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Hierarchical Plasma Shape, Position, and Current Control System for ITER

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Abstract—**This paper reports the results of the design and modeling of a hierarchical control system for ITER plasma shape, position and current. The system contains three hierarchy levels. The first level is a model correcting loop with a feedback constant matrix. The second level consists of two loops: a fast SISO plasma vertical position stabilization loop and a slow MIMO plasma current and shape control loop. Both loops have the controllers which were designed by an H_∞ loop shaping approach in the framework of the disturbance rejection configuration. The third level is a structure with a Model Predictive Controller for improving the plasma shape control process by a vertical position reference adjustment during a plasma discharge. The main idea of our proposal is to solve a tokamak plasma vertical position instability problem in cooperation with the plasma shape and current MIMO control loop. The plasma vertical position stabilization technique provides a more reliable control system in whole when compared with the known system approach when a plasma vertical speed is stabilized at about zero. The simulation results of the plasma magnetic hierarchical control system obtained on plasma-physics nonlinear DINA code for ITER are presented. The system was modeled on DINA code on the divertor phase of the plasma current ramp-up stage and on a DINA linearized model on the flat-top phase when a minor disruption occurs.**

I. Introduction

The maximization of the performance-to-cost ratio in modern tokamaks (leaders in the fusion world race) gives a vertically elongated plasma configuration. This causes plasma instability in the direction of the elongation. Because of this and the minimization of the power supply cost a plasma magnetic feedback control system must solve two problems simultaneously: 1) suppression of the plasma vertical instability and 2) control of plasma shape and current during plasma discharge [1]–[3].

The well known approach of plasma vertical speed stabilization about zero stated in ITER (International Thermonuclear Experimental Reactor, www.iter.org) documentation [3] gives a control system which is not strictly stable without a MIMO plasma shape and current control loop. The system is only well damped but is not stabilized relative to a vertical position behavior. In plasma magnetic control systems of such type the vertical stability problem is finally solved by the whole MIMO loop of the plasma shape control. The system operation with this configuration in the presence of the voltage and current saturation of the vertical stability power supply and current saturations of poloidal field coils is dangerous: the control system may lose stability.

In our study we suggest a plasma vertical position stabilization approach in cooperation with the plasma shape and current control. Such combined technique provides a more reliable system as a whole. If some malfunction occurs in the MIMO loop the system will keep stability because the plasma vertical position is stabilized and in this case the SISO loop is internally stable [4].

As one can see in section V, inflexible stabilization about some constant value of the plasma vertical position reduces reachability domain of the plasma shape and current control subsystem. To overcome this difficulty three-layer hierarchical control structure is proposed. The first level is a constant matrix feedback loop for plant behavior correction. The second level consists of the plasma vertical position stabilization loop and the plasma current and shape control loop mentioned above. The third level of the hierarchical structure has a MIMO Model Predictive Controller (MPC) for adjusting a reference for plasma vertical position stabilization subsystem. This hierarchical structure is making the MIMO system of the second level more flexible and this improves the plasma shape deviations behavior.

In this paper the results of the design and modeling of the proposed hierarchical plasma shape, position and current control system for ITER are presented. Section II shortly describes the tokamak plasma as a plant under control and the models of the tokamak plasma used in the research. Section III presents the plasma magnetic control problem statement in the case of the system hierarchical structure with the plasma vertical position stabilization. The synthesis methodology of an H_∞-robust plasma vertical position controller is given in section IV. The shape and current controller synthesis methodology is given in section V. The plasma vertical position reference adjustment technique is presented in section VI. Section VII presents and summarizes the simulation results obtained.

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II. Plant under Control

A. Tokamak plasma

Tokamak plasma is controlled by the set of superconductive coils generating poloidal magnetic field to avoid ablation of the component surface and plasma impurities due to the particle influx. This would lead to plasma disruption and consequent high electromagnetic load on the surrounding magnetic structures.

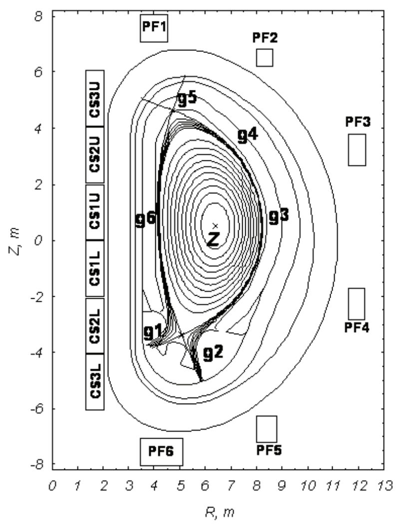

Fig. 1. ITER cross-section and location of PF coils and controlled gaps.

Fig. 1 shows a vertical cross-section of ITER with the location of the central solenoid (CS) coils, poloidal field (PF) coils, "separatrix – first wall" gaps and plasma equilibrium magnetic configuration [3]. The plasma shape control is performed by controlling 6 gaps from g_1 to g_6. Control signals are PF&CS coil voltages. As this takes place, CS1L and CS1U coils are connected in series [3].

Plasma control systems proceeded from very simple systems of a direct passive control when a tokamak vacuum vessel is surrounded by a copper shell to active feedback control systems with external power supplies (see short survey in [5]). In modern tokamaks multivariable plasma current, position, and shape magnetic control systems are used. They are operating in tracking modes in reference to plasma equilibrium parameters which are given by plasma discharge scenario [2].

B. Plant model and disturbance

Nonlinear DINA code [6] tuned for an ITER scenario is exploited for modeling the tokamak plasma discharge. The first principle equations used in DINA code for the plasma magnetic equilibrium description are the nonlinear Grad-Shafranov partial differential equation [7], [8] and the Kirchhoff vector differential equation of magnetically coupled circuits [8].

The linear plant model applied for the control system design was derived by the linearization procedure [9] about the reference of plasma equilibrium configuration at 100 s of an ITER discharge on DINA code. Finally the linear plant model is represented in the state space form [9] specifically:

$$\frac{dx}{dt} = Ax + B\delta U + E\frac{dw}{dt}, \;\; y = Cx + Fw \tag{1}$$

where $A \in R^{127\times127}$, $B \in R^{127\times11}$, $C \in R^{19\times127}$ are plant model matrices, x is the state vector (dim x = 127) which includes variations of currents in CS&PF coils δI_{PF} (11 states), currents in the elements of the plasma passive stabilization system and currents in the inner and outer shell of the vacuum vessel, y is the output vector including the vertical position displacement δZ, six gap displacements δg, the plasma current variation δI_{pl}, current variations δI_{PF} in CS&PF coils and the radial displacement δR_p. δU is the vector of control voltages. E, F are disturbance response matrices and $w = [\delta\beta_p \; \delta l_i]^{\mathrm{T}}$ is the disturbance vector.

The worst-case plasma disturbances are the Minor Disruptions characterized by instantaneous l_i drop $\delta l_i = 0.2(l_{i0} - 0.5)$ without recovery with simultaneous β_p drop with exponential recovery $\delta\beta_p = 0.2\beta_{p0}\exp(-t/3)$, $[t]$ = s. Here l_{i0} and β_{p0} are the values of l_i and β_p before perturbation where β_p is a ratio of a plasma gas-kinetic pressure to an external magnetic field pressure and l_i is a plasma internal inductance [7].

III. Plasma Control Problem Statement

There are two basic interconnected plasma magnetic control challenges in ITER:

1) Plasma vertical position stabilization: before the plasma shape and current control consideration the problem of the plasma vertical position stabilization is to be solved.

2) Plasma shape and current control: gaps and plasma current deviations should be minimized at the plasma current ramp-up stage, and minor disruption disturbance rejection is required on the plasma current flat-top stage.

To solve these challenges the control system structure shown in Fig. 2 is employed. The control signal δU represents the sum of the scalar fast vertical position stabilization signal δU_{VS} going via the special block K_z and the vector slow plasma shape and current control signal δU_M. The block $K_z = [0\;0\;0\;0\;0\;0\;-1\;-1\;1\;1\;0]^{\mathrm{T}}$ represents the connection of the vertical stability power supply to the PF2-PF5 coils [3].

The plasma vertical position stabilization technique provides strictly stable closed loop system with only 4 PF coils from 12 PF coils in the loop. This system is more reliable in whole in comparison with the known system structure [3] when plasma vertical speed is stabilized at about zero and vertical position stability problem is finally solved using all 12 control coils by the whole MIMO loop of the plasma shape control. The main design points for the control system development can be summarized as follows:

- Design an H_∞-robust SISO controller for plasma vertical position stabilization at about the reference value.
- Design an H_∞-robust MIMO controller for plasma shape and current deviation stabilization at about zero.
- Design an MPC for improving the plasma shape and current control loop operation by a vertical position reference value adjustment.

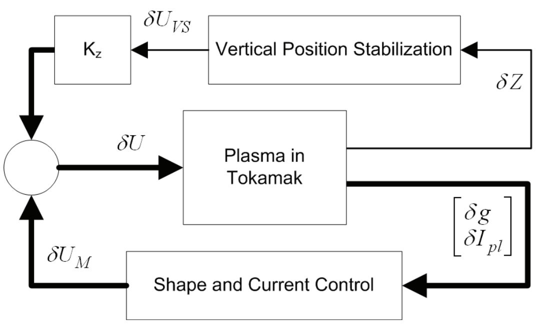

Fig. 2. Basic control system structure.

IV. Plasma Vertical Position Stabilization

A. Plant Dynamics Correction

First, the plasma vertical position stabilization problem is required to be solved. One cannot directly stabilize even the linearized plant model by the plasma vertical position output signal with the sufficient small settling time in the closed loop system. The scalar loop should be decoupled in the frequency domain towards the multivariable loop of the plasma shape and current control. Because of this the plant model is to be corrected by multivariable feedback. The main idea of the plant correction is to avoid the superconductive coils property: currents are integrals of the applied and induced voltages. This correction makes CS&PF currents deviations behavior slower in comparison with the plasma shape and current evolution and allows obtaining vertical position evolution reduced model with smaller order because zero poles (integrals) cannot be neglected otherwise. This approach is based on the methodology developed in [2], [5] when the state vector in (1) is separated into two parts: the vector of eddy currents in passive structures and the vector of CS&PF current variations. Then the fast eddy currents are neglected and a feedback constant matrix is designed to control only CS&PF currents.

The original model has 127 states and reduced model has 11 states. Both models have 11 inputs (PF voltages) and 11 outputs (PF current variations). Reduced plant model with a multivariable feedback constant matrix K_{fb} is as follows:

$$\frac{dx}{dt} = \hat{A}x + \hat{B}(r - K_{fb}\delta I_{PF}),\ \delta I_{PF} = \hat{C}x \qquad (2)$$

where r is the reference vector of 11 entries of the closed-loop system. Let the desired closed loop system matrix have a diagonal form $\Lambda = \mathrm{diag}\{-1\ \ldots -1\}$. Then using the reduced model in which all plant matrices $\hat{A}$, $\hat{B}$, and $\hat{C}$ have 11×11 size and are invertible one can obtain:

$$K_{fb} = -\hat{B}^{-1}(\hat{A} - \Lambda)\hat{C}^{-1}. \qquad (3)$$

With the use of (3) we can get from (2) a closed-loop reduced system model with the decoupled state equation:

$$\frac{dx}{dt} = \Lambda x + \hat{B}r,\ \delta I_{PF} = \hat{C}x.$$

B. SISO Loop Controller Design

Linear models obtained from nonlinear DINA code are not sufficiently precise because of the plasma complexity. To provide a satisfactory operation on nonlinear DINA code and on the original tokamak plasma the robust controllers are needed to be designed. Robust H_∞ open loop shaping design technique based on normalized coprime factorization (NCF) [10] was applied to find proper solutions.

The NCF synthesis technique gives a controller order which is equal to an augmented plant model order. Because of this the model reduction is required. The plant model G_{VS} of order 127 with the correction obtained above is separated into the two parts $G_{VS} = G_{US} + G_{ST}$, where G_{US} is the unstable part of order 1 and G_{ST} is the stable part with the rest order. Then, the stable part G_{ST} is reduced to $\hat{G}_{ST}$ by the balanced residualization procedure [4] and finally reduced model is $\hat{G}_{VS} = G_{US} + \hat{G}_{ST}$.

We reduced the corrected plant model stable part to order 3. Finally, the reduced plant model with the unstable mode has the order 4.

The reduced plant model is scaled by the input and output scaling factors $d_{IN} = 6000$ V and $d_{OUT} = 0.1$ m. So the reduced plant model takes the scaled form:

$$\delta\tilde{Z} = \tilde{G}_{VS}\tilde{u}_{VS}, \qquad (4)$$

where scaled output and input signals are $\delta\tilde{Z} = \delta Z / d_{OUT}$, $\tilde{u}_{VS} = \delta U_{VS} / d_{IN}$, and then the scaled SISO plant model is $\tilde{G}_{VS} = d_{IN}\hat{G}_{VS} / d_{OUT}$.

To perform the open loop shaping design the scaled reduced plant model $\tilde{G}_{VS}$ in (4), named nominal plant model in terms of H_∞ open loop shaping design procedure, is augmented with weighting functions $W_1(s) = 330/s$ and $W_2 = 1$ which have been chosen by trial and error procedure to satisfy the requirements on the frequency domain decoupling between the plasma vertical position loop and the plasma shape and current control loop:

$$G_W = W_2 \tilde{G}_{VS} W_1 = M_l^{-1} N_l$$

where the factors M_l and N_l are to satisfy the equality $M_l M_l^* + N_l N_l^* = I$. The perturbed plant with uncertain transfer functions Δ_M and Δ_N in the coprime factors M_l and N_l may be presented as follows: $G_P = (M_l + \Delta_M)^{-1}(N_l + \Delta_N)$. In that case robust stability margin satisfies the inequality $\left\| \Delta_N \quad \Delta_M \right\|_\infty < 1/\left\| Q \right\|_\infty$, and $Q = -\begin{bmatrix} K_\infty \\ I \end{bmatrix} \left(I + G_W K_\infty \right)^{-1} M_l^{-1}$ is known transfer function of the closed-loop "nominal plant – uncertainty" configuration control system [10]. The final feedback controller for the nominal plant model is

$$K_{VS} = \frac{W_1}{d_{OUT}} K_\infty d_{IN} W_2 . \qquad (5)$$

V. Plasma Shape and Current Control

For the plasma shape and current control system design the H_∞ loop shaping synthesis based on NCF technique is used as well. First, the reduction of the MIMO plant model G_M of order 193 (from 11 component input vector r to output vector $y = [\delta g^T \ \delta I_{pl}]^{\mathrm{T}}$) is required.

The reduced MIMO plant model $\hat{G}_M$ with order 50 is scaled by the input and output matrix scaling factors:

D_{IN} = diag{1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5} (kV),
D_{OUT} = diag{0.1 0.1 0.1 0.1 0.1 0.1 (m) 0.5 (MA)}.

In that case the reduced MIMO plant model takes the scaled form namely

$$\tilde{y} = \left[\delta \tilde{g}^T \quad \delta \tilde{I}_{pl} \right]^T = \tilde{G}_M \tilde{u}_M , \qquad (6)$$

where scaled output and input signals are $\tilde{y} = D_{OUT}^{-1} y$, $\tilde{u}_M = D_{IN}^{-1} r$, and then the scaled reduced MIMO plant model is $\tilde{G}_M = D_{OUT}^{-1} \hat{G}_M D_{IN}$.

Plant model (6) is augmented with matrix weighting functions:

$$W_1 = I_{11\times 11}, \; W_2(s) = \frac{10^5}{s} \mathrm{diag}\{1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 5\cdot 10^{-2}\} .$$

Robust stabilization approach based on the NCF of the augmented plant model gives the suboptimal stabilizing controller K_∞. Finally, the plasma shape and current controller K_{MIMO} is as follows:

$$K_{MIMO} = D_{IN} W_1 K_\infty W_2 D_{OUT}^{-1} \qquad (7)$$

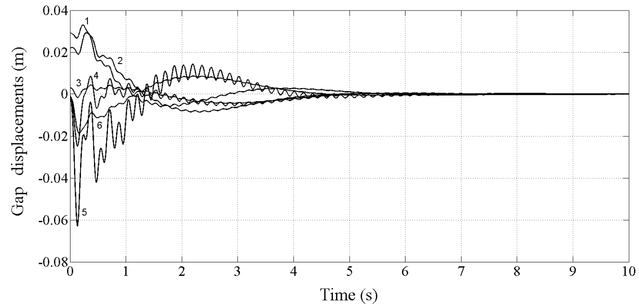

Fig. 3. Gap displacements with the vertical position stabilization about zero when the minor disruption occurs.

A minor disruption disturbance at plasma current flat-top stage (Fig. 3) was modeled with obtained controllers. There six gap displacements δg are shown as the results of modeling the closed-loop system with the plasma vertical position stabilization at about zero by controller K_{VS} and with controller K_{MIMO} in the plasma shape and current control loop.

VI. Plasma Vertical Position Reference Adjustment

In Fig. 3 one can see that the plasma vertical position stabilization at about zero provides the plasma shape and current control subsystem with large gap displacements of about 0.07 m and heavy oscillations. To overcome this shortcoming a control structure of the third hierarchy level is added. The basic idea is to adjust on-line the plasma vertical position by adding a reference value to improve the response of the plasma shape and current control loop. The plasma vertical position linked with the plasma shape is specified by magnetic surface geometry which is described by Grad-Shafranov equation [7], [8]. But in a dynamics evolution this link becomes "flexible" and we suppose that one can improve the plasma shape and current control system behavior with more flexible plasma vertical position stabilization way. To achieve this in cooperation with the plasma vertical position instability suppression loop the plasma vertical position reference is adjusted on-line.

The third level of the hierarchy structure has an MPC [11], [12] for adjusting the reference of the plasma vertical position stabilization subsystem. For the MPC synthesis a single-input and six-output discrete time plant model (from plasma vertical position reference displacement δZ_{REF} to output vector δg, with sampling time $T_s = 0.005$) is exploited. The MPC contains a state observer, thus the MPC order depends on a plant model order. Because of this we reduced the linear plant model up to 30 states.

A control objective is to minimize six gap displacements δg by the vertical position reference adjustment. The MPC consists of the observer designed for the reduced plant model and a structure realizing the following control law in discrete time:

$$\delta Z_{REF}(n) = K_x x(n) + (I + K_u)\delta Z_{REF}(n-1) + K_r \delta g_{REF}$$

where n is a current time step, x is the state estimation vector, δg_{REF} is the desired vector of plant output variables, K_x, K_u, K_r are controller parameter row-vectors obtained from reduced plant model by the MPC design procedure. The three-level hierarchical control system structure in general is shown in Fig. 6.

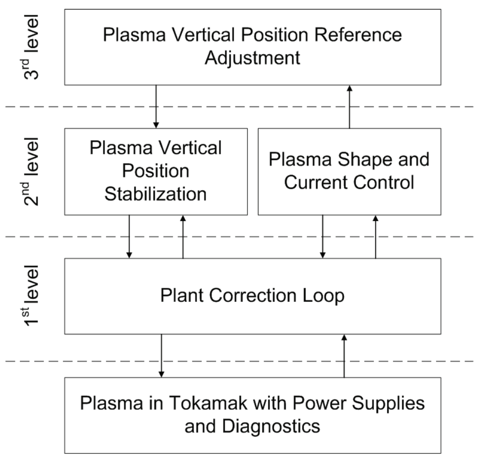

Fig. 4. Hierarchical control system structure.

VII. Simulation Results

A. Minor disruption on the current flat-top phase

The simulation results of the hierarchical control system at the minor disruption disturbance rejection on the plasma current flat-top phase (Fig. 5–7) were obtained on the DINA code linearized model at 100 s of the ITER plasma discharge.

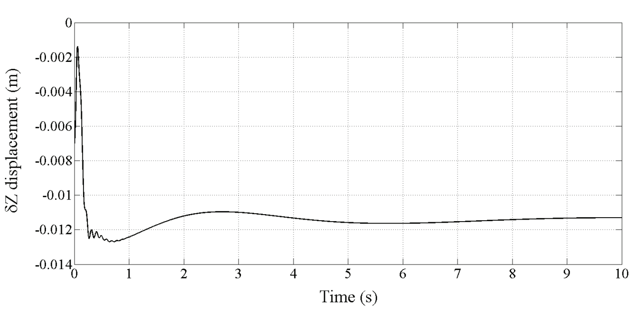

Fig. 5. Plasma vertical position variation δZ during minor disruption disturbance at 100 s.

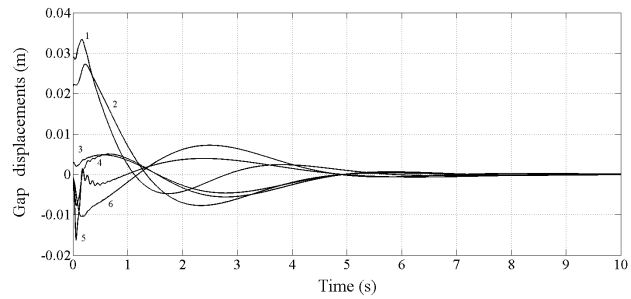

Fig. 6. Gap displacements δg during minor disruption disturbance at 100 s.

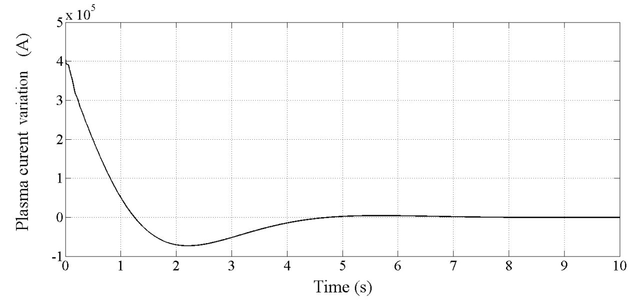

Fig. 7. Plasma current variation δI_{pl} during minor disruption disturbance at 100 s.

B. Plasma current ramp-up stage

The simulation results of the plasma shape and current control in tracking mode without any disturbances on the plasma current ramp-up stage (Fig. 8–11) were obtained on plasma-physics nonlinear DINA code tuned for the ITER scenario from 11.5 MA to 15 MA of the plasma current.

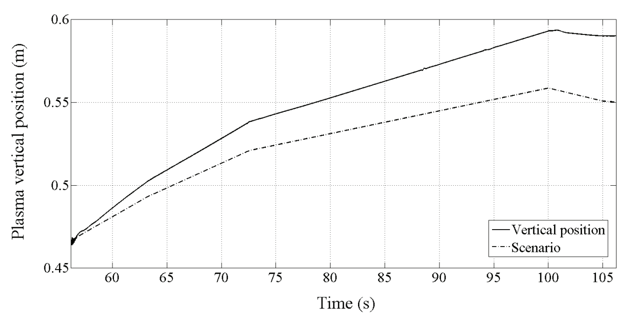

Fig. 8. Plasma vertical position and its scenario value on the plasma current ramp-up stage.

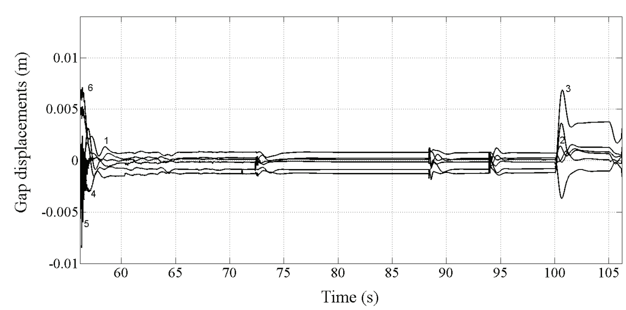

Fig. 9. Gap displacements on the plasma current ramp-up stage.

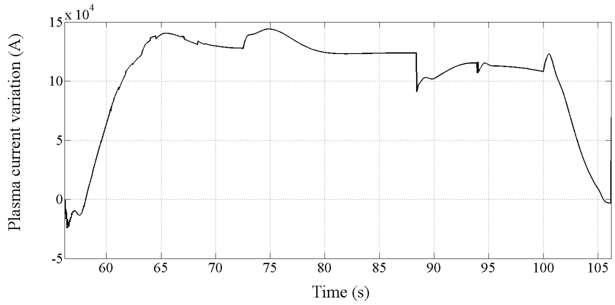

Fig. 10. Plasma current variation on the plasma current ramp-up stage.

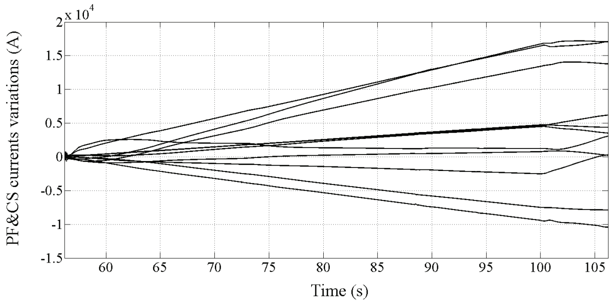

Fig. 11. CS&PF currents variations on the plasma current ramp-up stage.

In Fig. 8 one can see that the obtained plasma vertical position behavior differs from its scenario value because the vertical position reference adjusting is added to the original scenario on the hierarchy third level with the MPC. Fig. 11 shows that CS&PF currents do not match with their scenario values. This can be explained by the fact that the CS&PF currents control objective is not considered in this study and their scenario values were obtained on another plasma-physics code which differs from DINA code. A consideration of the CS&PF control problem forces us to search a trade-off between CS&PF currents tracking and plasma shape, position and current control [13]. To solve this problem we can adjust the CS&PF currents scenario as in [5], [14] using new obtained control currents in simulations done.

The tolerable control system performance was achieved in modeling of the control system on linear DINA model at the worst-case plasma minor disruption disturbance rejection (Fig. 5–7) with the largest gap displacement of 0.035 m and settling time of 8 s. The plasma shape and current control in tracking mode on plasma current ramp-up stage of nonlinear DINA code (Fig. 8–11) gave the control system performance with the largest gap displacement error of 0.015 m and the plasma current displacement overshoot of 150 kA (1%).

In the future development of our plasma control system approach we assume to apply the hierarchical control system with plasma vertical position adjustment throughout the whole plasma discharge from gas breakdown through limiter, divertor and plasma burning phases up to the end of plasma cooling.

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References

[1] M. Ariola and A. Pironti, *Magnetic Control of Tokamak Plasmas*, London: Springer-Verlag, 2008.

[2] Y. V. Mitrishkin, A. Y. Korostelev, V. N. Dokuka, and R. R. Khayrutdinov, "Design and Modeling of ITER Plasma Magnetic Control System in Plasma Current Ramp-Up Phase on DINA Code," in *Proc. Joint 48th IEEE Conf. on Decision and Control and 28 Chinese Control Conf.*, Shanghai, P.R. China, 2009, pp. 1354-1359.

[3] Plasma Performance Assessment. ITER_D_22HGQ7 v 3.0. Sept 2009.

[4] S. Skogestad and I. Postlethwaite *Multivariable Feedback Control (2nd ed.)*, John Wiley & Sons Ltd., 2005.

[5] Y. V. Mitrishkin, A. Y. Korostelyov, V. N. Dokuka, and R. R. Khayrutdinov, "Synthesis and modeling of two-level magnetic plasma control system of tokamak-reactor," *Plasma Physics*, vol. 37, no. 4, pp. 307-349, Apr. 2011 (in Russian).

[6] V. E. Lukash, V. N. Dokuka, and R. R. Khayrutdinov, "MATLAB system software-computing DINA complex for solving plasma control tasks," *Problems of atomic science and technology. Series: Nuclear Fusion,* vol. 1, pp. 40-49, 2004 (in Russian).

[7] J. A. Wesson, *Tokamaks.* Oxford: Clarendon Press, 2004.

[8] R. R. Khayrutdinov and V. E. Lukash, "Studies of Plasma Equilibrium and Transport in a Tokamak Fusion Device with the Inverse-Variable Technique," *Journal Comp. Physics*, 109, no. 2, pp. 193–201, 1993.

[9] Y. V. Mitrishkin, V. N. Dokuka, and R. R. Khayrutdinov "Linearization of ITER plasma equilibrium model on DINA code," in *Proc. 32nd EPS Plasma Physics Conference*, Tarragona, Spain, ID P5.080, June 2005.

[10] D. McFarlane and K. Glover, "A Loop Shaping Design Procedure Using H_∞ Synthesis," *IEEE Transactions of Automatic Control*, vol. 37, no. 6, pp. 759-768, 1992.

[11] L. Wang, *Model Predictive Control System Design and Implementation using MATLAB, Advances of Industrial control,* London: Springer-Verlag, 2009.

[12] Y. V. Mitrishkin and A. Y. Korostelev "A system with predictive model for plasma shape and current control in a tokamak," *Control Sciences*, issue. 5, pp. 19–25, 2008 (in Russian).

[13] V. N. Dokuka, A. A. Kadurin, Y. V. Mitrishkin, and R. R. Khayrutdinov, "Synthesis and modeling of the H_∞-system of magnetic control of the plasma in the tokamak-reactor," *Automation and Remote Control*, vol. 68, no. 8, pp. 1410-1428(19), Aug. 2007.

[14] Y. V. Mitrishkin, A. V. Kadurin, and A. Y. Korostelev, "Tokamak Plasma Shape and Current H_∞ Controller Design in Multivariable Cascade System," in *Proc. 18th IFAC World Congress,* Milan, Italy, 2011, pp. 3722-3727.