2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Discrete inversion based FDI for sampled LPV systems*

Z. Szabó ‡, A. Edelmayer and J. Bokor

Abstract— **The paper investigates the design problem for detection and isolation of faults in linear parameter varying (LPV) systems by means of dynamic inversion where the system matrix depends affinely from the parameters. A method for the construction of the inverse, relying on the concept of parameter varying invariant subspaces and related concepts of classical geometrical system theory is presented. A discretization method is proposed for the filter implementation that exploits the structure of the original LPV model, formulated in the continuous time, while maintaining the stability of the zero dynamics of the original system. The proposed method is illustrated through an application example concerning the detection of aileron and rudder faults on a commercial aircraft.**

I. Introduction

The basic objective of a fault detection methodology applied to dynamic systems is to provide techniques for detection and isolation of failed components. Using a mathematical model of the system it is possible to exploit the principle of analytical redundancy, which allows to check discrepancies between the real behavior of the system and its idealized mathematical description or model. Model-based FDI rely on analytical redundancy to generate fault indicators, named residuals.

There are many analytical redundancy methods available in the literature for linear and nonlinear systems. While recent nonlinear approaches are useful for the analysis, and partly design of detection filters, they are largely incapable for solving synthesis problems because of the computational burden they usually pose for the implementation.

Linear parameter varying (LPV) modeling is known to be a capable approach to alleviate this problem; it has been useful in many areas of control and filtering in treating nonlinear problems in the past years. The idea suggests that a broad class of nonlinear system models can be converted into a quasi-linear form, obtaining the so-called quasi-linear parameter varying (qLPV) representations, in which the state matrix depends affinely on a parameter vector. This approach is particularly appealing when the nonlinear plant can be considered as linear one assuming the presence of a set of time-varying parameters in the system matrix. The parameters are thought not necessarily known at design time but always measurable. Then, the natively nonlinear problem, embedded in the framework of the linear parameter varying model can be solved by using traditional linear techniques.

The paper considers the class of fault affected nonlinear systems that can be represented in the qLPV form:

$$\dot{x}(t) = A(\rho)x(t) + B_u(\rho)u(t) + L\nu(t), \tag{1}$$
$$y(t) = Cx(t), \tag{2}$$

where the system matrices A, B_u are parameter varying matrices whose entries can be dependent on the scheduling function ρ, which is determined by the measured variables y. C is the observation matrix, $x \in \mathcal{X} \subset \mathbb{R}^n$ is the state, $u \in \mathcal{U} \subset \mathbb{R}^{m_u}$ and $y \in \mathcal{Y} \subset \mathbb{R}^p$ is the input and output functions, respectively. The fault signals, which enter the state space in the directions L, are represented by the vector variable $\nu \in \mathcal{F} \subset \mathbb{R}^{m_f}$. Throughout the paper a polytopic parameter dependence is assumed, i.e.,

$$M(\rho(t)) = M_0 + \rho_1(t)M_1 + \ldots + \rho_N(t)M_N. \tag{3}$$

It is assumed that each parameter ρ_i ranges between its known extremal values $\rho_i(t) \in [\underline{\rho}_i, \overline{\rho}_i]$. The parameter set $(\rho_1(t), \cdots, \rho_N(t))$, $t \in [0, T]$, will be denoted by $\mathcal{P}$. For notational convenience the time dependency of the matrices $A(\rho) := A(\rho(y))$ will be dropped where it is possible.

In the continuous-time LPV setting a number of results for the solution of the FDI problem were obtained in the past years, see, e.g., [1], [2], [3], [4], [5], [6], [7], [8]. The continuous time approach, however, pose several limitations on the applicability and implementation of the methods. Moreover, continuous time LPV approaches do not directly support filter synthesis when measurement data are available at discrete time instants, by means of sampling.

On the other hand, a number of classical linear FDI methods seem particularly useful to nonlinear detection problems, if applied in the LPV formulation (see, e.g., [12], [13], [14]). A typical example is the inversion based detection filter design: in contrast to the discrete time case continuous time system inversion, see, e.g., [9], [10], [11], involves the knowledge of the derivatives of measurement variables in general. This, however, despite the innovative character of the solution, poses severe performance limitation on practical applications. In the discrete time framework these limitations could largely vanish.

Discretization of linear time invariant (LTI) systems is a fairly well understood area by now. Unfortunately, these results can not be directly transferred to the LPV domain due to the inherently time varying nature of the parameter varying principles of the modeling. Since the first attempts to the qLPV formulation of nonlinear problems were derived

* The research has been conducted as part of the European seventh framework work programme *Advanced Fault Diagnosis for Sustainable Flight Guidance and Control (ADDSAFE)* project under the grant agreement FP7-233815. This work was also supported by the Control Engineering Research Group, Hungarian Academy of Sciences at the Budapest University of Technology and Economics.

‡ Corresponding author. E-Mail: szaboz@decst.scl.sztaki.hu
The authors are with Computer and Automation Research Institute, Hungarian Academy of Sciences, Kende u. 13–17., H–1111 Budapest, Hungary.

in the continuous-time setting the current methodology of controller and filter design is based on a continuous-time state space synthesis, almost exclusively. Only a few papers deal with the LPV discretization problem, see [15], [16], [17], and none of them focus on the design specific issues of FDI filters.

The paper focuses on the inversion based detection filter design method. Discretization might destroy the relative degree of the plant, which is closely related to the invertibility conditions. Thus the straightforward approach, i.e., to perform the design on a discretized model, is not applicable in general. The first part of the paper selects a discretization method that fits the needs of the inversion based FDI filter design problem.

Since the class of discretization schemes that preserves the structures relevant to the invertibility imposes too strict limitations for practical purposes a novel approach is presented: the design is performed in continuous time while the zero dynamics and the directly invertible part of the systems are discretized separately. The advantage of the proposed discretization method is that exploits the structure of the continuous time problem and maintains the stability properties of the zero dynamics of the continuous system.

The structure of the paper is the following. Section II presents the possible qLPV discretization schemes. Section III provides a detailed analysis of inversion based filter design for the discretized qLPV system. The paper is concluded with an application example concerning the detection of aileron and rudder faults on a commercial aircraft.

II. DISCRETIZATION OF LPV SYSTEMS

Discretization of the qLPV system (1) can be related to the numerical integration of the general nonlinear time varying differential equations. The theory of numerical methods for the solution of general ordinary differential equations (ODEs) has reached certain maturity in the past decades, and algorithms, mainly based on the Runge-Kutta or linear multistep methods, have become widely available, see [18], [19]. Numerical solutions of the initial value problem for the system of ordinary differential equations discretize the equations in time and produce sequences of points that approximate the solutions over time.

Theory of dynamical systems focuses on the behavior of the solution trajectories in a long run, often investigating the intricate geometry in the structures formed by the trajectories. Geometric considerations have been introduced quite recently to the numerical analysis of ODEs: instead of concentrating only on the numerical approximation of a single solution trajectory, the discretization, as a numerical method, is considered as a discrete dynamical system which approximates the flow of the differential equation. It turned out that preservation of geometric properties of the flow not only produces an improved qualitative behavior, but also allows for a more accurate long-time integration, than using general purpose solution methods alone, see [20], [21], [22].

To stress the importance of this point recall that conventional discretization schemes neither preserve the relative degree of the continuous time system nor the stability of the zero dynamics it generates, in general, see [23], [24]. These factors, however, are strongly connected to invertibility, i.e., to the applicability of inversion based FDI approach for a given problem.

In the investigation of the problem the first observation that should be made is that the target of the discretization, i.e., the plant (design) or the controller/filter (implementation), imposes different requirements to the discretization scheme.

A. Discretization for implementation

Previous works on LPV discretization were mainly concerned with a two step approach primarily focusing on the implementation. By using this approach, the controller (filter) is designed, in the first step, in continuous time producing the continuous dynamics

$$\dot{\xi} = A_c(\rho)\xi + B_c(\rho)w. \tag{4}$$

The discretization is applied then to the qLPV system (4) in the second step.

The standing assumption here is that the qLPV system can be approximated by the replacement system

$$\dot{\bar{\xi}} = A_c(\rho_d)\bar{\xi} + B_c(\rho_d)w, \tag{5}$$

with sufficient accuracy, featuring a similar structure but a different scheduling variable ρ_d, i.e., the zero order hold (ZOH) approximation of the true scheduling variable ρ, obtained by the sampling process. Having a digital controller the input w is also a piecewise constant signal.

With these assumptions the discrete equivalent of Eq. (5) can be written as

$$\zeta_{k+1} = A_d(\rho_d, k)\zeta_k + B_d(\rho_d, k)w_k, \tag{6}$$

with $w_k = w(kT_s)$, where T_s is the sampling interval. This discretization corresponds to the formulation

$$\begin{aligned} A_d(\rho_d, k) &= e^{T_s A_c(\rho_k)}, \\ B_d(\rho_d, k) &= \int_0^{T_s} e^{\tau A_c(\rho_k)} B_c(\rho_k) d\tau, \end{aligned} \tag{7}$$

with $\rho_k = \rho(kT_S)$. If the inverse of $A_c(\rho_k)$ exists, one has

$$B_d(\rho_d, k) = A_c(\rho_k)^{-1}(e^{T_s A_c(\rho_k)} - I)B_c(\rho_k).$$

Matrix exponentiation is a costly operation, however. Moreover, since the method rests on strong nonlinear dependence of ρ, the preservation of the structure on the parameter dependence can not be ensured, therefore, it is not favored in the computation.

Under the standing assumption (5) the various discretization schemes differ in the type of approximation of the matrix exponent in (7). In the paper we rely on the explicit scheme derived from the Taylor approximation, i.e.,

$$\begin{aligned} A_d(\rho_d, k) &= I + \sum_{l=1}^{N_T} \frac{T_s^l}{l!} A_c^l(\rho_k), \\ B_d(\rho_d, k) &= T_s(I + \sum_{l=1}^{N_T-1} \frac{T_s^l}{(l+1)!} A_c^l(\rho_k))B_c(\rho_k)d\tau, \end{aligned} \tag{8}$$

with $N_T \leq 4$. It is not hard to figure out that this corresponds to the explicit Runge-Kutta method of order N_T. For the algorithmic details of other, possibly useful, schemes and properties of the approximation error, the interested reader is directed to [16].

Remark 1: Note that the standing assumption is reasonable for implementation. It is possible, however, to integrate (numerically) the system between two sampling times as an open loop nonlinear system using a suitably chosen fixed step size. The price for the possible increased accuracy is that the discretized system will have considerably more states, i.e., the increased computational load.

B. Discretization for design

For design purposes the plant itself is to be discretized. Thus, in contrast to the implementation oriented setting, one should also cope with the problem of the unknown initial condition. Therefore in addition to the standing assumption (5) a detectability condition is also required. Then, instead of the original system (1), one has to discretize an associated observer, e.g.,

$$\dot{\hat{x}}(t) = A_o(\rho)\hat{x}(t) + B_u(\rho)u(t) + B_y(\rho)y(t) + L\nu(t), \quad (9)$$

where $A_o(\rho) = A_c(\rho) - B_y(\rho)C$ and $\hat{x}(0) = 0$, with a suitable observer gain $B_y(\rho)$. The introduction of the fictitious system (9) has the additional benefit, that one can tune the numerical conditioning of the original system – by modifying the "spectrum" of $A_c(\rho)$, i.e., it can be considered as a preconditioning step for the discretization.

Remark 2: For a general nonlinear system described by a qLPV model and for a given sampling time the standing assumption (5) might not be hold. In that case one can apply a general discretization scheme (e.g., a 4^{th} order Runge-Kutta), between two sampling times of (9) with a sufficiently high gain.

Remark 3: Due to geometrical reasons system (9) has the same invertibilty properties as the original system, i.e., the same relative degree and zero dynamics.

III. INVERSION BASED FAULT DETECTION

The aim of this section is to give an overview of the conditions and technical considerations related to left invertibility of general input affine nonlinear and, in particular, linear time varying systems. As a starting point, consider the continuous time nonlinear system, which is affine in the faults as

$$\dot{x} = f(x) + \sum_{i=1}^{m_f} g_i(x)\nu_i, \quad (y_j)_{j=1,p} = (h_j(x))_{j=1,p}. \quad (10)$$

Note that, for the sake of simplicity, the control input u is considered zero in the following discussion by letting that the results can be extended for the presence of known $u(t)$, easily. It is reasonable to assume that the rank of $g = [g_i]$ is m_f and the rank of $h = [h_j]^T$ is p.

The concept of relative degree plays a key role in the invertibility for both linear and nonlinear systems. Recall that a system is said to have a vector relative degree $r = \{r_1, \cdots, r_p\}$ at x_0 if

i. $L_{g_j} L_f^k h_i(x) = 0$ for $j = 1, \cdots, m$, $i = 1, \cdots, p$, and $k < r_i - 1$.

ii. matrix $A(x) = \left[\; L_{g_j} L_f^{r_i - 1} h_i(x) \; \right]_{i=1,p,\, j=1,m}$ has rank m at x_0,

where $L_\varphi \lambda(x) = \sum_{i=1}^n \frac{\partial \lambda}{\partial x_i} \varphi_i(x)$ denotes the derivative of λ along φ, $L_\varphi^k \lambda = L_\varphi(L_\varphi^{k-1}\lambda)$. If the system has a well defined relative degree, the unknown inputs can be expressed from

$$\begin{bmatrix} {y_1}^{(r_1)} \\ \vdots \\ {y_p}^{(r_m)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1^1} h_1(x) \\ \vdots \\ L_f^{r_m^1} h_p(x) \end{bmatrix} + A(x) \begin{bmatrix} \nu_1 \\ \vdots \\ \nu_{m_f} \end{bmatrix}, \quad (11)$$

for details see [25].

The maximal controlled invariant distribution in $\text{Ker}\, dh$ is $V^* = \text{Ker span}\{\, dL_f^k h_i,\ i = \overline{1,p},\ k = \overline{0, r_i - 1}\}$ provided that $rank A(x) = m_f$, [26]. Our interest in V^* is motivated by its role played in the question of invertibility and the construction of the reduced inverse of controlled systems: let $\xi = (\xi^i)_{i=1,p} = \Xi(x)$ be the diffeomorphism defined by $\xi^i = (L_f^k h_i(x))_{k=0,r_i-1}$ (ξ contains the corresponding output derivatives), that can be extended to the whole state space as $\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \Phi(x) := \begin{bmatrix} \Xi(x) \\ \Lambda(x) \end{bmatrix}$.

Then the output of the dynamic inverse is given by the expression $\nu(t) = A^{-1}(\xi, \eta)(y^{(r)} - L_f^r h(\xi, \eta))$ with the detector dynamics $\dot{\eta} = \partial_x \Lambda f|_{\Phi^{-1}} + \partial_x \Lambda g \alpha|_{\Phi^{-1}}$ (zero dynamics), where $\alpha(x)$ is the solution of $A(x)\alpha(x) = \left[\; L_f^{r_i} h_i(x) \; \right]_{i=1,p}$, provided that the invertibility condition $span\{\, g_i(x) \,|\, i = 1, m\} \cap V^* = 0$ holds. Observe that the inverse does not inherit the structure of the original system, i.e., it is not necessarily input affine. For details, see [27].

In the geometrical approach of fault detection for LTI systems, certain *unobservability subspaces* play an important role, see [28], [29]. Concerning invertibility, these subspaces are related not only to the invertibility conditions but also to the practical construction of the inversion based filter, [9], [11]. The notion of unobservability subspaces extends to the *unobservability (co)distribution* for the larger class of nonlinear systems, [25], [30]. Due to the computational complexity involved, these general nonlinear methods have limited applicability in practice.

In the past years, the various LTI invariant subspaces of geometric theory were extended to qLPV systems by introducing the notion of *parameter varying invariant* subspaces. With the introduction of parameter varying invariant subspaces an important goal was to formulate conditions that lead to the construction of computationally tractable algorithms for affine parameter dependent problems, see [31].

A. Parameter varying invariant subspaces

The classical invariant subspace concept, which is the cornerstone of the classical LTI geometric theory, can be extended to qLPV systems in the following way:

Definition 1: A subspace $\mathcal{V}$ is called parameter-varying invariant subspace for the family of the linear maps $A(\rho)$

(or shortly $\mathcal{A}$-invariant subspace) if

$$A(\rho)\mathcal{V} \subset \mathcal{V} \qquad \text{for all } \rho \in \mathcal{P}, \text{ i.e., for all } t \in \mathcal{I}. \qquad (12)$$

Definition 2: Let $\mathcal{B}(\rho)$ denote Im $B(\rho)$. Then a subspace $\mathcal{V}$ is called a parameter-varying (A,B)-invariant subspace (or shortly $(\mathcal{A}, \mathcal{B})$-invariant subspace) if for all $\rho \in \mathcal{P}$ either of the following equivalent conditions holds :

$$A(\rho)\mathcal{V} \subset \mathcal{V} + \mathcal{B}(\rho); \qquad (13)$$

there exists a mapping $F \circ \rho : [0, T] \to \mathbb{R}^{m \times n}$ such that:

$$(A(\rho) + B(\rho)F(\rho))\mathcal{V} \subset \mathcal{V}. \qquad (14)$$

Analogously, one can introduce the notion of parameter-varying (C, A)-invariant subspaces (or shortly $(\mathcal{C}, \mathcal{A})$-invariant subspaces). Let us denote the maximal $\mathcal{A}$-invariant subspace contained in the constant subspace $\mathcal{K}$ by $\langle \mathcal{K} | A(\rho) \rangle$. For the qLPV case one may consider the following definitions:

Definition 3: A subspace $\mathcal{R}$ is called parameter varying controllability subspace if there exists a constant matrix K and a parameter varying matrix $F : [0, T] \to \mathbb{R}^{m \times n}$ such that

$$\mathcal{R} = \langle \mathcal{A} + B\mathcal{F} | \text{Im } BK \rangle, \qquad (15)$$

where $\mathcal{A} + B\mathcal{F}$ denotes the system $A(\rho) + BF(\rho)$.

A subspace $\mathcal{S}$ is called an unobservability subspace associated to an LPV system if there exists a constant matrix H and a parameter varying matrix $G : \mathcal{P} \to \mathbb{R}^{n \times p}$ such that

$$\mathcal{S} = \langle \text{Ker } HC | A(\rho) + G(\rho)C \rangle. \qquad (16)$$

As in the classical case, the family of controllability subspaces contained in a given subspace $\mathcal{K}$ has a maximal element $\mathcal{R}^*$ while the family of unobservability subspaces associated to an LPV system containing a given subspace $\mathcal{L}$ has a minimal element denoted by $\mathcal{S}^*$.

If the parameter functions are differential algebraically independent, then the parameter invariant subspaces, defined above, coincide with the corresponding invariant distribution or codistribution, respectively. The parameter varying versions of these invariant spaces are suitable objects to define the required decompositions, therefore, they can play the same role as their time invariant counterparts. To give sufficient conditions for the solution of observer-based filter design problems, therefore, it is enough to require that some decompositions of the state equations can be made.

A key observation concerning the subject of this paper is that the invariant subspaces which correspond to the discretized qLPV system, which can be obtained through (8), coincide with their continuous counterparts. This follows from the definition of these subspaces, and from their computational algorithm. For details see [31].

B. Inversion in continuous-time

Let us consider the qLPV system

$$\dot{x}(t) = A(\rho(t))x(t) + L(\rho(t))\nu(t), \quad y(t) = Cx(t),$$

and let $\mathcal{V}^*$ be the maximal $(\mathcal{A}, \mathcal{L})$-invariant subspace contained in Ker$C$. The invertibility conditions can be formulated as $dim\,\text{Im}L = m$ and $\mathcal{V}^* \cap \text{Im}L = 0$. If these conditions are fulfilled, one can always choose a coordinate transform $z = Tx$ where

$$T = \begin{bmatrix} \mathcal{V}^{*\perp} \\ \Lambda \end{bmatrix}, \quad \Lambda \subset (\text{Im}L)^{\perp},$$

i.e., the system will be decomposed as:

$$\dot{\xi} = A_{11}(t)\xi + A_{12}(t)\eta + \bar{L}\nu, \quad y = C_1\xi \qquad (17)$$

$$\dot{\eta} = A_{21}(t)\xi + A_{22}(t)\eta. \qquad (18)$$

The component ξ can be expressed as $\tilde{y} = \mathcal{S}\xi$, where $\tilde{y} = \left[y_1, \cdots, y_1^{(r_1)}, \cdots, y_p, \cdots, y_p^{(r_p)}\right]^T$. The dynamic inverse is:

$$\dot{\eta} = A_{22}\eta + A_{21}\mathcal{S}^{-1}\tilde{y}, \qquad (19)$$

$$\nu = F\eta + \bar{L}^{-1}\mathcal{S}^{-1}(\dot{\tilde{y}} - (\dot{\mathcal{S}}\mathcal{S}^{-1} + \mathcal{S}A_{11}\mathcal{S}^{-1})\tilde{y}), \qquad (20)$$

where F is a suitable feedback that makes $\mathcal{V}^*$ $(\mathcal{A} + \mathcal{B}F, \mathcal{L})$ invariant. The rows of the coordinate transform $\mathcal{S}$ can be determined by using the recursion

$$S_i^0(t) = c_i, \; S_i^{k+1}(t) = \dot{S}_i^k(t) + S_i^k(t)A_{11}(t), \; k \leq r_i, \qquad (21)$$

For details, see [2]. If additional outputs are also available such that the system with 'eliminated' unknown inputs is observable, (i.e., if by the elimination of the inputs through the algebraic relation $A(x)\nu = B(x)$, it is possible to construct a stable observer), then one can construct an inverse (not reduced). In such situations derivatives of the output are still needed but the stability of the zero dynamics does not play any role. For this class of problems the available additional degree of freedom makes the handling of the robustness issues possible, [9], [32].

C. Inversion of the sampled qLPV system

In obtaining an inversion based filter for the sampled system the first solution could be a direct application of the general scheme outlined in the previous section to the discrete-time system obtained using (8) with a suitable choice of the approximation order N_T to a system of type (9). This procedure, however, has the big disadvantage that the relative degree decreases with the order of the approximation scheme, hence the size of the zero dynamics increases. Moreover, the obtained discrete time system will be non-minimum phase, in general, even if the continuous time system was minimum phase.

There is only one discretization scheme, that preserves the relative degree, i.e., the explicit Euler scheme that corresponds to the choice $N_T = 1$ in (8). The numerical properties (accuracy, stability) of the explicit Euler scheme, however, are not advantageous. This fact motivates the introduction of a mixed discretization strategy.

The starting point of the proposed discretization scheme is the observation that unknown input depends on a dynamic component – zero dynamics – and a static one, corresponding to the the feedback $\nu = F(\rho)\eta + v$, where the static part v is obtained from (17), i.e., from the system

$$\dot{\xi} = A_{11}(\rho)\xi + \bar{L}\nu, \quad y = C_1\xi. \qquad (22)$$

In order to obtain v, the discretization should maintain the relative degree of this system. Therefore, the Euler discretization will be applied only for (22), i.e., after discretization one has the system

$$\zeta_{k+1} = A_d^e(\rho_k)\zeta_k + L_d v_k, \quad y_k = \bar{C}\zeta_k. \tag{23}$$

where $A_d^e(\rho_k) = I + T_s A_c(\rho_k)$ with $L_d = T_s \bar{L}$ and $\bar{C} = C_1$.

By a straightforward computation one has that the discrete state ζ_k can be expressed as $\tilde{y}_d(k) = \mathcal{S}_d(\rho, k)\zeta_k$, where

$$\tilde{y}_d(k) = \big[y_1(k), \cdots, y_1(k+r_1), \cdots, y_p(k), \cdots, y_p(k+r_p)\big]^T.$$

The rows of the coordinate transform $\mathcal{S}_d$ can be determined by using the recursion

$$\bar{S}_i^0 = I, \quad \bar{S}_i^{k+1} = \bar{A}_d^e(\rho_{k+1})\bar{S}_i^k, \ S_{d,i}^k = c_i \bar{S}_i^k, \ k \le r_i. \tag{24}$$

Hence, the value of the discrete state can be computed as $\zeta_k = \mathcal{S}_d(\rho, k)^{-1}\tilde{y}_d(k)$ while $v_k = L_d^{\{-1\}}(\zeta_{k+1} - A_d^e(\rho_k)\zeta_k)$.

Observe that $\tilde{y}_d(k)$ and $\mathcal{S}_d(\rho, k)$ requires samples forward in time up till $\max r_i$, i.e., the implementation will contain a delay of $\max r_i + 1$. This discretization actually leads to the approximation of the derivatives, that are involved in the continuous inverse, by a finite difference method. Since the discretized dynamics itself is not used explicitely the accuracy of the scheme is not crucial in this step: $\mathcal{S}(\rho)$ is approximated by $\mathcal{S}_d(\rho, k)$, hence the underlying numerical scheme should guarantee an acceptable performance only in a reduced time window ($\max r_i + 1$ steps). This can be achieved by reasonable choices of the sampling time.

Having the approximative values $\xi(kT_s) = \zeta_k$, the dynamical part η of the reconstructed input can be computed based on a discretization of (18), i.e., discretization of the zero dynamics. The advantage of the method is that the discretization scheme for this task can be chosen arbitrary. Since the sampling time T_s, i.e., the discretization step, is given, this freedom can be used to ensure the stability of the scheme (stability of the discretized dynamics), see [24].

Remark 4: In the continuous time setting the application of an inversion based scheme is often prevented by the unavailability of all the necessary derivative signals. The proposed scheme provides a remedy for this problem when the unavailable derivative signals can be replaced by their finite difference approximations.

IV. Simulation Example

As an illustrative example for the proposed LPV inversion scheme let us consider the following simplified linearized parameter varying model of the lateral dynamics of a commercial aircraft :

$$\dot{x}(t) = A(\rho)x(t) + B(\rho)u(t) + L_a\nu_a(t) + L_r\nu_r(t)$$
$$y(t) = Cx(t),$$

where ν_e and ν_s represent failures in the aileron and rudder actuation. The state components are the roll angle ϕ, side speed V_y, roll rate p and yaw rate r.

The control inputs are the left and right aileron and rudder signals while the measured outputs are the side speed and yaw rate. The components of scheduling signal ρ are the altitude ρ_h and calibrated air speed ρ_v normalized in the interval $[-1, 1]$, i.e., $A(\rho) = A_0 + \rho_h A_h + \rho_v A_v$.

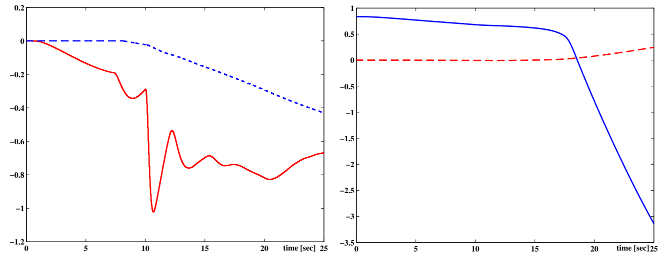

Fig. 1. Control inputs for ailerons and rudder (dashed) and the scheduling variables ρ_h (dashed) and ρ_v

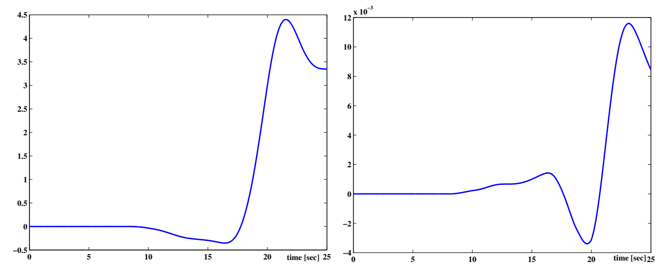

Fig. 2. Measured variables V_y and r

The control inputs, scheduling signals and the measured outputs are depicted on Figures 1 and 2, respectively.

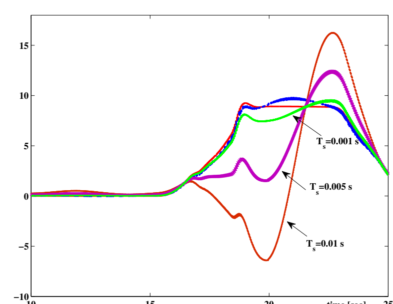

Fig. 3. Applied and reconstructed aileron faults

The zero dynamics is two dimensional and the designed continuous filter has the form

$$\dot{\eta} = A_i(\rho)\eta + B_{i,u}(\rho)u + B_{i,z}(\rho)y$$
$$\nu = C_i(\rho)\eta + D_{i,u}(\rho)u + D_{i,y}(\rho)y + D_{i,\dot{y}}(\rho)\dot{y}.$$

During the simulation the unmeasured derivatives were approximated by a finite difference scheme. For the discretized filter a second order scheme ($N_T = 2$) was used. The simulation results are depicted on Figure 3 and Figure 4. The red line is the fault signal the blue one is the reconstructed fault by using the continuous-time filter.

For a fixed order the accuracy of the discretized zero dynamics is affected by the sampling time. The simulation

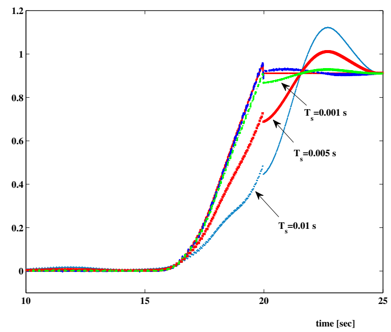

Fig. 4. Applied and reconstructed rudder faults

example demonstrates the role of accuracy in obtaining the zero dynamics component in the performance of the inversion based discretized detection filter.

V. CONCLUSIONS

The paper investigates the discretization of detection filter design for the detection and isolation of faults in qLPV systems by means of dynamic inversion where the system matrix depends affinely from the scheduling variables. The proposed method relies on the concept of parameter varying invariant subspaces and the results of classical geometrical system theory. The advantage of the proposed discretization method is that exploits the structure of the continuous time problem and maintains the stability properties of the zero dynamics of the continuous system.

REFERENCES

[1] S. Ganguli, A. Marcos, and G. Balas, "Reconfigurable LPV control design for Boeing 747-100/200 longitudinal axis," in *Proceedings of the 2002 American Control Conference*, vol. 5, 2002, pp. 3612–3617.

[2] Z. Szabo, J. Bokor, and G. Balas, "Inversion of LPV systems and its application to fault detection." in *Proceedings of the 5th IFAC Symposium on Fault Detection Supervision and Safety for Technical Processes, Washington, USA*, 2003, pp. 235 – 240.

[3] J. Bokor and G. Balas, "Detection filter design for LPV systems - a geometric approach," *Automatica*, vol. 40, pp. 511–518, 2004.

[4] I. Szaszi, A. Marcos, G. Balas, and J. Bokor, "LPV detection filter design for a Boeing 747-100/200 aircraft," *AIAA Journal of Guidance, Dynamics and Control*, vol. 28, no. 3, pp. 461–470, 2005.

[5] M. Rodrigues, D. Theilliol, S. Aberkane, and D. Sauter, "Fault tolerant control design for polytopic LPV systems," *Applied Mathematics and Computer Science*, vol. 17, no. 1, pp. 27–37, 2007.

[6] S. Armeni, A. Casavola, and E. Mosca, "Robust fault detection and isolation for LPV systems under a sensitivity constraint," *International Journal of Adaptive Control and Signal Processing*, vol. 23, no. 1, pp. 55–72, 2008.

[7] S. Grenaille, D. Henry, and A. Zolghadri, "A method for designing fault diagnosis filters for LPV polytopic systems," *Journal of Control Science and Engineering*, Article ID 231697, 2008.

[8] B. Kulcsár, J. Bokor, and J. Shinar, "Unknown input reconstruction for LPV systems," *International Journal of Robust and Nonlinear Control*, 2009.

[9] A. Edelmayer, J. Bokor, Z. Szabo, and F. Szigeti, "Input reconstruction by means of system inversion: a geometric approach to fault detection and isolation in nonlinear systems," *International Journal of Applied Mathematics and Computer Science*, vol. 14, no. 2, pp. 189–199, 2004.

[10] A. Edelmayer, J. Bokor, and Z. Szabo, "Robust detection and estimation of faults by exact fault decoupling and H^∞ disturbance attenuation in linear dynamical systems." in *Proceedings of the American Control Conference, Minneapolis, MN, USA*, 2006, pp. 5716–5721.

[11] A. Edelmayer, J. Bokor, and Z. Szabo, "Inversion-based residual generation for robust detection and isolation of faults by means of estimation of the inverse dynamics in linear dynamical systems," *International Journal of Control*, vol. 82, no. 8, pp. 1526–1538, 2009.

[12] J. Gertler, "Fault detection and isolation using parity relations," *Control Engineering Practice*, vol. 5, no. 5, pp. 653–661, 1997.

[13] J. Gertler, "Designing dynamic consistency relations for fault detection and isolation," *International Journal of Control*, vol. 73, no. 8, pp. 720–732, 2000.

[14] G. Conte, C. Moog, and A. M. Perdon, *Algebraic Methods for Nonlinear Control Systems*. Springer Verlag, 2006.

[15] P. Apkarian, "On the discretization of LMI-synthesized linear parameter-varying controllers," *Automatica*, vol. 33, no. 4, pp. 655–661, 1997.

[16] R. Tóth, F. Felici, P. Heuberger, and P. V. den Hof, "Crucial aspects of zero-order hold LPV state-space system discretization," in *Proceedings of the 17th World Congress The International Federation of Automatic Control Seoul, Korea*, 2008, pp. 4952–4957.

[17] R. Toth, M. Lovera, P. Heuberger, and P. V. den Hof, "Discretization of linear fractional representations of LPV systems," in *Proceedings of the 48th IEEE Conferenceon Decision and Control, Shanghai, China*, 2009, pp. 7424 – 7429.

[18] J. Butcher, *The Numerical Analysis of Ordinary Differential Equations.* Wiley, New York., 1987.

[19] E. Hairer, S. Norsett, and G. Wanner, *Solving Ordinary Differential Equations I. Nonstiff Problems.*, 2nd ed. Springer, 1993, vol. 8.

[20] E. Hairer, C. Lubich, and G. Wanner, *Geometric Numerical Integration Structure-Preserving Algorithms for Ordinary Differential Equations.*, Springer-Verlag., 2002, vol. 31.

[21] S. Blanes, F. Casas, and J. Ros, "High order optimized geometric integrators for linear differential equations," *BIT Numerical Mathematics*, vol. 42, no. 2, pp. 262–284, 2002.

[22] S. Blanes, F. Casas, J. Oteo, and J. Ros, "The Magnus expansion and some of its applications," *Physics Reports*, vol. 470, no. 5-6, pp. 151–238, 2009.

[23] J. P. Barbot, S. Monaco, and D. Normand-Cyrot, "A sampled normal form for feedback linearization," *Mathematics of Control, Signals, and Systems (MCSS)*, vol. 9, no. 2, pp. 162–188, 1996.

[24] N. Kazantzis and C. Kravaris, "System-theoretic properties of sampled-data representations of nonlinear systems obtained via taylor-lie series," *International Journal of Control*, vol. 67, no. 6, pp. 997–1020, 1997.

[25] A. Isidori, *Nonlinear Control Systems*. Springer–Verlag, 1989.

[26] H. Nijmeijer, "Invertibility of affine nonlinear control systems: a geometric approach," *Systems and Control Letters*, vol. 2, pp. 163–168, 1991.

[27] A. Edelmayer, J. Bokor, and Z. Szabo, "A geometric view on inversion-based detection filter design in nonlinear systems," in *Proceedings of the 5th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes, Washington, USA*, pp. 783–788.

[28] W. M. Wonham, *Linear Multivariable Control – A Geometric Approach*. Springer-Verlag, New York (Third Edition), 1985.

[29] G. Basile and G. Marro, "A new characterization of some structural properties of linear systems: unknown-input observability, invertibility and functional controllability," *International Journal on Control*, vol. 17, pp. 931–943, 1989.

[30] C. De Persis and A. Isidori, "On the observability codistributions of a nonlinear system," *Systems and Control Letters*, vol. 40, no. 5, pp. 297–304, 2000.

[31] G. Balas, J. Bokor, and Z. Szabo, "Invariant subspaces for LPV systems and their applications." *IEEE Transactions on Automatic Control*, vol. 48, no. 11, pp. 2065–2069, 2003.

[32] A. Edelmayer, J. Bokor, and Z. Szabo, "Exact fault and disturbance decoupling by means of direct input reconstruction and estimation of the inverse dynamics," in *Proceedings of the Mediterranean Conference on Control and Automation (MED'07), Athens, Greece*, 2007, pp. 719–723.