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Experimental Joint Stiffness Identification Depending on Measurements Availability

A. Janot, M. Gautier, A. Jubien and P.O. Vandanjon

Abstract— **This paper addresses the important topic of joint flexibility identification. Three dynamic models depending on measurements availability are compared. The parameters are estimated by using the ordinary least squares of an over linear system obtained from the sampling of the dynamic model along a closed loop tracking trajectory. An experimental setup exhibits the experimental identification results.**

I. INTRODUCTION

ACCURATE dynamic robots models are needed to control and simulate their motions. Identification of rigid robots has been widely investigated in the last decades. The usual identification process is based on the inverse dynamic model and the ordinary or weighted least squares estimation. This method has been performed on several prototypes and industrial robots with accurate results [1][2][3][4][5].

Identification of flexibilities is complex because only a subset of state variables is measured [6] and one can not use directly linear regressions [5]. This can be solved by adding sensors [7] and/or external excitations [8]. However, this solution is harder to apply because it can be quite expansive and the experiments can be quite involved with additional mounting sensors and signals to measure. In [9], the authors use the System Identification Toolbox for Matlab [10][11] to identify both joint and structural flexibilities of one axis of an industrial robot. Inertia and stiffness parameters are well identified, but friction repartition and data filtering are not addressed.

In [12] and [13], the authors have calculated some minimal identification models depending on the measurements availability, and they have developed a methodology to tune the bandpass filtering for joint stiffness identification. Experimental results are convincing, but the regroupings of friction parameters are not discussed and the results obtained with the different minimal models are not compared.

Tacking all these remarks into account, this paper deals with joint stiffness identification performed on an experimental setup. Three minimal identification models depending on the measurements availability are developed. The identification results are compared and the regroupings of friction parameters are presented and experimentally validated.

This paper is divided into five sections. Section II describes the experimental setup and its modeling. Section III presents three minimal identification models depending on the measurements availability. Section IV is devoted to the identification based on the inverse dynamic model. Finally, experimental results obtained with the experimental setup show the efficiency of the different methods.

II. MODELING OF A FLEXIBLE JOINT ROBOT

A. Experimental setup

The EMPS is a high-precision linear Electro-Mechanical Positioning System (see Fig. 1). It is a standard configuration of a drive system for prismatic joint of robots or machine tools. It is connected to a dSPACE digital control system for easy control and data acquisition using Matlab and Simulink software.

Fig. 1. EMPS prototype to be identified

Its main components are:

- A Maxon DC motor equipped with an incremental encoder. This DC motor is position controlled with a PD controller.
- A Star high-precision low-friction ball screw drive positioning unit. An incremental encoder at its extremity supplies information about the angular position of the screw.
- A load in translation.
- An accelerometer placed on the load supplies information about the load acceleration.

These components are presented Fig. 2.

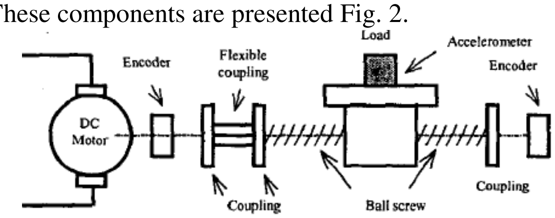

Fig. 2. EMPS Components

All variables and parameters are given in ISO units on the load side.

M. Gautier is with the Institut de Recherche en Communications et Cybernétique de Nantes (IRCCyN) and with the University of Nantes, 44321 Nantes France (phone: +33(0)240376960; fax: +33(0)240376930; e-mail: Maxime.Gautier@irccyn.ec-nantes.f r).

A. Janot is with the ONERA, DCSD, 31055, Toulouse, France, Alexandre.Janot@onera.fr, A. Jubien is with the IRCCyN, mail@anthonyjubien.com and PO. Vandanjon is with IFSTTAR, 44341, Bouguenais, France, pierre-olivier.vandanjon@ifsttar.fr .

B. Rigid inverse dynamic model

In this case, the system is modeled with one inertia and frictions. The inverse dynamic model (IDM) expressing the motor torque according to the state and its derivatives is:

$$\tau_1 = ZZ_{1R}\ddot{q}_1 + F_{v1R}\dot{q}_1 + F_{c1R}sign\left(\dot{q}_1\right) \quad (1)$$

Where, q_1, $\dot{q}_1$, $\ddot{q}_1$ are respectively the motor position, velocity and acceleration; τ_1 is the motor torque; ZZ_{1R} is the total inertia; F_{v1R} and F_{c1R} are the total viscous and Coulomb friction parameters.

C. The flexible inverse dynamic model

In this case, the mechanical system can be modeled with two inertias, a spring and a structural damping, Fig. 3.

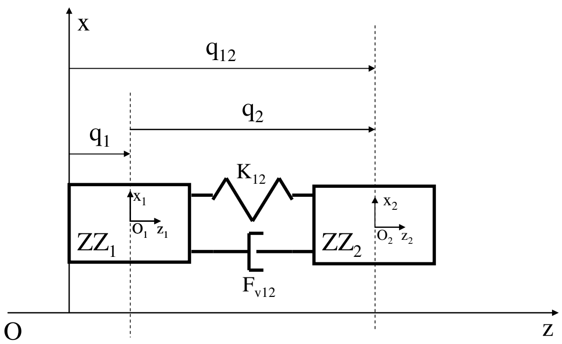

Fig. 3. EMPS modeling and DHM frames

With the Newton – Euler equations, we obtain the following IDM [5]:

$$\begin{aligned} \tau_1 &= ZZ_1\ddot{q}_1 + F_{v1}\dot{q}_1 + F_{c1}sign(\dot{q}_1) - K_{12}q_2 - F_{12}\dot{q}_2 \\ 0 &= ZZ_2\ddot{q}_{12} + F_{v2}\dot{q}_{12} + F_{c2}sign(\dot{q}_{12}) + K_{12}q_2 + F_{12}\dot{q}_2 \end{aligned} \quad (2)$$

Where: q_1, $\dot{q}_1$, $\ddot{q}_1$ are respectively the motor position, velocity and acceleration; τ_1 is the motor torque; q_{12}, $\dot{q}_{12}$, $\ddot{q}_{12}$ are respectively the load position, velocity and acceleration; q_2, $\dot{q}_2$, $\ddot{q}_2$ are respectively the elastic DOF position, velocity and acceleration with, $q_{12} = q_1 + q_2$, $\dot{q}_{12} = \dot{q}_1 + \dot{q}_2$ and $\ddot{q}_{12} = \ddot{q}_1 + \ddot{q}_2$; ZZ_1 is the motor inertia, F_{v1} and F_{c1} are respectively the viscous and Coulomb motor friction parameters; ZZ_2 is the load inertia, F_{v2} and F_{c2} are respectively the viscous and Coulomb load friction parameters; K_{12} is the stiffness and F_{12} the damping.

The dynamic model (2) can be written in a linear relation to the dynamic parameters as follows:

$$Y = D_{STD}\chi_{STD} \quad (3)$$

With: $Y = \left(\tau_1 \quad 0\right)^T$ $\chi_{STD} = \left(ZZ_1 \quad F_{v1} \quad F_{c1} \quad K_{12} \quad F_{12} \quad ZZ_2 \quad F_{v2} \quad F_{c2}\right)^T$

$$D_{STD} = \begin{pmatrix} \ddot{q}_1 & \dot{q}_1 & sign(\dot{q}_1) & -q_2 & -\dot{q}_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_2 & \dot{q}_2 & \ddot{q}_{12} & \dot{q}_{12} & sign(\dot{q}_{12}) \end{pmatrix}$$

There are 8 parameters to be identified called standard parameters. The flexible IDM can be written as follows:

$$\Gamma = M\left(q\right)\ddot{q} + N\left(q,\dot{q}\right) + Kq + B\dot{q} \quad (4)$$

With: $q = \begin{pmatrix} q_1 \\ q_{12} \end{pmatrix}$ $\dot{q} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_{12} \end{pmatrix}$ $\ddot{q} = \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_{12} \end{pmatrix}$ $\Gamma = \begin{pmatrix} \tau_1 \\ 0 \end{pmatrix}$ $M\left(q\right) = \begin{pmatrix} ZZ_1 & 0 \\ 0 & ZZ_2 \end{pmatrix}$

$N\left(q,\dot{q}\right) = \begin{pmatrix} F_{v1}\dot{q}_1 + F_{c1}sign\left(\dot{q}_1\right) \\ F_{v2}\dot{q}_{12} + F_{c2}sign\left(\dot{q}_{12}\right) \end{pmatrix}$ $K = \begin{pmatrix} K_{12} & -K_{12} \\ -K_{12} & K_{12} \end{pmatrix}$ $B = \begin{pmatrix} F_{v12} & -F_{v12} \\ -F_{v12} & F_{v12} \end{pmatrix}$

The direct dynamic model (DDM) is then described by:

$$M\left(q\right)\ddot{q} = \Gamma - N\left(q,\dot{q}\right) - Kq - B\dot{q} \quad (5)$$

Now, we present the minimal identification models depending on the available measurements.

III. Different minimal identification models depending on the available measurements

A. Identification model using motor and load positions

This is the idealistic case. The minimal model corresponds to the standard model given by (3). So, we have:

$$D_1 = D_{STD}\text{ , } \chi_1 = \chi_{STD} \text{ and } y_1 = \left(\tau_1 \quad 0\right)^T \quad (6)$$

B. Identification model with load acceleration and motor position

This is a realistic case in industrial applications because accelerometers are often used to identify flexibilities. Since the load acceleration is measured, we could integrate this signal twice to get the load position. But, difficulties arise with the estimation of initial conditions. An efficient way consists in using the derivative of the motor torque. $\dot{q}_{12}$ being not accessible, F_{c2} is regrouped with F_{c1}.

The non linear function *sign* in (2) is a problem for derivative computation. The friction torque described by viscous and Coulomb parameters is not valid for velocities close to zero [5]. Thus, they are eliminated making the derivative of the function *sign* null. Finally, to identify F_{c1R}, the rigid model is introduced. The minimal identification model is thus given by:

$$D_2 = \begin{pmatrix} \ddddot{q}_1 & \dddot{q}_1 & 0 & -\ddot{q}_2 & -\dddot{q}_2 & 0 & 0 \\ 0 & 0 & 0 & \ddot{q}_2 & \dddot{q}_2 & \ddddot{q}_{12} & \dddot{q}_{12} \\ \ddot{q}_1 & \dot{q}_1 & sign(\dot{q}_1) & 0 & 0 & 0 & 0 \end{pmatrix}, y_2 = \begin{pmatrix} \ddot{\tau}_1 \\ 0 \\ \tau_1 \end{pmatrix}$$

and $\chi_2 = \left(ZZ_1 \quad F_{v1} \quad F_{c1R} \quad K_{12} \quad F_{v12} \quad ZZ_2 \quad F_{v2}\right)^T$ (7)

C. Identification model with only motor position

This is the common case in industrial applications. Equation (2) must be rewritten. The flexible DOF q_2 must be written according to q_1 and its derivatives. To make it possible, F_{v2} and F_{c2} are regrouped with F_{v1} and F_{c1} respectively. Hence:

$$-K_{12}q_2 - F_{v12}\dot{q}_2 = \tau_1 - ZZ_1\ddot{q}_1 - F_{v1R}\dot{q}_1 - F_{c1R}sign(\dot{q}_1) \quad (8)$$

$$\ddot{q}_2 = -\ddot{q}_1 + \left(-K_{12}q_2 - F_{v12}\dot{q}_2\right)/ZZ_2 \quad (9)$$

This gives:

$$\ddot{q}_2 = -\ddot{q}_1 + \left(\tau_1 - ZZ_1\ddot{q}_1 - F_{v1R}\dot{q}_1 - F_{c1R}sign(\dot{q}_1)\right)/ZZ_2 \quad (10)$$

By eliminating velocities close to zero and by derivating (10) once and (8) twice, we obtain:

$$\dddot{q}_2 = -\dddot{q}_1 + \left(\dot{\tau}_1 - ZZ_1\dddot{q}_1 - F_{v1R}\ddot{q}_1\right)/ZZ_2 \quad (11)$$

$$\ddot{\tau}_1 = ZZ_1\ddddot{q}_1 + F_{v1R}\dddot{q}_1 - K_{12}\ddot{q}_2 - F_{v12}\dddot{q}_2 \quad (12)$$

Finally, the minimal identification model is given by:

$$y_3 = \tau_1,\ D_3 = \begin{pmatrix}-\ddot{\tau}_1 & -\dot{\tau}_1 & \ddddot{q}_1 & \dddot{q}_1 & \ddot{q}_1 & \dot{q}_1 & sign(\dot{q}_1)\end{pmatrix}$$

$$\chi_3 = \begin{pmatrix}b_2 & b_1 & a_4 & a_3 & a_2 & a_1 & a_0\end{pmatrix} \quad (13)$$

With: $b_2 = ZZ_2 / K_{12}$, $b_1 = F_{v12} / K_{12}$, $a_4 = ZZ_1 ZZ_2 / K_{12}$,
$a_3 = \left(F_{v1R}ZZ_2 + F_{v12}ZZ_2 + F_{v12}ZZ_1\right) / K_{12}$
$a_2 = F_{v12}F_{v1R} / K_{12} + ZZ_1 + ZZ_2$, $a_1 = F_{v1}$, $a_0 = F_{c1R}$.
The dynamic parameters are calculated as follows:

$ZZ_1 = a_4 / b_2$, $ZZ_2 = a_2 - \left(b_1 a_1 + a_4 / b_2\right)$

$K_{12} = \left(a_2 - \left(b_1 a_1 + a_4 / b_2\right)\right) / b_2$ $F_{v12} = \left(a_2 - \left(b_1 a_1 + a_4 / b_2\right)\right) b_1 / b_2$

IV. Identification Method and Data Filtering

A. Identification method

The identification method developed for the manipulator robots is applied for joint stiffness identification. The vector χ is estimated with ordinary least squares (OLS) technique from an over determined system built from the sampling of (6), (7) and (13):

$$Y = W\chi + \rho \quad (14)$$

Where: Y is the (rx1) measurement vector, W the (rxb) regressor, χ is the (bx1) vector of parameters to be identified and ρ is the (rx1) residual vector. We have $r = n * n_e$, where n_e is the number of collected samples.

The unicity of the OLS solution is ensured if W is a full rank matrix i.e. if $rank(W) = b$. To avoid rank deficiency, only the b base parameters must be considered [14][15] and trajectories must be exciting enough [16][17].

The detailed calculation of the standard deviation $\sigma_{\hat{\chi}_j}$ and the relative standard derivation $100 * \left|\sigma_{\hat{\chi}_j} / \hat{\chi}_j\right|$ for $\hat{\chi}_j \neq 0$ can be found in [3].

Calculating the OLS solution of (14) from noisy discrete measurements or estimations of derivatives may lead to bias because W may be correlated to ρ. However, it has been shown that the OLS estimation is as consistent as sophisticated methods such as instrumental variable method provided that a well tuned bandpass filtering is performed [18]. Then, it is essential to filter data in Y and W before computing the OLS solution.

B. Data filtering

Velocities and accelerations are estimated by means of a band pass filtering of the positions. This band pass filtering is obtained with the product of a low pass filter in both forward and reverse direction (Butterworth) and from a derivative filter obtained by central difference algorithm, without phase shift. The magnitude of the frequency response is given by:

$$\left|H(j\omega)\right| = 1 \Big/ \left(1 + \left(j\omega / \omega_{butter}\right)^{2n_{butter}}\right) \quad (15)$$

Where n_{butter} is the filter order and ω_{butter} is the cut-off frequency. n_{butter} is fixed according to the maximum derivatives order, n_{mdo}, in the minimal identification model. The cut-off frequency ω_{butter} of the low pass filter must be chosen to avoid any magnitude distortion on the filtered signals in the range $\begin{bmatrix}0 & \omega_{dyn}\end{bmatrix}$ defined by the dynamics to be identified. Details about the choice of ω_{butter} and n_{butter} can be found in [12][13].

To eliminate high frequency noises and torque ripples, a parallel decimation is performed on Y and the columns of W. This low pass decimate filter resamples each signal at a lower rate. It keeps one sample over n_d because no information is contained in the range $\begin{bmatrix}\omega_{dyn} & \omega_s / 2\end{bmatrix}$, where ω_s is sampling frequency. Details about data decimation can be found in [3].

V. Experimental Validation

A. Data acquisition and "rigid" nominal values

Motor and load positions are measured by means of high precision encoders working in quadrature count mode and with an accuracy of 100000 counts per revolution. The sample acquisition frequency for joint position and current reference (drive force) is 1 KHz.

We calculate the motor torque using the relation:

$$\tau_1 = G_\tau v_\tau \quad (16)$$

where v_τ is the current reference of the amplifier current loop, and G_τ is the gain of the joint drive chain, which is taken as a constant in the frequency range of the robot because of the large bandwidth (700 Hz) of the current loop.

The first natural frequency, ω_n, is of 30Hz. This was verified with appropriate mechanical experiments such as blocked output test (see [4]). The cut-off frequency of the Butterworth filter is fixed at $2 * \pi * 60\ rad / s$ and the cut-off frequency of the decimate filter is fixed at 60Hz. We keep one sample over 12.

The system is position controlled with a PD controller, the bandwidth of the closed loop is tuned at 30Hz to identify the dynamic parameters.

Exciting trajectories consist of trapezoidal velocity with pulses: trapezoidal velocity excites very well inertia and friction parameters while pulses excite flexibility. We have $cond(W)=30$ implying that the dynamic parameters are well excited and can be identified [16][17] with a good accuracy.

The "rigid" identified values are summed up in Table 1.

TABLE 1. DIDIM IDENTIFIES VALUES WITH THE RIGID MODEL

Parameter	$\hat{\chi}_j$	$2*\sigma_{\hat{\chi}_j}$	$100*\left|\sigma_{\hat{\chi}_j}/\hat{\chi}_j\right|$
ZZ_{1R}	106	0.44	0.21
F_{v1R}	208	3.5	0.84
F_{c1R}	20.0	0.35	0.88

B. *Experimental identification results with no additional mass on the load*

With the first minimal identification model described by (6), the maximum derivatives order (n_{mdo}) is 2. According to [12], $n_{butter}=4$. With the second and third identification models described by (7) and (13), $n_{mdo}=4$. Hence, $n_{butter}=6$. The results are summed up in Table 2, Table 3, Table 4 and Table 5. In addition, the estimated natural frequency and $\left\|(Y-W\hat{\chi})\right\|/\left\|Y\right\|$, the relative norm of the residue, are given.

Cross tests validations have been performed. They consist in simulating the EMPS with the identified values and in integrating the DDM (5). In any case, the estimated torque follows closely the measured one (see Fig. 4, Fig. 5 and Fig. 6). Furthermore, the relative norm of the error between the measured torque and the simulated one, $\left\|(Y-\hat{Y}_S)\right\|/\left\|Y\right\|$, is computed and summed up in Table 2, Table 3 and Table 4.

TABLE 2: OLS IDENTIFIED VALUES WITH THE FIRST MINIMAL IDENTIFICATION MODEL

Parameter	$\hat{\chi}_j$	$2*\sigma_{\hat{\chi}_j}$	$100*\left|\sigma_{\hat{\chi}_j}/\hat{\chi}_j\right|$
ZZ_1	72.3	0.35	0.24
F_{v1}	92.0	3.37	1.83
F_{c1}	10.0	0.30	1.50
K_{12}	$8.0\ 10^5$	$6.4\ 10^3$	0.41
F_{v12}	126.0	34.90	13.84
ZZ_2	34.8	0.36	0.52
F_{v2}	110.0	3.35	1.52
F_{c2}	10.4	0.30	1.45
Estimated natural frequency: 29.0Hz			
$\left\|(Y-W\hat{\chi})\right\|/\left\|Y\right\|=7\%$, $\left\|(Y-\hat{Y}_S)\right\|/\left\|Y\right\|=7\%$			
$ZZ_1+ZZ_2=107Kg$, $F_{v1}+F_{v2}=202Ns/m$, $F_{c1}+F_{c2}=20.4N$			

TABLE 3: OLS IDENTIFIED VALUES WITH THE SECOND MINIMAL IDENTIFICATION MODEL

Parameter	$\hat{\chi}_j$	$2*\sigma_{\hat{\chi}_j}$	$100*\left|\sigma_{\hat{\chi}_j}/\hat{\chi}_j\right|$
ZZ_1	69.6	0.33	0.24
F_{v1}	170.0	5.03	3.00
F_{c1R}	22.8	0.45	1.95
K_{12}	$8.3\ 10^5$	$3.0\ 10^3$	0.18
F_{v12}	430.0	19.10	2.21
ZZ_2	35.5	0.19	0.25
F_{v2}	-29.8	20.75	34.90
Estimated natural frequency: 29.0Hz			
$\left\|(Y-W\hat{\chi})\right\|/\left\|Y\right\|=11\%$, $\left\|(Y-\hat{Y}_S)\right\|/\left\|Y\right\|=11\%$			
$ZZ_1+ZZ_2=105.1Kg$, $F_{v1}=170Ns/m$, $F_{c1}=22.8N$			

TABLE 4: OLS IDENTIFIED VALUES WITH THE THIRD MINIMAL IDENTIFICATION MODEL

Parameter	$\hat{\chi}_j$	$2*\sigma_{\hat{\chi}_j}$	$100*\left|\sigma_{\hat{\chi}_j}/\hat{\chi}_j\right|$
b_2	$3.81\ 10^{-5}$	$6.00\ 10^{-7}$	0.78
b_1	$-3.42\ 10^{-5}$	$8.42\ 10^{-5}$	123.0
a_4	0.0028	$3.25\ 10^{-5}$	0.59
a_3	-0.059	0.006	5.12
a_2	107.0	0.42	0.20
a_1	210.0	4.05	1.08
F_{c1R}	20.0	0.40	1.02
Estimated natural frequency: 29.0Hz			
$\left\|(Y-W\hat{\chi})\right\|/\left\|Y\right\|=11\%$, $\left\|(Y-\hat{Y}_S)\right\|/\left\|Y\right\|=11\%$			
$ZZ_1+ZZ_2=106Kg$, $F_{v1}=210Ns/m$, $F_{c1}=20N$			

TABLE 5: PHYSICAL PARAMETERS WITH OLS IDENTIFIED VALUES

Parameter	$\hat{\chi}_j$
ZZ_1	71.0
F_{v1R}	210.0
F_{c1R}	20.0
K_{12}	$8.5\ 10^5$
F_{v12}	-30.0
ZZ_2	35.0

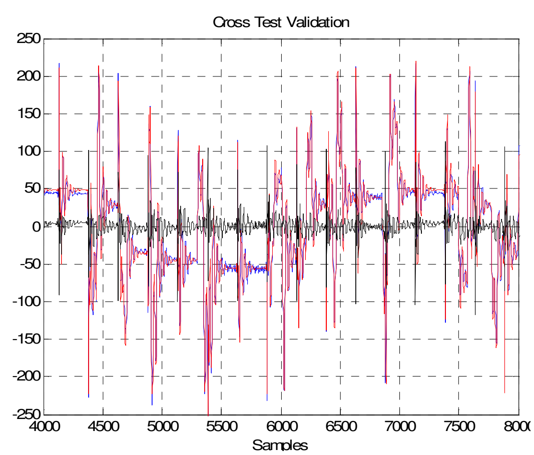

Fig. 4. Cross test validation with the first minimal identification model. Blue: measurement, Red: simulated torque, Black: error.

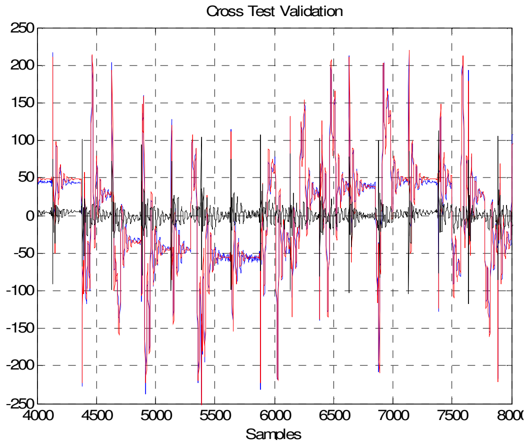

Fig. 5. Cross test validation with the second minimal identification model. Blue: measurement, Red: simulated torque, Black: error.

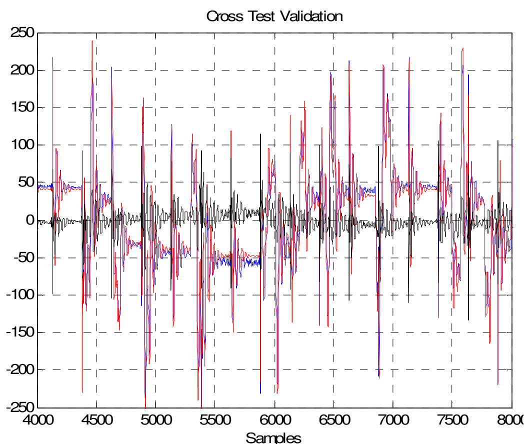

Fig. 6. Cross test validation with the third minimal identification model. Blue: measurement, Red: simulated torque, Black: error.

The most accurate identification results are obtained with the first model thanks to motor and load positions measurements. The relative norm of the residue like the relative norm of the error between the measured torque and the simulated one is small (less than 10%). Unlike chirp signals ([9][12][13]), the applied exciting trajectories enable us to identify friction repartition. Furthermore, by adding inertia, viscous and Coulomb friction parameters, we retrieve the "rigid" values. Finally, we get an accurate flexible dynamic model and the values identified with the first model are our references.

The second minimal model provides good results though we have regrouped the parameter F_{c2} with F_{c1}. The estimation of F_{v1} is close to F_{v1R}. This is due to the fact that F_{v2} is not well identified because it has large relative deviation and when removed from the model, the other values and the residue norm do not vary significantly (less than 1%). Its contribution to dynamics is negligible. The identified values of ZZ_1, ZZ_2 and K_{12} are very close to those summed up in Table 2. We can accurately identify the first natural frequency with this model, but we can not identify the frictions repartition.

Good results are obtained with the third model though F_{v2} and F_{c2} can not be identified. The identified values of ZZ_1, ZZ_2 and K_{12} are very close to those summed up in Table 2 and Table 3. As stated for the second model, the first natural frequency can be accurately identified unlike the friction repartition.

C. *Experimental results with an additional mass of 10Kg on the load*

As a final test, the experimental identification is performed while an extra mass of 10Kg is added on the load. If the identification process is well designed, then the variation observed on ZZ_2 must be close to 10Kg and the variations observed on the other parameters must be insignificant.

The identified values, the estimated natural frequency, the relative norm of the residue and the relative norm of the error between the measured torque and the simulated one are summed up Table 6. Since standard deviations and relative standard derivations are very close to those exposed in Table 2, Table 3 and Table 4, they are missing. The variations of the estimations are given in Table 7.

TABLE 6: IDENTIFIED VALUES WITH AN EXTRA MASS OF 10KG

Parameter	$\hat{\chi}_j$: 1st model	$\hat{\chi}_j$: 2nd model	$\hat{\chi}_j$: 3rd model
ZZ_1	73.1	69.0	70.0
F_{v1}	92.9	165.0	220.0
F_{c1}	10.2	23.0	21.0
K_{12}	$8.2\ 10^5$	$8.2\ 10^5$	$8.4\ 10^5$
F_{v12}	116.0	440.0	-65.1
ZZ_2	44.3	44.5	45.5
F_{v2}	115.0	-12.3	X
F_{c2}	11.0	X	X
ω_n	27 Hz	27 Hz	27 Hz
$\|(Y-W\hat{\chi})\|/\|Y\|$	7%	11%	11%
$\|(Y-\hat{Y}_s)\|/\|Y\|$	7%	11%	11%

TABLE 7: VARIATIONS OF ESTIMATIONS

Parameter	$\hat{\chi}_j$: 1st model	$\hat{\chi}_j$: 2nd model	$\hat{\chi}_j$: 3rd model
ΔZZ_1	**0.8**	**-0.6**	**-1.0**
ΔF_{v1}	0.9	-5.0	10.0
ΔF_{c1}	0.2	0.2	1.0
ΔK_{12}	**$0.2\ 10^5$**	**$-0.1\ 10^5$**	**$-0.1\ 10^5$**
ΔF_{v12}	-10.0	10.0	-35.1
ΔZZ_2	**9.5**	**9.0**	**10.5**
ΔF_{v2}	5.0	-17.5	X
ΔF_{c2}	0.6	X	X

For each minimal identification model, variations observed on ZZ_2 are close to 10Kg whereas variations observed on the other parameters are practically insignificant. Of course, these variations can not be perfectly null because of noises and experiment conditions.

Direct and cross test validations have been performed. As done for the previous experiments, cross test validations consist in simulating the EMPS. The results are very close to those illustrated Fig. 4, Fig. 5 and Fig. 6.

All these experimental results mean that the identification is of good quality and the identification process described along this paper is suitable to joint stiffness identification.

VI. CONCLUSION

Experimental joint stiffness identification has been performed with three minimal identification models depending on the measurement availability:
- Motor and load positions,
- Motor position and load acceleration,
- Motor position only.

As expected, the most accurate identification results are obtained when both motor and load positions are measured.

The two other minimal identification models give good results though the parameters F_{v2} and F_{c2} have been regrouped with F_{v1} and F_{c1} respectively. Indeed, the identified values of inertia and stiffness parameters are very close to those identified with the first model. Finally, these methods can be used as an alternative to the first one to identify the first natural frequency. But, they can not distinguish motor frictions from load frictions.

The main weakness of models described by (7) and (13) is the presence of derivatives orders greater than two. Motor position must be accurate enough and bandpass filtering must be well tuned. Difficulties arise for low encoder resolutions, typically less than 1000 counts per revolution. In most of cases, remember that the sampling rate of the controller is fixed by the manufacturer, the choice being very limited. Hence, the method to calculate the optimal sampling rate presented in [12] cannot be easily applied.

Future works concern the use of the instrumental variable method (IV) to identify parameters with only motor position or with poor encoder resolutions. As shown in [18], the IV method has been recently extended to identify rigid robots and it acts as a natural adaptive filter. This could be helpful in experimental joint stiffness identification, especially with poor encoder resolutions.

They concern also the extension of the Direct and Inverse Dynamic Identification Models technique (DIDIM, see [19]) to joint stiffness identification. This method, needing only torque/force data, has been validated on a 6 DOF rigid robot. It would be interesting to extend it in order to identify joint stiffness with only one measurement.

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