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Receding horizon control for constrained networked systems subject to data-losses

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Abstract— **We propose a novel receding horizon strategy for Networked Control Systems described by uncertain polytopic linear plants subject to time-varying delays and data-losses. We make use of sequences of pre-computed inner approximations of the one-step ahead state prediction sets on-line exploited as target sets for state predictions. The present approach is capable to deal with data-loss events of arbitrarily length without compromising closed-loop stability and constraints fulfilment.**

I. Introduction

Technological advances are delivering devices endowed with sensing and communication capabilities which can be ubiquitously embedded in the physical world. A networked control system (NCS) consists of numerous physical and computing elements called agents, which have interactions and dependencies, supported by overlapping network resources. Due to these features the study of stability analysis and control design of NCSs is attracting considerable attention in literature, see in [7], [13] and references therein.

Time-delay systems [12] are one of the starting points for analyzing the delay effect in the NCS framework. However a NCS is different from a traditional time-delay system where the delay is simply assumed to be constant or bounded. A network-induced latency is instead variable or even unbounded making the analysis and control design more challenging tasks. Recent contributions on the linear time-invariant case can be found in [13].

Noticeable contributions on feedback control strategies for NCS exploit several approaches, see e.g.[2], [14], [9], [10]. Of interest here is a contribution on a constrained control methodology which is based on a receding horizon strategy for nonlinear networked systems [5]. The motivation for considering this problem is provided by control under wireless and asynchronous measurement sampling. In order to regulate the state of the system towards an equilibrium point while minimizing a given performance index, a Lyapunov-based model predictive controller is designed by explicitly taking into account data losses, both in the optimization problem formulation and in the controller implementation. The proposed scheme allows an explicit characterization of the stability region and guarantees that such a set is invariant for the closed-loop system under data-losses if the maximum time, in which the loop is open, is shorter than a given constant that depends on the parameters of the system and on the Lyapunov-based controller.

Moving from these considerations we will focus here on a novel discrete time receding horizon strategy for NCSs which are described by means of uncertain polytopic linear plants subject to time-varying delays and data-losses. The main motivation behind this approach relies on the fact that the measurement and control commands need to be sent over communications links which bring to varying transmission delays and packet dropouts between plant-controller and vice-versa. The proposed strategy is obtained by interlacing two “ingredients”. First, the use of sequences of pre-computed inner approximations of the one-step ahead state prediction sets are on-line exploited as target sets for the actual state prediction vector to compute the commands to be applied to the plant in a receding horizon philosophy. Then, the time-varying delays and data-losses occurrences are taken into account by resorting to both Independent-of-Delay (**IOD**) and Delay Dependent (**DD**) stability concepts that are used to initialize the one-step controllable sequences. The theoretical results are proved through a final example.

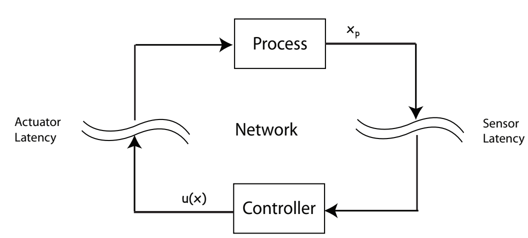

Fig. 1. Networked control system

II. Problem formulation

In what follows we will refer to the networked scheme depicted in Fig. 1 where delay effects are taken into consideration from the sensor and the actuator sides. The process is described by a multi-model discrete-time linear system

$$x_p(t+1) = \Phi(\alpha(t))x_p(t) + G(\alpha(t))u(t) \qquad (1)$$

where $t \in \mathbb{Z}_+ := \{0,1,...\}$, $x_p(t) \in \mathbf{R}^n$ denotes the state plant and $u(t) \in \mathbf{R}^m$ the control input. The possibly time-varying vector $\alpha(t) \in \mathbf{R}^l$ is assumed to belong to the unit simplex

$$\mathcal{P}_l := \left\{ \alpha \in \mathbf{R}^l : \sum_{i=1}^{l} \alpha_i = 1,\ \alpha_i \geq 0 \right\} \quad (2)$$

and the system matrices $\Phi(\alpha)$ and $G(\alpha)$ belongs to

$$\Sigma(\mathcal{P}_l) := \left\{ (\Phi(\alpha), G(\alpha)) = \sum_{i=1}^{l} \alpha_i (\Phi_i, G_i),\ \alpha \in \mathcal{P}_l \right\} \quad (3)$$

where the pairs (Φ_i, G_i) denote the vertices of the polytope $\Sigma(\mathcal{P}_l)$. Moreover, the control input is subject to

$$u(t) \in \mathcal{U},\ \forall t \geq 0, \quad \mathcal{U} := \{u \in \mathbf{R}^m \,|\, u^T u \leq \bar{u}\}, \quad (4)$$

with $\bar{u} > 0$ and $\mathcal{U}$ a compact subset of $\mathbf{R}^m$ containing the origin as an interior point.

The network latency is modelled as a time-varying delay $\tau(t)$, where data can be lost at the plant-controller and controller-plant links. Due to the presence of a double latency an unavoidable time misalignment exists between the measured plant state which is sent to the controller and the model state which is exploited by the regulation algorithm to compute the input (see Fig. 1). To this end we will represent the model by means of the following time updating law

$$x(t+1) = \Phi(\alpha(t))x(t) + G(\alpha(t))u(t) \quad (5)$$

Then, on the basis of the *a-priori* information available on the communication channel two situations could arise:

- each time-delay occurrence is bounded, $\tau(t) \leq \bar{\tau}$ and no data-loss events occur. The upper bound $\bar{\tau}$ represents the maximum allowable transmission interval (MATI) [14];
- there exists a time instant $\bar{t}$ such that $\tau(\bar{t}) > \bar{\tau}$: the state measurement will be no longer available for feedback.

Therefore, the problem we want to solve can be stated as:
Network Constrained Stabilization (NCS) problem - Given the networked system in Fig. 1 with the plant described by (1)-(3), determine a state-feedback strategy

$$u(t) = g(x_p(t - \tau(t))),\ u(t) \in \mathcal{U} \quad (6)$$

which asymptotically stabilizes the closed-loop system regardless of any time-delay occurrence $\tau(t)$. □

In the sequel, the problem will be addressed by adopting a dual-mode model-based control approach. First, a stabilizing state-feedback control law (6) for (1)-(3) is off-line computed by resorting to **DD** and **IOD** stability concepts. Then, the working region of the algorithm is off-line enlarged by deriving sets of states that can be steered into the target set in a finite number of steps. On-line, at each time t and given the delayed state obtained by (1), a receding horizon control strategy is obtained by checking the “smallest” ellipsoidal set (**DD** or **IOD** region) which includes the delayed state.

III. Off-line phase

In this section, the key aspects to develop the control strategy in a networked context are outlined and discussed.

A. Constrained DD and IOD stabilization problems

We are here interested in determining the conditions under which a constant state-feedback control law of the form

$$u(t) = K_{DD} x_p(t - \tau(t)) \quad (7)$$

satisfies the prescriptions of the **NCS problem** for the regulated plant

$$x_p(t+1) = \Phi(\alpha(t))x_p(t) + G(\alpha(t)) K_{DD} x_p(t - \tau(t)) \quad (8)$$

To this end we will consider an auxiliary state which is capable to trace all the delayed state informations $y(t) = x_p(t+1) - x_p(t)$ and a descriptor form of (1) can be obtained

$$\begin{cases} x_p(t+1) & = & y(t) + x_p(t) \\ 0 & = & -y(t) + \Phi(\alpha(t))x_p(t) - x_p(t) \\ & & -G(\alpha(t)) K_{DD} x_p(t - \tau(t)) \end{cases} \quad (9)$$

By the augmented state $\bar{x}(t) = \left[x_p^T(t)\, y^T(t)\right]^T$, we have

$$E\,\bar{x}(t+1) = \mathcal{A}_{DD}\,\bar{x}(t) + \mathcal{B}_{DD} \sum_{j=t-\tau(t)}^{t-1} y(j) \quad (10)$$

with $E = \text{diag}\{I, 0\}$,

$$\mathcal{A}_{DD} = \begin{bmatrix} I & I \\ \Phi(\alpha) - I - G(\alpha)K_{DD} & -I \end{bmatrix},\ \mathcal{B}_{DD} = \begin{bmatrix} 0 \\ G(\alpha)K_{DD} \end{bmatrix}$$

and by resorting to the **DD** Lyapunov-Krasovskii functional

$$\begin{array}{l} V(t) = \bar{x}^T(t) E P_{DD} E \bar{x}(t) + \sum_{m=-\bar{\tau}}^{-1} \sum_{j=t+m}^{t-1} y^T(j)\,[R+Q]\,y(j) \\ P_{DD} = P_{DD}^T \geq 0, \quad R = R^T \geq 0, \quad Q = Q^T \geq 0 \end{array} \quad (11)$$

it can be proved, by means of standard technicalities (see [6] for details), that the constrained **DD** feedback control law (7) asymptotically stabilizes the plant if the following matrix inequalities, evaluated over the polytope vertices (3), in the unknowns K_{DD}, P_{DD}, Q and R are satisfied

$$\begin{bmatrix} E^T P_{DD} E - \mathcal{S}_{DD} & 0 & \mathcal{A}_{DD}^T P_{DD} \\ 0 & \tau_{max}(R+Q) & \mathcal{B}_{DD}^T P_{DD} \\ P_{DD}\mathcal{A}_{DD} & P_{DD}\mathcal{B}_{DD} & P_{DD} \end{bmatrix} \geq 0 \quad (12)$$

$$\begin{bmatrix} \bar{u}^2 E^T P E & \begin{bmatrix} K_{DD}^T \\ 0 \end{bmatrix} \\ \begin{bmatrix} K_{DD} & 0 \end{bmatrix} & I \end{bmatrix} \geq 0 \quad (13)$$

where $\mathcal{S}_{DD} \triangleq \text{diag}\{0, \tau_{max}(R+Q)\}$. Moreover, by using the following **IOD** Lyapunov-Krasovskii functional

$$V(t) = \bar{x}(t)\,E\,P_{IOD}\,E\,\bar{x}(t) + \sum_{j=t-\bar{\tau}}^{t-1} x^T(j)\,S\,x(j), \quad (14)$$

with $S = S^T \geq 0$, the constrained **IOD** feedback control law

$$u(t) = K_{IOD} x_p(t - \tau(t)) \quad (15)$$

stabilizes the plant

$$x_p(t+1) = \Phi(\alpha(t))x_p(t) + G(\alpha(t)) K_{IOD} x_p(t - \tau(t)) \quad (16)$$

if

$$\begin{bmatrix} E^T P_{IOD} E - \mathcal{S}_{IOD} & \mathcal{A}_{IOD}^T P_{IOD} \\ P_{IOD}\mathcal{A}_{IOD} & P_{IOD} \end{bmatrix} \geq 0 \quad (17)$$

$$\begin{bmatrix} \bar{u}^2 E^T P_{IOD} E & \begin{bmatrix} K_{IOD}^T \\ 0 \end{bmatrix} \\ \begin{bmatrix} K_{IOD} & 0 \end{bmatrix} & I \end{bmatrix} \geq 0 \qquad (18)$$

where $\mathcal{S}_{IOD} \triangleq \operatorname{diag}\{S, 0\}$, $\mathcal{A}_{DD} = \begin{bmatrix} I & I \\ (\Phi(\alpha(t)) - I + G(\alpha(t))K_{IOD}) & -I \end{bmatrix}$. Hence, the ellipsoidal sets $\mathcal{E}_{DD} := \text{Proj}_x\{\bar{x} \in \mathbf{R}^{2n} \,|\, \bar{x}^T E^T P_{DD} E^T \bar{x} \leq 1\} = \{x \in \mathbf{R}^n \,|\, x^T Q_{DD} x \leq 1\} \subset \mathbf{R}^n$, $\mathcal{E}_{IOD} := \text{Proj}_x\{\bar{x} \in \mathbf{R}^{2n} \,|\, \bar{x}^T E^T P_{IOD} E^T \bar{x} \leq 1\} = \{x \in \mathbf{R}^n \,|\, x^T Q_{IOD} x \leq 1\} \subset \mathbf{R}^n$ arising from the inequalities (12), (13) and (17), (18) are robust positively invariant regions for the state evolutions of the closed-loop systems (8), (16) and the input constraints (4) are satisfied, viz. $K_{DD}\mathcal{E}_{DD} \subset \mathcal{U}$, $K_{IOD}\mathcal{E}_{IOD} \subset \mathcal{U}$.

From now on we will assume that there exists a **DD** pair $(K_{DD}, \mathcal{E}_{DD})$, with $\mathcal{E}_{DD} \neq \emptyset$, and an **IOD** pair $(K_{IOD}, \mathcal{E}_{IOD})$, with $\mathcal{E}_{IOD} \neq \emptyset$, such that the closed-loop system

$$x_p(t+1) = \begin{cases} \Phi(\alpha(t))x_p(t) + G(\alpha(t))K_{DD}x_p(t-\tau(t)), \tau(t) \leq \tau_{max}; \\ \Phi(\alpha(t))x_p(t) + G(\alpha(t))K_{IOD}x_p(t-\tau(t)), \tau(t) \leq \bar{\tau}, \end{cases} \qquad (19)$$

complies with the prescriptions of the **NCS problem**.

B. One-step ahead Ellipsoidal controllable sets

Given the plant (1) and assuming a time-delay free scenario it is possible to compute the sets of states i-step controllable to a given target set $\mathcal{T}$ as follows:

$$\begin{array}{l} \mathcal{T}_0 := \mathcal{T} \\ \mathcal{T}_i := \{x_p : \exists u \in \mathcal{U} : \forall \alpha \in \mathcal{P}_l, \Phi(\alpha)x_p + G(\alpha)u \in \mathcal{T}_{i-1}\} \end{array} \qquad (20)$$

where $\mathcal{T}_i$ is the set of states that can be steered into $\mathcal{T}_{i-1}$ using a single move with a causal control [3].

To generalize such a concept to the proposed framework, it is important to notice that the one-step state predictions need to be evaluated on the basis of the model (5). Therefore, when on-line exploited, the predictions could be different from those generated by using the process description (1) due to the presence of time-delay occurrences, see Fig. 2. There,

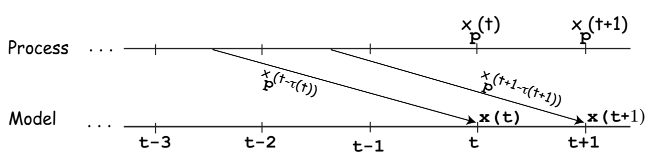

Fig. 2. Process/model discrepancy

at the generic time instant t, the model (5) uses as current state the measurement generated $\tau(t)$ instants before, i.e. $x(t) = x_p(t - \tau(t))$, while the process state is $x_p(t)$. Since in general $x(t) \neq x_p(t)$, there exists a unavoidable discrepancy between the process (1) and the model (5) that, if not properly treated, can lead to erroneous input computations. There is in fact no guarantee that if $x(t) \in \mathcal{T}_i$ the same holds true for $x_p(t)$. A possible way to comply with the above reasoning is to determine the sequence of sets $\mathcal{T}_i$ by resorting to the following one-step transition map, valid for $0 \leq \tau(t) \leq \tau_{up}$, where τ_{up} could be either τ_{max} or $\bar{\tau}$,

$$x(t+1) = \Phi(\alpha(t))x(t - \tau(t)) + G(\alpha(t))u(t) \qquad (21)$$

where the delayed state $x(t - \tau(t))$ is instrumental to take care at each instant t the difference between the state measurement $x(t)$ and the *real* plant state $x_p(t)$.
Therefore, the sequence of controllable sets should be derived by explicitly considering time-delay occurrences, i.e.

$$\begin{array}{rl} \mathcal{T}_i := & \{x : \exists u \in \mathcal{U} : \forall \alpha \in \mathcal{P}_l, \\ & \Phi(\alpha)x(t-\tau(t)) + G(\alpha)u \in \mathcal{T}_{i-1}, \forall \tau(t) \in [0,\ \tau_{up}]\} \end{array} \qquad (22)$$

The latter means that if $x(t) \in \mathcal{T}_i$ with $x(t) \neq x_p(t)$, the same holds for $x_p(t)$, and there exists a command $u(t)$ that drives $x(t+1)$ into $\mathcal{T}_{i-1}$ for all $\tau(t) \in [0,\ \tau_{up}]$. To recast such an idea into a computable scheme, explicit time-delay dependencies in the auxiliary model (21) need to be derived.
This can be done as follows: by re-writing w.l.o.g. (21) as

$$x(t+1) = \Phi(\alpha(t))x(t) + \Phi(\alpha(t))x(t-\tau(t)) + G(\alpha(t))u(t) \qquad (23)$$

and by considering the auxiliary state $y(t)$, the following descriptor form results

$$\begin{bmatrix} x(t+1) \\ 0 \end{bmatrix} = \begin{bmatrix} y(t) + x(t) \\ -y(t) + \Phi(\alpha(t))x(t) \\ +\Phi(\alpha(t))x(t-\tau(t)) + G(\alpha(t))u(t) - x(t) \end{bmatrix} \qquad (24)$$

By noticing that $x(t-\tau(t)) = x(t) - \sum_{j=t-\tau(t)}^{t-1} y(j)$, by imposing $y(t-1) = y(t-2) = \cdots = y(t-\tau_{up})$ (worst-case scenario) and by defining the augmented state $x_{aug}(t) = \left[x^T(t)\ y^T(t)\ x(t-\tau_{up})^T(t)\ y(t-\tau_{up})^T(t)\right]^T$, we have

$$\bar{E}_{aug}\bar{x}_{aug}(t+1) = \bar{\Phi}(\alpha(t))_{aug}\bar{x}_{aug}(t) + \bar{G}(\alpha(t))_{aug}u(t) \quad (25)$$

with $\bar{E}_{aug} = \operatorname{diag}\{I, 0, \tau_{up}I, 0\}$, $\bar{\Phi}(\alpha(t)) = \begin{bmatrix} I & I \\ 2\Phi(\alpha(t)) - I & -I \end{bmatrix}$ and

$$\bar{\Phi}(\alpha(t))_{aug} = \begin{bmatrix} I & I & 0 & 0 \\ 2\Phi(\alpha(t)) - I & -I & 0 & 0 \\ 0 & 0 & I & I \\ 0 & 0 & \Phi(\alpha(t)) - \tau_{up}I & -I \end{bmatrix}$$

$$\bar{G}(\alpha(t))_{aug} = \begin{bmatrix} 0 & G(\alpha(t)) & 0 & 0 \end{bmatrix}^T$$

Therefore, the following recursions hold true

$$\begin{array}{l} \mathcal{T}_i := \text{Proj}_x\{\bar{x}_{aug} \in \mathbf{R}^{4n} : \exists u \in \mathcal{U} : \forall \alpha \in \mathcal{P}_l, \\ \text{Proj}_x\{\bar{\Phi}(\alpha)_{aug}\bar{x}_{aug} + \bar{G}(\alpha)_{aug}u\} \in \mathcal{T}_{i-1}\} \end{array} \qquad (26)$$

C. Off-line time-delays and data-losses management

Let us start by considering time-delay occurrences within $[0,\ \bar{\tau}]$. By resorting to the ideas developed in Section III, the time-delay can be managed by computing two one-step sequences of controllable ellipsoidal regions with $N+1$ elements ($N > 0$) $\{\mathcal{T}_i^{DD}\}_{i=0}^N$ and $\{\mathcal{T}_i^{IOD}\}_{i=0}^N$, such that

$$\mathcal{T}_0^{DD} \subseteq \bigcup_{i=0}^{N} \mathcal{T}_i^{IOD} \qquad (27)$$

The key idea can be stated as follows: the above two sequences are achieved on the hypothesis that the time-delay occurrence is $\tau(t) \leq \tau_{max}$ and $\tau_{max} < \tau(t) \leq \bar{\tau}_{max}$ for $\{\mathcal{T}_i^{DD}\}_{i=0}^N$ and for $\{\mathcal{T}_i^{DD}\}_{i=0}^N$, respectively. Now, at each time instant and on the basis of the information $\tau(t)$, if the current measurement $x_p(t-\tau(t))$ belongs to $\mathcal{T}_i^{IOD}$ (resp. $\mathcal{T}_i^{IOD}$), there exists a command u_i, compatible with (4), capable to drive the state to $\mathcal{T}_{i-1}^{DD}$ (resp. $\mathcal{T}_{i-1}^{IOD}$). Therefore, there exists an admissible control strategy which steers in a finite number of steps any initial state $x(0) \in \bigcup_{i=0}^{N} \mathcal{T}_i^{DD}$ $\left(\text{resp. } \bigcup_{i=0}^{N} \mathcal{T}_i^{IOD}\right)$ to the terminal (target) set $\mathcal{T}_0^{DD}$ (resp. $\mathcal{T}_0^{IOD}$).

Let us now consider a data-loss event occurrence:

Definition 1: There exists an arbitrarily large time period $\Delta_t := [\bar{t}_{on},\ \bar{t}_{fin}]$ such that $\tau(t) > \bar{\tau},\ \forall t \in \Delta_t$.

Under data-losses, the combined use of the sequences $\{\mathcal{T}_i^{DD}\}_{i=0}^N$ and $\{\mathcal{T}_i^{IOD}\}_{i=0}^N$ is not able to deal with all the time-varying delay occurrences. In fact, due to its unpredictable nature, it may happen that when the current state $x(t)$ lies in any set of the ellipsoids sequences $\{\mathcal{T}_i^{DD}\}_{i=0}^N$ and $\{\mathcal{T}_i^{IOD}\}_{i=0}^N$, then there is no guarantee on the size of Δ_t. In fact let us assume that at the generic time instant t $x(t) \in \mathcal{T}_i^{DD}$, $i < N$, then the maximum size of Δ_t is equal to i, i.e. only i-time steps without state measurements are allowable. There always exists an input virtual sequence, namely $\{u_i, u_{i-1}, \ldots, u_1\}$, such that the state trajectory is driven to $\mathcal{T}_0^{DD}$ at the $(i+1)-th$ time step, where the control action is generated by K^{DD} which requires a state measurement. The same reasoning applies for the sequence $\{\mathcal{T}_i^{IOD}\}_{i=0}^N$. Such a drawback is here overcome by computing the **DD** and **IOD** sequences under the following condition:

Statement 1: Let $x^+ := \Phi(\alpha)x,\ \forall \alpha \in \mathcal{P},\ \forall x \in \mathcal{T}_0^{DD}$ and $\forall x \in \mathcal{T}_0^{IOD}$ be the one-step state evolution under zero-input $u \equiv 0_m$, then

$$x^+ \subseteq \left(\bigcup_{i=0}^{N} \mathcal{T}_i^{DD}\right) \bigcup \left(\bigcup_{i=0}^{N} \mathcal{T}_i^{IOD}\right) \tag{28}$$

If (28) holds, one has that at the $(i+1)-th$ time step the zero input $u \equiv 0_m$ can be applied in place of the feedback gain K^{DD} when no state measurements are available.

IV. On-line phase

The on-line phase is devoted to consider time-delay occurrences by taking advantage of the MPC philosophy.

At each instant t the algorithm derives first the set containing $x_p(t-\tau(t))$ and three scenarios could arise:

- **a)** If $\tau(t) \leq \tau_{max}$: the smallest index i such that $x_p(t-\tau(t)) \in \mathcal{T}_i^{DD}$ is selected;
- **b)** If $\tau_{max} < \tau(t) \leq \bar{\tau}$: if $x_p(t-\tau(t)) \in \mathcal{T}_i^{IOD}$ then the set $\mathcal{T}_i^{IOD}$ is selected, otherwise determine the smallest index i such that $x_p(t-\tau(t)) \in \mathcal{T}_i^{DD}$;
- **c)** If $\tau(t) > \bar{\tau}$: the state $x_p(t-\tau(t))$ is not available for checking its membership to **IOD** or to **DD** sets.

Let $x(t) = x_p(t-\tau(t))$ be the delayed state and $x_{-1}(t)$ the state measurement stored at the previous time instant $t-1$. Then, an admissible input $u(t)$ is computed by minimizing a given performance index $J_{i(t)}(x(t),u)$ (see Appendix):

- If **a)** holds true then

$$u(t) = \arg\min J_{i(t)}(x(t),u) \quad \text{s.t.} \tag{29}$$

$$\Phi_j x(t) + G_j u \in \mathcal{T}_{i(t)-1}^{DD},\ u \in \mathcal{U},\ j = 1,\ldots,l \tag{30}$$

- The case **b)** gives rise to the following situations:
  - $x(t) \in \mathcal{T}_i^{IOD}$:

$$u(t) = \arg\min J_{i(t)}(x(t),u) \quad \text{s.t.} \tag{31}$$

$$\Phi_j x(t) + G_j u \in \mathcal{T}_{i(t)-1}^{IOD},\ u \in \mathcal{U},\ j = 1,\ldots,l \tag{32}$$

  - $x(t) \in \mathcal{T}_i^{DD}$:

$$u(t) = \arg\min J_{i(t)}(x(t),u), \quad \text{s.t.} \tag{33}$$

$$\Phi_j x(t) + G_j u \in \mathcal{T}_{i(t)-1}^{DD}, u \in \mathcal{U}, j = 1,\ldots,l, \tag{34}$$

$$\Phi_j x + G_j u \in \mathcal{T}_{i(t)-1}^{DD}, u \in \mathcal{U}, j = 1,\ldots,l, \forall x \in \mathcal{T}_{i(t)}^{DD} \tag{35}$$

- The case **c)** envisages the following events:
  1) if $x_{-1}(t) \in \mathcal{T}_{i+1}^{DD}$:

$$[\hat{x}(t),\, u(t)] = \arg\min J_{i(t)}(\hat{x},u) \quad \text{s.t.} \tag{36}$$

$$\Phi_j \hat{x} + G_j u \in \mathcal{T}_{i(t)-1}^{DD}, \hat{x} \in \mathcal{T}_{i(t)}^{DD}, u \in \mathcal{U}, j = 1,\ldots,l, \tag{37}$$

$$\Phi_j x + G_j u \in \mathcal{T}_{i(t)-1}^{DD}, u \in \mathcal{U}, j = 1,\ldots,l, \forall x \in \mathcal{T}_{i(t)}^{DD} \tag{38}$$

  2) if $x_{-1}(t) \in \mathcal{T}_{i+1}^{IOD}$, then solve (36)-(38) with $\mathcal{T}_i^{IOD}$;
  3) if $x_{-1}(t) \in \mathcal{T}_0^{DD}$ or $x_{-1}(t) \in \mathcal{T}_0^{IOD}$, then use the couple $(\hat{x}(t) = x_{-1}(t), u \equiv 0_m)$.

Remark 1- The scenario **a)** is addressed by following the delay-free MPC scheme in [1] because the bank of precomputed ellipsoids $\{\mathcal{T}_i^{DD}\}_{i=0}^N$, based on the pair $(K_{DD}, \mathcal{E}_{DD})$, can be on-line exploited for the one-step state predictions regardless of any time-delay occurrence. The situation becomes more cumbersome when the case **b)** is taken into consideration. If the delayed state measurement belongs to some $\mathcal{T}_i^{DD}$, the free-delay scheme cannot be directly applied: in fact if the scheme exploited for **a)** would be applied, after N steps the state will belong to $\mathcal{T}_0^{DD}$ where we should have to consider the law $u(\cdot) = K_{DD}x(\cdot)$ that is not designed for managing time-delays greater than τ_{max}. Then the above situation can be considered as a data-loss event and $u(t)$ computed by imposing that $\Phi_j x(t) + G_j u$ belongs to $\mathcal{T}_{i-1}^{DD}, \forall x \in \mathcal{T}_i^{DD}$ i.e. solve the optimization (33)-(35). Finally, when the new state measurement belongs to some $\mathcal{T}_i^{IOD}$ or $\tau(t) \leq \tau_{max}$, the optimizations (29)-(30) or (31)-(32) are respectively solved (see details in Appendix). □

Remark 2- Let us consider the data-loss scenario **c)**. By noticing that the state at the actual time instant t has been generated starting from the measurement $x_{-1}(t)$ at $t-1$ by applying an admissible input $u(t-1)$. Then, if $x_{-1}(t) \in \mathcal{T}_{i+1}^{DD}$ this implies that $x(t) \in \mathcal{T}_i^{DD}$, $i > 0$. Now the *real* value of $x(t)$ is unknown but a *worst-case* approach can be used to determine a *virtual* state $\hat{x}(t) \in \mathcal{T}_i^{DD}$ such that the computed one-step state evolution $x(t+1)$ is the *worst* under the minimization of the cost $J_{i(t)}(\hat{x},u)$. The latter translates into the solution of the optimization (36)-(38). A slight difference arises when $x_{-1}(t) \in \mathcal{T}_0^{DD}$ or $x_{-1}(t) \in \mathcal{T}_0^{IOD}$, because $\mathcal{T}_0^{DD}$ and $\mathcal{T}_0^{IOD}$ are robust positively invariant sets and therefore $x_{-1}(t)$ directly represents the worst case for the one-step state ahead prediction. Specifically, when the measurement $x_{-1}(t)$ belongs to $\mathcal{T}_0^{DD}$ or $\mathcal{T}_0^{IOD}$, in virtue of (28), the zero-input $u(t) \equiv 0_m$ is used such that $\Phi(\alpha)x_{-1}(t)$ belongs to some $\mathcal{T}_i^{DD}$ or $\mathcal{T}_i^{IOD}$. In principle the use of $u(t) \equiv 0_m$ cannot take place sequentially in time, but if $x^+ \subseteq \mathcal{T}_0^{DD}$ (resp. $\mathcal{T}_0^{IOD}$) an iterative application is admissible until x^+ escapes from the terminal ellipsoid or a time-delay latency $\tau(t) \le \bar{\tau}$ occurs. □

V. RHC ALGORITHM

The above developments allow to synthesize the following Receding-Horizon control strategy.

Assumption - At the initial time instant $t = 0$, the process (1) and the model (5) are synchronized. □

NCS-MPC-Algorithm -

Off-line -

0.1 *Given the scalars τ_{max} and $\bar{\tau}$, compute the nonempty robust invariant ellipsoidal regions $\mathcal{T}_0^{DD} \subset \mathbf{R}^n$, $\mathcal{T}_0^{IOD} \subset \mathbf{R}^n$ and the stabilizing state feedback gains K^{DD}, K^{IOD};*

0.2 *Generate the sequences of N one-step controllable sets $\mathcal{T}_i^{DD}$ and $\mathcal{T}_i^{IOD}$ complying with (27) and (28);*

0.3 *Store the ellipsoids $\{\mathcal{T}_i^{DD}\}_{i=0}^N$ and $\{\mathcal{T}_i^{IOD}\}_{i=0}^N$.*

On-line -

1.1 *Let $x(t) = x_p(t-\tau(t))$ be the most recent available state measurement. Check $x(t)$:*

1.2.1 *if $x(t) \in \mathcal{T}_i^{DD}$ (resp. $\mathcal{T}_i^{IOD}$) with $\mathcal{T}_i^{DD}$ the set imposed at $t-1$ by the input $u(t-1)$, then $x(t)$ is an admissible measurement;*

1.2.2 *else discard $x(t)$. If a new measure is available goto Step 1.2.1, otherwise consider a data-loss event.*

1.3 a- *If $\tau(t) \le \tau_{max}$ find $i(t) := \min\{i : x(t) \in \mathcal{T}_i^{DD}\}$*

1) *If $i(t) = 0$ then $u(t) = K^{DD}x(t)$*

2) *else solve (29)-(30);*

b- *If $\tau_{max} < \tau(t) \le \bar{\tau}$ then*

1) *If there exists $i(t) := \min\{i : x(t) \in \mathcal{T}_i^{IOD}\}$ then*

∗ *If $i(t) = 0$ then $u(t) = K^{IOD}x(t)$*

∗ *else solve (31)-(32);*

2) *else find $i(t) := \min\{i : x(t) \in \mathcal{T}_i^{DD}\}$ and solve (33)-(35);*

c- *If $\tau(t) > \bar{\tau}$*

1) *If $i(t) := \min\{i : x_{-1}(t) \in \mathcal{T}_i^{DD}\}$ then*

∗ *If $i(t) = 0$ then $\hat{x}(t) = x_{-1}(t)$, $u(t) = 0_m$*

∗ *else solve (36)-(38);*

2) *else if $i(t) := \min\{i : x_{-1}(t) \in \mathcal{T}_i^{IOD}\}$ then*

∗ *If $i(t) = 0$ then $\hat{x}(t) = x_{-1}(t)$, $u(t) = 0_m$*

∗ *else solve (36)-(38) with $\mathcal{T}_i^{IOD}$;*

1.4 – *If $0 \le \tau(t) \le \bar{\tau}$ then apply $u(t)$ from step* **a-** *or* **b-***;*

– *else apply $(\hat{x}(t), u(t))$ from step* **c-** *and update $x_{-1}(t+1) = \Phi(\alpha(t))\hat{x}(t)$.*

1.5 *$t := t+1$; goto 1.1.*

The next proposition proves feasibility retention and closed-loop stability of the proposed **MPC-NCS-Algorithm**.

Proposition 1: Let the sequences of sets $\mathcal{T}_i^{DD}$ and $\mathcal{T}_i^{IOD}$ be non-empty and $x(0) \in \left(\bigcup_i \mathcal{T}_i^{DD}\right) \cup \left(\bigcup_i \mathcal{T}_i^{IOD}\right)$. Then, the **MPC-NCS-Algorithm** always satisfies the constraints and ensures robust stability.

Moreover if the sequence of data-losses is finite, there exists a finite time $\bar{t}$ such that $x(t) \in \mathcal{T}_0^{DD} \cup \mathcal{T}_0^{IOD}$, $\forall t \ge \bar{t}$.

Proof - The proof follows by using similar arguments of [1] and by collecting the discussions in **Remarks** 1 - 2. □

VI. ILLUSTRATIVE EXAMPLE

The aim of this section is to test the effectiveness of the proposed MPC strategy to deal with time-varying and data-loss scenarios. We consider the uncertain multi-model process described by the following matrices:

$$\Phi(\alpha) = \begin{bmatrix} 1 & 0 \\ 0 & 1.01+\alpha \end{bmatrix}, \; G(\alpha) = \begin{bmatrix} -0.02 \\ -0.01+\alpha \end{bmatrix},$$

with $|\alpha| \le 0.08$. The objective is to regulate the state trajectory to the origin in the presence of the following input saturation constraint $u^2(t) \le 10$, $\forall t$. For this simulation the time-delay occurrences are depicted in Fig. 3 with $\tau_{max} = 15$ and $\bar{\tau} = 25$. These numerical values have been approximated by feasibility checks on the **DD** conditions (12)-(13) and **IOD** conditions (17)-(18). Moreover, it has been supposed that data-loss events occur within $\Delta_t = [370,\ 630]$.

First, the **DD** and **IOD** terminal pairs are here reported:

$$K_{DD} = [-0.6837\ 20.5616],\ Q_{DD} = \begin{bmatrix} 7.7159 & 0.4011 \\ 0.4011 & 0.0329 \end{bmatrix}$$

$$K_{IOD} = [-0.3489\ 15.5650],\ Q_{IOD} = \begin{bmatrix} 7.3568 & 0.3086 \\ 0.3086 & 0.0225 \end{bmatrix}$$

Then two ellipsoidal families $\{\mathcal{T}_i^{DD}\}_{i=0}^N$ and $\{\mathcal{T}_i^{IOD}\}_{i=0}^N$, $N = 70$, have been computed under the requirements (27)-(28) and the initial state has been set to $x(0) = [7.987,\ 0.3]^T$. The state trajectory (continuous line) is depicted in Fig. 4 along with the pre-computed regions. Figs. 4-6 show the capability of the scheme to manage data-loss occurrences (grey-zone in Fig. 3). Notice that the no data-loss phases, although subject to large time-varying delays, give rise to *normal* behaviours as enlighten by considering the performance results (Fig. 5). On the other hand, the data-loss phase shows the specific merits of the proposed algorithm. At the time step 370, the actual state lies in $\mathcal{T}_0^{IOD}$ (Fig. 6) and by virtue of the constraint (28) the zero-input is applied. From now on an iterative use of $u = 0$ is admissible because the *virtual* state remains inside $\mathcal{T}_0^{IOD}$. At $t = 428$ the state evolution x^+ lies in $\mathcal{T}_2^{IOD}$, (see Fig. 6) and a new command input needs to be computed as it results in Fig. 5.

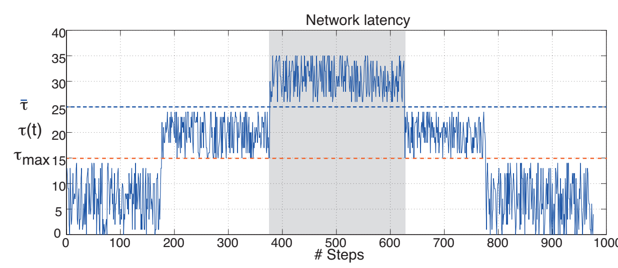

Fig. 3. Network latency

VII. CONCLUSIONS

In this paper, a novel discrete time receding horizon strategy for uncertain networked systems subject to input saturations and data-losses has been proposed. The key idea was to develop a control strategy based on set-invariance concepts

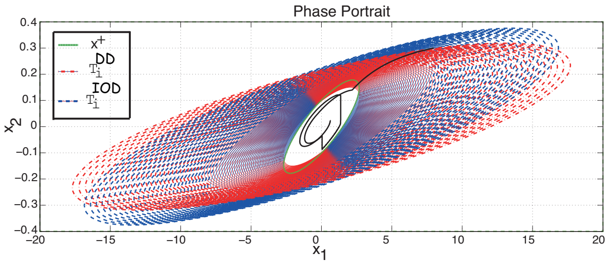

Fig. 4. Phase portrait and constraints (27)-(28) satisfaction

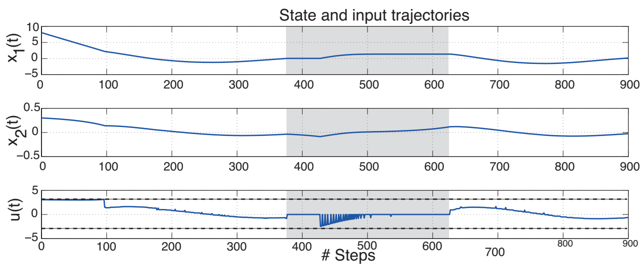

Fig. 5. Regulated state evolutions and commands

and ellipsoidal calculus to properly manage the absence on state measurements due to large network delays. First, the off-line phase has been built up in order to avoid any critical situation by imposing constraints on the construction of the one-step controllable sets. Then, the on-line optimization problems have been defined by accounting for both different time-delay occurrences and data-loss scenarios.

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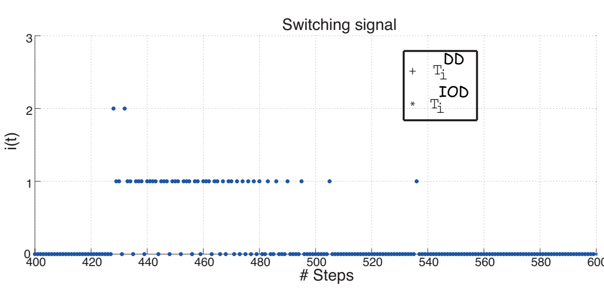

Fig. 6. Switching signal: zoomed graph on the time window [400, 600] # Steps

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APPENDIX - LMIs FOR (33)-(35) and (36)-(38)

First, let us consider the following running cost

$$J_{i(t)}(x(t),u)=\max_j \|\Phi_j x(t)+G_j u\|^2_{P^{DD}_{i(t)-1}} \qquad (39)$$

with $P^{DD}_{i(t)-1}>0$ the shaping matrix of $\mathcal{T}^{DD}_{i(t)-1}$.

Theorem 1: The optimization problem (33)-(35) can be solved by the following semi-definite programming problem:

$$\min_u \gamma_x^j \text{ s.t.} \qquad (40)$$

$$\begin{bmatrix} \gamma_x^j & (\Phi_j x(t)+G_j u)^T \\ & (P^{DD}_{i-1})^{-1}(t) \end{bmatrix} \geq 0, \; j=1,\ldots,l \qquad (41)$$

$$\begin{bmatrix} \bar{u} & u^T \\ & I \end{bmatrix} \geq 0 \qquad (42)$$

$$\begin{bmatrix} \gamma_x^j-\lambda_j & -u^T L_j^T \\ & I \end{bmatrix} \geq 0, \; j=1,\ldots,l \qquad (43)$$

where L_j is the Cholesky factor of $L_j^T L_j = G_j^T P^{DD}_{i-1}(t) G_j + G_j^T P^{DD}_{i-1}(t) \left(-\Phi_j^T P^{DD}_{i-1}(t)\Phi_j+\lambda_j P_i^{DD}(t)\right)^{-1} P^{DD}_{i-1}(t) G_j$ and the multipliers λ_j are computed *mutatis mutandis* as in [4]. Then, the following index is used

$$J_{i(t)}(\hat{x},u)=\max_{\left\{\begin{array}{c}\hat{x}\in\mathcal{T}_i^{DD}(t),\\ j=1,\ldots,l\end{array}\right.} \|\Phi_j\hat{x}+G_j u\|^2_{P^{DD}_{i(t)-1}} \qquad (44)$$

Theorem 2: The optimization problem (36)-(38) can be solved by the following semi-definite programming problem:

$$\min_{\hat{x},u} \gamma_x^j \qquad (45)$$

$$\text{s.t.} \begin{bmatrix} \gamma_x^j & (\Phi_j\hat{x}+G_j u)^T \\ & (P^{DD}_{i-1})^{-1}(t) \end{bmatrix} \geq 0, \; j=1,\ldots,l \qquad (46)$$

$$\begin{bmatrix} \bar{1} & \hat{x}^T \\ & (P_i^{DD})^{-1}(t) \end{bmatrix} \geq 0 \qquad (47)$$

$$(42)\text{-}(43)$$

Proofs of Theorems 1-2 follow similar arguments as in [4] and here omitted for the sake of space.