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A Formation Flying Algorithm for Autonomous Underwater Vehicles

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Abstract— **A control algorithm for the problem of formation flying for autonomous underwater vehicles is presented. Using a restrictive communication scheme and LQ optimal control, the control algorithm respects the limitations of underwater communication and on-board computing power. The influence of restricting communications on the performance of the control algorithm is illustrated by a simulation.**

I. Introduction

This paper describes a case study for coordination control, involving several autonomous underwater vehicles (AUVs): One AUV or surface vehicle should track an external reference signal, and two AUVs should follow the first vehicle in formation. This case study is strongly related to the problem statement of formation flying for AUVs formulated in [7]. The similar problem of coordinated path following control for AUVs is discussed in e.g. [2], and other approaches to formation flying using leader-follower structures are found in e.g. [1], [5].

The purpose of this case study is, on the one hand, to illustrate the theory of coordination control developed in [6], [3], [4], and on the other hand, to provide a computationally efficient control algorithm for the problem of formation flying for AUVs.

The control problem considered in this paper consists of three tracking problems, coupled by the formation to be kept, and subject to fixed bounds on the speed and acceleration of each vehicle, random waves and currents, and errors and delays in the communication among the vehicles.

Our approach adopts the linearized version of the model from [7]. In [7], a more general version of this problem is formulated, and solved using moving-horizon model predictive control on a linearized version of the model. While this approach leads to very good control laws, the online computations necessary for implementing these control laws exceed the on-board processing power of the AUVs considered in this setting.

The novelty of our approach lies in restricting the communication among the AUVs to a minimum by imposing a hierarchical structure on the set of vehicles, and then using LQ optimal control to solve each tracking problem separately. The navigation and communication constraints are taken into account after finding the optimal control laws. This leads to a control law which can be implemented with very little computational effort.

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In a simulation study, we compare the performance of our control law to the centralized case, in which the communication among the vehicles is not restricted. Our approach leads to a slightly higher total cost for the overall tracking problem, while decreasing the total amount of information to be communicated considerably. Moreover, our approach is easily extendable to larger groups of AUVs because the total amount of information communicated among the vehicles increases linearly with the number of vehicles, while this increase is exponential in the centralized approach.

II. Description of the Setting

The setting considered here concerns three vehicles, two of which are AUVs, and one may be either an AUV or a surface vehicle. The main goal is to have one vehicle track an externally given reference signal, while the other two vehicles (the AUVs) follow this vehicle in a fixed formation. This setting is illustrated in Figure 1.

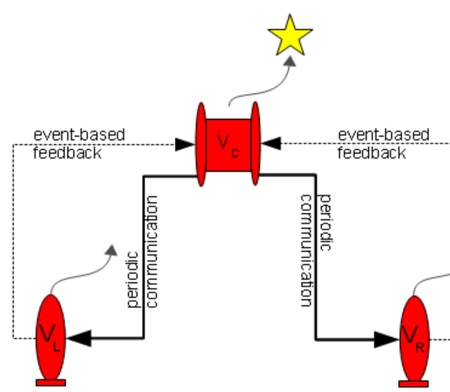

Fig. 1. Setting

The external reference signal may belong to a fourth vehicle, or be the solution of another control problem, e.g. a search mission. In the setting considered here, the vehicle following this signal can observe the current reference position at all times.

The vehicle following the external reference signal will be called the coordinating vehicle (V_C). V_C regularly sends its position to the other two vehicles (V_1 and V_2). These vehicles use this information to follow V_C - this is modeled as a tracking problem for each follower vehicle, with as reference signal the trajectory of V_C, shifted in space by a fixed amount.

All vehicles are subject to currents and disturbances, and their velocities and accelerations are bounded in norm. Because of these restrictions, it may not always be possible for the vehicles to successfully track their reference trajectories.

This leads to two possible problems: The follower vehicles V_1 and V_2 might fail to stay in formation (in the worst case, they might get lost), and the vehicles might collide. These two problems necessitate some form of communication from V_1 and V_2 to V_C in the case that the control objectives cannot be met. Since underwater communication is extremely limited, we opt for a form of event-based communication from V_1 and V_2 to V_C in exceptional circumstances: At each time step, each follower vehicle checks whether its distance to its reference position exceeds a fixed limit. This can be done internally and without additional communication since their reference position is communicated by V_C anyway. In the event that a vehicle exceeds the limit, it sends its actual position to the coordinating vehicle V_C, which then takes measures to avoid collisions, or one vehicle being left behind.

Underwater communication is modeled as being subject to random delays and packet losses. All messages sent are time-stamped, which means that at the time a message is received, the recipient knows when the message was sent. The corresponding observer can then recompute its current estimate, starting from the time given in the time stamp. The clock drift among the different vehicles is bounded for missions of limited duration, and will be ignored here.

In the setting described here, the coordinating vehicle V_C has to communicate its position regularly, while the other vehicles V_1 and V_2 do not. This means that V_C needs to use much more of its resources for communication. One possible option for ensuring that the resources of all vehicles are used in a more balanced way is to switch roles among the vehicles from time to time. In the case that V_C is a different type of vehicle than V_1 and V_2 (e.g. V_C is a ship, or an underwater vehicle with more energy available), this imbalance in the communication requirements is actually desirable.

For the purpose of comparing performances, a second setting will be considered, in which all vehicles can communicate with one another at all times. However, the communication is subject to the same delays and packet losses as described above.

III. Model with Communication Constraints

For ease of implementation, all dynamics involved will be approximated by discrete-time linear dynamical systems, as derived in [7]. To justify this choice, we note that a linearizing feedback is commonly applied to the AUVs by a lower-level controller. The approximation errors are modeled as disturbances, together with possible currents and other external disturbances. All disturbances are modeled as being zero-mean disturbances in the long run.

The following notation will be used:

- V_C: coordinating vehicle
- $V_1,\ V_2$: vehicles following V_C
- R_C: external reference signal to be tracked by V_C
- $R_1,\ R_2$: reference signals to be tracked by V_1 and V_2
- $p \in \mathbb{R}^3$: position
- $s \in \mathbb{R}^3$: velocity
- $a \in \mathbb{R}^3$: acceleration
- $w \in \mathbb{R}^3$: disturbances
- $\hat{p},\ \hat{s} \in \mathbb{R}^3$: observer estimates for position and velocity
- $\Delta_1, \Delta_2 \in \mathbb{R}^3$: desired relative positions of V_1 and V_2 with respect to the position of V_C
- $\tau \in \mathbb{R}$: a time constant

For each vehicle, the acceleration is the control input. The disturbances are modeled as velocities, and affect only the change in position, not the change in velocity.

These variables and their interconnections, in the case with communication constraints, are illustrated in Figure 2.

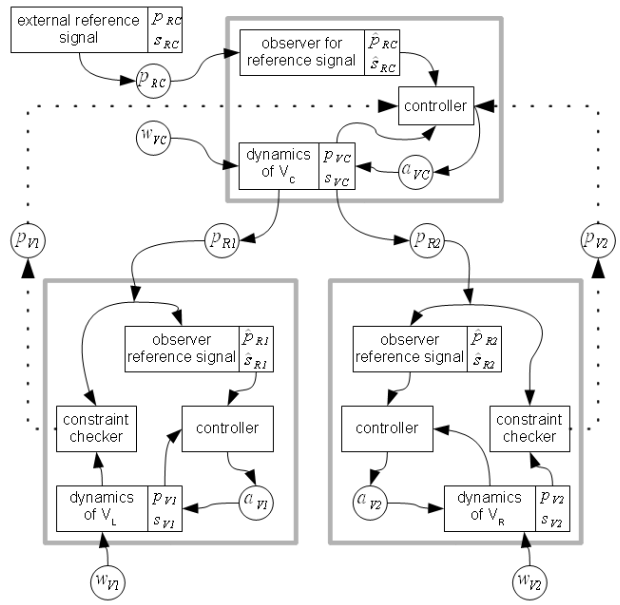

Fig. 2. Modeling scheme

For the external reference system R_C, we use an internal model with the following dynamics:

$$\begin{bmatrix} p_{R_C} \\ s_{R_C} \end{bmatrix}(t+1) = \begin{bmatrix} I & I \\ 0 & \frac{\tau-1}{\tau} I \end{bmatrix} \begin{bmatrix} p_{R_C} \\ s_{R_C} \end{bmatrix}(t) + \begin{bmatrix} 0 \\ \frac{1}{\tau} I \end{bmatrix} a_{R_C}(t) + \begin{bmatrix} I \\ 0 \end{bmatrix} w_{R_C}(t).$$

The state variables of this internal model are the position p_{R_C} and velocity s_{R_C} of R_C, and the acceleration a_{R_C} is the control input. The disturbance w_{R_C} is an uncontrollable input; including w_{R_C} in the dynamics of the reference system is realistic if R_C is an actual vehicle or target to be tracked, it does not make sense if R_C is a virtual system (e.g. the solution of a control problem).

All vehicles V_1, V_2 and V_C have the following dynamics, derived in [7]:

$$\begin{bmatrix} p_{V_j} \\ s_{V_j} \end{bmatrix}(t+1) = \begin{bmatrix} I & I \\ 0 & \frac{\tau-1}{\tau} I \end{bmatrix} \begin{bmatrix} p_{V_j} \\ s_{V_j} \end{bmatrix}(t) + \begin{bmatrix} 0 \\ \frac{1}{\tau} I \end{bmatrix} a_{V_j}(t) + \begin{bmatrix} I \\ 0 \end{bmatrix} w_{V_j}(t),$$

with $j = 1, 2, C$. Again, the state consists of the position and velocity of the vehicle (thus the state space is $\mathbb{R}^6$), the acceleration is the control input, and the disturbance is the uncontrollable input.

At each time step, V_C observes the current position $p_{R_C}(t) = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} p_{R_C} \\ s_{R_C} \end{bmatrix}(t)$ of the external reference signal. The reference trajectories R_1 and R_2 are related to the position of the coordinating vehicle V_C as follows:

$$p_{R_1}(t) = p_{V_C}(t) + \Delta_1, \quad p_{R_2}(t) = p_{V_C}(t) + \Delta_2.$$

The observer dynamics for all three observers are

$$\begin{bmatrix}\hat{p}_{R_j}\\ \hat{s}_{R_j}\end{bmatrix}(t+1)=\begin{bmatrix}I-G^p_{R_j} & I\\ -G^s_{R_j} & \frac{\tau-1}{\tau}I\end{bmatrix}\begin{bmatrix}\hat{p}_{R_j}\\ \hat{s}_{R_j}\end{bmatrix}(t)+\begin{bmatrix}G^p_{R_j}\\ G^s_{R_j}\end{bmatrix}p_{R_j}(t),$$

$j=1,2,C$, with p_{R_j} denoting the observations of the actual reference positions. The error dynamics are

$$\begin{bmatrix}p^{err}_{R_j}\\ s^{err}_{R_j}\end{bmatrix}(t+1)=\begin{bmatrix}I-G^p_{R_j} & I\\ -G^s_{R_j} & \frac{\tau-1}{\tau}I\end{bmatrix}\begin{bmatrix}p^{err}_{R_j}\\ s^{err}_{R_j}\end{bmatrix}(t)+\begin{bmatrix}0\\ \frac{1}{\tau}I\end{bmatrix}a_{R_j}(t)+\begin{bmatrix}I\\ 0\end{bmatrix}w_{R_j}(t),$$

where a_{R_C} and w_{R_C} are the acceleration and disturbance of the external reference signal, and $a_{R_1}=a_{R_2}=a_{V_C}$ and $w_{R_1}=w_{R_2}=w_{V_C}$ because the reference systems of the follower vehicles have the same dynamics as the coordinating vehicle. a_{R_1}, a_{R_2}, w_{R_1} and w_{R_2} only play a role in the observer errors; they are not used on-board by V_1 and V_2. $G^p_{R_j}$ and $G^s_{R_j}$ are appropriate observer gains.

Combining these variables and dynamics, we arrive at the open-loop system given in Table I. This is an affine system because the last term, involving Δ_1 and Δ_2, is constant.

The state variables p_{V_1}, s_{V_1}, $\hat{p}_{R_1}$, $\hat{s}_{V_1}$ belong to vehicle V_1, the variables p_{V_2}, s_{V_2}, $\hat{p}_{R_2}$, $\hat{s}_{R_2}$ are the state variables of vehicle V_2, and the state variables p_{V_C}, s_{V_C}, $\hat{p}_{V_C}$, $\hat{s}_{V_C}$ belong to the coordinating vehicle V_C. For each vehicle, the state space dimension is 12, and the state space of the overall system has dimension 36.

The internal model used for the external reference signal is not included in this open-loop system because the state variables of the external reference signal are not located in either of the vehicles. The accelerations a_{V_1}, a_{V_2}, a_{V_C} are the control inputs, the variables w_{V_1}, w_{V_2}, w_{V_C}, p_{R_C} are the external inputs, and Δ_1, Δ_2 are fixed parameters.

The open-loop system in Table I is a coordinated affine system, see [6], [3] and [4]. The coordinating vehicle corresponds to the coordinator of a coordinated system, and the follower vehicles correspond to the subsystems. Coordinated systems have the property that the coordinator influences the subsystems, while the subsystems have no influence on the coordinator, or on each other. In this case study, this corresponds to the coordinating vehicle sending its position to the other vehicles regularly. The event-based feedback from the other vehicles to the coordinating vehicle does not comply with the structure of a coordinated system, and hence the closed-loop system will only correspond to a coordinated system during the time intervals between two occurrences of this event-based feedback.

IV. CONTROL

In the formulation of the control problem, we have to consider the following control objectives:

- For each vehicle we have a tracking problem: for $j=1,2,C$, vehicle V_j should track its reference signal R_j.
- The vehicles should never collide.

Possible solutions of the control problem are constrained by the fact that in practice, the velocities and accelerations of all vehicles are bounded in norm. The positions of the vehicles may also be constrained, e.g. by obstacles or if they should stay within a certain region. This will not be taken into account here.

The combined consideration of both control objectives and the constraint leads to a very difficult control problem. Finding an optimal control law (if one exists) would involve on-line computations of a complexity that is not feasible for the type of vehicles considered here (see [7]). Hence, our approach is to treat the objectives and constraint one-by-one; this does not lead to an optimal control law, but to an admissible control law that performs well, and that can be implemented with limited on-board computing power.

In the following, we start by solving the tracking problems for the vehicles, first for the setting with communication constraints, and then for the setting without communication constraints. We then augment the optimal control law found for the tracking problem in such a way that the bounds on the speed and acceleration are achieved. Finally we consider the problems of stability and collision: In the case with communication constraints, we have to utilize the event-based feedback from V_1 and V_2 to V_C in order to avoid collisions.

A. The tracking problem, with communication constraints

First we only look at the tracking problem, ignoring the collision problem and bounds. Each vehicle V_j tries to track its observed reference position, while avoiding excessive control efforts. The tracking problem for each vehicle V_j can be formulated as an LQ optimal control problem (see e.g. [8]):

$$\min_{a_{V_j}}\sum_{t=t_0}^{\infty}\left\|p_{V_j}(t)-\hat{p}_{R_j}(t)\right\|^2+\alpha\left\|a_{V_j}(t)\right\|^2,\quad j=1,2,C.$$

Here, $\alpha\in\mathbb{R}$ is a parameter weighing the cost of acceleration against the cost of deviating from the reference trajectory.

The infinite-horizon formulation is chosen for simplicity, and all disturbances are ignored for now, since otherwise and without discounting, the cost would be infinite.

The difference vector $\begin{bmatrix}p_{V_j}-\hat{p}_{R_j}\\ s_{V_j}-\hat{s}_{R_j}\end{bmatrix}$ has dynamics

$$\begin{bmatrix}p_{V_j}-\hat{p}_{R_j}\\ s_{V_j}-\hat{s}_{R_j}\end{bmatrix}(t+1)=\begin{bmatrix}I & I\\ 0 & \frac{\tau-1}{\tau}I\end{bmatrix}\begin{bmatrix}p_{V_j}-\hat{p}_{R_j}\\ s_{V_j}-\hat{s}_{R_j}\end{bmatrix}(t)+\begin{bmatrix}0\\ \frac{1}{\tau}I\end{bmatrix}a_{V_j}(t)+\begin{bmatrix}I\\ 0\end{bmatrix}w_{V_j}(t)+\begin{bmatrix}G^p_{R_j}\\ G^s_{R_j}\end{bmatrix}(\hat{p}_{R_j}-p_{R_j})(t),$$

where p_{R_j} denote the observations of the actual reference position. For $\tau<1$, this system is controllable (see e.g. [8]).

Now the tracking problem for each vehicle can easily be solved off-line, leading to an optimal feedback

$$a_{V_j}(t)=\begin{bmatrix}F^p & F^s\end{bmatrix}\begin{bmatrix}p_{V_j}-\hat{p}_{R_j}\\ s_{V_j}-\hat{s}_{R_j}\end{bmatrix}.$$

The corresponding closed-loop system for each vehicle is then

$$\begin{bmatrix}p_{V_j}-\hat{p}_{R_j}\\ s_{V_j}-\hat{s}_{R_j}\end{bmatrix}(t+1)=\begin{bmatrix}I & I\\ \frac{1}{\tau}F^p & \frac{\tau-1}{\tau}I+\frac{1}{\tau}F^s\end{bmatrix}\begin{bmatrix}p_{V_j}-\hat{p}_{R_j}\\ s_{V_j}-\hat{s}_{R_j}\end{bmatrix}(t)+\begin{bmatrix}I\\ 0\end{bmatrix}w_{V_j}(t)+\begin{bmatrix}G^p_{R_j}\\ G^s_{R_j}\end{bmatrix}(\hat{p}_{R_j}-p_{R_j})(t).$$

TABLE I

THE OPEN-LOOP SYSTEM

$$
\left[\begin{array}{c} p_{V_1} \\ s_{V_1} \\ \hat{p}_{R_1} \\ \hat{s}_{R_1} \\ \hline p_{V_2} \\ s_{V_2} \\ \hat{p}_{R_2} \\ \hat{s}_{R_2} \\ \hline p_{V_C} \\ s_{V_C} \\ \hat{p}_{R_C} \\ \hat{s}_{R_C} \end{array}\right](t+1)=
\left[\begin{array}{cccc|cccc|cccc}
I & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\tau-1}{\tau}I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & I-G^p_{R_1} & I & 0 & 0 & 0 & 0 & G^p_{R_1} & 0 & 0 & 0 \\
0 & 0 & -G^s_{R_1} & \frac{\tau-1}{\tau}I & 0 & 0 & 0 & 0 & G^s_{R_1} & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & I & I & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\tau-1}{\tau}I & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & I-G^p_{R_2} & I & G^p_{R_2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -G^s_{R_2} & \frac{\tau-1}{\tau}I & G^s_{R_2} & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & I & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\tau-1}{\tau}I & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I-G^p_{R_C} & I \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -G^s_{R_C} & \frac{\tau-1}{\tau}I
\end{array}\right]
\left[\begin{array}{c} p_{V_1} \\ s_{V_1} \\ \hat{p}_{R_1} \\ \hat{s}_{R_1} \\ \hline p_{V_2} \\ s_{V_2} \\ \hat{p}_{R_2} \\ \hat{s}_{R_2} \\ \hline p_{V_C} \\ s_{V_C} \\ \hat{p}_{R_C} \\ \hat{s}_{R_C} \end{array}\right](t)
$$

$$
+\left[\begin{array}{c|c|c}
0 & 0 & 0 \\
\frac{1}{\tau}I & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\hline
0 & 0 & 0 \\
0 & \frac{1}{\tau}I & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\hline
0 & 0 & 0 \\
0 & 0 & \frac{1}{\tau}I \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right]
\left[\begin{array}{c} a_{V_1} \\ \hline a_{V_2} \\ \hline a_{V_C} \end{array}\right](t)+
\left[\begin{array}{c|c|c|c}
I & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\hline
0 & I & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\hline
0 & 0 & I & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & G^p_{R_C} \\
0 & 0 & 0 & G^s_{R_C}
\end{array}\right]
\left[\begin{array}{c} w_{V_1} \\ \hline w_{V_2} \\ \hline w_{V_C} \\ p_{R_C} \end{array}\right](t)+
\left[\begin{array}{c|c}
0 & 0 \\
0 & 0 \\
G^p_{R_1} & 0 \\
G^s_{R_1} & 0 \\
\hline
0 & 0 \\
0 & 0 \\
0 & G^p_{R_2} \\
0 & G^s_{R_2} \\
\hline
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{array}\right]
\left[\begin{array}{c} \Delta_1 \\ \hline \Delta_2 \end{array}\right]. \tag{1}
$$

For the treatment of the constraints in Section IV.C, we need to rewrite the closed-loop system in terms of the original state variables:

$$
\begin{bmatrix} p_{V_j} \\ s_{V_j} \\ \hat{p}_{R_j} \\ \hat{s}_{R_j} \end{bmatrix}(t+1) =
\begin{bmatrix}
I & I & 0 & 0 \\
\frac{1}{\tau}F^p & \frac{\tau-1}{\tau}I+\frac{1}{\tau}F^s & -\frac{1}{\tau}F^p & -\frac{1}{\tau}F^s \\
0 & 0 & I-G^p_{R_j} & I \\
0 & 0 & -G^s_{R_j} & \frac{\tau-1}{\tau}I
\end{bmatrix}
\begin{bmatrix} p_{V_j} \\ s_{V_j} \\ \hat{p}_{R_j} \\ \hat{s}_{R_j} \end{bmatrix}(t)
+ \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix} w_{V_j}(t) + \begin{bmatrix} 0 \\ 0 \\ G^p_{R_j} \\ G^s_{V_j} \end{bmatrix} p_{R_j}(t).
$$

The matrices characterizing the tracking problem are the same for all vehicles, and hence the feedback matrices F^p and F^s are also the same for all vehicles.

Since the reference trajectories p_{R_1} and p_{R_2} depend on the closed-loop dynamics of V_C, and observer estimates of these reference trajectories influence the control problems for V_1 and V_2, solving the tracking problem for each vehicle independently does not lead to a centralized optimum: The sum of the tracking costs for all vehicles can be decreased further by solving the combined optimization problem for all vehicles at once. However, for implementing the centralized optimum, the current states of all vehicles would need to be communicated. This alternative is used for testing the performance of our approach, and is described in the following subsection.

B. The tracking problem, without communication constraints

In this subsection, the same open-loop system for the motion of the vehicles is used. All communications are subject to the same uncertainties as in the setting with communication constraints. However, in this setting we do not impose any constraints on communication among the vehicles.

In this setting, each vehicle has observers for the states of all other vehicles, so in other words each vehicle keeps a copy of the whole system in memory, with exact values for its own state, and observers for the states of the other vehicles. The control feedback for the tracking problem is the same for all vehicles: They all solve the combined tracking problem

$$
\min_{a_{V_1},a_{V_2},a_{V_C}} \sum_{t=t_0}^{\infty} \left\| \begin{bmatrix} p_{V_1}(t)-p_{V_C}(t)-\Delta_1 \\ p_{V_2}(t)-p_{V_C}(t)-\Delta_2 \\ p_{V_C}(t)-\hat{p}_{R_C}(t) \end{bmatrix} \right\|^2 + \alpha \left\| \begin{bmatrix} a_{V_1}(t) \\ a_{V_2}(t) \\ a_{V_C}(t) \end{bmatrix} \right\|^2 .
$$

The solution of this LQ-problem is

$$
\begin{bmatrix} a_{V_1} \\ a_{V_2} \\ a_{V_C} \end{bmatrix} =
\begin{bmatrix}
F_{11} & F_{12} & F_{13} & F_{14} & F_{15} & F_{16} \\
F_{21} & F_{22} & F_{23} & F_{24} & F_{25} & F_{26} \\
F_{31} & F_{32} & F_{33} & F_{34} & F_{35} & F_{36}
\end{bmatrix}
\begin{bmatrix} p_{V_1}-p_{V_C}-\Delta_1 \\ s_{V_1}-s_{V_C} \\ p_{V_2}-p_{V_C}-\Delta_2 \\ s_{V_2}-s_{V_C} \\ p_{V_C}-\hat{p}_{R_C} \\ s_{V_C}-\hat{s}_{R_C} \end{bmatrix},
$$

where $F_{11},\ldots,F_{36} \in \mathbb{R}^{3\times 3}$ can be found off-line.

Each vehicle has its own copy of the overall closed-loop system, with observer estimates for the states of the other vehicles.

C. The bounds on velocity and acceleration

Since the norm of the acceleration for each vehicle is penalized in the cost function of the tracking problem, the accelerations found from the state feedbacks for the two settings above will usually be small in norm. However, this does not guarantee that they stay within fixed bounds. Moreover, the velocities of the vehicles are not bounded as a result of the state feedbacks found above, and we might need a fixed bound on the speed of each vehicle for a realistic model of the settings.

With upper bounds $a_{\max} \in \mathbb{R}$ and $s_{\max} \in \mathbb{R}$ on the acceleration and speed of each vehicle, we define $\lambda_s \in \mathbb{R}$ by

$$\lambda_s = \frac{1}{\|a_{V_j}\|^2} \Big((1-\tau) a_{V_j}^T s_{V_j} + \\ + \sqrt{(\tau-1)^2 a_{V_j}^T (s_{V_j} a_{V_j}^T - a_{V_j} s_{V_j}^T) s_{V_j} + \tau^2 s_{\max}^2 \|a_{V_j}\|^2} \Big).$$

This variable is used for limiting the speed of the vehicle, and is derived from requiring that the velocity $\bar{s}_{V_j}(t+1)$ obtained by applying the corrected input $\lambda_s a_{V_j}(t)$ satisfies

$$\|\bar{s}_{V_j}(t+1)\|^2 = \|\frac{\tau-1}{\tau} s_{V_j}(t) + \frac{1}{\tau}\ \lambda_s a_{V_j}(t)\|^2 = s_{\max}^2.$$

A simple (but not necessarily optimal) way of implementing a fixed upper bound on the acceleration and speed of each vehicle is to use the following control input:

$$\bar{a}_{V_j}(t) = \min\left\{\lambda_s, \frac{a_{\max}}{\|a_{V_j}(t)\|}, 1\right\} * a_{V_j}(t),$$

where $a_{V_j}(t)$ is the optimal control feedback found in the previous two subsections, depending on the setting. This control law satisfies the bounds $\|\bar{a}_{V_j}(t)\| \leq a_{\max}$ and $\|\bar{s}_{V_j}(t+1)\| \leq s_{\max}$.

D. Stability and the collision constraint

In practice the speed and the acceleration of an AUV are bounded. This means that, even though both of the closed-loop systems derived in the previous subsections are output stable with respect to the output

$$y(t) = \begin{bmatrix} p_{V_1}(t) - p_{V_C}(t) - \Delta_1 \\ p_{V_2}(t) - p_{V_C}(t) - \Delta_2 \\ p_{V_C}(t) - \hat{p}_{R_C}(t) \end{bmatrix},$$

the closed-loop systems together with the constraints $\|a\| \leq a_{\max}$ and $\|s\| \leq s_{\max}$ might not be output stable.

This is interpreted as follows: If the external reference signal moves at a speed higher than $s_{\max}$ then V_C is not able to track the reference signal, and $p_{V_C} - \hat{p}_{R_C}$ increases. There is nothing that can be done about this. Another possibility is that the followers V_1 and V_2 cannot track their reference positions, because they are subjected to strong disturbances and cannot accelerate enough to compensate for that. This may lead to a follower being left behind, or a collision of two vehicles. This can be avoided if V_C is informed about the positions of V_1 and V_2, at least in the case that V_1 or V_2 are deviating too much from their reference positions. For this potential problem, we suggest three possible solutions:

- V_C receives feedback from V_1 and V_2 regularly, and includes these positions into its local tracking problem. The deviation from the formation will be small, but this involves more communication than necessary.
- V_C receives feedback from V_1 or V_2 only in the case that a follower vehicle is too far from its reference position, i.e. if $\|p_{V_j} - (\hat{p}_{V_C} + \Delta_j)\| \geq r$ for some fixed $r > 0$. In this way, the communication from V_1 and V_2 to V_C is kept minimal. If the safety regions of radius r around V_1 and V_2 are chosen far enough from each other then this approach avoids collision.
- We set the maximum speed of V_C well below the actual maximum speed of V_1 and V_2. The follower vehicles have a better chance at tracking their reference signal. No additional communication is necessary, however V_C cannot fly at its maximum speed, and hence might have more difficulties tracking the external reference signal.

In this paper we choose the second option: At each time instant, the follower vehicles check whether their position deviates from their observed reference position by more than r. If that is the case, they send their position p_{V_j} to V_C.

There are several possibilities for V_C to use this information in order to help the follower vehicle get back into formation. One option, which turned out to be successful in simulations, is to have the V_C track the signal

$$\hat{p}_{R_C} + \frac{(\hat{p}_{V_1} - \Delta_1 - p_{V_C})\mathbb{I}_1 + (\hat{p}_{V_2} - \Delta_2 - p_{V_C})\mathbb{I}_2}{W}$$

instead of the signal $\hat{p}_{R_C}$, where $\mathbb{I}_j = 1$ if V_C received p_{V_j} from V_j during this time step, and $\mathbb{I}_j = 0$ otherwise. The second term is a weighted average of the deviations of the vehicle positions from their reference positions, with weight parameter $W > 0$. This average deviation has to be computed by V_C. At most times, V_C does not know the positions of the follower vehicles because the follower vehicles are within a radius r of their reference positions. In this case, the tracking signal is $\hat{p}_{R_C}$.

The collision problem is automatically solved by our approach if the distance between the uncertainty regions $\mathbb{D}_r(p_{V_j})$ for the two follower vehicles is large enough - this can be made more precise by taking into account the maximum speed and acceleration.

E. The control algorithm

To summarize the control algorithm described in the previous subsections: For the case with communication constraints, we have

$$a_{V_j} = \begin{bmatrix} F^p & F^s \end{bmatrix} \begin{bmatrix} p_{V_j} - \hat{p}_{R_j} \\ s_{V_j} - \hat{s}_{R_j} \end{bmatrix},\ j = 1, 2,$$

$$a_{V_C} = \begin{bmatrix} F^p & F^s \end{bmatrix} \begin{bmatrix} p_{V_j} - \hat{p}_{R_j} - \frac{(\hat{p}_{V_1} - \Delta_1 - p_{V_C})\mathbb{I}_1 + (\hat{p}_{V_2} - \Delta_2 - p_{V_C})\mathbb{I}_2}{W} \\ s_{V_j} - \hat{s}_{R_j} \end{bmatrix}.$$

For the case without communication constraints, we found

$$\begin{bmatrix} a_{V_1} \\ a_{V_2} \\ a_{V_C} \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} & F_{15} & F_{16} \\ F_{21} & F_{22} & F_{23} & F_{24} & F_{25} & F_{26} \\ F_{31} & F_{32} & F_{33} & F_{34} & F_{35} & F_{36} \end{bmatrix} \begin{bmatrix} p_{V_1} - p_{V_C} - \Delta_1 \\ s_{V_1} - s_{V_C} \\ p_{V_2} - p_{V_C} - \Delta_2 \\ s_{V_2} - s_{V_C} \\ p_{V_C} - \hat{p}_{R_C} \\ s_{V_C} - \hat{s}_{R_C} \end{bmatrix},$$

with observer values where actual values are not available.

For both cases, the control feedback to be implemented is then given by

$$\bar{a}_{V_j}(t) = \min\left\{\lambda_s, \frac{a_{\max}}{\|a_{V_j}(t)\|}, 1\right\} * a_{V_j}(t), \quad j = 1, 2, C.$$

As discussed in the previous subsections, this control law meets the control objectives and satisfies the constraint. Since the feedback and observer gains can be computed offline, the computational burden on the AUVs is very low.

V. Simulation results

We test the performance of the control law and communication scheme described above using MATLAB simulations. Simulation 1 implements the control law with communication constraints, on the linearized version of the model and with noise. Simulation 2 implements the system without communication constraints.

A. Settings and parameters

Our simulations ran over 1000 time steps, each of length $1s$. For the external reference trajectory we chose a circular path, starting at a distance of $40m$ from the vehicles. For the vehicles, we used $s_{\max} = 3m/s$, $a_{\max} = 0.3m/s^2$ and $\tau = 5$. The disturbances were chosen to be Gaussian with mean 0 and $\sigma = 0.3$, and we used uncertainty radius $r = 7m$ around the follower vehicles. Messages were modeled to arrive with a probability of 0.9, and with an average delay of $2.4s$. The weights for the tracking problems were chosen to be $\alpha = 10$ and $W = 7$.

B. Performance and comparison

Both simulations show that the control objectives and constraints are met. Figure 3 illustrates this for Simulation 1: While the distances of the vehicles to their observed reference positions quickly drop below $10m$, the distances between the three vehicles stay between $20m$ and $40m$ at all times. Feedback from V_1 and V_2 occured at 110 time steps.

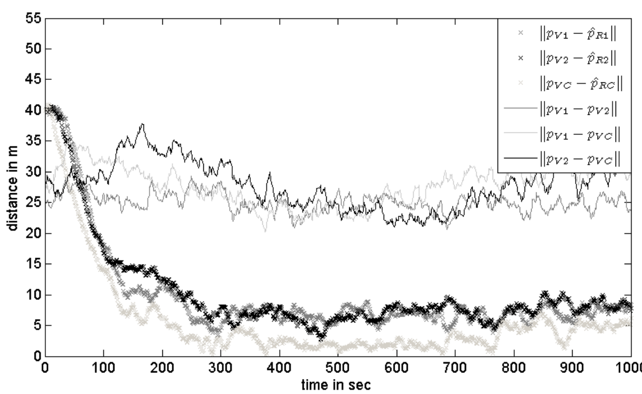

Fig. 3. Simulation results

For comparing performances, we evaluate the cost function from the tracking problems (note that the cost function is the same in both cases). Based on one representative run, we found that the total costs are:

- Simulation 1: $1.37 * 10^5$,
- Simulation 2: $1.23 * 10^5$.

This means that our control law with communication constraints leads to an increase by around 11%, compared to the control law with unconstrained communication.

We can take into account communication costs by specifying a fixed cost C_C per message broadcast by the coordinating vehicle V_C (a message is an element of $\mathbb{R}^3$ or $\mathbb{R}^2$), and a fixed cost C_F per message broadcast by one of the follower vehicles V_1 and V_2. Now the total communication costs are:

- Simulation 1: $1000C_C + 110C_F$,
- Simulation 2: $2000C_C + 2000C_F$.

The communication costs for Simulation 1 depend strongly on the disturbances, and on the radius r of the uncertainty regions.

VI. Concluding remarks

In this paper, we described a control algorithm and a communication scheme for the problem of formation flying for AUVs. This approach is implementable with low onboard computing power, and it requires very little communication among the vehicles. In a simulation, we compared the performances of this approach and a similar approach with unlimited communication. While the total cost increased slightly with our communication scheme, the total amount of communication decreased considerably.

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References

[1] R. Cui, S. S. Ge, B. V. E. How and Y. S. Choo. Leader-follower formation control of underactuated autonomous underwater vehicles. In *Ocean Engineering*, Vol. 37, nNo. 17-18, pp. 1491 1502, 2010.

[2] R. Ghabcheloo, A. Aguiar, A. Pascoal, C. Silvestre, I. Kaminer, J. Hespanha. Coordinated path-following in the presence of communication losses and time delays. *SIAM - Journal on Control and Optimization*, Vol. 48, No. 1, pp. 234-265, 2009.

[3] P.L. Kempker, A.C.M. Ran and J.H. van Schuppen. Construction of a Coordinator for Coordinated Linear Systems. In *Proc. European Control Conference (ECC.2009)*, Budapest, 2009.

[4] P.L. Kempker, A.C.M. Ran and J.H. van Schuppen. Controllability and observability of coordinated linear systems. Submitted.

[5] G. Lafferriere, A. Williams, J. Caughman and J. J. P. Veerman. Decentralized control of vehicle formations. In *Systems & Control Letters*, 54 (2005), pp. 899-910.

[6] A.C.M. Ran and J.H. van Schuppen. Control for Coordination of Linear Systems. In *Proc. International Symposium on the Mathematical Theory of Networks and Systems (MTNS.2008)*, Joe Ball, et al., Virginia Institute of Technology, Blacksburg, 2008.

[7] J. Sousa, F. Pereira et al. C4C project - Task WP2.3 Single vehicle operations. *C4C Deliverable*, FP7-ICT-2007.3.7, `www.c4c-project.eu/uploads/files/d-wp2-3.pdf`.

[8] H.L. Trentelman, A.A. Stoorvogel, and M.L.J. Hautus. Control Theory for Linear Systems. *Communications and Control Engineering*, Springer, London, 2001.