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The Informative Enough Property of the Data Set in a Networked Control System

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Abstract—**Problems related to the identifiability of Networked Control Systems (NCSs) are studied in this paper. The *informative enough* property is a necessary condition to ensure a system's identifiability. In this paper, it is proved that the data set which is not *informative enough with respect to some model sets* in a conventional closed-loop system will become *informative enough with respect to these model sets* in an NCS, due to the random delay and packet dropout introduced by network transmission. The result shows that the random network influence of NCSs which is usually treated as being harmful can bring benefit to the identifiability of closed-loop system identification.**

I. INTRODUCTION

NETWORKED Control Systems (NCSs) are kind of closed-loop control systems, where transmission between sensor and controller, and transmission between controller and actuator are through shared digital communication network [1]–[4]. In spite of many advantages introduced by the network [5]–[8], the band-limited network paths are often unreliable due to the network-induced delay and packet dropout occurring in both the sensor-to-controller path (S-C path) and the controller-to-actuator path (C-A path) [2]–[4], [6].

It is known that for a linear controlled system within a conventional closed-loop, the condition that the data set is *informative enough with respect to some model sets* [9], [10] is a necessary condition to ensure the system's identifiability, but cannot be satisfied unless the reference input is persistent exciting [9], [10], or some additional constraint conditions are added [11]–[15].

The purpose of this paper is to prove that the closed-loop data set of an NCS becomes *informative enough with respect to these model sets* due to the introduction of random network-induced delay and packet dropout even when those constraint conditions mentioned above are not satisfied. The result shows that although the random network influence of an NCS is usually treated as being harmful, it can bring benefit to the identifiability of closed-loop system identification.

Our idea is motivated by the results in [11] and [13], which shows that the *informative enough* property of a closed-loop data set can be guaranteed as long as there are switches between different feedback laws, but the switches must be very seldom and each feedback law is occupied with a nonzero proportion of the total time. In an NCS, the feedback controller together with the S-C path and C-A path can be viewed equivalently as a time-varying controller, so it is intuitively to speculate that the random network-induced delay and packet dropout of an NCS may bring benefit to the closed-loop data set from the viewpoint of *informative enough*. However, the frequency-domain analysis in [11] and [13] cannot be applied to NCS as the equivalent time-varying feedback law caused by the delay and dropout are random and vary very fast, which violates the precondition in [11] and [13] that the switches between different feedback laws must be very seldom, and prevents the power spectrum analysis in [9] from being used. Therefore, in this paper, we prove that the data set of an NCS is *informative enough with respect to some model sets* through analysis directly in time-domain.

Although the identification of a system operating in networked environment has been reported by [16]–[21], they all make the assumption that the system is identifiable as a precondition, and focus on how to overcome the problem of incomplete data set caused by delay and dropout, without directly analyzing the influence on the information content in closed-loop data set brought by the random network transmission.

There are also some results related to the topic of this paper for closed-loop systems other than NCSs. Reference [22] inserts an adaptive quantizer into the feedback path, and uses the quantization error as exciting signal to offer identifiability. However, it analyzes the persistency and statistical property of the quantization error only through simulation and physical experiment. Reference [23] proves that the estimation of least-squares method is consistent if the output data of the closed-loop system is over-sampled. Reference [24] further proves the identifiability of that problem in frequency domain by using the lifting technique, and the concepts of bifrequency map and bispectrum.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Preliminaries

In this section, some preliminary concepts which are given by [9] and necessary to our analysis will be briefly introduced, including Definition 2.1, Definition 4.1, and Definition 8.1 in [9].

Definition 1 [9]**:** A signal $\{x(k)\}$ is quasi-stationary if it

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satisfies:

(i) $|\mathrm{E}x(k)|$ is bounded, $\forall k$;

(ii) $|\mathrm{E}x(k_1)x(k_2)|$ is bounded, $\forall k_1, k_2$;

(iii) $\lim_{L\to\infty}\frac{1}{L}\sum_{k=1}^{L}\mathrm{E}x(k)x(k+\tau)$ exists, and only depends on τ, $\forall\tau$.

The symbol $|\cdot|$ and E in (i), (ii), and (iii) represent the "absolute value" and the "mathematical expectation" respectively.

Definition 2 [9]**:** For a signal $\{x(k)\}$, $\bar{\mathrm{E}}x(k)$ is defined as

$$\bar{\mathrm{E}}x(k) = \lim_{L\to\infty}\frac{1}{L}\sum_{k=1}^{L}\mathrm{E}x(k). \tag{1}$$

Definition 3: According to (4.118) and Definition 4.1 of [9], a candidate model set can be defined as

$$M^* = \left\{W_{y,i}(z^{-1}), W_{u,i}(z^{-1}) \middle| i \in D_M\right\}$$

where all the models in M^* are linear, stable, and in the predictor form. D_M is defined as an index set.

It is worth noting that, according to (4.115) of [9], there is a one-to-one relationship between a system model and its predictor form.

Definition 4 [9]**:** A quasi-stationary data set $Z^\infty = \{y(i), u(i) | i \ge 1\}$ is *informative enough with respect to* M^* which is defined in Definition 3 if, for any two models $\{W_{y,1}(z^{-1}), W_{u,1}(z^{-1})\}$ and $\{W_{y,2}(z^{-1}), W_{u,2}(z^{-1})\}$ in M^*,

$$\bar{\mathrm{E}}\left[\Delta W_y(z^{-1})y(k) + \Delta W_u(z^{-1})u(k)\right]^2 = 0 \tag{2}$$

implies $\Delta W_y(e^{-j\omega}) \equiv 0$ and $\Delta W_u(e^{-j\omega}) \equiv 0$ for almost all ω, where

$$\begin{aligned}\Delta W_y(z^{-1}) &= W_{y,1}(z^{-1}) - W_{y,2}(z^{-1}) = \sum_{i=1}^{+\infty} r_y(i)\cdot z^{-i}\\ \Delta W_u(z^{-1}) &= W_{u,1}(z^{-1}) - W_{u,2}(z^{-1}) = \sum_{i=1}^{+\infty} r_u(i)\cdot z^{-i}.\end{aligned} \tag{3}$$

Remark 1: According to Definition 3, the models in both $\{W_{y,1}(z^{-1}), W_{u,1}(z^{-1})\}$ and $\{W_{y,2}(z^{-1}), W_{u,2}(z^{-1})\}$ are stable, which implies that $\Delta W_y(z^{-1})$ and $\Delta W_u(z^{-1})$ are stable, therefore, according to Chap. 15.3.2 in [25], we know that

$$\lim_{i\to 0} r_y(i) = 0$$
$$\lim_{j\to 0} r_u(j) = 0.$$

B. NCS model

An NCS shown in Fig.1 is considered in this paper, where the actuator and the sensor are clock-driven with a fixed sampling interval.

1) Process and controller

Since the actuator and the sensor are assumed to be clock-driven, the discrete process from $u(k)$ to $y(k)$ can be described as

$$y(k) = G_0(z^{-1})u(k) + H_0(z^{-1})e_0(k) \tag{4}$$

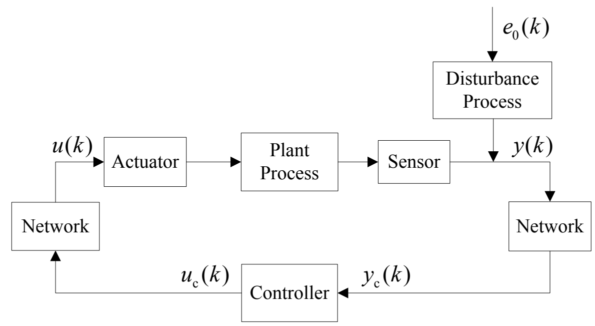

Fig. 1. Networked Control System

with

$$G_0(z^{-1}) = \sum_{i=1}^{+\infty} g_i \cdot z^{-i}, \quad H_0(z^{-1}) = \sum_{i=0}^{+\infty} h_i \cdot z^{-i} \tag{5}$$

where $u(k) \in \mathrm{R}$ and $y(k) \in \mathrm{R}$ are the input of the actuator and the output of the sensor at time instant k respectively, $y_c(k) \in \mathrm{R}$ and $u_c(k) \in \mathrm{R}$ are the input and output of the controller at time instant k respectively, $H_0(z^{-1})e_0(k)$ is random disturbance.

Assumption 1: As in [9], suppose that $\{e_0(k)\}$ is a sequence of independent identically distributed (i.i.d.) random variables with zero mean value and variance λ^2, and $H_0(z^{-1})$ is an inversely stable, monic filter with $h_0 = 1$.

Suppose that the controller is given by

$$u_c(k) = F_y(z^{-1})y_c(k) \tag{6}$$

where

$$F_y(z^{-1}) = \sum_{i=0}^{+\infty} f_i \cdot z^{-i} \tag{7}$$

and $f_0 \ne 0$.

2) Network transmission

Suppose at each time instant k, the network-induced delay and packet dropout happen randomly in both the S-C path and the C-A path, then there could be either no input or multiple inputs to both the controller and the actuator during $(k, k+1]$. There are different ways to handle that problem [3], [6], e.g., the controller and actuator could always use the latest data in their buffers. In this paper, we do not make any assumption on the mechanisms adopted by the controller and the actuator, i.e., any mechanism can be selected, such as those mentioned in [3] and [6].

Let $S(k)$ denote the joint transmission state of S-C path and C-A path at time instant k, i.e.

$$\begin{cases} S(k) = 0, & \text{if both } y(k) \text{ and } u_c(k) \text{ are delivered normally}\\ S(k) = 1, & \text{otherwise (i.e., if at least one of } y(k) \text{ and } u_c(k) \text{ is}\\ & \text{not delivered normally due to delay or dropout).}\end{cases}$$

Then we make the following assumptions on the network transmission:

Assumption 2:

(i) Suppose that the network-induced delay and packet dropout happen independently with the i.i.d. random sequence $\{e_0(k)\}$.

(ii) $\lim_{L\to\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\text{Prob}\{S(k)=0\}\right\}=1-p_d$ and $\lim_{L\to\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\text{Prob}\{S(k)=1\}\right\}=p_d$ with $0<p_d<1$, where $\text{Prob}\{S(k)=0\}$ and $\text{Prob}\{S(k)=1\}$ denote the probabilities of "$S(k)=0$" and "$S(k)=1$" respectively.

Remark 2: The reason why we only use one statistical parameter p_d to describe the behaviors of delay and packet dropout of the network and do not make discrimination between them is that p_d is useful and sufficient to the proof of Theorem 1 in Sec. III.

Assumption 3: The NCS composed of (4), (6), and network transmission satisfying Assumption 2 is stable.

3) The available closed-loop data

Since this paper focuses on how the random delay and dropout of network influence the information content in closed-loop data set from the viewpoint of *informative enough*, we use the closed-loop data set at the plant side, i.e. $Z^L=\{y(i),u(i)\,|\,1\le i\le L\}$, for off-line identification. Z^L is not available at the controller side but can be obtained off-line by reading the storages which are added at both the sensor and the actuator. It is worth noticing that if network-induced delay or packet dropout happens in the C-A path at time instant k, $u(k)$ could be "0" or the "latest data in the buffer" depending on the different mechanism of actuator. Further, we make the following assumption on the data set:

Assumption 4: Suppose that both $\{y(k)\}$ and $\{u(k)\}$ are quasi-stationary.

Remark 3: The quasi-stationary properties of $\{y(k)\}$ and $\{u(k)\}$ depend on the statistical properties of both $\{e_0(k)\}$ and network transmission in a complicated way, therefore, Assumption 4 need to be further analyzed and justified in the future.

C. Problem formulation

When the network transmission is reliable, i.e. $S(k)\equiv 0, \forall k$, the NCS in Fig.1 will become a conventional closed-loop system. It is well known that the data set Z^∞ is not *informative enough with respect to* M^* in this case unless additional constraint conditions of the feedback controller [11]–[15] are added, e.g., there are switches between different feedback laws, but the switches must be very seldom and each feedback law is occupied with a nonzero proportion of the total time [11], [13], or the orders of the controller must be greater than the orders of the process model when the process can be discretized as ARX or ARMAX [12], [14], etc..

Therefore, we make the following assumption:

Assumption 5: The data set Z^∞ is not *informative enough with respect to* M^* when $S(k)\equiv 0, \forall k$.

In this paper, we will prove that the data set which is not *informative enough with respect to* M^* in a conventional closed-loop system becomes *informative enough with respect to* M^* in an NCS defined in Sec. II.B with Assumptions 1–5, due to the random delay and dropout introduced by the network transmission.

III. Information content in the closed-loop data set of an NCS

In this section, according to Definition 4, we will prove that the data set which is not *informative enough with respect to* M^* in conventional closed-loop system will become *informative enough with respect to* M^* due to the introduction of the random delay and dropout defined in Sec. II.B. Before the proof, two lemmas are first given for the sake of convenience.

Considering that the network-induced delay and packet dropout are random, the feedback controller together with the S-C path and the C-A path can be viewed equivalently as a time-varying controller, then $u(k)$ and $y(k)$ in Fig.1 can be written as the outputs of the following time-varying closed-loop systems excited by $\{e_0(k)\}$

$$y(k)=\sum_{j=0}^{+\infty}\left[p_{y,k}(j)\cdot e_0(k-j)\right] \tag{8}$$

$$u(k)=\sum_{j=0}^{+\infty}\left[p_{u,k}(j)\cdot e_0(k-j)\right] \tag{9}$$

where the time-varying coefficients $\{p_{y,k}(j)\},\{p_{u,k}(j)\}$ are determined by the process model (4), the controller model (6), and the network transmission conditions. Motivated by [11] and [13], the structure of time-varying controller may bring more constraint conditions about $\Delta W_y(z^{-1})$ and $\Delta W_u(z^{-1})$. These conditions are found through the following lemma.

Lemma 1: The coefficients in the closed-loop models (8) and (9) satisfy

$$p_{y,k}(0)=1, \forall k \tag{10}$$

$$p_{u,k}(0)=\begin{cases} f_0, & \forall k, \text{if } S(k)=0 \\ 0, & \forall k, \text{if } S(k)=1 \end{cases}. \tag{11}$$

Proof:

The coefficients $p_{y,k}(0)$ and $p_{u,k}(0)$ represent the influences of $e_0(k)$ on $y(k)$ and $u(k)$ respectively. Then, (10) and (11) are obtained through analyzing how $e_0(k)$ influences $y(k)$ and $u(k)$.

According to (4), there is

$$y(k)=e_0(k)+G_0(z^{-1})u(k)+\left[H_0(z^{-1})-1\right]e_0(k). \tag{12}$$

Then according to (5) and (9), $G_0(z^{-1})u(k)$ is only influenced by $e_0(j)$ for j up to $k-1$. Similarly, according to (5) and since $h_0=1$ according to Assumption 1, $\left[H_0(z^{-1})-1\right]e_0(k)$ is also only influenced by $e_0(j)$ for j up to $k-1$. Then by comparing (12) with (8), it can be concluded that (10) holds.

According to (6) and (7), there is

$$u_c(k) = f_0 \cdot y_c(k) + \left[F_y(z^{-1}) - f_0\right] y_c(k). \tag{13}$$

If $S(k)=0$, there are $y_c(k)=y(k)$ and $u(k)=u_c(k)$, then by substituting (8), (10) into (13), $u(k)$ can be written as

$$\begin{aligned} u(k) = f_0 \cdot e_0(k) + f_0 \cdot \sum_{j=1}^{+\infty}\left[p_{y,k}(j)\cdot e_0(k-j)\right] \\ + \sum_{i=1}^{+\infty} f_i \cdot y_c(k-i). \end{aligned} \tag{14}$$

Obviously, both the second and third terms of the right side of (14) are only influenced by $e_0(j)$ for j up to $k-1$. Then by comparing (14) with (9), it can be concluded that $p_{u,k}(0)=f_0$, if $S(k)=0$ holds. On the other hand, if $S(k)=1$, then no matter in which path the delay or dropout happens, $u(k)$ at actuator cannot get the information about $e_0(k)$ at time instant k according to (4), (6) and Fig. 1, therefore $u(k)$ can be viewed as linear combination of $e_0(j)$ for j up to $k-1$. Then by comparing this conclusion with (9), we can conclude that $p_{u,k}(0)=0$, if $S(k)=0$ holds. The lemma is proved. □

Remark 4:

(i) From Lemma 1, it can be seen that receiving a packet with a delay or not receiving the packet has the same influence on $p_{u,k}(0)$. Because no matter the network-induced delay or the dropout happens in the S-C path or the C-A path at time instant k, i.e. $S(k)=1$, there is always that $u(k)$ cannot get the information of $y(k)$, which means that $u(k)$ is not influenced by $e_0(k)$, then $p_{u,k}(0)=0$ according to (9).

(ii) The reasons why we only analyze $p_{y,k}(0), p_{u,k}(0)$ in Lemma 1 are: they are useful and sufficient to the proof of the *informative enough* property which will be given in Theorem 1; the analysis of other coefficients in $\{p_{y,k}(i)|\forall k\}, \{p_{u,k}(i)|\forall k\}, i\ge 2$ is much more difficult due to the complicated network transmission.

In order to utilize Lemma 1, (2) is rewritten into an equivalent form (i.e. (15)) by applying (3), (8), and (9).

Lemma 2: Equation (2) is equivalent to

$$\lim_{L\to\infty}\left\{\sum_{m=1}^{+\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\mathrm{E}\left[q_k(m)\right]^2\right\}\right\}=0 \tag{15}$$

where $q_k(m)$ is defined as

$$q_k(m)=\sum_{i=1}^{m}\left[r_y(i)\cdot p_{y,k-i}(m-i)+r_u(i)\cdot p_{u,k-i}(m-i)\right]. \tag{16}$$

Proof:

According to (3), (2) is equivalent to

$$\bar{\mathrm{E}}\left[\sum_{i=1}^{+\infty} r_y(i)\cdot y(k-i)+\sum_{i=1}^{+\infty} r_u(i)\cdot u(k-i)\right]^2=0$$

which is further equivalent to

$$\begin{aligned}\bar{\mathrm{E}}\Bigg\{\sum_{i=1}^{+\infty}\sum_{j=0}^{+\infty}\Big\{\left[r_y(i)\cdot p_{y,k-i}(j)+r_u(i)\cdot p_{u,k-i}(j)\right] \\ \cdot e_0(k-i-j)\Big\}\Bigg\}^2=0\end{aligned} \tag{17}$$

by substituting the closed-loop models (8) and (9) into it. Define

$$w(k)= \sum_{i=1}^{+\infty}\sum_{j=0}^{+\infty}\left\{\left[r_y(i)\cdot p_{y,k-i}(j)+r_u(i)\cdot p_{u,k-i}(j)\right]\cdot e_0(k-i-j)\right\} \tag{18}$$

and $m=1,2,3,\cdots$, the coefficient of $e_0(k-m)$ in $w(k)$ is

$$\sum_{i=1}^{m}\left[r_y(i)\cdot p_{y,k-i}(m-i)+r_u(i)\cdot p_{u,k-i}(m-i)\right]$$

then (18) is equivalent to

$$w(k)= \sum_{m=1}^{+\infty}\left\{\sum_{i=1}^{m}\left[r_y(i)\cdot p_{y,k-i}(m-i)+r_u(i)\cdot p_{u,k-i}(m-i)\right]\right\}\cdot e_0(k-m)$$

therefore, (17) can be written as

$$\begin{aligned}\bar{\mathrm{E}}\Bigg\{\sum_{m=1}^{+\infty}\Bigg\{\sum_{i=1}^{m}\left[r_y(i)\cdot p_{y,k-i}(m-i)+r_u(i)\cdot p_{u,k-i}(m-i)\right]\Bigg\} \\ \cdot e_0(k-m)\Bigg\}^2=\bar{\mathrm{E}}\left[\sum_{m=1}^{+\infty} q_k(m)\cdot e_0(k-m)\right]^2=0\end{aligned} \tag{19}$$

where $q_k(m)$ is defined as

$$q_k(m)=\sum_{i=1}^{m}\left[r_y(i)\cdot p_{y,k-i}(m-i)+r_u(i)\cdot p_{u,k-i}(m-i)\right].$$

According to (1), (19) can be further written as

$$\lim_{L\to\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\mathrm{E}\left[\sum_{m=1}^{+\infty} q_k(m)\cdot e_0(k-m)\right]^2\right\}=0. \tag{20}$$

Since $\{p_{y,k}(j)\}, \{p_{u,k}(j)\}$ are only dependent on random network transmission, and deterministic dynamics of the process (4) and the controller (6), Assumption 2.(i), i.e. the assumption that the network transmission is independent with $\{e_0(k)\}$, means that $\{q_k(m)\}$ is independent with $\{e_0(k)\}$. In addition, $\{e_0(k)\}$ is a sequence of i.i.d. random variables according to Assumption 1. Then we have

$$\begin{aligned}&\mathrm{E}\left[\sum_{m=1}^{+\infty} q_k(m)\cdot e_0(k-m)\right]^2 \\ &=\sum_{i=1}^{+\infty}\sum_{j=1}^{+\infty}\mathrm{E}\left[q_k(i)\cdot q_k(j)\cdot e_0(k-i)\cdot e_0(k-j)\right] \\ &=\sum_{i=1}^{+\infty}\sum_{j=1}^{+\infty}\mathrm{E}\left[q_k(i)q_k(j)\right]\cdot \mathrm{E}\left[e_0(k-i)e_0(k-j)\right] \\ &=\lambda^2\cdot\sum_{m=1}^{+\infty}\mathrm{E}\left[q_k(m)\right]^2.\end{aligned} \tag{21}$$

According to (21), (20) is equivalent to

$$\lim_{L\to\infty}\left\{\sum_{m=1}^{+\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\mathrm{E}\left[q_k(m)\right]^2\right\}\right\}=0.$$

The lemma has been proved. □

Theorem 1: For the NCS defined in Sec. II.B, with Assumptions 1–5, the closed-loop data set Z^{∞} is *informative enough with respect to* M^*.

Proof:

In this proof, (15) which is equivalent to (2) according to Lemma 2, will be proved to lead to $r_y(i) = r_u(i) = 0$, $\forall i < \infty$, which is equivalent to $\Delta W_y(z^{-1}) \equiv 0$ and $\Delta W_u(z^{-1}) \equiv 0$ according to (3) and Remark 1. Then according to Definition 4, the theorem can be proved.

Equation (15) leads to

$$\lim_{L\to\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\mathrm{E}\left[q_k(m)\right]^2\right\} = 0, \quad \forall m < \infty. \tag{22}$$

In the following, we will prove $r_y(i) = r_u(i) = 0, \forall i < \infty$ from (22) by utilizing mathematical induction.

Let $m = 1$. Equation (22) leads to

$$\lim_{L\to\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\mathrm{E}\left[q_k(1)\right]^2\right\} = 0. \tag{23}$$

Equation (16) leads to

$$q_k(1) = r_y(1)\cdot p_{y,k-1}(0) + r_u(1)\cdot p_{u,k-1}(0). \tag{24}$$

Then by applying (10) and (11) of Lemma 1 to (24), there is

$$q_k(1) = \begin{cases} r_y(1) + r_u(1)\cdot f_0, \ \forall k, \text{ if } S(k-1) = 0 \\ r_y(1), \qquad \forall k, \text{ if } S(k-1) = 1. \end{cases} \tag{25}$$

According to (25), we have: $\forall k$

$$\begin{aligned}\mathrm{E}\left[q_k(1)\right]^2 &= \left[r_y(1) + r_u(1)\cdot f_0\right]^2\cdot \mathrm{Prob}\left\{S(k-1)=0\right\} \\ &\quad + \left[r_y(1)\right]^2\cdot \mathrm{Prob}\left\{S(k-1)=1\right\}.\end{aligned} \tag{26}$$

Substituting (26) into (23), and according to Assumption 2.(ii), we have

$$\begin{aligned}&\lim_{L\to\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\mathrm{E}\left[q_k(1)\right]^2\right\} \\ &= \lim_{L\to\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\left\{\left[r_y(1)+r_u(1)\cdot f_0\right]^2\cdot \mathrm{Prob}\left\{S(k-1)=0\right\}\right\}\right\} \\ &\quad + \lim_{L\to\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\left\{\left[r_y(1)\right]^2\cdot \mathrm{Prob}\left\{S(k-1)=1\right\}\right\}\right\} \\ &= \left[r_y(1)+r_u(1)\cdot f_0\right]^2\cdot \lim_{L\to\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\mathrm{Prob}\left\{S(k-1)=0\right\}\right\} \\ &\quad + \left[r_y(1)\right]^2\cdot \lim_{L\to\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\mathrm{Prob}\left\{S(k-1)=1\right\}\right\} \\ &= \left[r_y(1)+r_u(1)\cdot f_0\right]^2\cdot\left(1-p_d\right) + \left[r_y(1)\right]^2\cdot p_d = 0.\end{aligned} \tag{27}$$

Considering that there is $0 < p_d < 1$ according to Assumption 2.(ii), then (27) leads to

$$\begin{aligned} r_y(1) + r_u(1)\cdot f_0 &= 0 \\ r_y(1) &= 0. \end{aligned} \tag{28}$$

Since $f_0 \neq 0$, (28) yields $r_y(1) = r_u(1) = 0$.

For $2 \le m < \infty$, suppose that $r_y(i) = r_u(i) = 0$ holds for $\forall i, 1 \le i \le m-1$. Equation (22) leads to

$$\lim_{L\to\infty}\left\{\frac{1}{L}\sum_{k=1}^{L}\mathrm{E}\left[q_k(m)\right]^2\right\} = 0. \tag{29}$$

According to (16), there is

$$q_k(m) = r_y(m)\cdot p_{y,k-m}(0) + r_u(m)\cdot p_{u,k-m}(0). \tag{30}$$

Again, applying (10) and (11) of Lemma 1 to (30), there is

$$q_k(m) = \begin{cases} r_y(m) + r_u(m)\cdot f_0, \ \forall k, \text{ if } S(k-m) = 0 \\ r_y(m), \qquad \forall k, \text{ if } S(k-m) = 1. \end{cases} \tag{31}$$

Similarly, according to (31), we have: $\forall k$

$$\begin{aligned}\mathrm{E}\left[q_k(m)\right]^2 &= \left[r_y(m) + r_u(m)\cdot f_0\right]^2\cdot \mathrm{Prob}\left\{S(k-m)=0\right\} \\ &\quad + \left[r_y(m)\right]^2\cdot \mathrm{Prob}\left\{S(k-m)=1\right\}.\end{aligned} \tag{32}$$

Substituting (32) into (29), and use a derivation similar to(27)–(28), we can conclude that $r_y(m) = r_u(m) = 0$.

So far, it has been proved that (15) leads to $r_y(i) = r_u(i) = 0, \forall i < \infty$. Therefore, we can say that (15), which has been proved to be equivalent to (2) in Lemma 2, implies $\Delta W_y(e^{-j\omega}) \equiv 0$ and $\Delta W_u(e^{-j\omega}) \equiv 0$ according to (3) and Remark 1. Then according to Definition 4, we can conclude that the closed-loop data set Z^{∞} of the NCS defined in Sec. II.B with Assumptions 1–5 is *informative enough with respect to* M^*. □

Remark 5: From (25) and (28), it can be seen that their second equations play a key role in obtaining $r_y(1) = r_u(1) = 0$. However, if there is no abnormal network transmission (i.e., there is neither delay nor dropout in both paths), p_d defined in Assumption 2.(ii) will be zero, then both the second equation of (25) and (28) will not exist. Therefore, it is actually the random network transmission that makes the closed-loop data set become *informative enough with respect to* M^*.

IV. Simulation

In this section, a simulation example is given to show the benefit to the information content in closed-loop data set brought by random network transmission with the assumption that process (4) is a Single-In-Single-Out (SISO) autoregressive model with external input (ARX), whose definition can be found in (4.33) of [9]. Since the ARX model structure is identifiable [9], the *informative enough* property of closed-loop data set is equivalent to the parameter identifiability of closed-loop system in this case.

A. Closed-loop system model

Suppose that the process (4) is given by

$$y(k) = \frac{\sum_{i=1}^{4} b_i\cdot z^{-i}}{1+\sum_{i=1}^{6} a_i\cdot z^{-i}}u(k) + \frac{1}{1+\sum_{i=1}^{6} a_i\cdot z^{-i}}e_0(k)$$

where $\{e_0(k)\}$ is a sequence of i.i.d. random variables with zero mean value and variance 0.01, and the parameters to be identified are specified as

$$\boldsymbol{\theta}_0 = [a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4]^{\mathrm{T}}$$
$$= [0.202, 0.27, 0.54, 0.65, -0.06, -0.07, 0.02, 0.03, 0.05, 0.06]^{\mathrm{T}}.$$

Suppose that the controller (6) is selected as

$$F_y(z^{-1}) = \frac{10}{1+0.5z^{-1}}.$$

B. Network transmission

Suppose that the random delay is bounded by 3 sampling intervals, and a packet with delay longer than 3 sampling intervals will be dropped. Let $t_{\mathrm{sc}}(k) \in \{-1, 0, 1, 2, 3\}$ and $t_{\mathrm{ca}}(k) \in \{-1, 0, 1, 2, 3\}$ denote the states of network transmission in S-C path and C-A path at time instant k respectively, where "-1", "0", "1", "2", and "3" represent the states of "dropout", "successful delivery", "1-step delay", "2-step delay", and "3-step delay" respectively.

Let $\mathbf{P}_{\mathrm{sc}}(k)$ and $\mathbf{P}_{\mathrm{ca}}(k)$ denote the probability distributions of $t_{\mathrm{sc}}(k)$ and $t_{\mathrm{ca}}(k)$ at time instant k respectively, i.e.

$$\mathbf{P}_{\mathrm{sc}}(k) = [\mathrm{Prob}\{t_{\mathrm{sc}}(k) = -1\}, \mathrm{Prob}\{t_{\mathrm{sc}}(k) = 0\}, \mathrm{Prob}\{t_{\mathrm{sc}}(k) = 1\}, \mathrm{Prob}\{t_{\mathrm{sc}}(k) = 2\}, \mathrm{Pr}\{t_{\mathrm{sc}}(k) = 3\}]$$

$$\mathbf{P}_{\mathrm{ca}}(k) = [\mathrm{Prob}\{t_{\mathrm{ca}}(k) = -1\}, \mathrm{Prob}\{t_{\mathrm{ca}}(k) = 0\}, \mathrm{Prob}\{t_{\mathrm{ca}}(k) = 1\}, \mathrm{Prob}\{t_{\mathrm{ca}}(k) = 2\}, \mathrm{Prob}\{t_{\mathrm{ca}}(k) = 3\}].$$

Since the network-induced delay or the packet dropout is often modeled as a Bernoulli process or a Markov chain [6]–[8], we consider the following two cases of network transmission in simulation respectively.

1) Bernoulli process

Suppose that $\{t_{\mathrm{sc}}(k)\}$ and $\{t_{\mathrm{ca}}(k)\}$ are both Bernoulli processes with the following probability distributions respectively

$$\mathbf{P}_{\mathrm{sc}}(k) = [0.02, 0.8, 0.08, 0.06, 0.04]$$
$$\mathbf{P}_{\mathrm{ca}}(k) = [0.01, 0.82, 0.09, 0.05, 0.03].$$

2) Markov chain

Suppose that $\{t_{\mathrm{sc}}(k)\}$ and $\{t_{\mathrm{ca}}(k)\}$ are both Markov chains with the following initial probability distributions and probability transfer matrices respectively

$$\mathbf{P}_{\mathrm{sc}}(1) = [0.02, 0.8, 0.08, 0.06, 0.04]$$
$$\mathbf{P}_{\mathrm{ca}}(1) = [0.01, 0.82, 0.09, 0.05, 0.03]$$

$$\mathbf{M}_{\mathrm{sc}} = \begin{bmatrix} 0.1 & 0.65 & 0.05 & 0.08 & 0.12 \\ 0.02 & 0.8 & 0.08 & 0.06 & 0.04 \\ 0.03 & 0.75 & 0.1 & 0.08 & 0.04 \\ 0.04 & 0.72 & 0.08 & 0.1 & 0.06 \\ 0.08 & 0.68 & 0.04 & 0.08 & 0.12 \end{bmatrix}$$

$$\mathbf{M}_{\mathrm{ca}} = \begin{bmatrix} 0.08 & 0.74 & 0.04 & 0.06 & 0.08 \\ 0.01 & 0.82 & 0.09 & 0.05 & 0.03 \\ 0.02 & 0.8 & 0.08 & 0.06 & 0.04 \\ 0.04 & 0.78 & 0.06 & 0.08 & 0.04 \\ 0.05 & 0.76 & 0.06 & 0.05 & 0.08 \end{bmatrix}.$$

The mechanisms of the controller and the actuator are both selected as: using the latest data in their buffers [3], [6].

C. Identification experiments and simulation results

For comparison, three scenarios are considered in simulation, including (i) there is neither delay nor dropout in network transmission, i.e. the NCS in Fig.1 becomes a conventional closed-loop system; (ii) the network transmission follows the Bernoulli assumption in B.1); (iii) the network transmission follows the Markov assumption in B.2).

In simulation, the data length is selected as 10000, and least-squares method is adopted for parameter identification. Let $\mathbf{H} \in \mathbf{R}^{10000\times 10}$ denote the data matrix in least-squares method (please refer to [26] for its definition).

In simulation, it is very easy to verify that there is $\mathrm{rank}(\mathbf{H}^{\mathrm{T}}\mathbf{H}) = 7 < 10$ for *Scenario* 1 which shows that the solution of least-squares estimation is not unique [26]. However, there is $\mathrm{rank}(\mathbf{H}^{\mathrm{T}}\mathbf{H}) = 10$ for both *Scenario* 2 and 3, which means that the random network transmission makes the solutions become unique.

The estimation results for *Scenarios* 2 and 3 are further given in Table I, with "*T*", "*S*2", and "*S*3" denoting the true parameters and the estimated parameters in *Scenarios* 2 and 3 respectively. From Table I, it can be seen that the estimation results of *Scenario* 2 and 3 are satisfying and close to the true values.

TABLE I
SIMULATION RESULTS

	*a*1	*a*2	*a*3	*a*4	*a*5
T	0.202	0.27	0.54	0.65	-0.06
*S*2	0.1970	0.2611	0.5287	0.6451	-0.0665
*S*3	0.1960	0.2686	0.5445	0.6495	-0.059
	*a*6	*b*1	*b*2	*b*3	*b*4
T	-0.07	0.02	0.03	0.05	0.06
*S*2	-0.0638	0.0201	0.0299	0.0488	0.0579
*S*3	-0.0662	0.0194	0.0310	0.0513	0.0594

T represents the true parameters, *S*2 and *S*3 represent the estimated parameters in *Scenario* 2 and 3 respectively.

The simulation results illustrate that the data set which is not *informative enough with respect to some model sets* in a conventional closed-loop system will become *informative enough with respect to these model sets* in an NCS.

V. CONCLUSION

In this paper, it has been proved that the data set which is not *informative enough with respect to some model sets* in a conventional closed-loop system will become *informative enough with respect to these model sets* in an NCS, as a result of the random network-induced delay and packet dropout in data transmission. The result shows that the random network influence of NCSs which is usually treated as being harmful can bring benefit to the identifiability of closed-loop system identification.

There still remain some interesting problems to be studied in the future, e.g., extending the result to the case that at least one of the sensor and actuator is event-driven, extending the result to the Multiple-In-Multiple-Out (MIMO) case, evaluating how the network-induced delay and packet dropout influence the convergence and consistency of a specific identification method, such as least-squares method, etc..

VI. References

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