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Aggregated Modeling of Thermostatic Loads in Demand Response: A Systems and Control Perspective

Karanjit Kalsi, *Member, IEEE,* Forrest Chassin, David Chassin, *Senior Member, IEEE*

Abstract— **Demand response is playing an increasingly important role in smart grid research and technologies being examined in recently undertaken demonstration projects. The behavior of load as it is affected by various load control strategies is important to understanding the degree to which different classes of end-use load can contribute to demand response programs at various times. This paper focuses on developing aggregated control models for a homogeneous population of thermostatically controlled loads. The different types of loads considered in this paper include, but are not limited to, water heaters and HVAC units. The effects of demand response and user over-ride on the load population dynamics are investigated. The controllability of the developed lumped models is validated which forms the basis for designing different control strategies.**

I. INTRODUCTION

Renewable energy resources are generally regarded as the most important near to mid-term solution to the carbon-emission problem of electricity production worldwide. The most common of these are hydroelectric, wind, and solar. The main disadvantage of the latter two renewable resources is the intermittency associated with them. More specifically, they depend on highly variable prime movers, particularly over very short time spans. One proposed approach to address the problem of renewable intermittency is to increase the energy storage capacity of the system as a whole.

More recently, the development of the Smart Grid concept has given rise to the notion that instead of trying to regulate the system by controlling generation, the focus should be to control demand to the extent possible. Controlling loads is usually called demand response, and there are several types of demand response. Direct load control programs can also be effective in providing peak load management and are used by many utilities [1]. However, utilities are well aware that these strategies have adverse effects on customers and only use them *in extremis*.

Another approach that has been tested ([1], [3]) and shows promise is the use of real-time prices combined with enabling technology to "signal" loads when supply is scarce (prices are high) or supply is plentiful (prices are low). When prices are high, loads adjust in a non-intrusive way to reduce power drawn, and conversely increase power drawn when prices are low. If prices are updated frequently and load controls are highly interactive, then loads can become as effective at following intermittent generation as standard generation is at following changing load, without affecting consumers.

Dynamic modeling of thermostatically controlled loads was first studied in [4] and [5]. In [4], aggregate load models are designed to study the effects of cold load pickup after a service interruption. Functional models of devices, which account for factors such as weather and human behavior, are developed in [5]. A model of a large number of similar devices is then obtained through statistical aggregation of the individual component models. In [6] and [7], aggregated dynamic homogenous and non-homogeneous models are developed for thermostatic loads.

In this paper, aggregated control models of thermostatically controlled devices are developed from state transition diagrams. The state transition approach to aggregate load modeling is different from the conventional aforementioned approaches. First, a model of the physical dynamics of thermostatically controlled loads is developed. From this model, a state transition diagram is created to represent the natural state dynamics of a large population of devices. After having obtained the state transition diagram, a state space model of the dynamics, both with and without demand response programs is derived. The state space control model provides a framework to develop new control and state estimation strategies for these devices, which are subjected to various demand response programs. The robustness of the models and controller design is tested using different levels of user over-ride.

The paper is organized as follows. Section II will discuss the physical model of thermostatic loads. In Sections III and IV, aggregate models for heating/cooling loads and electric water heaters will be derived from the physical models of the devices. In Section V, the controllability of these models is discussed, and simulation results will be presented.

II. PHYSICAL DYNAMICS OF THERMOSTATICALLY CONTROLLED LOADS

Residential electric end-use loads can be divided into two principal classes, one for all non-thermostatic loads, such as lights and plugs, and one for all thermostatic loads, such as water heating and air-conditioning. All loads retain an underlying demand function that drives the overall

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K.Kalsi, F. Chassin and D. Chassin are with Pacific Northwest National Laboratory, Richland, WA 99354 USA.

requirement for energy. The distinction made between thermostatic and non-thermostatic load has to do with how that function is satisfied and how power is used to satisfy it [9]. As a general rule, non-thermostatic loads tend to have load shapes where the instantaneous power drawn itself is a non-recurring function of the load state of the individual device. In contrast, thermostatic loads at the level of a single device give rise to pulse sequences where the pulse width and frequency arise from the control hysteresis and the heat balance in the load, while the power drawn changes between two or more fixed values. In this paper, the focus is primarily on the latter type of load. The thermal model, adapted from [10], representing such loads is shown in Fig 1.

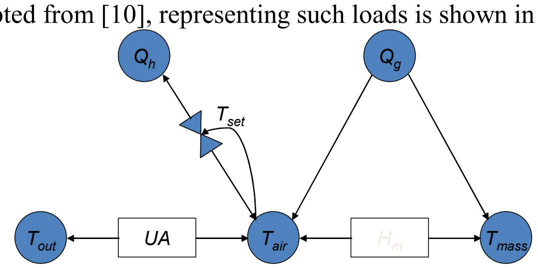


Fig. 1 Thermal model of home heating/cooling system (H_m is ignored in simplified models)

For the purposes of this paper it is assumed that the thermal coupling H_m between air and mass is perfect, although it is a relatively simple matter to model the air-mass coupling as described in [10]. The solution for this particular model can be given in terms of the heating and cooling rates when the heating/cooling system is on and off as follows:

$$r_{off} = \frac{UA(T_{out} - T_{set}) + Q_g}{C_a + C_m} \quad ; \quad r_{on} = r_{off} + \frac{Q_h}{C_a + C_m} \tag{1}$$

where UA is the conductance of the building envelope, T_{out} is the outdoor air temperature, T_{set} is the indoor air thermostat setpoint, Q_g is the heat gain from solar and internal loads, C_a is the thermal mass of the air, C_m is the thermal mass of the building materials and furnishings, and r_{on} and r_{off} are the rates of heating/cooling and coasting air temperatures.

The rates computed thus are given in ^{o}F/h and, given the thermostat deadband L needed for adequate control hysteresis, the time for the *on* and *off* cycles is

$$t_{off} = \frac{L}{r_{off}} \quad ; \quad t_{on} = \frac{L}{r_{on}} \tag{2}$$

The duty cycle φ can then be computed as

$$\varphi = \frac{t_{on}}{t_{on} + t_{off}} = \frac{r_{off}}{r_{on} + r_{off}} \tag{3}$$

In (3), without the loss of generality, it can be assumed that $r_{on} + r_{off} = 1$, in which case $\varphi = r_{off}$ and $1 - \varphi = r_{on}$, a condition necessary to ensure numerical stability in certain finite differences implementation of state models. In the following sections, state space models will be constructed based on this normalized physical model.

III. AGGREGATED MODELS FOR HEATING/COOLING UNITS

A. State space model without demand response

The state dynamics of heating/cooling units can be derived from a state transition diagram. The cycling behavior of a population of thermostatic loads can be described using a state queuing model [8]. The units can be in either the *on* or *off* state. Each of these states can be further divided into two sub-states, which are the loads being *satisfied* or *unsatisfied*. The device can therefore occupy any one of these four states. The objective of the heating unit, for example, is to keep the room temperature at a certain value. The satisfied state is when the room temperature is above the setpoint temperature and when the temperature falls below the setpoint, the unit is said to be in the unsatisfied state.

The heating units, in the *on* state, heat at a certain rate, r_{on}, and move from the unsatisfied to the satisfied state. When the upper control point of the thermostat is reached, the units move from the on satisfied to the off satisfied state. The units then coast at a rate r_{off} to the *off* unsatisfied state after which they turn on and continue the cycle. The state diagram illustrating the heating cycle of a HVAC system is shown in Fig. 2.

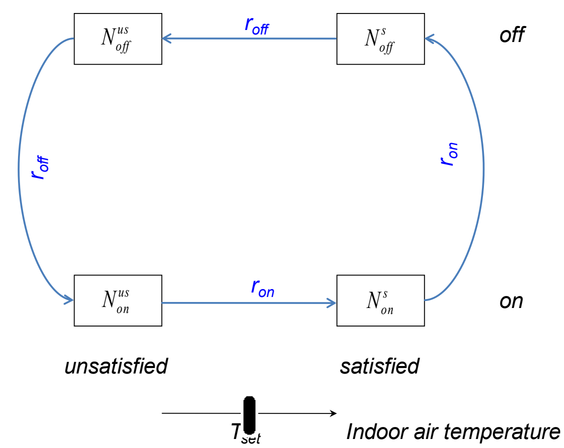


Fig. 2 State transition diagram for heating/cooling units (heating cycle)

The dynamics of device occupancy in each state based on the state transition diagram in Fig. 2 are given as

$$\begin{aligned}
\dot{N}^{us}_{off} &= -\varphi N^{us}_{off} + \varphi N^{s}_{off} \\
\dot{N}^{s}_{off} &= -\varphi N^{s}_{off} + (1-\varphi) N^{s}_{on} \\
\dot{N}^{us}_{on} &= \varphi N^{us}_{off} - (1-\varphi) N^{us}_{on} \\
\dot{N}^{s}_{on} &= (1-\varphi) N^{us}_{on} - (1-\varphi) N^{s}_{on}
\end{aligned} \tag{4}$$

Using (4), the state space representation is given by

$$\dot{x}(t) = Ax(t) \tag{5}$$

where,

$$x(t) = \begin{bmatrix} N^{us}_{off} \\ N^{s}_{off} \\ N^{us}_{on} \\ N^{s}_{on} \end{bmatrix}; \quad A = \begin{bmatrix} -\varphi & \varphi & 0 & 0 \\ 0 & -\varphi & 0 & 1-\varphi \\ \varphi & 0 & \varphi - 1 & 0 \\ 0 & 0 & 1-\varphi & \varphi - 1 \end{bmatrix} \tag{6}$$

B. State space model with demand response and user over-ride

The above model needs to be modified to include demand response and user over-ride. The fraction of on devices that are curtailed by the utility is given by γ and the fraction of these curtailed devices that are turned back on by the consumer is given by α. The device occupancy dynamics in each state are then modified as

$$\begin{aligned}
\dot{N}_{off}^{us} &= -\varphi N_{off}^{us} + \varphi N_{off}^{s} - \alpha s_2(t) \\
\dot{N}_{off}^{s} &= -\varphi N_{off}^{s} + (1-\varphi) N_{on}^{s} \\
\dot{N}_{on}^{us} &= \varphi N_{off}^{us} - (1-\varphi) N_{on}^{us} - \gamma s_1(t) \\
\dot{N}_{on}^{s} &= (1-\varphi) N_{on}^{us} - (1-\varphi) N_{on}^{s} - \gamma s_1(t)
\end{aligned} \tag{7}$$

where $s_1(t)$ and $s_2(t)$ are the utility curtailment and user over-ride signals respectively. The state space model for the above system can be represented as

$$\dot{x}(t) = Ax(t) + B_1 u_1(t) + B_2 u_2(t) \tag{8}$$

where A is the same state transition matrix defined in (6), $u_1 = s_1(t)$ is the utility input (control input), $u_2 = s_2(t)$ is the user input which is unknown to the system, and B_1, B_2 are utility and user input matrices defined as

$$\begin{aligned}
B_1 &= \begin{bmatrix} 0 & 0 & -\gamma & -\gamma \end{bmatrix}^T \\
B_2 &= \begin{bmatrix} -\alpha & 0 & 0 & 0 \end{bmatrix}^T
\end{aligned} \tag{9}$$

The user-control input, u_2, is assumed to be unknown since the utility cannot, in advance, know how the consumer will respond to curtailment. However, the consumer input could still be controlled, to some extent, by using real-time pricing of energy, based on the current demand. In this case, both the utility and user inputs can be used as control inputs.

IV. AGGREGATED MODELS FOR WATER HEATERS

A. State space model without demand response

The cycling behavior of the water heaters is the same as the heating/cooling units described in Section III. The main difference is the effects of consumer demand and the stalling phenomena of the water heater.

When a consumer draws hot water, no matter how long, the water heater will turn on if it was off. This is because the hot water taken from the top of the tank is replaced by cold water at the bottom of the tank, where the thermostat is located. The fraction of off devices subjected to consumer demand is represented by η.

The layers of hot and cold water are separated by a thermocline in the tank. Depending on the relative rates of heat in and out of the tank, the thermocline will either rise or fall. When these rates are approximately equal, the thermocline is stationary and the device is stalled, in which case it will remain in its current on state. The fraction of devices that stall is represented by ρ. The state transition diagram is given in Fig. 3.

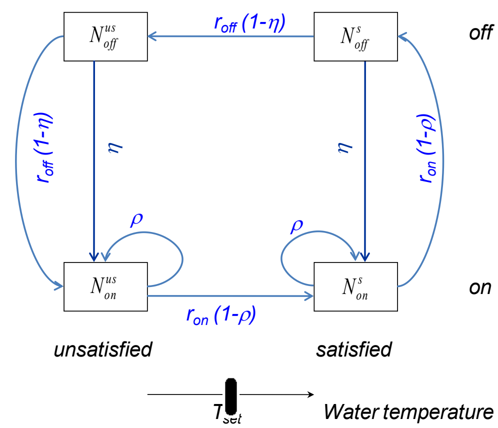


Fig. 3 State transition diagram for water heaters

In the off state, the number of devices that turn on due to consumer demand is ηN_{off} so the fraction of devices that can continue to cycle is $(1\text{-}\eta)N_{off}$. The fraction of on devices that are not stalled and can continue to cycle is $(1\text{-}\rho)N_{on}$. The dynamics of device occupancy in each state are given as

$$\begin{aligned}
\dot{N}_{off}^{us} &= -\varphi(1-\eta) N_{off}^{us} + \varphi(1-\eta) N_{off}^{s} - \eta N_{off}^{us} \\
\dot{N}_{off}^{s} &= -\varphi(1-\eta) N_{off}^{s} + (1-\varphi)(1-\rho) N_{on}^{s} - \eta N_{off}^{s} \\
\dot{N}_{on}^{us} &= \varphi(1-\eta) N_{off}^{us} - (1-\varphi)(1-\rho) N_{on}^{us} + \eta N_{off}^{us} \\
\dot{N}_{on}^{s} &= (1-\varphi)(1-\rho) N_{on}^{us} - (1-\varphi)(1-\rho) N_{on}^{s} + \eta N_{off}^{s}
\end{aligned} \tag{10}$$

The state transition matrix is given as

$$A = \begin{bmatrix}
-\varphi(1-\eta)-\eta & \varphi(1-\eta) & 0 & 0 \\
0 & -\varphi(1-\eta)-\eta & 0 & (1-\varphi)(1-\rho) \\
\varphi(1-\eta)+\eta & 0 & -(1-\varphi)(1-\rho) & 0 \\
0 & \eta & (1-\varphi)(1-\rho) & -(1-\varphi)(1-\rho)
\end{bmatrix} \tag{11}$$

B. State space model with demand response and user over-ride

The demand response and user over-ride effects are modeled in the same fashion as described in Section III B. The state transition matrix A is the same as defined in (11). The utility and user control input matrices B_1 and B_2 are the same as that of the heating/cooling units given in (9). Once again, the control input to the system could either be just the utility signal or both the utility and user by using real-time pricing of energy.

V. RESULTS & DISCUSSION

In this section, the state space models of the HVAC and water heater units developed in Section IV will be simulated, and the effects of utility control and user over-ride discussed.

A. Results for HVAC units

First, the state dynamics of the HVAC system without any demand response is considered. The initial state of the system is chosen to be $x(0) = [0.25\ 0.25\ 0.25\ 0.25]^T$. It can be seen from Fig. 4 that the system states eventually converge to their steady state (natural equilibrium) value $x_{eq} = [0.13\ 0.13\ 0.37\ 0.37]^T$. This indicates that starting from any random initial state, the population of loads will converge to their equilibrium state, since the natural system described by (6) is stable. Next, it is assumed that 50% of the loads are curtailed (γ = 0.5) and 10% of the users over-ride the demand response signal (α = 0.1). The demand response and user over-ride signals are taken as step inputs, which as shown in Fig. 5, de-stabilize the system, thereby indicating a need for utility control. Even for smaller fractions of demand response and consumer over-ride, the open-loop system is still unstable. It can easily be verified that the state space model given in (8) and (9) is controllable *i.e. rank* ($[B\ AB\ A^2B \ldots A^{n-1}B]) = n$. Since the system is controllable, the utility input is taken as a control input and a proportional state feedback controller $u_1 = -K(x-x_{eq})$, with the control gain K designed so as to drive the system to its natural equilibrium state. The user over-ride signal is viewed as an unknown input or a persistent disturbance to the system. The system state responses under different levels of utility over-ride (10% and 20%) are shown in Fig. 6 (a) and (b) respectively. The results indicate that the controlled closed-loop system is robust (states converge to steady state values) under different consumer over-ride levels. Furthermore, it can be seen that the states converge to a steady state value which is not their natural equilibrium point. This is, as expected, due to the presence of the unknown persistent user over-ride signal.

As discussed earlier, when real-time pricing information is available, both the utility and consumer inputs can be taken as control inputs. In this case, as seen from Fig. 7, the populations of loads converge to their natural equilibrium state. During cold load pickup (initial state $x(0) = [1\ 0\ 0\ 0]^T$) and also when the system starts from its equilibrium state ($x(0) = [0.13\ 0.13\ 0.37\ 0.37]^T$), the controlled system states converge to their steady state values as seen from Fig. 8 (a) and (b).

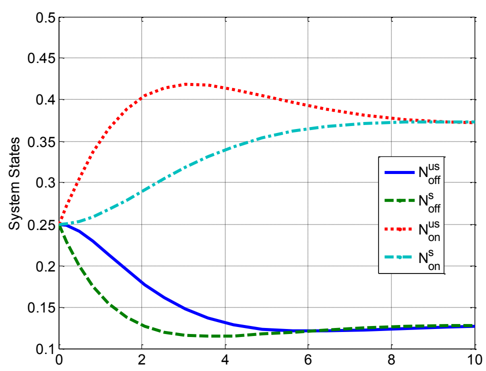


Fig. 4 State dynamics of HVAC system without demand response

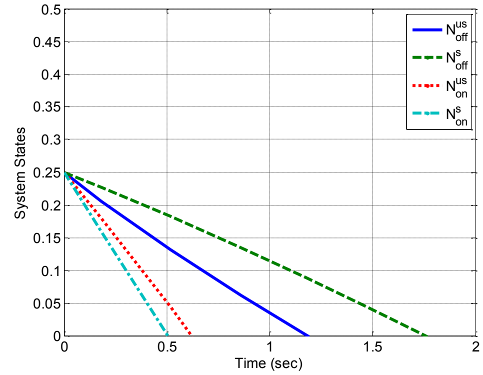
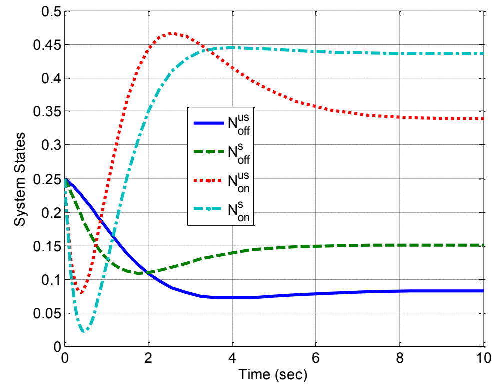
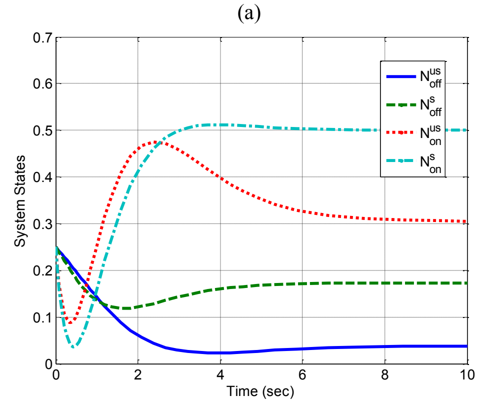


Fig. 5 State dynamics of HVAC system with step demand response and no control



(a)



(b)

Fig. 6 State dynamics of HVAC system with utility control (a) 10% user over-ride (b) 20% user over-ride

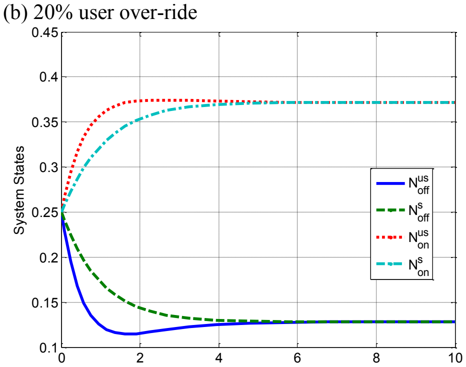
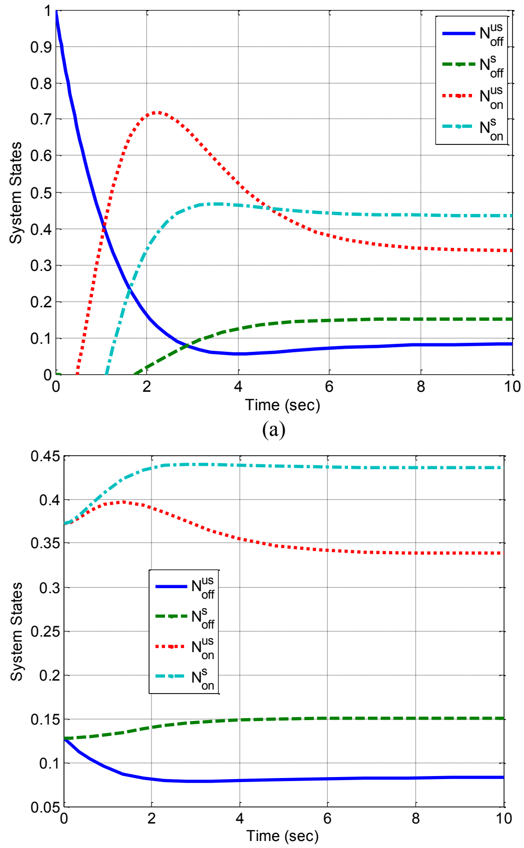


Fig. 7 State dynamics of HVAC system with utility and user control



(a)

(b)

Fig. 8 State dynamics of HVAC system under different initial conditions (a) cold load pickup (b) equilibrium state

B. *Results for water heater units*

It is initially assumed that the water heater units do not have any demand response. The initial state of the system is chosen to be $x\,(0) = [0.25\ 0\ 0.25\ 0.5]^T$. The fraction of off devices subjected to consumer demand and the fraction of stalled devices are both chosen to be 10% ($\eta = \rho = 0.1$). With these parameters, it can be seen from Fig. 9 that the system states converge to their steady state (equilibrium) value x_{eq}= [0.09 0.11 0.32 0.37]^T. As in the case of the HVAC system, it is assumed that 50% of the loads are curtailed ($\gamma = 0.5$) and 10% of the users over-ride the demand response signal ($\alpha = 0.1$). The demand response and user over-ride signals are taken as step inputs which, as shown in Fig. 10, de-stabilize the system. It can be verified that the system is controllable and hence a utility controller can be designed.

The system response under different levels of utility over-ride (10% and 20%) is shown in Fig. 11 (a) and (b) respectively. The results indicate that the controlled closed-loop system states converge to their steady state values under different consumer over-ride levels. The evolution of the load populations when the fraction of devices being subject to consumer demand and the fraction of stalled devices are increased to 20% is shown in Fig. 12 (a) and (b) respectively. The results further validate the robustness of the system and control design. Once again, when both the utility and consumer inputs are taken as control inputs, as seen from Fig. 13, the populations of loads converge to their natural equilibrium state.

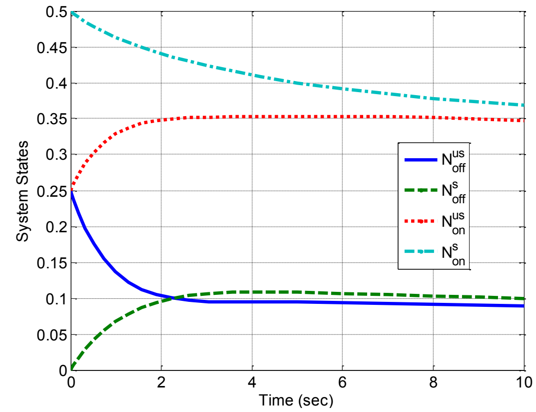


Fig. 9 State dynamics of water heater units without demand response

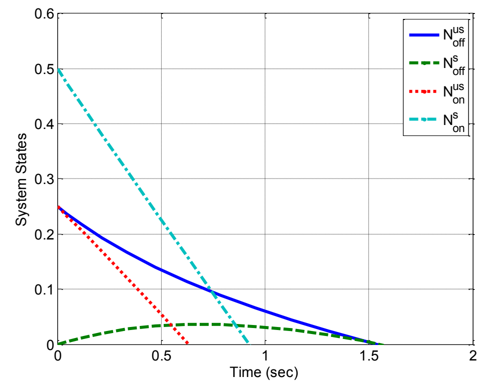
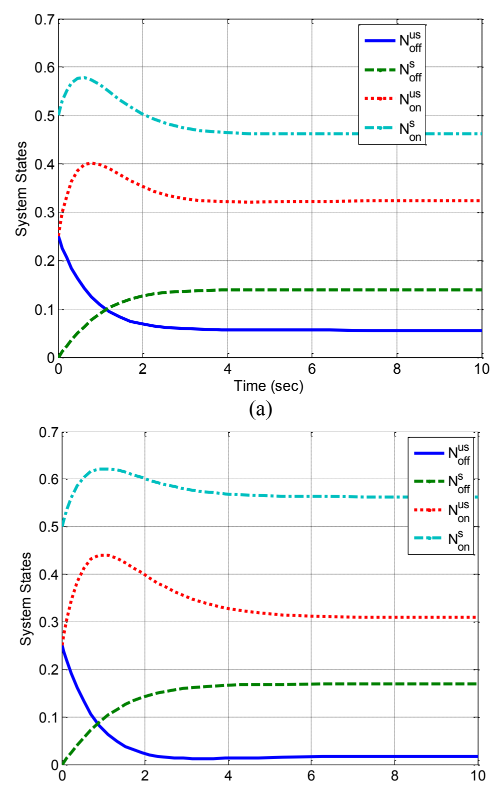


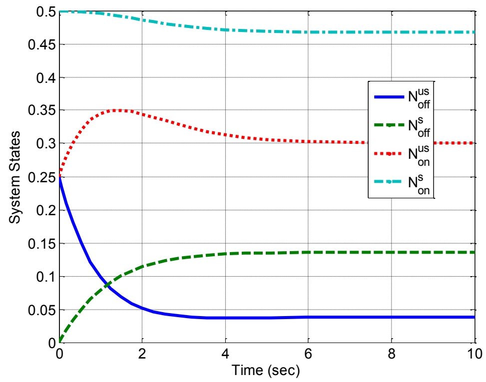
Fig. 10 State dynamics of water heater units with demand response and no control



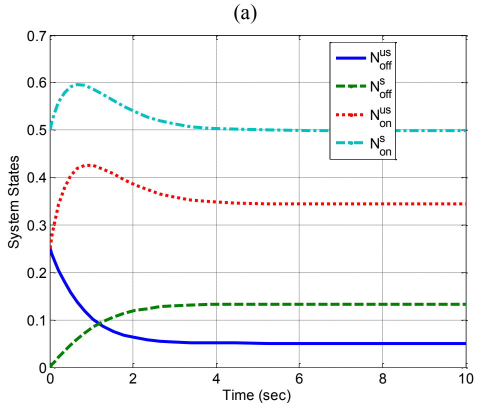
(a)

(b)

Fig. 11 State dynamics of water heater units with utility control (a) 10% user over-ride (b) 20% user over-ride



(a)



(b)

Fig. 12 State dynamics of water heater units with utility control (a) 20% consumer demand (b) 20% stalled devices

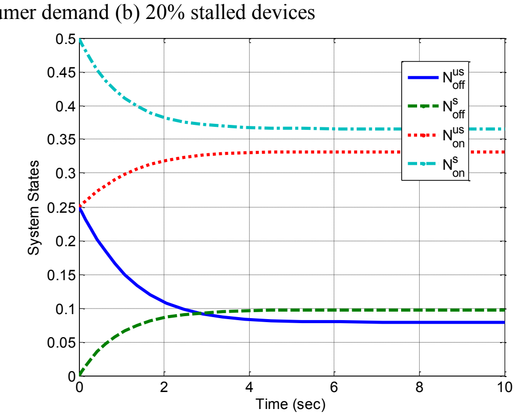


Fig. 13 State dynamics of water heater units with both utility and user control

Utility control of demand response is typically accomplished today by making a once-daily determination of whether to deploy the demand resources available. This single impulse direct load control is known to be stable and is believed to be controllable (although it has not been demonstrated formally to our knowledge).

The control based model developed in this paper addresses whether a continuous load control signal, regardless of the method of communicating the control objective, can be expected to meet utility expectations for controllability and observability. The results suggest that the design of load control systems is neither obvious nor intuitive. Furthermore, continuous load control systems should not be treated the same way as single impulse direct load control, and the design of consumer override controls needs to consider the impact it may have on the stability and controllability of the load control strategies.

VI. CONCLUSIONS

In this paper, aggregate load models have been developed for thermostatically controlled loads by considering the state dynamics of a large homogenous population of such devices. It has also been shown that these models are fully controllable, and with the proper controller design, the system can be forced to any final steady state from any initial state. This opens up the possibility of controlling load to match the intermittent generation of renewable sources, reducing the need for costly energy storage.

Further work would be to extend this to end uses which are not thermostatically controlled. One such example is microwaves, which are controlled by timers and consumer behavior. Another type of device to consider would be clothes dryers which, in addition to timers and consumer behavior, are also controlled by a thermostat. Based on this work, it would also be possible to develop model reference adaptive controllers to track the response of various system inputs and design a state estimator that would give knowledge of the state populations from the total load time series.

VII. ACKNOWLEDGEMENT

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