_k)$$

where $f \in \mathcal{R}^1$ is the analog frequency output, u is a control signal, d is a \mathcal{L}_2 disturbance and K_{VCO} and B_w are constant. R and C represent the resistive and capacitive components, respectively, of the circuit, and b is an offset.

This model is approximately discretized, in such a way that the whole Digitally-Controlled Oscillator (DCO) is modeled by :

$$f_{k+1} = (1 - \rho T_s)f_k + K_{DCO}T_s u_{k-h} + bT_s + B_w T_s d_k$$

where T_s is the sampling period, K_{DCO} is the gain of the whole DCO and $\rho \triangleq \frac{1}{RC}$.

Defining $e_k \triangleq f_r - f_k$, where f_r is the reference signal, the error equation is

$$e_{k+1} = (1 - \rho T_s)e_k - K_{DCO}T_s u_{k-h} + \rho T_s f_r - bT_s - B_w T_s d_k. \quad (1)$$

A controller proposed for this system is given by

$$u_{k-h} = \text{sat}_{\bar{u}_m}^{\bar{u}_M} \{u_{k-h-1} + Kx_{k-h}\}, \quad (2)$$

where h is the size of the constant delay introduced by the sensor, $x_{k-h} \triangleq [e_{k-h} \quad e_{k-h-1}]^T$ and $K \triangleq [K_1 \quad K_2]$. Note that the controller proposed presents a simple enough structure to be implemented in hardware. This property is very important in SoCs [11].

III. PROBLEM STATEMENT

The problem statement is formulated. The closed-loop system (1) is rewritten in a state-space form taking into account the delay, saturation, uncertain parameters and any exogenous disturbance.

Parameters R , C , T_s and K_{DCO} can be considered uncertain. Their corresponding uncertainty intervals are

- $R \in [R^m, R^M]$,
- $C \in [C^m, C^M]$,
- $T_s \in [T_s^m, T_s^M]$,
- $K_{DCO} \in [K_{DCO}^m, K_{DCO}^M]$.

Consequently, the main objective is to find the optimal gain K , in such a way that, Control (2) is robust with respect to delays as well as parameter uncertainties. Likewise, this optimal K must guarantee asymptotic stability and disturbance rejection for the known constant delay h .

A. Alternative representation for the saturated control (2) and the error equation (1)

Firstly, some lemmas are given to rewrite the saturated control (2) in an alternative form.

Define $\chi \triangleq \begin{bmatrix} u_{k-1-h_1} \\ x_{k-h} \end{bmatrix}$. Note, from Eq.(1), u_{k-1-h_1} directly depends on $x_k = [e_k, \quad e_{k-1}]$.

Lemma 1: [12], let $K, G \in \mathbb{R}^{1 \times 2}$ be given. For all, $\chi_{k-h} \in \mathbb{R}^{1 \times 3}$, if $\chi_{k-h} \in \{\chi_{k-h} \in \mathbb{R}^{1 \times 3} : [1 \quad G]\chi_{k-h} \in [1 \quad N]\}$, then

$$\text{sat}_1^N \{[1 \quad G]\chi_{k-h}\} \in \mathcal{Co} \{[1 \quad K]\chi_{k-h}, \quad [1 \quad G]\chi_{k-h}\}.$$

□

Lemma 2: Assume that there exists $G \in \mathbb{R}^{1 \times 2}$, $c > 0$ and $\Psi \triangleq \text{diag}\{\rho, P_1\}$, where $P_1^\dagger > 0 \in \mathbb{R}^2$, $\rho > 0 \in \mathbb{R}^1$ such that for any $\chi_{k-h} \in \mathfrak{X}$, where

$$\mathfrak{X} = \{\chi_{k-h} : \chi_{k-h}^T \Psi \chi_{k-h} \leq c^{-1}\}, \quad (3)$$

$^\dagger P_1$ is a positive matrix defined to guarantee system stability.

then, $1 < u_{k-1-h_1} + Gx_{k-h} < N$, and Control (2) admits the following representation

$$\begin{aligned} u_{k-h_1} &= [\alpha_k(u_{k-1-h_1} + Kx_{k-h}) \\ &\quad + (1 - \alpha_k)(u_{k-1-h_1} + Gx_{k-h})] \\ &= [u_{k-1-h_1} + \alpha_k Kx_{k-h} + (1 - \alpha_k)Gx_{k-h}] \\ &= [u_{k-1-h_1} + \bar{u}_{k-h}], \end{aligned}$$

where $\bar{u}_{k-h} \triangleq (\alpha_k K + (1 - \alpha_k)G)x_{k-h}$ with $\alpha_k \in [0, 1]$, for all $k > 0$. \square

Then, Eq. (1) can be rewritten as

$$\begin{aligned} e_{k+1} &= (1 - \rho T_s)e_k - bT_s + \rho T_s f_r - B_w T_s d_k \\ &\quad - K_{DCO} T_s (u_{k-1-h_1} + \bar{u}_{k-h}) \end{aligned} \quad (4)$$

B. State-space representation

The saturated control law and error equation, as redefined above, allow system (1) rewrite it in a state-space form. From Eq. (1),

$$u_{k-h} = \frac{-e_{k+1} + (1 - \rho T_s)e_k + \rho T_s f_r - bT_s - B_w T_s d_k}{K_{DCO} T_s},$$

and, therefore

$$u_{k-h-1} = \frac{-e_k + (1 - \rho T_s)e_{k-1} + \rho T_s f_r - bT_s - B_w T_s d_{k-1}}{K_{DCO} T_s}$$

which substituted in (4) gives

$$\begin{aligned} e_{k+1} &= (2 - \rho T_s)e_k + (\rho T_s - 1)e_{k-1} - K_{DCO} T_s \bar{u}_{k-h} \\ &\quad + T_s B_w (d_k - d_{k-1}) \end{aligned} \quad (5)$$

From Lemma 1 and 2, Eq. (5) can be rewritten in the following matrix form:

$$x_{k+1} = Ax_k + B\bar{u}_{k-h} + \bar{B}_w w_k \quad (6)$$

where $w_k \triangleq [d_k \quad d_{k-1}]^T$,

$$A \triangleq \begin{bmatrix} 2 - \rho T_s & \rho T_s - 1 \\ 1 & 0 \end{bmatrix}, \quad B \triangleq \begin{bmatrix} -K_{DCO} T_s \\ 0 \end{bmatrix},$$

$$\bar{B}_w \triangleq \begin{bmatrix} T_s b_w & -T_s b_w \\ 0 & 0 \end{bmatrix}.$$

C. Stability and disturbance rejection problem

Equation (6) can be rewritten in the following explicit closed-loop form, in such a way that a H_∞ problem can be formulated.

$$x_{k+1} = Ax_k + B(\alpha_k K + (1 - \alpha_k)G)x_{k-h} + \bar{B}_w w_k, \quad (7)$$

$$x_l = \phi_l, \quad \forall l \in [-h, 0] \quad (8)$$

$$z_k = I_2 x_k, \quad (9)$$

where $x_k, z_k, w_k \in \mathcal{R}^2$ are the state vector, controlled output and exogenous disturbance input, respectively. ϕ_l is the initial condition and $h \geq 0 \in \mathcal{R}$ is a fixed and known delay.

Problem 1: The problem is to find $\mathfrak{X}(P_1, c)$, vector G and K , such that

- a) Lemma 2 holds and, hence, the closed-loop system (7), and

- b) there exists a Lyapunov-Krasovskii functional $V_k > 0$ such that $V_{k+1} - V_k$ along the solution of (7) fulfills

$$V_{k+1} - V_k < 0, \quad (10)$$

when the system is not perturbed, and for any disturbance input, there exists a minimum disturbance attenuation, $\gamma^* \geq 0$, such that, for all $\gamma \geq \gamma^*$ the \mathcal{L}_2 gain between the disturbance vector w_k and the output vector z_k is less or equal to γ . i.e.

$$\begin{aligned} \|z_k\|_2^2 - \gamma^2 \|w_k\|_2^2 &< 0, \quad \forall w_k \in \mathcal{L}_2 \\ \text{for } \phi_l &= 0, \quad -h \leq l \leq 0. \end{aligned} \quad (11)$$

The solution to this problem guarantees the asymptotic convergence, system stability as well as the disturbance rejection for the time-delay system (7)–(9).

IV. H_∞ CONTROL DESIGN

In order to cope with Problem 1 a mathematical manipulation of Eq. (7) is performed via a descriptor model transformation [13]. The descriptor approach is a variable change, which makes easier to work with Lyapunov-Krasovskii functional [8].

A. Descriptor model transformation

Equation (7) is manipulated in order to achieve the previous objectives. A descriptor model transformation is applied.

Considering $y_k \triangleq x_{k+1} - x_k$, $\psi_k \triangleq \sum_{i=k-h}^{k-1} y_i$, Eq. (7) is rewritten in the descriptor form [13]:

$$\begin{bmatrix} x_{k+1} \\ 0 \end{bmatrix} = \begin{bmatrix} y_k + x_k \\ -y_k + Ax_k - x_k + B(\alpha_k K + (1 - \alpha_k)G)x_{k-h} + \bar{B}_w w_k \end{bmatrix}$$

From $x_{k-h} = x_k - \psi_k$, this system can be compactly written as:

$$E\bar{x}_{k+1} = \bar{A}\bar{x}_k - \begin{bmatrix} 0 \\ B(\alpha_k K + (1 - \alpha_k)G) \end{bmatrix} \psi_k + \begin{bmatrix} 0 \\ \bar{B}_w \end{bmatrix} w_k, \quad (12)$$

where

$$\bar{A} \triangleq \begin{bmatrix} I_2 & I_2 \\ A + B(\alpha_k K + (1 - \alpha_k)G) - I_2 & -I_2 \end{bmatrix}$$

$$E \triangleq \text{diag}\{I_2, 0_2\}, \quad \bar{x}_k \triangleq \begin{bmatrix} x_k \\ y_k \end{bmatrix}.$$

B. Condition for state-space representation

Condition a) of Problem 1 is satisfied, if

$$1 < u_{k-1-h_1} + Gx_{k-h} < N, \quad \forall \chi_{k-h} \in \mathfrak{X} \quad (13)$$

given in (3) is guaranteed.

Subtracting $\frac{N+1}{2}$ in inequality (13) and from [14], it is seen that, it is necessary satisfy

$$\begin{aligned} 2N - 2 &> N(1 + cx_{k-h}^T P_1 x_{k-h} + cu_{k-1-h}^T \rho u_{k-1-h}) - 2 \\ &> 4u_{k-1-h} + 4Gx_{k-h} - 2(N+1) \end{aligned} \quad (14)$$

From Lemma 2, relationships (14) correspond to

$$\begin{bmatrix} 1 \\ u_{k-1-h_1} \\ x_{k-h}^T \end{bmatrix}^T \begin{bmatrix} 3N & -2 & -2G \\ -2 & cN\rho & 0 \\ -2G^T & 0 & cNP_1 \end{bmatrix} \begin{bmatrix} 1 \\ u_{k-1-h_1} \\ x_{k-h} \end{bmatrix} > 0 \quad (15)$$

This inequality is satisfied if

$$\begin{bmatrix} c & -2 & -2Y \\ -2 & \rho 3N^2 & 0 \\ -2Y^T & 0 & 3N^2\bar{P}_1 \end{bmatrix} > 0 \quad (16)$$

Note that this LMI is equivalent to (15) by means of employing the Schur's complement, defining $Y \triangleq GQ_1$ with $Q_1 \in \mathbb{R}^2$ which is Hermitian, applying $\bar{P}_1 = Q_1 P_1 Q_1$ and pre- and post-multiplying by $\text{diag}\{1, 1, Q_1\}$.

C. Control design

Now, condition b) of Problem 1 can be formulated in terms of Linear Matrix Inequalities (LMIs) [8]. Fulfillment of conditions (10) and (11) is looked for.

Assumption 1: There exists a Lyapunov function V_k , with condition (10) and a γ , such that,

$$V_{k+1} - V_k + z_{k+1}^T z_{k+1} - \gamma^2 (w_k^T w_k) \leq \zeta^T \Gamma \zeta < 0. \quad (17)$$

where $\zeta \triangleq [\bar{x}_k \ \psi_k \ w_k]^T$ is an augmented state vector and $\Gamma \in \mathcal{R}^{4 \times 4}$ is a symmetric matrix.

Define $P \triangleq \begin{bmatrix} P_1 & P_2 \\ P_2 & 0 \end{bmatrix}$, being P_2 Hermitian.

The function Γ is obtained defining the next Lyapunov-Krasovskii candidate

$$V_k = V_{1,k} + V_{2,k} + V_{3,k}, \quad (18)$$

being

$$V_{1,k} = \bar{x}_k^T EPE\bar{x}_k \quad (19)$$

$$V_{2,k} = \sum_{n=1}^h \sum_{i=k-n}^{k-1} y_i^T R y_i, \quad R > 0 \quad (20)$$

$$V_{3,k} = \sum_{i=k-h}^{k-1} x_i^T S x_i, \quad S > 0, \quad (21)$$

where $V_{1,k}$ guarantees asymptotic stability of system (12) without delays. Delay-dependent as well as delay-independent criteria are considered in $V_{2,k}$ and $V_{3,k}$, respectively [8], [15].

Next, a sufficient condition for asymptotic stability and disturbance rejection is derived.

Theorem 1: Consider system (7)–(9) with energy-bounded w_k and control law $\bar{u}_{k-h} = \alpha_k K x_{k-h} + (1 - \alpha_k) G x_{k-h}$ where $h > 0 \in \mathcal{R}$ is a known constant delay and $K, G \in \mathcal{R}^{1 \times 2}$. If there exist $S, R \in \mathcal{R}^{2 \times 2}$ such that

$$P_1 > 0, \quad (22)$$

$$\Gamma_k < 0 \quad (23)$$

LMIs (16)

where Γ_k is defined in (24) found at the top of next page, then the equilibrium of the closed-loop system (7)–(9) is

asymptotically stable and there is a value γ^* such that for $\gamma < \gamma^*$ condition (11) is fulfilled.

Proof: The goal is to satisfy $V_{k+1} - V_k + z_k^T z_k - \gamma^2 w_k^T w_k < 0$ for both disturbance rejection and asymptotic stability of the equilibrium for system (12).

Lyapunov-Krasovskii method yields:

$$\begin{aligned} V_{1,k+1} - V_{1,k} &= \bar{x}_{k+1}^T EPE\bar{x}_{k+1} - \bar{x}_k^T EPE\bar{x}_k \\ &= \{\bar{x}_k^T \bar{A}^T - \psi_k^T [0 \ \alpha_k K^T B^T + (1 - \alpha_k) G^T B^T] + \\ &+ w_k [0 \ \bar{B}_w^T]\} P \left\{ \bar{A}\bar{x}_k - \begin{bmatrix} 0 \\ \alpha_k BK + (1 - \alpha_k) BG \end{bmatrix} \psi_k \right. \\ &+ \left. \begin{bmatrix} 0 \\ \bar{B}_w \end{bmatrix} w_k \right\} - \bar{x}_k^T EPE\bar{x}_k = \bar{x}_k^T [\bar{A}^T P \bar{A} - EPE] \bar{x}_k \\ &- \bar{x}_k^T \bar{A}^T P \begin{bmatrix} 0 \\ \alpha_k BK + (1 - \alpha_k) BG \end{bmatrix} \psi_k \\ &- \psi_k^T [0 \ \alpha_k K^T B^T + (1 - \alpha_k) G^T B^T] P \bar{A} \bar{x}_k \\ &- \bar{x}_k^T \bar{A}^T P \begin{bmatrix} 0 \\ \bar{B}_w \end{bmatrix} w_k + w_k [0 \ \bar{B}_w^T] P \bar{A} \bar{x}_k. \end{aligned}$$

From (20) and Jensen Inequality [16]:

$$\begin{aligned} V_{2,k+1} - V_{2,k} &= h y_k^T R y_k - \sum_{n=1}^h y_{k-n}^T R y_{k-n} \\ &\leq \bar{x}_k^T \begin{bmatrix} 0 & 0 \\ 0 & hR \end{bmatrix} \bar{x}_k - \frac{1}{h} \psi_k^T R \psi_k \end{aligned}$$

Finally,

$$\begin{aligned} V_{3,k+1} - V_{3,k} &= x_k^T S x_k - x_{k-h}^T S x_{k-h} \\ &= x_k^T S \psi_k + \psi_k^T S x_k - \psi_k^T S \psi_k \end{aligned}$$

These developed expressions are applied to inequality (17), in such a way that the LMIs (23) are obtained. ■

V. ROBUST CONTROL

The uncertain parameters given in Section III are now taken into account, at the same time that stability as well as disturbance rejection for the time-delay Vdd-hopping system are ensured. For this reason, Theorem 1 is extended in the case of polytopic uncertainties.

Denote $\Omega \triangleq [A \ B \ \alpha_k]$ and assume that $\Omega \in \mathcal{Co}\{\Omega_j, \ j = 1, \dots, 16\}$, namely

$$\Omega = \sum_{j=1}^n \lambda_j \Omega_j, \quad 0 \leq \lambda_j \leq 1, \quad \sum_{j=1}^n \lambda_j = 1$$

and being the vertices of the polytope described by $\Omega_j = [A^{(j)} \ B^{(j)} \ \alpha_k^{(j)}]$ for $j = 1, \dots, 16$.

Pre- and post-multiplying LMI (23) by $Q = \text{diag}\{Q_1, Q_1, Q_1, I_2\}$ and taking $Q_1 = P_2^{-1} > 0$ and $\bar{P}_1 = Q_1 P_1 Q_1$, $\bar{R} = Q_1 R Q_1$, $\bar{S} = Q_1 S Q_1$, the following sufficient condition is achieved.

Theorem 2: Consider system (7)–(9) with energy-bounded w_k and control law $\bar{u}_{k-h} = \alpha_k K x_{k-h} + (1 - \alpha_k) G x_{k-h}$ for $\alpha_k \in [0, 1]$, where $h \geq 0 \in \mathcal{R}$ is a known constant delay and $K, G \in \mathcal{R}^{1 \times n}$. If there exist $T, Y \in$

$$\Gamma_k \triangleq \begin{bmatrix} \bar{A}^T P \bar{A} - E P E + \text{diag}\{I_2, hR\} & -\bar{A}^T P \begin{bmatrix} 0 \\ B\alpha_k K + (1 - \alpha_k)G \end{bmatrix} + \begin{bmatrix} S \\ 0 \end{bmatrix} & \bar{A}^T P \begin{bmatrix} 0 \\ \bar{B}_w \end{bmatrix} \\ * & * & 0 \\ * & * & -\gamma^2 I_2 \end{bmatrix} \quad (24)$$

$\mathcal{R}^{n \times 1}$ and $Q_1 \in \mathcal{R}^{n \times n}$ with $K = TQ_1^{-1}$, $G = YQ_1^{-1}$ and $\bar{R}, \bar{S} > 0 \in \mathcal{R}^{n \times n}$ for $j = 1, \dots, 16$ such that

$$\bar{P}_1 > 0 \quad (25)$$

$$\bar{\Gamma}^{(j)} < 0 \quad j = 1, \dots, 16, \quad (26)$$

LMI (16)

being $\bar{\Gamma}^{(j)}$ defined in (27) found at the top of next page, are satisfied. Then, in the vertices i and j , the equilibrium is asymptotically stable as well as the disturbances are rejected in the entire polytope.

Proof: This is an extension of Theorem 1 for polytopic uncertainties with some mathematical manipulations. Therefore, this theorem proof follows Theorem 1 proof. ■

Remark 1: This robust control tuning method is conservative due to the definition of matrix P , as well as, the attraction domain, \mathfrak{X} .

Corollary 1: Gain K , obtained from T and Q_1 in Theorem 2, fulfills Theorem 1 and consequently guarantees both robust stability and robust disturbance rejection for a fixed delay.

VI. OPTIMAL AND ROBUST CONTROL

In order to obtain a maximum attraction domain, i.e. minimum c , and for the disturbances to be effectively rejected (i.e. minimum γ), an optimal control problem is formulated.

Assumption 2: There exists a functional cost

$$J \triangleq x_{k+1}^T P_1 x_{k+1} + \|z_{k+1}\|_2^2 - \gamma^2 \|w_k\|_2^2$$

The first term on the right side quantifies the size of the attraction domain and the other terms the disturbance attenuation.

Lemma 3: Suppose that Assumptions 1 and 2 are fulfilled, Then the optimal controller gain K for Problem 1 can be found by:

Minimize $-J$
subject to:

$$\bar{\Gamma}^{(j)} < 0$$

$$\bar{P}_1 > 0.$$

$$\text{LMIs}^{(j)}(16) \quad j = 1, \dots, 16.$$

Proof: The optimal Problem 1 is solved by Lemma 1 if condition (11) is fulfilled [17].

For $w_k \neq 0$ and under zero initial conditions

$$V_{k+1} - V_k \leq -z_{k+1}^T z_{k+1} + \gamma^2 w_k^T w_k.$$

The summation of both side is

$$V_{k+1} - V_0 \leq -\sum_{k=0}^{k+1} z_{k+1}^T z_{k+1} + \gamma^2 \sum_{k=0}^k w_k^T w_k.$$

For $k \rightarrow \infty$, under the zero initial condition $V_0 = 0$ and the positive definitiveness of the Lyapunov function, it is proved that

$$\sum_{k=0}^{\infty} z_{k+1}^T z_{k+1} \leq \gamma^2 \sum_{k=0}^{\infty} w_k^T w_k \Rightarrow \|z_{k+1}\|_2^2 \leq \gamma^2 \|w_k\|_2^2. \quad \blacksquare$$

Corollary 2: The optimal gain K , obtained applying Lemma 3 guarantees both robust stability and robust disturbance rejection and provides a maximum attraction domain.

VII. OPTIMAL AND ROBUST CONTROL RESULT

In this section, the robust control gains for Control (2) are computed. For the control signal $\bar{u}_m = 0.6V$ and $\bar{u}_M = 1.1V$, $b = -26MHz$ and $h = 1$. $b_w = 10$ was taken, for any exogenous disturbance. The uncertain parameters take the following ranges:

- $C \in [2, 4]nF$,
- $R \in [5, 10]\Omega$,
- $K_{DCO} \in [10, 1000]MHz/LSB$,
- $T_s \in [20, 30]ns$.

From Lemma 3, it comes that

$$K_1 = 0.8 \quad K_2 = -0.8.$$

Likewise $\gamma = 0.025$, $c = 0.9$, $G = [0.0747 \quad -0.0732]$ and $P_1 = \begin{bmatrix} 0.0062 & -0.0062 \\ -0.0062 & 0.0062 \end{bmatrix}$.

Note that even if the control constant tuning is conservative, there is a feasible solution.

VIII. SIMULATION RESULTS.

In this section, some simulations show the robustness of the optimal and robust saturated control law proposed for the Clock Generator. For these simulations, the data given above are taken.

Figure 2 shows some simulation results when the system parameters are changed:

- syst 1:** $K_{DCO} = 0.04 \frac{MHz}{LSB}$, $R = 5\Omega$, $C = 2 \cdot 10^{-9}F$, $T_s = 20ns$,
- syst 2:** $K_{DCO} = 0.14 \frac{MHz}{LSB}$, $R = 5\Omega$, $C = 2 \cdot 10^{-9}F$, $T_s = 20ns$,
- syst 3:** $K_{DCO} = 0.04 \frac{MHz}{LSB}$, $R = 10\Omega$, $C = 410^{-9}F$, $T_s = 20ns$,
- syst 4:** $K_{DCO} = 0.04 \frac{MHz}{LSB}$, $R = 5\Omega$, $C = 2 \cdot 10^{-9}F$, $T_s = 30ns$.

‘syst 1’, ‘syst 3’ and ‘syst 4’ are superposed. Note that the four systems converge to the set-point, showing the system robustness.

Finally, note in Fig. 3 that the control law rejects an added exogenous disturbance, as expected.

$$\bar{\Gamma}^{(j)} \triangleq \begin{bmatrix} \bar{\Gamma}_1^{(j)} & \bar{\Gamma}_2^{(j)} & -\alpha_k^{(j)} B^{(j)} T - (1 - \alpha_k^{(j)}) B^{(j)} Y + \bar{S} & \bar{B}_w Q_1 \\ * & \bar{P}_1 - 2Q_1 + h\bar{R} & -\alpha_k^{(j)} B^{(j)} T - (1 - \alpha_k^{(j)}) B^{(j)} Y & \bar{B}_w Q_1 \\ * & * & -\frac{\bar{R}}{h} - \bar{S} & 0 \\ * & * & * & -\gamma^2 Q_1 \end{bmatrix}, \quad j = 1, \dots, 16. \quad (27)$$

where

$$\begin{aligned} \bar{\Gamma}_1^{(j)} &\triangleq Q_1 A^{(j)T} + A^{(j)} Q_1 - 2Q_1 + \alpha_k^{(j)} T^T B^{(j)T} + (1 - \alpha_k^{(j)}) Y^T B^{(j)T} + \alpha B_k^{(j)} T + (1 - \alpha_k^{(j)}) B^{(j)} Y + I_2 I_2^T \\ \bar{\Gamma}_2^{(j)} &\triangleq \bar{P}_1 + Q_1 A^{(j)T} - 2Q_1 + \alpha_k^{(j)} T^T B^{(j)T} + (1 - \alpha_k^{(j)}) Y^T B^{(j)T}, \end{aligned}$$

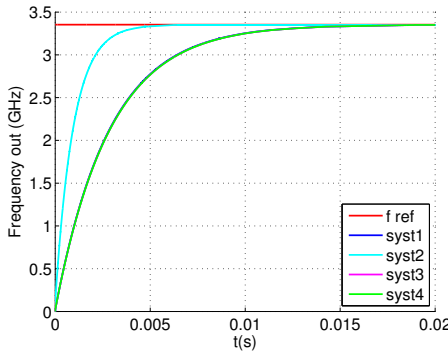


Fig. 2: Evolution of the output frequency for four different systems.

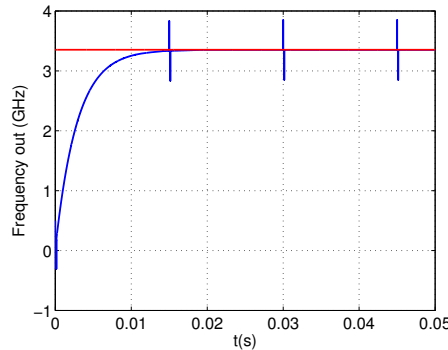


Fig. 3: Exogenous disturbance added to ‘syst 1’.

IX. CONCLUSIONS

In this paper an optimal and robust saturated controller was designed for a time-delay Clock Generator. The system is rewritten in a suited state-space representation, such that an optimal and robust H_∞ problem can be formulated to tune the control gains. This problem is dealt with Lyapunov-Krasovskii theory, which provides some stability conditions through Linear Matrix Inequalities (LMIs). Consequently, robust equilibrium stability as well as robust disturbance rejection under parameter uncertainties are ensured for the time-delay system. The method also takes into account the control saturation estimating a maximum attraction domain. The closed-loop system robustness is exemplified by means of simulations.

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