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3D Path Planning in a Threat Environment

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Abstract— **We address optimal path planning in three dimensional space for an unmanned aerial vehicle (UAV) in the stationary risk environment. We separate the task into two stage, in the first one we determine the risk optimal 2D path for fixed time problem. Then we solve the series of BVPs (Boundary Value Problems) with different UAV speeds and determine the admissible 2D path, which satisfies the time and risk constraints. In the last step one takes into account the relief along the chosen path and determine the approximated 3D path, which minimizes 2D threat along the path and satisfies other constraints.**

I. INTRODUCTION

Problem of the path planning in a threat environment is well known but stills to be in the focus of research related particularly to the mission planning of autonomous UAV. The main difficulty of the problem is the absence of exact information about the probabilities distributions of risks, such as possibility of detection of the UAV by the enemy sensor or/and radars, hitting the UAV by means of air defense, collision with obstacles, which are rather difficult to identify in exact mathematical terms. Meanwhile all these problems have to be solved by a mission planer usually in short time of the operation planning. One can mention the article [7], where authors search for the optimal 2D path of the minimal risk of the UAV detection in the presence of multiple radars. Their approach is based on the search for on-line solution with the aid of the spline trajectoty approximation. So, the idea of our work is to join together exact mathematical tools and interactive approaches. In the first stage one have to take into account the spatial distribution of the risks centers and to find the path which comes from initial to terminal point at given speed and minimizes the total specific risk of the mission failure. Then the choice of the speed will change the flight time, real value of the probability of the mission failure, and dynamic parameters such as the angular and linear accelerations along the paths. This is enough for the mission planer to make a decision and get the optimal 2D path of UAV.

Then he has to take into account the relief along the path. Of course, it would be nice to design the path in 3D immediately, taking into account the relief as a threat, distributed along the altitude, but this 3D optimal control problem becomes rather unstable and gives rather sophisticated solutions which don't have the real physical meaning [15]. So the easy approach is to get the altitude curve along the desired path and manually put some values of the altitudes desired from the planner point of view, afterwards the trajectory may be represented as some polynomial, approximating the 3D path and used by autopilot for UAV control. Our approach is focused on numerical procedures, so in the next Section 2 we consider the different threats' description and the problem statement. In Section 3 we consider the decomposition of the general problem into series of more simple, but realistic from applied point of view problems. At first stage we discuss the solution of the optimal control problem which leads to 2D BVP (Boundary Value Problem). In Section 4 we use this solution to get the admissible path and calculate 3D path and to approximate it by polynomial or spline. Discussion of the results are given in Section 5.

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II. PROBLEM STATEMENT. DESCRIPTION OF THREATS' RELIEF.

Problem of the trajectory planning is known long ago [8], [16] and is still in the focus of researchers' efforts [1], [5], [6], [12], [17], [18] particularly with respect to the autonomous UAV mission planning.

From the very beginning [8], the problem is stated as a problem of determining the path satisfies given initial and terminal conditions, which minimizes some integral functional (the integral of the hazard rate along the path), which corresponds to the probability of the mission failure in the case of Markov hazard model [4].

A. Models for threats' relief

Application of the optimal control methods requires the hazard relief and its derivatives. Typical risk distribution is described by following parameters:

- coordinates of threats' centers $(x_i, y_i), i = 1, ..., M$
- spatial distribution of the hazard rate $f_i(x, y)$, where (x, y) are the coordinates in the plane.

In the literature one can find different functions f_i used for the description of the risk distribution:

- **gaussian:** [4]

$$f_i(x,y) = \alpha_i\, exp\left\{-[\beta_i(x-x_i)^2 + \gamma_i(x-x_i)^2]\right\},$$

$\alpha_i, \beta_i, \gamma_i > 0$, unfortunately the rate of decrement of this function does not corresponds any physical phenomenon;
- **rational:** [6]

$$f_i(x,y) = \alpha_i \frac{V_i^m}{\rho_i(x,y)},$$

where $\alpha_i > 0$, V_i is a relative velocity of the UAV, and $i-th$ center of threat, $m > 0$, $\rho_i(x,y)$ is a distance between the UAV and $i-th$ center of threat. This model is much more realistic from physical point of view, and even gives the possibility to solve the problem of the optimal control explicitly [6] for one center of threat. However, the case of multiple threats' centers needs numerical solution and the singularity of f_i at the $i-th$ threats' center, creates very serious difficulties;
- **modified rational:** [18]

$$f_i(x,y) = \alpha_i(\theta) \frac{V_i^m}{(\rho_i(x,y))^n},$$

where the coefficient $\alpha_i(\theta)$ depends the UAV orientation and is equivalent to the effective reflection coefficient and $n = 2$ corresponds to a passive sensor, and $n = 4$ corresponds to a radar.
The last model is rather difficult for explicit solution, however in some cases admits it [18]. Meanwhile generally it has to be treated numerically with the same singularity problems as the ordinary rational model.
- **rational smooth:** basing on our research [12], [15] we choose a smoothed threats' rate distribution in the following form

$$f_i(x,y) = \frac{a_i}{(b_i + c_i(x-x_i)^2 + d_i(y-y_i)^2)^n},$$

with all a_i, b_i, c_i, d_i positive. This model is smooth enough to be treated numerically and has a real physical rate of decrease for different n, (see the case of modified rational risk distribution above).

The general stationary threats' relief is taken in the form

$$f(x,y) = \sum_{i=1}^{M} f_i(x,y), \tag{1}$$

which reflects the independence of threats' sources [4]. We assume also that the height relief is also given either as some function

$$h = h(x,y), \tag{2}$$

or in the form of digital map, giving the value of heights $h_j = h(x_j, y_j)$ in some set of points $(x_j, y_j), j = 1,..,N$.

B. Model of motion and the cost function

We assume the motion with constant speed. This problem, named as Markov-Dubins problem in 3D [14], and is described by the following equations

$$\begin{aligned} \dot{x}(t) &= V\cos\gamma(t)\cos\theta(t) \\ \dot{y}(t) &= V\sin\gamma(t)\cos\theta(t) \\ \dot{z}(t) &= V\sin\theta(t), \end{aligned} \tag{3}$$

where V is a constant speed, and the controls $\gamma(t) \in [-\pi, \pi]$, $\theta(t)[-\pi/2, \pi/2]$ are the yaw and pitch angles, respectively. In some works [5] the controls are the derivatives of yaw and pitch angles, however, such optimal control problem becomes very difficult for numerical solution. Meanwhile the value of speed V is still to be a control parameter as well and gives the possibility to adjust the angular velocities and acceleration in reasonable limits.

The purpose of the optimal control problem, is to find the path, satisfying the initial and terminal conditions

$$\begin{aligned} A &= (x(0), y(0)) = (x_0, y_0), \\ B &= (x(T), y(T)) = (x_T, y_T) \end{aligned} \tag{4}$$

conditions, the height constraints $z(t) \geq h(x(t), y(t))$, and perhaps another side constraints

$$z(t) \leq \tilde{h}(x(t), y(t)),$$

and providing the minimum to the cost function

$$J[x(\cdot), y(\cdot)] = \int_0^T f(x(t), y(t))dt \to \min_{x(\cdot), y(\cdot), T, V}. \tag{5}$$

Here T and V are the flight time and speed, respectively, and play role of control parameters.

So even in this very simple statement the problem belongs to the class of free boundary optimal control problem with phase constraints. From theoretical viewpoint it would be rather challenging to treat this problem numerically in general statement, however, the real world applications need more simple and realistic approach.

One can assume various simplifications of the problem, for example, the using of penalization approach to take into account the terminal conditions and the height constraints. We used this methodology in [15], where we take the cost function in the form

$$\begin{aligned} &J[x(\cdot), y(\cdot)] = \int_0^T [f(x(t), y(t)) + \phi(x(t), y(t), z(t)]dt + \\ &k[(x(T) - x_T)^2 + (y(T) - y_T)^2 + (z(T) - z_T)^2] \\ &\to \min_{x(\cdot), y(\cdot), z(\cdot), T, V}, \end{aligned} \tag{6}$$

with $k \gg 1$ and function

$$\phi(x,y,z) = \beta\left[\frac{1}{z - h(x,y)} + \frac{1}{h(x,y) + h_0 - z}\right], \tag{7}$$

which penalizes the inclination from the given height limits. However, the corresponding boundary value problem was rather unstable and the numerical procedure usually fails to converge [15].

III. Reduction to 2D optimal control problem

We decompose the problem by solving at the first stage the 2D optimal control problem with only yaw angle as a control and not taking into account the altitude constraints. So the reduced optimal control problem is:

A. Statement of 2D Optimal control problem

Dynamical model:

$$\begin{array}{l} \dot{x}(t) = V\cos\gamma(t) \\ \dot{y}(t) = V\sin\gamma(t). \end{array} \tag{8}$$

Control: $\gamma(t) \in [-\pi, \pi]$.
Parameters: $V \in [V_1, V_2]$, $\quad T \in [T_1, T_2]$.
Cost function:

$$J[x(\cdot), y(\cdot)] = \int_0^T f(x(t), y(t))dt \to \min_{T,V,x(\cdot),y(\cdot)}. \tag{9}$$

Initial and terminal conditions:

$$(x(0), y(0)) = (x_0, y_0), \quad (x(T), y(T)) = (x_T, y_T). \tag{10}$$

However, even this problem is difficult for solution since the free boundary (non fixed values of (T, V)) implies the additional terminal condition in the from of nonlinear equation [2] §2.7, §2.8.

B. Reduction to the auxiliary problem in the fixed time interval

Assume that the initial and terminal conditions (4), the relief of the hazard rate (1), the time of flight $T \in [T_1, T_2]$, and admissible speed $V \in [V_1, V_2]$ are given and satisfy the constraints

$$L = \sqrt{(x_T - x_0)^2 + (y_T - y_0))^2} \le V_2 T_1,$$
$$L = \sqrt{(x_T - x_0)^2 + (y_T - y_0))^2} \le V_1 T_2,$$

which are necessary for the existence of admissible solution.

For given (T, V), corresponding to admissible path, make a change of variable $s = t/T$, in (9), which gives

$$J = T\int_0^1 f(x(sT), y(sT))ds = T\int_0^1 f(\tilde{x}(s), \tilde{y}(s))ds, \tag{11}$$

where

$$\tilde{x}(s) = x(sT), \quad \tilde{y}(s) = y(sT)$$

satisfy the same terminal conditions

$$\begin{array}{l} (\tilde{x}(0), \tilde{y}(0)) = (x(0), y(0)) \\ (\tilde{x}(1), \tilde{y}(1)) = (x(T), y(T)), \end{array} \tag{12}$$

and the system of equations

$$\begin{array}{l} \dot{\tilde{x}}(s) = TV\cos\gamma(sT) = \tilde{V}\cos\tilde{\gamma}(s), \\ \dot{\tilde{y}}(s) = TV\sin\gamma(sT) = \tilde{V}\sin\tilde{\gamma}(s). \end{array} \tag{13}$$

Here the specified speed $\tilde{V}$ (indeed the length of the curve joining points A and B) and the time T satisfy the constraints

$$\tilde{V} \in [V_1 T, V_2 T], \quad \text{where} \quad T \in [T_1, T_2], \tag{14}$$

and the new control

$$\tilde{\gamma}(s) \in [-\pi, \pi]. \tag{15}$$

So one can state the auxiliary control problem of minimizing the auxiliary performance criterion

$$J^{aux} = \int_0^1 f(\tilde{x}(s), \tilde{y}(s))ds, \tag{16}$$

with dynamic equations (13), (15), where the specified speed $\tilde{V}$ satisfies the constraints (14).

C. Optimal path in the original problem

The minimal value of J^{aux} depends on specified speed $\tilde{V}$ which have to be chosen from constraints (14). For given real speed V we have the following relation

$$J[V, T, x(\cdot), y(\cdot)] = \frac{\tilde{V}}{V} J^{aux}[\tilde{V}, \tilde{x}(\cdot), \tilde{y}(\cdot)],$$

which gives the following relation between the solutions of auxiliary and original problems.

Proposition 1:

$$\begin{array}{l} \min\limits_{V,T,x(\cdot),y(\cdot)} J[V, T, x(\cdot), y(\cdot)] = \\ \min\limits_{V}\left[\min\limits_{\tilde{V},\tilde{x}(\cdot),\tilde{y}(\cdot)} \dfrac{\tilde{V}}{V} J^{aux}[\tilde{V}, \tilde{x}(\cdot), \tilde{y}(\cdot)]\right] = \\ \min\limits_{V}\left[\dfrac{1}{V}\min\limits_{\tilde{V}}\left[\tilde{V}\min\limits_{\tilde{x}(\cdot),\tilde{y}(\cdot)} J^{aux}[\tilde{V}, \tilde{x}(\cdot), \tilde{y}(\cdot)]\right]\right]. \end{array} \tag{17}$$

Remark 1: In (17) $\tilde{V}$ must satisfy (14).

Remark 2: Proposition 1 gives the following algorithm for the solution of the Problem III-A.

Description of the algorithm

1) Step 1: for given admissible (T, V) determine the limits of possible values of $\tilde{V}$.
2) Step 2: choose some admissible $\tilde{V}$ and solve the problem of $J^{aux} \to \min\limits_{\tilde{x}(\cdot),\tilde{y}(\cdot)}$ and determine the product $\tilde{V}\min J^{aux}(\tilde{V})$.
3) Step 3: repeat the step 2 for other admissible values of $\tilde{V}$, obtain the set of the BVP solutions and determine $\min\limits_{\tilde{V}}\left[\tilde{V}\min\limits_{(\tilde{x},\tilde{y})} J^{aux}(\tilde{V})\right]$.
4) Take the corresponding

$$\tilde{x}(s), \tilde{y}(s), \tilde{\gamma}(s), \tilde{V}$$

and determine the optimal path in Problem III-A as follows

$$x(t) = \tilde{x}(t/T), \quad y(t) = \tilde{y}(t/T), \quad \gamma(t) = \tilde{\gamma}(t/T),$$

where

$$T = \tilde{V}/V, \quad \min J = T\min J^{aux}.$$

D. Solution of the auxiliary problem with the aid of the maximum principle

We solve the auxiliary problem with the aid of penalization approach, so we consider the performance criterion

$$\tilde{J}^{aux} = \int_0^1 f(\tilde{x}(s), \tilde{y}(s))ds + \Phi(\tilde{x}(1), \tilde{y}(1))$$

$$= \int_0^1 f(\tilde{x}(s), \tilde{y}(s))ds + k\left[(\tilde{x}(1) - x_T)^2 + (\tilde{y}(1) - y_T)^2\right], \tag{18}$$

with sufficiently large $k \gg 1$, this permits to solve the problem without terminal conditions.

Introduce a Hamiltonian

$$H(\tilde{x}, \tilde{y}, \tilde{\gamma}, \psi_x, \psi_y) = \psi_x \tilde{V} \cos\tilde{\gamma} + \psi_y \tilde{V} \sin\tilde{\gamma} - f(\tilde{x}, \tilde{y}). \tag{19}$$

The optimal trajectory satisfies the following necessary optimality condition

Proposition 2: **Maximum principle.** Suppose that $(\tilde{x}, \tilde{y}, \tilde{\gamma})$ is the optimal path and the corresponding control in the problem (13), (15),(18). Then there exist the set of adjoint variables $\psi_x(s), \psi_y(s)$, such that $\psi_x^2(s) + \psi_y^2(s) \neq 0$, and satisfying the system of equations

$$\begin{aligned} \dot{\psi}_x(s) &= \left.\frac{\partial f(x,y)}{\partial x}\right|_{(\tilde{x}(s),\tilde{y}(s))}, \\ \dot{\psi}_y(s) &= \left.\frac{\partial f(x,y)}{\partial y}\right|_{(\tilde{x}(s),\tilde{y}(s))}, \end{aligned} \tag{20}$$

with terminal conditions

$$\psi_x(1) = -k(\tilde{x}(1) - x_T), \quad \psi_y(1) = -k(\tilde{y}(1) - y_T), \tag{21}$$

such that for almost all $s \in [0, 1]$ the optimal control provides the maximum to the Hamiltonian (19)

$$\tilde{\gamma}(s) = \underset{\tilde{\gamma}}{argmax}\, H(\tilde{x}(s), \tilde{y}(s), \tilde{\gamma}, \psi_x(s), \psi_y(s)), \tag{22}$$

which gives

$$\cos\tilde{\gamma}(s) = \frac{\psi_x(s)}{\sqrt{\psi_x^2(s) + \psi_y^2(s)}},$$

$$\sin\tilde{\gamma}(s) = \frac{\psi_y(s)}{\sqrt{\psi_x^2(s) + \psi_y^2(s)}}.$$

Boundary Value Problem

So the optimal path is the solution of the boundary value problems (BVP) for the system of ODE

$$\dot{\tilde{x}}(s) = \frac{\tilde{V}\psi_x(s)}{\sqrt{\psi_x^2(s) + \psi_y^2(s)}}, \quad \dot{\tilde{y}}(s) = \frac{\tilde{V}\psi_y(s)}{\sqrt{\psi_x^2(s) + \psi_y^2(s)}}, \tag{23}$$

and (20) for adjoint variables $(\psi_x(s), \psi_y(s))$ with initial $(x(0), y(0)) = (x_0, y_0)$ and terminal conditions (21).

We find the solution of this BVP with the aid of MathLab numerically.

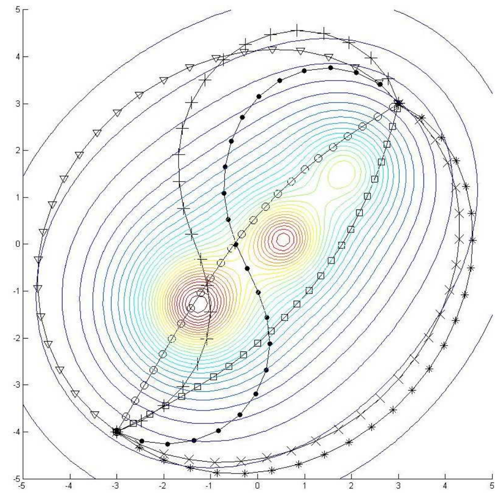

Fig. 1. Set of solutions of BVP for the paths from $A = (-3, -4)$ to $B = (3, 3)$ corresponding to different values of the specified speed $\tilde{V} \in [9.2, 14.56]$

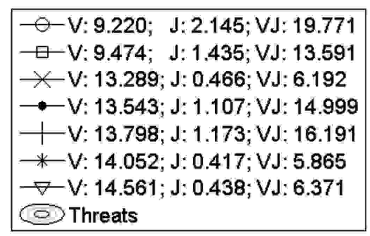

Fig. 2. Description of solutions of BVP for the path from $A = (-3, -4)$ to $B = (3, 3)$ corresponding to different values of the specified speed

E. Example od the solution of 2D BVP

The family of the BVP solutions corresponding to various values of specified speed $\tilde{V}$ is shown in Fig. 1. The best path, corresponding to the minimum value of risk is the path corresponding to the minimum value of the product $\tilde{V}J^{aux}$, which gives $\tilde{V} = 13.289$, $J^{aux} = 0.466$, $\tilde{V}J^{aux} = 6.192$. In this example the hazard rate has the following distribution

$$\begin{aligned} f(x,y) &= \frac{4.0}{(x+1.3)^2 + (y+1.3)^2 + 1.0} \\ &+ \frac{2.0}{(x-1.9)^2 + (y-1.6)^2 + 1.0} \\ &+ \frac{1}{(x-0.4)^2 + (y-0.1)^2 + 1.0} \\ &+ \frac{1}{(y-0.1)^2 + (x-0.6)^2 + 0.5}. \end{aligned}$$

Since $\tilde{V}$ is the length of the curve, then, for example, with $V = 2$ we get $T \approx 6.6$ and the risk integral $J = TJ^{aux} \approx 3.1$. In the Markov hazard rate model the probability of the mission successful accomplishment is a function of a type $P = \exp\{-J\}$ [4], so one can evaluate this probability as $P \approx 0.045$. Increasing of the speed up to $V = 4$ gives the probability $P \approx 0.21$.

Fig. 2 gives the description of various solutions of BVP, corresponding to different velocities.

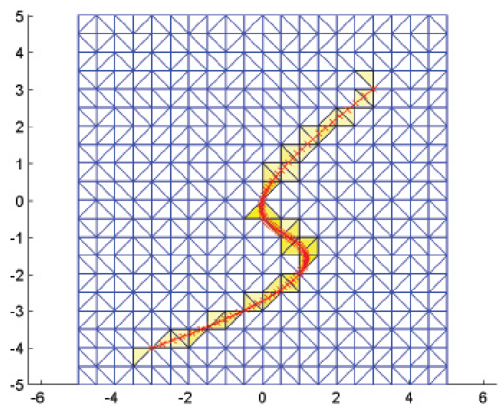

Fig. 3. Set of triangles (yellow color) corresponding to choosen path connecting points $A = (-3, -4)$ and $B = (3, 3)$.

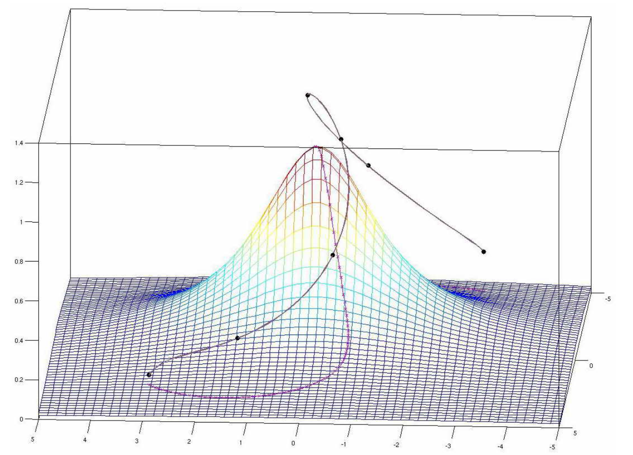

Fig. 4. The finally calculated 3D trajectory with heights above given terrain relief ($\bullet - \bullet - \bullet$..) and the original trajectory along given relief ($\times - \times - \times$..)

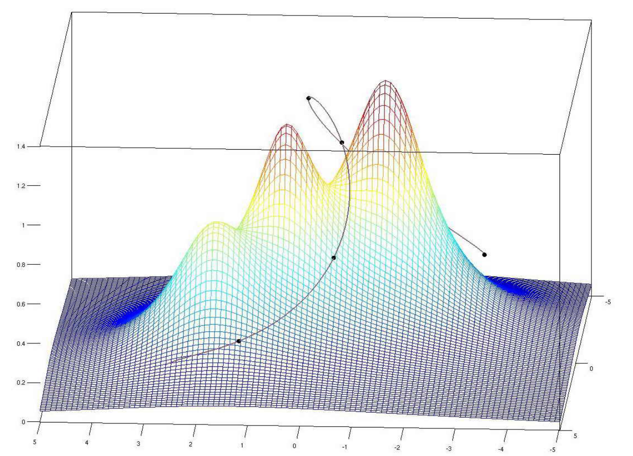

Fig. 5. The finally calculated 3D trajectory in given threats' relief

IV. Calculation of 3D trajectory

A. Delaunay triangulation for relief representation

Calculation of 3D trajectory needs the knowledge of the terrain relief. One can imagine that in the real situation of the path planning the digital map, giving the values of the terrain heights for some set of points will be given. However, for calculation of the height distribution along some 2D trajectory one need to know the height value in some intermediate points. Typical and well known approach is based on so called *Delaunay triangulation* $DT(P)$ for a set P of points in the plane. By definition the triangulation is such that no point in P is inside the circumcircle of any triangle in $DT(P)$. Delaunay triangulations maximize the minimum angle of all the angles of the triangles in the triangulation; they tend to avoid skinny triangles. The triangulation was invented by Boris Delaunay in 1934 [3]. The Delaunay triangulation permits to design the relief approximation even in the case of nonuniform distribution of points P, however, for our example we use the $DT(P)$ for uniformly distributed set of points P in the vertexes of rectangular mesh. There exists MatLab program which design the $DT(P)$ for an arbitrary set of of points in the plane.

For given 2D curve $(x(t), y(t))$ and designed triangulation for the set of points (x_i, y_i) one need to determine such i that $x(t) \in [x_i, x_{i+1}]$ and $y(t) = [y_i, y_{i+1}]$. At the next step it is necessary to determine the appropriate triangle for any give point $(x(t), y(t))$. For any given $(x(t), y(t))$ there is MatLab procedure which gives the triplet $A = (X_A, Y_A)$, $B = (X_B, Y_B)$ and $C = (X_C, Y_C)$ such that $(x(t), y(t)) \in \Delta ABC$.

We use the approximation of the height as a convex combination of the heights in the appropriate triangle vertexes. That is for given point $(x(t), y(t))$ and corresponding DT(P) we determine the coefficients of the convex combination from the system

$$\begin{aligned}&(x(t), y(t)) = \\ &\alpha_1(t)(x_A, y_A) + \alpha_2(t)(x_B, y_B) + \alpha_3(t)(x_C, y_C), \\ &\alpha_1(t) + \alpha_2(t) + \alpha_3(t) = 1\end{aligned}$$

Then approximated height is calculated as follows

$$\begin{aligned}&\tilde{h}(x(t), y(t)) = \\ &\alpha_1(t)h(x_A, y_A) + \alpha_2(t)h(x_B, y_B) + \alpha_3(t)h(x_C, y_C).\end{aligned}$$

B. 3D trajectory in given relief

For practical realization of this trajectory we defined heights above the relief along the 2D trajectory and approximate the resulting 3D trajectory by polynomial. Resulting 3D trajectory in model terrain relief

$$h(x, y) = \frac{1}{1 + 0.5x^2 + (y + 1)^2}$$

and in threats' relief are shown in Fig. 4 and Fig. 5, respectively.

C. Polynomial approximation of 3D trajectory

To calculate the trajectory we use the polynomial approximation of 3D curve. The equation of motion with the constant speed is given by the following system of equations

$$\begin{aligned} \dot{\bar{x}}(t) &= \frac{V\dot{x}(\mu(t))}{\sqrt{(\dot{x}(\mu(t)))^2 + (\dot{y}(\mu(t)))^2 + (\dot{z}(\mu(t)))^2}}, \\ \dot{\bar{y}}(t) &= \frac{V\dot{y}(\mu(t))}{\sqrt{(\dot{x}(\mu(t)))^2 + (\dot{y}(\mu(t)))^2 + (\dot{z}(\mu(t)))^2}}, \\ \dot{\bar{z}}(t) &= \frac{V\dot{z}(\mu(t))}{\sqrt{(\dot{x}(\mu(t)))^2 + (\dot{y}(\mu(t)))^2 + (\dot{z}(\mu(t)))^2}}, \\ \dot{\mu}(t) &= \frac{V}{\sqrt{(\dot{x}(\mu(t)))^2 + (\dot{y}(\mu(t)))^2 + (\dot{z}(\mu(t)))^2}}. \end{aligned} \tag{24}$$

In (24) $V = L/T$, where T is a given time of flight from A to B,

$$L = \int_0^1 \sqrt{(\dot{x}(s))^2 + (\dot{y}(s))^2 + (\dot{z}(s))^2} ds$$

is the path length and $x(s), y(s), z(s)$ is a polynomial approximation of 3D trajectory.

V. Conclusions

So in this article we present a way of calculation of the nominal trajectory of UAV in real terrain and threats' relief. The work is motivated by necessity of the development simple methods applicable in real field path planning. Moreover, the path planning in dangerous environment is the necessary step, preceeding the development of the trajectory admissible from the point of view of the maximal acceleration values, which can be easily adjusted by the choice of the speed. Moreover, this admissible or reference trajectory is necessary for development of the stabilization system. Further works will be devoted to the realization of such trajectory under random perturbations. The problem of stabilization of the UAV at the chosen path needs the solution of hybrid stochastic control problem, where it is necessary to identify the type of atmospheric perturbation, evaluate the perturbation magnitude and design the corresponding control law. We are going to describe the perturbation in terms of rarely appearing zones, appearing as changes of the state of Markov chain, such that it changes the stochastic model of the UAV motion. The estimation procedure must includes the mode estimation (in terms of conditional probabilities) and the states-velocities estimation which are used for the control law design. For small UAVs the most important types of such perturbations zones are: the zone of turbulence and/or the wind shift. These perturbations need the fast in-flight identification and using the corresponding control algorithm. One can assume that appearence of such zones can be described by a controlled Markov chain, there the role of control is not only the compensation of perturbation, but also selection of the speed, which from one side can help to overcome the perturbation zone faster, but at the same time increases the perturbation magnitudes and its influence. We are going to investigated these topics in further works. The basic approach to this class of problem is the using of general filtering scheme for the system driven by Markov chains [11] and then the application of the suboptimal estimation with the sub-optimal estimation of the change of state. Some successful examples of the application of this methodology has been presented in [9], [10].

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