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Application of Takagi-Sugeno observers for state estimation in a quadrotor

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Abstract— **In this paper, the validity of Takagi-Sugeno observers to estimate the angular positions and speeds in the experimental platform of a quadrotor will be assessed. Takagi-Sugeno observers are compared to observers based on the linearized model designed with the same optimization criteria and design parameters. Experimental results confirm that Takagi-Sugeno models and observers behave similarly to linear ones around the linearization point, and have a better performance over a larger operating range.**

I. Introduction

Quadrotor setups have gained popularity as a platform for testing advanced control techniques, see [1], [2] and [3]. Their first-principle rigid-body model is nonlinear and nonlinearities also arise in the propeller. Hence, quadrotors are a sensible benchmark for nonlinear control and observation techniques.

This paper presents the design of an observer for a quadrotor, implemented using nonlinear Takagi-Sugeno (TS) models. The TS fuzzy model-based approach has been chosen due to its efficiency with complex non-linear systems in a wide range of application areas, e.g. [4] and [5]. The designed observer is tested in a 3DOF quadrotor, and its performance is compared with a linear observer designed in a similar manner.

The design of state observers for non-linear systems using Takagi-Sugeno (TS) models has been actively considered during the last decades [6], [7]. TS models are currently being used for a large class of physical and industrial processes, such as electrical machines and robot manipulators [8], [9].

A large class of nonlinear systems can be represented or well approximated by TS fuzzy models [10], which in theory can approximate a general nonlinear system to an arbitrary degree of accuracy [11]. The TS fuzzy model consists of a fuzzy rule base. The rule antecedents partition a given subspace of the model variables into fuzzy regions, while the consequent of each rule is usually a linear or affine model, valid locally in the corresponding region.

For a TS fuzzy model, well-established methods and algorithms can be used to design observers that estimate unmeasurable states. Several types of observers have been developed for TS fuzzy systems, among which: fuzzy Thau-Luenberger observers [12], [13], reduced-order observers [14], [15], and sliding-mode observers [16]. These observers are designed such that the estimation error dynamics are asymptotically stable. In general, the design methods lead to a Linear Matrix Inequality (LMI) feasibility problem, which is easy to solve.

The paper is organized as follows: Section II presents the sector nonlinearity approach that will be used for obtaining the TS representation of the quadrotor's model and conditions for observer design. The platform and the mathematical model of the quadrotor are described in Section III. The TS modeling of the quadrotor is realized in Section IV and the observers are designed in Section V. Section VI presents experimental results and finally Section VII provides the conclusions and the future works.

II. Preliminaries: TS Models and Observers

Consider the non-linear system

$$\begin{aligned} x(k+1) &= f(z(k))x(k) + g(z(k))u(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

with f and g smooth non-linear matrix functions, $x \in \mathscr{R}^n$ the state vector, $u \in \mathscr{R}^{n_u}$ the input vector, $y \in \mathscr{R}^{n_y}$ the measurement vector, z some vector function of x, y, and u, all variables assumed to be bounded on a compact set $\mathscr{C}_{xyu}$.

1) Takagi-Sugeno Models: The sector-nonlinearity technique [17], [18] can be applied to the above system in order to obtain a so-called TS model. Basically, let $\mathrm{nl}_j(\cdot) \in [\underline{\mathrm{nl}}_j, \overline{\mathrm{nl}}_j]$, $j = 1, 2, \ldots, p$ be the set of bounded non-linearities in f and g, i.e., components of either f or g. An exact TS fuzzy representation of (1) can be obtained by constructing first the weighting functions

$$w_0^j(\cdot) = \frac{\overline{\mathrm{nl}}_j - \mathrm{nl}_j(\cdot)}{\overline{\mathrm{nl}}_j - \underline{\mathrm{nl}}_j} \quad w_1^j(\cdot) = 1 - w_0^j(\cdot)$$

for each nonlinearity $j = 1, 2, \ldots, p$, and defining the membership functions as

$$h_i(z) = \prod_{j=1}^{p} w_{i_j}^j(z_j) \tag{2}$$

with $i = 1, 2, \cdots, 2^p$, $i_j \in \{0, 1\}$. These membership functions are normal, i.e., $h_i(z) \geq 0$, $i = 1, 2, \ldots, r$, and $\sum_{i=1}^{r} h_i(z) = 1$, $r = 2^p$, where r is the number of rules.

Using the membership functions defined in (2), an exact representation of (1) is given as:

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$$x(k+1) = \sum_{i=1}^{r} h_i(z(k))(A_i x(k) + B_i u(k))$$
$$y(k) = Cx(k) \tag{3}$$

with r the number of local linear models, A_i and B_i matrices of proper dimensions, with $i = 1, 2, \ldots, r$, and h_i defined as in (2).

2) TS Observers: In general, an observer designed for the model (3) has the form

$$\widehat{x}(k+1) = \sum_{i=1}^{r} h_i(\widehat{z}(k)) \Big(A_i \widehat{x}(k) + B_i u(k) + L_i (y(k) - \widehat{y}(k)) \Big)$$
$$\widehat{y}(k) = C\widehat{x} \tag{4}$$

where $\widehat{z}$ denotes the estimated scheduling vector and L_i, $i = 1, \ldots, r$, are the observer gains. The observer design problem is to calculate the values of L_i such that the estimation error converges to zero. Approaches to TS observer designs in literature have been considered in [19], [20], [21].

The estimation error is $e(k) = \widehat{x}(k) - x(k)$ and the dynamics can be written as

$$\begin{aligned} e(k+1) = & \sum_{i=1}^{r} h_i(\widehat{z})(A_i - L_i C) e(k) \\ & + \sum_{i=1}^{r} (h_i(z) - h_i(\widehat{z})) \Big(A_i x(k) + B_i u(k) \Big) \end{aligned} \tag{5}$$

In order for the estimation error to converge to zero, the observer gains L_i are computed such that the first term of (5) converges to zero and such that the disturbance due to the second term, $h_i(z) - h_i(\widehat{z})$ becomes zero as $\widehat{z}$ approaches z. In general, it holds that there exists a $\mu > 0$ so that for all k,

$$\| \sum_{i=1}^{r} (h_i(z(k)) - h_i(\widehat{z}(k))) A_i x(k) \| \leq \mu \| e(k) \|$$

Using the above condition, the estimation error dynamics (5) is asymptotically stable, i.e., the estimation error converges to zero if there exists a positive definite matrix P such that [22]

$$\begin{pmatrix} P - \mu^2 I & * & * \\ P(A_i - L_i C) & P & * \\ 0 & P & I \end{pmatrix} > 0 \qquad i = 1, \ldots, r \tag{6}$$

The inequalities above can be transformed into the following LMI problem: *Find a positive definite matrix P and matrices M_i, where $M_i = PL_i, i = 1, \ldots, r$, such that*

$$\begin{pmatrix} P - \mu^2 I & * & * \\ PA_i - M_i C & P & * \\ 0 & P & I \end{pmatrix} > 0 \qquad i = 1, \ldots, r \tag{7}$$

The gains L_i of the Takagi-Sugeno observer can be obtained by solving the LMI in (7). The design details of this observer will be presented in Section V, after presenting the dynamic model of the quadrotor experimental platform.

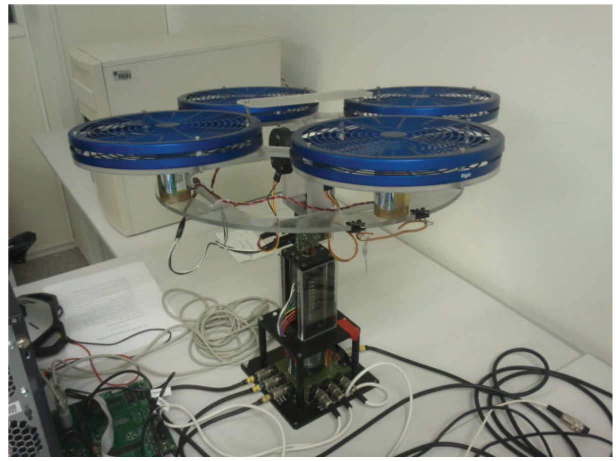

Fig. 1. The Quanser Helicopter

III. Experimental Platform

The three degrees of freedom (3DOF) Hover system consists of a frame with 4 propellers mounted on a 3 DOF pivot joint, such that the body can freely move in roll, pitch and yaw, see Fig. 1. The propellers generate a lift force that can be used to control the pitch and roll angles. The total torque generated by the propeller motors causes a yaw to the body as well. Two propellers in the system are counter-rotating propellers, such that the total torque in the system is balanced when the thrusts of the 4 propellers are approximately equal.

The sensors of the platform are encoders that measure the position of the three axes of the system ϕ, θ and ψ. The control inputs are the voltages applied to each of the 4 propellers.

The communications between the computer and the platform were made with a PMC I/O target. The non-linear model of the platform is presented in the following equations, as given in [23].

$$\begin{aligned} \ddot{\phi} &= \frac{J_r \dot{\theta}}{I_{xx}} K_v (V_1 + V_3 - V_2 - V_4) + \frac{I_{yy} - I_{zz}}{I_{xx}} \dot{\theta} \dot{\psi} + u_1 \\ \ddot{\theta} &= \frac{J_r \dot{\phi}}{I_{xx}} K_v (-V_1 - V_3 + V_2 + V_4) + \frac{I_{zz} - I_{xx}}{I_{yy}} \dot{\psi} \dot{\phi} + u_2 \\ \ddot{\psi} &= \frac{I_{xx} - I_{yy}}{I_{zz}} \dot{\theta} \dot{\phi} + u_3 \end{aligned} \tag{8}$$

where

$$\begin{aligned} u_1 &= \frac{blK_v^2 (V_2^2 - V_4^2)}{I_{xx}} \\ u_2 &= \frac{blK_v^2 (V_3^2 - V_1^2)}{I_{yy}} \\ u_3 &= \frac{dK_v^2 (V_1^2 - V_2^2 + V_3^2 - V_4^2)}{I_{zz}} \end{aligned}$$

are the net torques from the propellers' actuation, which can be computed from the input voltage commands. For observer design purposes, in what follows, the input signals will be considered to be the transformed signals u_i.

The symbols used and their values, where applicable, are given in Table I (extracted from [24]).

The input voltages V_i, $i = 1, 2, 3, 4$, are limited by the drivers, $V_i \in [V_{\min}, V_{\max}]$, with $V_{\min} = -10\,\text{V}$ and $V_{\max} = 10\,\text{V}$. We considered that the angular velocities $\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$

TABLE I

VARIABLES AND PARAMETERS.

Symbol	Meaning	Value Hover	Units
ϕ	Roll angle	Measured	rad
θ	Pitch angle	Measured	rad
ψ	Yaw angle	Measured	rad
V_i	Voltage applied to propeller i	Known input	V
K_v	Transformation constant	54.945	rad s/V
J_r	Rotators inertia	$6 \cdot 10^{-5}$	kgm^2
I_{xx}	Inertia X-axis	0.0552	kgm^2
I_{yy}	Inertia Y-axis	0.0552	kgm^2
I_{zz}	Inertia Z-axis	0.1104	kgm^2
b	Thrust coefficient	$3.935139 * 10^{-6}$	N/Volt
d	Drag coefficient	$1.192464 * 10^{-7}$	Nm/Volt
l	Distance from pivot to motor	0.1969	m
m	Mass	2.85	kg
g	Acceleration due to gravity	9.81	m/s^2
T_s	Sampling time	0.005	s

are bounded, $\dot{\phi}, \dot{\theta}, \dot{\psi} \in [d\alpha_{\min}, d\alpha_{\max}]$, with $d\alpha_{\min} = -\pi/4$ rad/s and $d\alpha_{\max} = \pi/4$ rad/s. The maximum pitch and roll angles are assumed to be $\pi/2$ rad, while the maximum yaw angle is also considered to be π rad.

IV. TS MODELING OF THE 3DOF HOVER

In this section an exact TS representation of the discretized 3DOF model is developed. The TS model will be used later to design the non-linear observer for the hover system.

The gyroscopic effects in the roll and pitch dynamics contain the term $K_v(V_1+V_3-V_2-V_4)$, which is the sum of the (known) inputs. This term is denoted by $u_g = K_v(V_1+V_3-V_2-V_4)$. Furthermore, to simplify the notations, the terms containing the moments of inertia of the 3DOF quadrotor are denoted as $I_{xyz} = \frac{I_{xx}-I_{yy}}{I_{zz}}$, $I_{yzx} = \frac{I_{yy}-I_{zz}}{I_{xx}}$, and $I_{zxy} = \frac{I_{zz}-I_{xx}}{I_{yy}}$.

With the notations presented above, the model (8) is rewritten as

$$\begin{aligned} \ddot{\phi} &= \frac{J_r\dot{\theta}}{I_{xx}}u_g + I_{yzx}\dot{\theta}\dot{\psi} + u_1 \\ \ddot{\theta} &= -\frac{J_r\dot{\phi}}{I_{xx}}u_g + I_{zxy}\dot{\psi}\dot{\phi} + u_2 \\ \ddot{\psi} &= I_{xyz}\dot{\theta}\dot{\phi} + u_3 \end{aligned} \tag{9}$$

The state vector x is defined as $x = (\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi})^T$. Then, one possible[1] representation of (9) is

$$\begin{aligned} \dot{x} &= A_c(x)x + B_c u \\ y &= Cx \end{aligned}$$

with

$$A_c(x) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{J_r}{I_{xx}}u_g & 0 & I_{yzx}x_4 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{J_r}{I_{xx}}u_g & 0 & 0 & 0 & I_{zxy}x_2 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & I_{xyz}x_4 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$B_c = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

[1] Due to the multiplication of the angular velocities, the matrix $A_c(x)$ can be defined in several ways.

where x_i denotes the ith variable of the state vector x.

Since the variables are measured in discrete time, a discrete-time observer will be designed. It is assumed that the sampling time is small enough such that an Euler discretization can be effectively used for the model (9). Consequently, the non-linear discrete-time model is

$$\begin{aligned} x(k+1) &= A_d(x(k))x(k) + B_d u(k) \\ y(k) &= Cx(k) \end{aligned} \tag{10}$$

with

$$A_d(x(k)) = \begin{pmatrix} 1 & T_s & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & T_s\frac{J_r}{I_{xx}}u_g(k) & 0 & T_sI_{yzx}x_4(k) \\ 0 & 0 & 1 & T_s & 0 & 0 \\ 0 & -T_s\frac{J_r}{I_{xx}}u_g(k) & 0 & 1 & 0 & T_sI_{zxy}x_2(k) \\ 0 & 0 & 0 & 0 & 1 & T_s \\ 0 & T_sI_{xyz}x_4(k) & 0 & 0 & 0 & 1 \end{pmatrix} \tag{11}$$

$$B_d = T_s B_c$$

To obtain an exact fuzzy representation of the non-linear model (10), the sector non-linearity approach [18] is used.

The non-constant terms in the matrix $A_d(x(k))$ are $u_g(k)$, $x_4(k)$, and $x_2(k)$, therefore $z(k) = (u_g(k), x_2(k), x_4(k))^T$. Each of these terms are bounded and their weighting functions are constructed[2] as follows:

1) The bounds on the term $u_g(k)$ can be computed based on the bounds of the voltage input and are $u_{g,\min} = 4K_vV_{\min}$ and $u_{g,\max} = 4K_vV_{\max}$. The weighting functions are $w_1^0 = \frac{u_{g,\max}-u_g(k)}{u_{g,\max}-u_{g,\min}}$ and $w_1^1 = 1 - w_1^0$. The term $u_g(k)$ is expressed as $u_g(k) = u_{g,\min}w_1^0 + u_{g,\max}w_1^1$.
2) The bounds of $x_4(k)$ are the bounds of the angular velocity, $d\alpha_{min}$ and $d\alpha_{\max}$. The weighting functions are $w_2^0 = \frac{d\alpha_{\max}-x_4(k)}{d\alpha_{\max}-d\alpha_{\min}}$ and $w_2^1 = 1 - w_2^0$. The term $x_4(k)$ is expressed as $x_4(k) = d\alpha_{\min}w_2^0 + d\alpha_{\max}w_2^1$.
3) $x_2(k)$ is also angular velocity, and its bounds and weighting functions are the same as for $x_4(k)$. Thus, the weighting functions are $w_3^0 = \frac{d\alpha_{\max}-x_2(k)}{d\alpha_{\max}-d\alpha_{\min}}$ and $w_3^1 = 1 - w_3^0$. The term $x_2(k)$ is expressed as $x_2(k) = d\alpha_{\min}w_3^0 + d\alpha_{\max}w_3^1$.

As shown above, there are three non-linearities. For each of these nonlinearities we have 2 weighting functions, and therefore the fuzzy model will have $2^3 = 8$ rules. The membership functions are computed as (2), and the corresponding local linear models are obtained by substituting the corresponding values into the A_d matrix. For instance, the first membership function and the corresponding local matrix are

$$h_1(z(k)) = w_1^0w_2^0w_3^0$$

$$A_1 = \begin{pmatrix} 1 & T_s & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & T_s\frac{J_r}{I_{xx}}u_{g,\min} & 0 & T_sI_{yzx}d\alpha_{\min} \\ 0 & 0 & 1 & T_s & 0 & 0 \\ 0 & -T_s\frac{J_r}{I_{xx}}u_{g,\min} & 0 & 1 & 0 & T_sI_{zxy}d\alpha_{\min} \\ 0 & 0 & 0 & 0 & 1 & T_s \\ 0 & T_sI_{xyz}d\alpha_{\min} & 0 & 0 & 0 & 1 \end{pmatrix}$$

Each of the local models is observable given the available measurements.

[2] Note that the multiplication with a constant of a non-linearity does not affect the weighting functions.

V. OBSERVER DESIGN

A. TS Observer

To design a TS observer, it is assumed that both the state and the measurements are corrupted by noise, i.e., the system equations can be written as

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{r} h_i(z(k))(A_i x(k)) + Bu + v(k) \\ y(k) &= Cx(k) + \eta(k) \end{aligned} \tag{12}$$

where $v(k)$ and $\eta(k)$ are the state transition and measurement noises, respectively.

The fuzzy observer is

$$\begin{aligned} \widehat{x}(k+1) &= \sum_{i=1}^{r} h_i(\widehat{z}(k))(A_i\widehat{x}(k) + L_i(y(k) - \widehat{y}(k))) + Bu \\ \widehat{y}(k) &= C\widehat{x}(k) \end{aligned} \tag{13}$$

and the error dynamics are

$$\begin{aligned} e(k+1) = &\sum_{i=1}^{r} h_i(\widehat{z}(k))\Big((A_i - L_iC)e(k) + (I \ - L_i)\begin{pmatrix} v(k) \\ \eta(k) \end{pmatrix}\Big) \\ &+ \sum_{i=1}^{r}(h_i(z(k)) - h_i(\widehat{z}(k)))A_i x(k) \end{aligned}$$

with

$$\| \sum_{i=1}^{r}(h_i(z(k)) - h_i(\widehat{z}(k)))A_i x(k)\| \le \mu \|e(k)\|$$

Our goal is to design the observer gains L_i, such that the effect on the disturbances $v(k)$ and $\eta(k)$ on $De(k)$ is minimized, where D is a known matrix. This can be written as:

$$\|e^T De\|_2 \le \gamma^2 \|\omega^T I \omega\|_2 \tag{14}$$

After some algebraic manipulations (details are omitted for brevity) it can be proven that the effect of the disturbances is minimized and the estimation error converges with a desired desired convergence rate β, if the observer gains are obtained by solving *minimize* $\gamma > 0$, *find* $P = P^T > 0$, L_i, $i = 1, 2, \ldots, r$ so that

$$\begin{pmatrix} (1-2\beta T_s)P - \mu^2 I & * & * & * & * \\ 0 & \gamma I & * & * & * \\ P(A_i - L_iC) & P(Q \ - L_iR) & P & * & * \\ 0 & 0 & P & I & * \\ D & 0 & 0 & 0 & \gamma I \end{pmatrix} > 0 \tag{15}$$
$$i = 1, \ldots, r$$

where β is the equivalent desired convergence rate for the continuous system, Q is the covariance matrix of the state noise and R is the covariance matrix of the measurement noise. To transform equation (15) into an LMI a change of variable $M_i = PL_i$ is performed. The obtained LMI is:

$$\begin{pmatrix} (1-2\beta T_s)P - \mu^2 I & * & * & * & * \\ 0 & \gamma I & * & * & * \\ PA_i - M_iC & PQ \ - M_iR & P & * & * \\ 0 & 0 & P & I & * \\ D & 0 & 0 & 0 & \gamma I \end{pmatrix} > 0 \tag{16}$$
$$i = 1, \ldots, r$$

For this platform it has been considered that:

$$\begin{aligned} Q &= diag(0.0001, 1, 0.0001, 1, 0.0001, 1) \\ R &= diag(8 \cdot 10^{-4}, 8 \cdot 10^{-4}, 8 \cdot 10^{-4}) \\ D &= diag(10, 0.038, 10, 0.038, 10, 0.038) \\ \beta &= 2.25 \end{aligned}$$

and the value $\mu = 0.003$ has been computed from the knowledge of h_i and the validity range of the TS model.

In total 8 observer gains have been obtained. For instance, the gain matrix for the first rule is:

$$L_{TS,1} = \begin{pmatrix} 1.1799 & 0.0000 & -0.0000 \\ 35.9791 & -0.4295 & 0.1508 \\ -0.0000 & 1.1799 & 0.0000 \\ 0.4299 & 35.9791 & -0.1509 \\ 0.0000 & -0.0000 & 1.1918 \\ 0.0001 & -0.0002 & 38.3548 \end{pmatrix}$$

B. Linear Observer

To design a linear observer, first the non-linear model, presented in (11), is linearised around $x = 0$, obtaining

$$\begin{aligned} x(k+1) &= A_0 x(k) + B_d u(k) + \nu(k) \\ y(k+1) &= Cx(k+1) + \eta(k) \end{aligned} \tag{17}$$

where A_0 is the local state matrix, B_d is the input matrix, C is the measurement matrix, and $\nu(k)$ and $\eta(k)$ having the same interpretation as in (12).

A deterministic linear observer is considered. The resulting equation is:

$$\widehat{x}(k+1) = A_0\widehat{x}(k) + B_d u(k) + L_L(y(k) - C\widehat{x}(k))$$

where L_L denotes the observer gain. This gain is computed by solving the matrix inequality (16), similarly to the TS observer design. Hence, the linear observer uses only one observer gain and one vertex model whereas the TS one uses eight gains and eight vertex models.

The obtained linear gain is

$$L_L = \begin{pmatrix} 1.0509 & -0.0000 & -0.0000 \\ 10.1863 & -0.0000 & -0.0000 \\ -0.0000 & 1.0509 & 0.0000 \\ -0.0000 & 10.1863 & 0.0000 \\ -0.0000 & 0.0000 & 1.0509 \\ -0.0000 & 0.0000 & 10.1863 \end{pmatrix}$$

VI. EXPERIMENTAL RESULTS

As the open-loop system is unstable, an LQR controller, designed on the linearised system, was implemented to stabilise the closed-loop.

The inputs of this control are the angular positions of roll, pitch and yaw, which are measured by the encoders in the experimental platform, and the angular velocities of the three degrees of freedom. As the objective of this paper is not designing a high-performance controller, but a high-performance observer, suffice to say that is a state feedback controller.

Input-output data have been generated by inserting sinusoidal and step references to this basic stabilising loop.

As there is no direct access to the real state variables, a noncausal zero-phase filter, incorporating numerical differentiation in the speed estimation case (`filtfilt` function of Matlab®) has been used to compute the "real" value. The results given by the Takagi-Sugeno and linear observers have been compared to the results of the noncausal filter to compute the (approximate) error.

With the objective of validating the TS observer, the system has been subjected to an excitation achieving large enough angular speeds for the nonlinear terms to be significant. Hence, a sinusoidal excitation was introduced in ψ from second 5 till 40 and a reference in θ and ϕ changes every 5

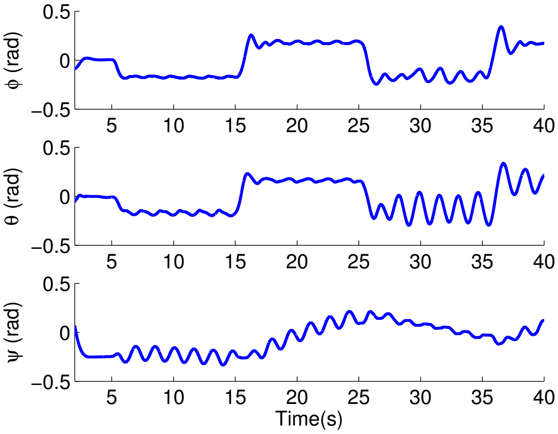

Fig. 2. Measurement data of the platform

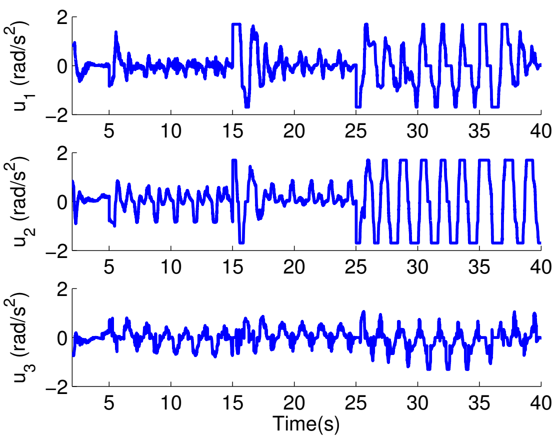

Fig. 3. Input data of the platform

seconds from 10 to -10 degrees. The initial conditions were close to the linearization point, and in the first 5 seconds no input excitation has been applied. The input-output data collected appear in Fig. 2 and 3. This data confirms that the system states satisfy the bounds from Section III.

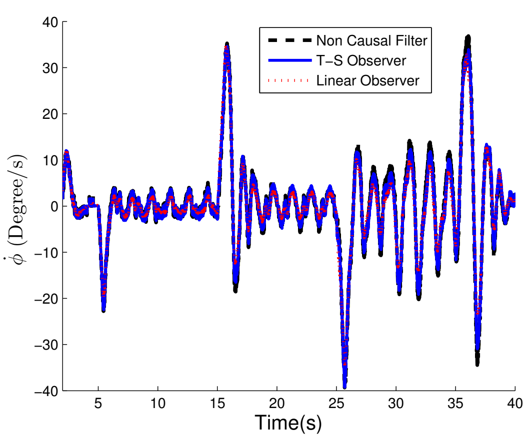

Fig. 4. Estimations of the full experiment (Velocity)

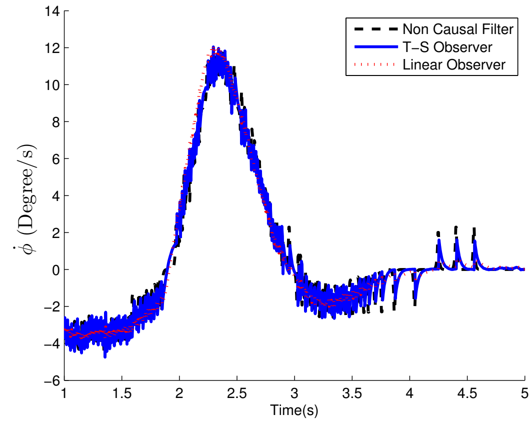

Fig. 5. Zoom in the time space [1 5]s

The estimation results for the noncausal filter and the Takagi-Sugeno and linear observers are shown in Fig. 4. Note that the position estimates are actually very precise as a direct low-noise encoder output is available. As intuitively expected, speed estimation is less precise and the differences between the observer alternatives in the speed case will be discussed below.

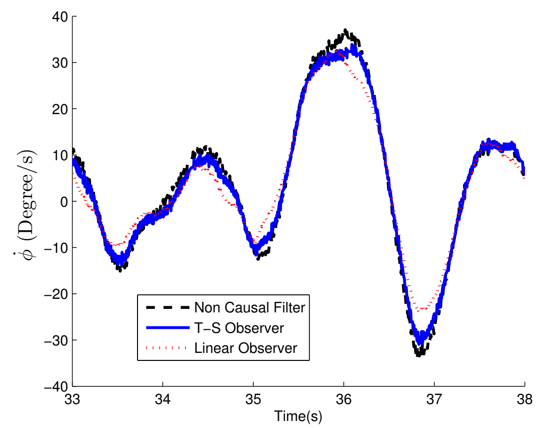

Fig. 6. Zoom in the time space [33 38]s

Fig. 5 shows the first 5 seconds of the experiment, when there is no yaw excitation. It can be seen that the three observers estimate the velocity in a similar way, possibly because the linearized model is reasonably valid. Fig. 6, a zoom in of the experiment in a zone where there was a ψ excitation and reference change in ϕ and θ (from 33 to 38 seconds), shows a clear difference between the estimations of the different observers.

To have a better understanding of the platform estimation improvement, the ISE (Integral Squared Error) of the observers estimation error (as compared to the non causal filter output) has been computed, and the result is presented in Fig. 7.

Fig. 7 shows that although in the first seconds the linear

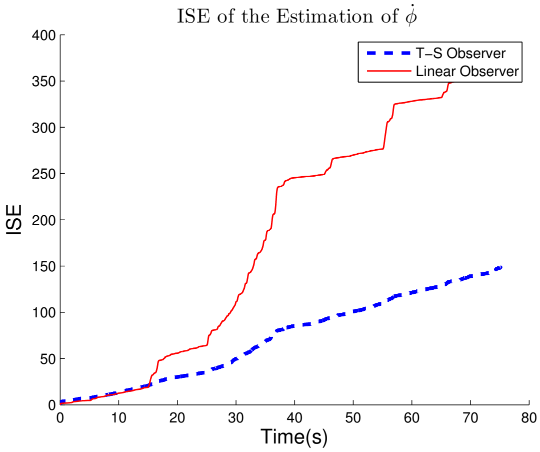

Fig. 7. ISE of the Linear and TS observers of $\dot{\phi}$. Linear observer (Solid Line) and TS observer (Dashed line)

observer has less error than the TS, when the non-linear stimulations (ψ sinusoidal and reference changes) affect the system, the linear observer error increases significantly. The ISE of the attitude of the quadrotor is shown in Table II. It is clear that the error of the linear observer is larger than the error in the TS.

TABLE II
ISE OF ATTITUDE ESTIMATION

	ISE TS	ISE Linear
$\dot{\phi}$	148.6471	360.7385
$\dot{\theta}$	533.3477	858.5370
$\dot{\psi}$	535.5539	1175.7
Combination of velocity	1217.5487	2394.237
Combination of position	0.2681	0.2858

VII. CONCLUSIONS

An LMI-based Takagi-Sugeno nonlinear observer has been designed for attitude and rotational speed estimation in a quadrotor. The experimental results presented show that a better estimation is obtained with the TS observer when the operating range is far away from the point of linearization of a similarly designed linear observer. In this way, the theoretical advantages of the TS framework are confirmed in a real experiment.

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