2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

On the Performance Limit of Sensor Localization

Baoqi Huang, Tao Li, Brian D.O. Anderson, Changbin Yu

Abstract— **In this paper, we analyze the performance limit of sensor localization from a novel perspective. We consider distance-based single-hop sensor localization with noisy distance measurements by Received Signal Strength (RSS). Differently from the existing studies, the anchors are assumed to be randomly deployed, with the result that the trace of the associated Cramér-Rao Lower Bound (CRLB) matrix becomes a random variable. We adopt this random variable as a scalar metric for the performance limit and then focus on its statistical attributes. By the Central Limit Theorems for U-statistics, we show that as the number of anchors goes to infinity, this scalar metric is asymptotically normal. In addition, we provide the quantitative relationship among the mean, the standard deviation, the number of anchors, parameters of communication channels and the distribution of the anchors. Extensive simulations are carried out to confirm the theoretical results. On the one hand, our study reveals some fundamental features of sensor localization; on the other hand, the conclusions we draw can in turn guide us in the design of wireless sensor networks.**

I. Introduction

Location information plays a vital role in the applications of sensor networks, for it is useful to report the geographic origin of events, to assist in target tracking, to achieve geographic aware routing, to manage sensor networks, to evaluate their coverage, and so on. A sensor network generally consists of two kinds of nodes: anchors and sensors. Anchor positions are known *a priori* (e.g., through GPS or manual configurations), while sensor positions are unknown and need to be determined through certain procedures of localization. Up to now, considerable efforts have been invested in developing sensor localization algorithms.

Apart from designing sensor localization algorithms, the analysis of localization performance also gains much attention. Performance studies specific to sensor localization algorithms are realized to evaluate and compare different sensor localization algorithms. More importantly, the performance limit of sensor localization, namely the lower bound for location estimate errors produced by all localization algorithms, provides a theoretically optimal performance no matter what sensor localization algorithm is applied, and thus reflects fundamental impacts of various factors on sensor localization in an algorithm-independent manner. Due to the essence of Cramér-Rao Lower Bound (CRLB), it has been widely used to characterize the performance limit of sensor localization [1].

Most of the existing CRLB analysis is based on given sensor-anchor geometries. In this paper, we analyze the performance limit of single-hop sensor localization from a novel perspective. As commonly used in the literature, we adopt the trace of the associated CRLB matrix as a scalar metric for the performance limit of sensor localization [1]. However, differently from existing CRLB studies which require exact sensor-anchor geometries to compute the deterministic CRLB, we assume that a fixed number of sensors and anchors are randomly deployed in a two-dimensional plane with distance measurements from Received Signal Strength (RSS). Consequently, the trace of the associated CRLB matrix becomes a random variable with respect to the sensor-anchor geometries, and we focus on the statistical attributes of the trace of the CRLB.

The motivations of our study are as follows. In a mobile environment, such as ad-hoc networks, target tracking, Simultaneous Localization and Mapping (SLAM) [2], mobile anchors assisting in sensor localization [3] and so on, it is trivial to concentrate on the localization performance in one particular time instant, whereas it is attractive to grasp the average localization performance over a period of time and in a wide region. Hopefully, this can be solved by our statistically modeling method. Furthermore, the advantages of our study include: (i) it provides some knowledge about how the scalar metric, equivalently the minimal mean square estimation error (MSE), is distributed over all possible sensor-anchor geometries; (ii) the mean of the scalar metric reveals how the average minimal MSE with respect to all possible sensor-anchor geometries evolves with the number of anchors, the parameters of communication channels and the measurement noises; (iii) the ratio of the standard deviation to the mean indicates the sensitivity of the minimal MSE to sensor-anchor geometries; (iv) it not only provides insights into single-hop sensor localization including source localization and target tracking as specific cases, but also as a prototype paves the way for dealing with more complicated scenarios of sensor localization. In summary, statistical sensor-anchor geometry modeling is a powerful method for investigating the performance limit of sensor localization, which is an essential problem for sensor networks. To the best of our knowledge, this method has never been considered.

Essentially, this scalar metric is a function of U-statistics.

B. Huang is with the Australian National University and NICTA Ltd., Canberra, A.C.T., Australia, and also with Shandong Computer Science Center, Jinan, China. Email:Baoqi.Huang@anu.edu.au

B.D.O. Anderson is with the Australian National University and NICTA Ltd., Canberra, A.C.T., Australia. Email:Brian.Anderson@anu.edu.au

T. Li is with the Key Laboratory of Systems and Control, Institute of Systems Science, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China Email:litao@amss.ac.cn

C. Yu is with the Australian National University and NICTA Ltd., Canberra, A.C.T., Australia, and also with Shandong Computer Science Center, Jinan, China. Email:Brad.Yu@anu.edu.au. Correspondence: RSISE Building 115, The Australian National University, Canberra ACT 0200, Australia, Tel: +61 2 61258670, Fax: +61 2 61258660.

In statistical theory, U-statistics introduced by the seminal paper [4] are a class of important statistics, and are of great significance in estimation theory in that asymptotic properties of both estimators and test statistics have been derived by using the Central Limit Theorems for U-statistics. Based on the theory of U-statistics, we show that as the number of anchors goes to infinity, this scalar metric is asymptotically normal. We provide the quantitative relationship among the mean, the standard deviation, the number of anchors, parameters of communication channels and the distribution of the anchors. Since our results are based on an asymptotic analysis, the conditions under which our results approximate the real situations well are identified.

The remainder of this paper is organized as follows. The next section introduces the problem formulation. Section III presents the main results about statistical attributes of performance limits. Finally, we conclude this paper and shed light on future work in Section V.

II. Problem Formulation

In this section, we formulate the scalar metric of the performance of single-hop sensor localization using RSS measurements and define a random sensor-anchor geometry model. Throughout this paper, we shall use the following mathematical notations: $(\cdot)^T$ denotes transpose of a matrix or a vector; $Tr(\cdot)$ denotes the trace of a square matrix; $Pr\{\cdot\}$ denotes the probability of an event; $E(\cdot)$ denotes the expected value of a random variable; $Var(\cdot)$ denotes the variance; $Std(\cdot)$ denotes the standard deviation.

A. One-hop Sensor Localization Using RSS Measurements

In a two-dimensional plane, consider a single sensor (or source, target) located at the origin and N distance (or angle) measurements made to this sensor at N known locations, as illustrated in Figure 1. Here, the N known locations are abstracted as anchors and are labeled $1, \cdots, N$ with the i-th anchor's location denoted by $\mathbf{s}_i = [x_i, y_i]^T$. The true distance between the sensor and the i-th anchor is denoted by $d_i = \|\mathbf{s}_i\|$. The true angle subtended at the sensor by the i-th anchor and the positive x-axis is denoted θ_i.

For a specific localization problem, the precise locations of the N anchors, i.e. $[x_i, y_i]^T$, are given in advance; pair-wise distance measurements $\{\hat{d}_i, i = 1, \cdots, N\}$ between the sensor and the anchors are made and obey certain error models. Then, the aim of single-hop sensor localization is finding an estimate of the true sensor position using the observable set of distance measurements $\{\hat{d}_i, i = 1, \cdots, N\}$. In this paper, we consider the performance limit of sensor localization over a family of random anchor locations other than a specific localization problem with given anchor locations.

Let the sensor be a transmitter and the N anchors be receivers. Define $\{P_i, i = 1, \cdots, N\}$ to be the measured received signal powers at the N anchors transmitted by the sensor. We make the following assumptions:

Assumption 1: The wireless channel satisfies the log-normal (shadowing) model and the received powers $\{P_i, i = 1, \cdots, N\}$ at the N anchors are statistically independent.

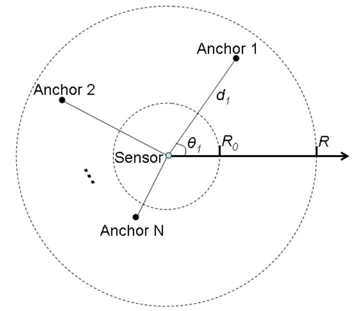


Fig. 1. Localizing a sensor using N anchors.

Remark 1: Assumption 1 is the basis for converting the RSS measurements (i.e. received powers) to distance estimates [5], and is commonly made in studies on RSS-based sensor localization (e.g. [1], [6]). It follows that $P_i(\text{dBm}) = 10\log_{10} P_i$ are Gaussian

$$P_i(\text{dBm}) = \overline{P_0}(\text{dBm}) - 10\alpha \log_{10} \frac{d_i}{R_0} + Z, \qquad (1)$$

where $\overline{P_0}(\text{dBm})$ is the mean received power in dBm at a reference distance R_0, α is the path-loss exponent, and Z is a random variable representing the shadowing effect, normally distributed with mean zero and variance σ_{dB}^2 (in dBm). As pointed out in [7], due to the fact that the log-normal model does not hold for $d_i = 0$, the close-in distance R_0 is introduced as the known received power reference point, and is virtually the lower bound on practical distances used in the wireless communication system. Further, $\overline{P_0}(\text{dBm})$ is computed from the free space path loss formula (see, e.g. [7]).

B. A Random Sensor-Anchor Geometry Model

Assumption 2: The N anchors are randomly and uniformly distributed inside the annulus centered at the sensor and defined by radii R_0 and R ($R > R_0 > 0$).

Remark 2: In Assumption 2, R is the upper bound on practical distances which is normally restricted by the factors determining path loss attenuations; R_0, though representing the lower bound, is mainly devised to avoid the inconvenience in calculations, and theoretically speaking, any arbitrarily small positive number can be the lower bound. By Assumption 2, each possible sensor-anchor geometry is as probable as another, in the sense that the sensor-anchor geometry follows a "uniform" distribution. Furthermore, it is easy to show that $\{d_i, i = 1, \cdots, N\}$ and $\{\theta_i, i = 1, \cdots, N\}$ are mutually independent.

C. The Scalar Metric

The probability density function (pdf) of P_i can be formulated as follows

$$f_P(P_i) = \frac{10}{(\ln 10)\sqrt{2\pi}\sigma_{dB} P_i} \exp\left\{-\frac{b}{2}\left(\ln \frac{d_i}{\tilde{d}_i}\right)^2\right\}, \qquad (2)$$

where $b = \left(\frac{10\alpha}{\sigma_{dB}\ln 10}\right)^2$ and $\tilde{d}_i = d_0\left(\frac{\overline{P_0}}{P_i}\right)^{\frac{1}{\alpha}}$.

For the purpose of computing the CRLB for sensor localization using the RSS measurements, we formulate the Fisher information matrix (FIM) F_{RSS} as

$$F_{RSS} = b \begin{pmatrix} \sum_{i=1}^{N} \frac{\cos^2 \theta_i}{d_i^2} & \sum_{i=1}^{N} \frac{\cos\theta_i \sin\theta_i}{d_i^2} \\ \sum_{i=1}^{N} \frac{\cos\theta_i \sin\theta_i}{d_i^2} & \sum_{i=1}^{N} \frac{\sin^2 \theta_i}{d_i^2} \end{pmatrix}. \quad (3)$$

A detailed derivation can be found in [1]. If F_{RSS} is non-singular, the CRLB, denoted C_{RSS}, is just the inverse of F_{RSS}. Then, we define $Tr(C_{RSS})$ to be a metric for the performance limit of localizing the sensor and have

$$Tr(C_{RSS}) = \frac{1}{b} \left(\frac{\sum_{i=1}^{N} \frac{1}{d_i^2}}{\sum_{1 \le i < j \le N} \frac{\sin^2(\theta_i - \theta_j)}{d_i^2 d_j^2}} \right). \quad (4)$$

Since $\{d_i, i = 1, \cdots, N\}$ and $\{\theta_i, i = 1, \cdots, N\}$ are random variables, $Tr(C_{RSS})$ is obviously a random variable.

D. U-statistics

U-statistics are very natural in statistical work, particularly in the context of independent and identically distributed (i.i.d.) random variables, or more generally for exchangeable sequences, such as in simple random sampling from a finite population. The origins of the U-statistics theory are traceable to the seminal paper [4], which proved the Central Limit Theorems for U-statistics. Following the publication of this seminal paper, the interest in this class of statistics steadily increased, crystallizing into a well-defined and vigorously developing line of research in probability theory. Its formal definition is presented as follows:

Definition 1: Let $\{X_i, i = 1, \cdots, N\}$ be i.i.d. p-dimensional random vectors. Let $h(x_1, \cdots, x_r)$ be a Borel function on $\mathbb{R}^{r \times p}$ for a given positive integer r $(\le N)$ and be symmetric in its arguments. A U-statistic U_N is

$$U_N = \frac{r!(N-r)!}{N!} \sum_{1 \le i_1 < \cdots < i_r \le N} h(X_{i_1}, \cdots, X_{i_r}) \quad (5)$$

and $h(x_1, \cdots, x_r)$ is called the kernel of U_N.

It is obvious that $Tr(C_{RSS})$ involves the ratio of two U-statistics according to (4), which motivates us to study $Tr(C_{RSS})$ through an asymptotic analysis based on the theory of U-statistics.

III. Main Results

Due to the complexity of $Tr(C_{RSS})$, it is very difficult to give its accurate distribution directly. As such, we endeavor to present an asymptotic analysis at first. Due to the space limit, proofs are omitted.

A. Theories

According to (4), a key property of $Tr(C_{RSS})$ is that it is the ratio of two sums of random variables, which can be processed by using the following lemma.

Lemma 1: Given $\{X_i^{(1)}, i = 1, \cdots, N\}$ and $\{X_i^{(2)}, i = 1, \cdots, N\}$ where

- $\{X_i^{(1)}, i = 1, \cdots, N\}$ are i.i.d. random variables with bounded values;
- $\{X_i^{(2)}, i = 1, \cdots, N\}$ are i.i.d. random variables with bounded values;
- $\{X_i^{(1)}, i = 1, \cdots, N\}$ and $\{X_i^{(2)}, i = 1, \cdots, N\}$ are mutually independent,

define vectors $X_i = [\ X_i^{(1)} \quad X_i^{(2)}\ ]^T$ $(i = 1, \cdots, N)$ and two sequences of random variables

$$T_N = \frac{1}{N} \sum_{i=1}^{N} X_i^{(1)}, \quad (6)$$

$$S_N = \frac{2}{N(N-1)} \sum_{1 \le i < j \le N} \Big[X_i^{(1)} X_j^{(1)} \times \sin^2(X_i^{(2)} - X_j^{(2)}) \Big]. \quad (7)$$

Then, as $N \to \infty$,

$$\frac{T_N}{S_N} = \frac{1}{m_1 m_2} + \frac{2\sigma_1^2}{N m_1^3 m_2} + M_N + R_N \quad (8)$$

where $m_1 = E(X_1^{(1)})$, $\sigma_1 = Std(X_1^{(1)})$, $m_2 = E(\sin^2(X_1^{(2)} - X_2^{(2)}))$,

$$M_N = \frac{2}{N} \sum_{i=1}^{N} g_1(X_i) + \frac{2}{N(N-1)} \sum_{1 \le i < j \le N} g_2(X_i, X_j), \quad (9)$$

$$g_1(X_i) = \frac{m_1 - X_i^{(1)}}{2 m_1^2 m_2}, \quad (10)$$

$$g_2(X_i, X_j) = \frac{1}{m_1 m_2} - \frac{X_i^{(1)} + X_j^{(1)}}{m_1^2 m_2} + \frac{2 X_i^{(1)} X_j^{(1)}}{m_1^3 m_2} - \frac{X_i^{(1)} X_j^{(1)} \sin^2(X_i^{(2)} - X_j^{(2)})}{m_1^3 m_2^2}, \quad (11)$$

and R_N is the remainder term. For any $\varepsilon > 0$, R_N satisfies

$$Pr\{|N R_N| \ge \varepsilon\} = O(N^{-1}), \quad (12)$$

$$Pr\{|N(\ln N) R_N| \ge \varepsilon\} = o(1), \quad (13)$$

In Lemma 1, by letting $X_i^{(1)} = \frac{1}{d_i^2}$ and $X_i^{(2)} = \theta_i$, we have $m_2 = 0.5$, and

$$m_1 = 2 \left(\frac{\ln \frac{R}{R_0}}{R^2 - R_0^2} \right), \quad (14)$$

$$\sigma_1 = \sqrt{\frac{1}{R_0^2 R^2} - \left(\frac{2 \ln \frac{R}{R_0}}{R^2 - R_0^2} \right)^2}, \quad (15)$$

and our main result is further summarized as follows.

Theorem 1: Let m_1 and σ_1 be defined by (14) and (15). Define a sequence of random variables

$$W_N = \left(\frac{\sqrt{N}(N-1) b m_1^2}{4\sigma_1} \right) Tr(C_{RSS}) - \frac{\sqrt{N} m_1}{\sigma_1} - \frac{2\sigma_1}{\sqrt{N} m_1}. \quad (16)$$

Then, as $N \to \infty$, W_N converges in distribution to a standard normal random variable.

Remark 3: In view of the linear relationship between W_N and $Tr(C_{RSS})$, it is clear that $Tr(C_{RSS})$ is asymptotically normal. Therefore, for a sufficiently large N, the distribution of $Tr(C_{RSS})$ can be approximated by the normal distribution

$$\mathcal{N}\left(\frac{4}{(N-1)bm_1}\left(1+\frac{2\sigma_1^2}{Nm_1^2}\right),\left(\frac{4\sigma_1}{\sqrt{N}(N-1)bm_1^2}\right)^2\right). \tag{17}$$

Most importantly, the above normal random variable makes it possible for us to analytically study the performance limit, i.e. $Tr(C_{RSS})$. Firstly, we can obtain a comprehensive knowledge about how $Tr(C_{RSS})$ is statistically distributed and how $Tr(C_{RSS})$ is affected by N. Secondly, using the normal distribution function from (17), we can compute the probability that $Tr(C_{RSS})$ is below a given threshold for a known value of N; in turn, we can determine a threshold such that $Tr(C_{RSS})$ is below the threshold with a certain confidence level, say 0.99; in addition, we can find the minimum N such that $Tr(C_{RSS})$ is below a given threshold with a certain confidence level. Such analysis is undoubtedly helpful for the design and deployment of sensor networks. Thirdly, the moments of $Tr(C_{RSS})$ can be approximated by the corresponding moments of the normal variable defined by (17), namely,

$$E(Tr(C_{RSS})) \approx \frac{4}{(N-1)bm_1}+\frac{8\sigma_1^2}{N(N-1)bm_1^3}, \tag{18}$$

$$Std(Tr(C_{RSS})) \approx \frac{4\sigma_1}{\sqrt{N}(N-1)bm_1^2}, \tag{19}$$

which characterize the relationship among the mean and standard deviation of $Tr(C_{RSS})$, the number of anchors, noise statistics of the RSS measurements and the spatial distributions of the anchors.

A natural question arises as to how large N should be to obtain a good approximation; this gives rise to the convergence rate study. In the literature of U-statistics, the Berry-Esseen bound was developed for characterizing the convergence rates of U-statistics [8], [9]. Because W_N is affine to a U-statistic (i.e. M_N), we propose the following theorem describing the convergence rate of W_N in the way similar to the Berry-Esseen bound.

Theorem 2: Use the notations in Theorem 1 and define

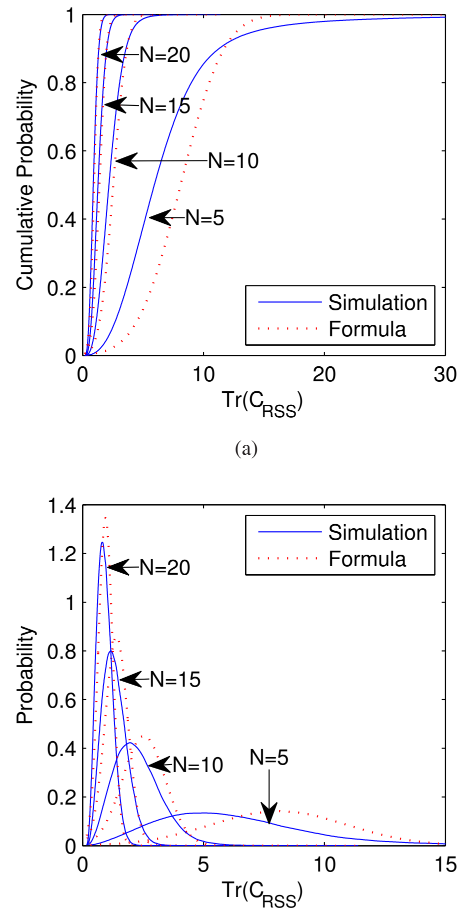
$$\nu_3 = E\left(\left(\frac{1}{d_1^2}-m_1\right)^3\right). \tag{20}$$

Then, as $N \to \infty$,

$$\sup_x |F_N(x)-\Phi(x)| \le \left|\left(\frac{\nu_3+\frac{2\sigma_1^4}{m_1}}{6\sigma_1^3}\right)\frac{(x^2-1)e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\right| N^{-\frac{1}{2}}+O(N^{-1}) \tag{21}$$

where $F_N(x)$ is the distribution function of W_N and $\Phi(x)$ is the standard normal distribution function.

Remark 4: Theorem 2 shows that as $N \to \infty$, the density of W_N converges to standard normality with the rate $O(N^{-\frac{1}{2}})$. Additionally, it can be verified that the coefficient associated with $N^{-\frac{1}{2}}$ is a function of the ratio $\frac{R}{R_0}$; that is to say, the convergence rate of the density of W_N is not determined by the individual values of R_0 and R, but by the ratio $\frac{R}{R_0}$.



(b)

Fig. 2. The distribution functions and pdfs of $Tr(C_{RSS})$ with $R_0 = 1$m, $R = 10$m, $\alpha = 2.3$ and $\sigma_{dB} = 3.92$.

IV. Simulations

In this subsection, we would like to carry out simulations to verify Theorem 1 and the approximations given in (18) and (19). The parameters α, σ_{dB} and R_0 describing the wireless channel are set as 2.3, 3.92 and 1 m, respectively, which are measured in a practical environment [1].

Firstly, we plot in Fig. 2(a) the actual distribution functions of $Tr(C_{RSS})$ (with the legend "Simulation") and the normal distribution function (17) (with the legend "Formula") for $N = 5, 10, 15, 20$. As can be seen, when $N = 5$, the discrepancy between them is quite obvious; when $N = 10$, the discrepancy becomes very small; when $N = 15$ or 20, the discrepancy is negligible. The discrepancy reduces with N increasing as illustrated in Fig. 2(a), and arises for two reasons: the intrinsic error in approximating a U-statistic by normality, and the existence of the remainder term R_N which obeys $Pr\left\{|R_N| \ge \frac{\varepsilon}{N}\right\} = O(N^{-1})$, see (12), and though nonzero is neglected in the calculation. Furthermore, we plot the corresponding pdfs in Fig. 2(b). It can be seen that the overall shapes of the actual pdfs (with the legend

"Simulation") are quite similar to those of normality, and the discrepancy in between reduces with N increasing. These observations are consistent with and in turn demonstrate Theorem 1.

Secondly, we plot the means and the standard deviations of $Tr(C_{RSS})$ from both simulations and the formulas (18) and (19) in Figs. 3(a), 3(c), 3(b) and 3(d). It is evident that the larger is N, the more precise are the formulas. When $N = 5$, the standard deviation attains comparatively large values, and the associated surface in Fig. 3(b) is non-smooth; the most probable reason is that the actual standard deviation is infinite for a N as small as 5.

For better comparison, we define the *relative error* to be the ratio of the difference between the quantity from the simulations and that from the corresponding formula to the former one, and plot them in Fig. 3(e) and 3(f). It can be seen that: (i) the mean is underestimated by (18) when R is small, say $R = 2$m, but is overestimated by (18) when R is large, say $R = 10$m, while the associated absolute value of the relative error decreases with N increasing in most cases; (ii) the standard deviation of $Tr(C_{RSS})$ is underestimated by (19) and the associated absolute value of the relative error decreases with increasing N and R; (iii) suppose the absolute value of the relative error below 10% is acceptable: when $R = 2$m, (18) is applicable if $N \geq 6$, but (19) is not applicable even if $N = 20$; when $R = 10$m, both (18) and (19) are applicable if $N \geq 11$.

In what follows, we present some useful remarks on the properties of sensor localization provided that (18) and (19) are applicable. It is notable that in (18) and (19), the mean and standard deviation of $Tr(C_{RSS})$ normalized by R^2 (or R_0^2) are dependent upon the ratio $\frac{R}{R_0}$; hence, we simplify the discussion involving R_0 and R by letting $R_0 = 1$ m and only concentrating on R.

Remark 5: Equation (18) quantitatively characterizes the average performance limit over all possible sensor-anchor geometries and is indicative for evaluating the average localization performance over a period of time and/or in a wide region. In addition, because the mean is in inverse proportion to N, a *critical* value N^* differing from the parameters R_0, R, σ_{dB} and α can be determined, such that having more anchors than N^* contributes little to the quality of sensor localization.

Remark 6: It can be easily deduced that both (18) and (19) monotonically decrease with R decreasing, as illustrated in Fig. 3(c) and 3(d); the reason is that long distance measurements from RSS suffer greater errors, and thus produce worse localization performance. Therefore, given a fixed N, distance measurements from a sensor are better made at locations as close to the sensor as possible. Moreover, it turns out that using more distance measurements spread over a wide range is not necessarily better than using fewer distance measurements but spread in a narrow range in terms of the average performance limit. For instance, $E(Tr(C_{RSS}))$ is approximately 0.52431 m^2 given $N = 15$ and $R = 6$ m, while a smaller mean which is approximately 0.43174 m^2 can be achieved given $N = 10$ and $R = 4$ m. Thus, tradeoff should be made between the number of anchors (i.e. N) and their spreading (i.e. R_0 and R) in sensor localization.

Remark 7: Though we discuss the impacts of N and R separately, the variables are correlated in some situations, and so the impacts are related. Normally, increasing all the transmission powers in a wireless sensor network enlarges the communication coverage of every node, and both N and R for localizing one sensor tend to rise, but $Tr(C_{RSS})$ and its mean will definitely decrease according to [1].

Remark 8: The dispersion of $Tr(C_{RSS})$ reflects its sensitivity to sensor-anchor geometries. Specifically, with a large dispersion, the chance of having two different sensor-anchor geometries to lead to a big difference in the resulting values of $Tr(C_{RSS})$ is large, implying a large sensitivity, and we should be careful about sensor-anchor geometries; by contrast, with a small dispersion, the chance is certainly small, so is the sensitivity, and there is less reason to worry about sensor-anchor geometries even if the anchors are randomly deployed. Given a random variable, the coefficient of variation, defined to be the ratio of its standard deviation to its mean, is a normalized measure of dispersion of its distribution. Therefore, the coefficient associated with $Tr(C_{RSS})$ has the order of $O(N^{-\frac{1}{2}})$ and the less is the coefficient, the smaller is the sensitivity. In particular, if the coefficient equals its minimum, i.e. 0, all the sensor-anchor geometries will result in one unique value of $Tr(C_{RSS})$, so that the minimum sensitivity is attained. Alternatively, we can observe the sensitivity from Fig. 2(b): the range of $Tr(C_{RSS})$ with a non-trivial probability becomes narrower and narrower with N increasing, implying that the sensitivity is reducing.

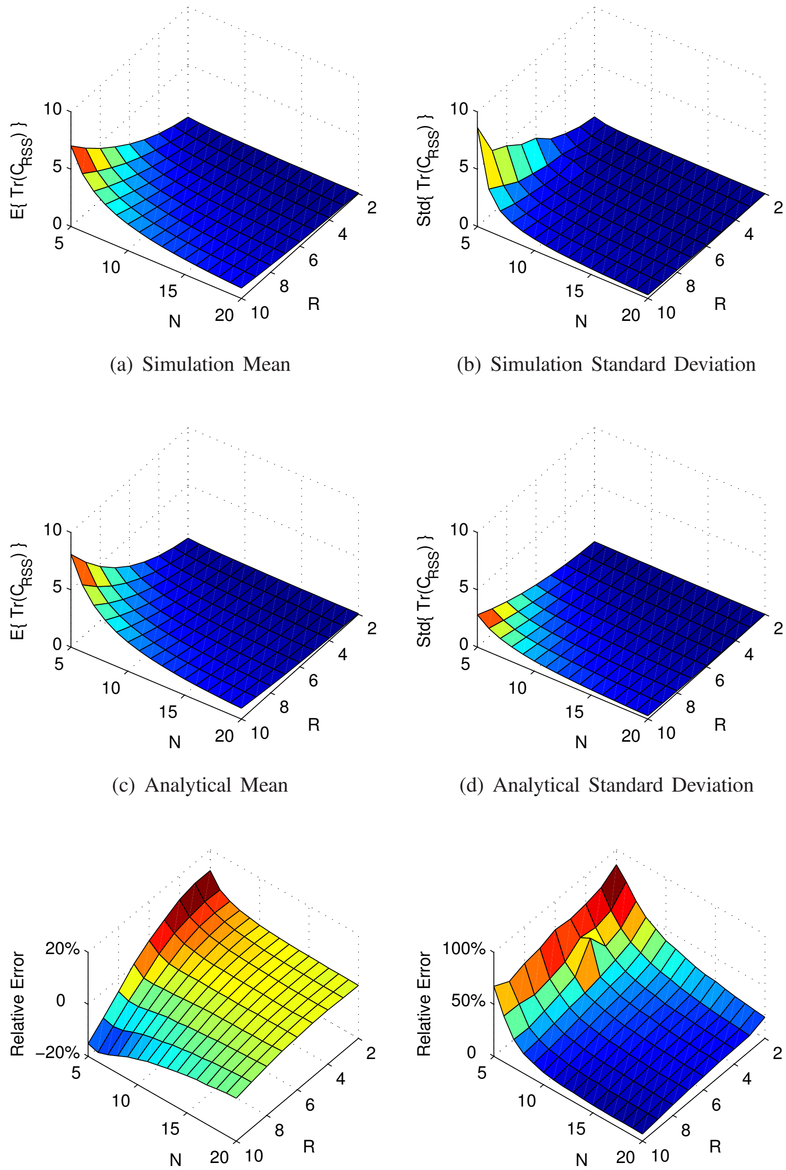
V. Conclusion and Future Work

In this paper, we investigated the performance limit of single-hop sensor localization with the RSS measurements by statistically sensor-anchor geometry modeling. That is, the positions of anchors are assumed to be random and the statistical attributes of the trace of the CRLB matrix embodies essential features of sensor localization. With strict mathematical proofs, we showed that the trace of the CRLB matrix is asymptotically normal. Based on this study, we analyzed the features of sensor localization and carried out extensive simulations.

In future work, we would like to take into account other distributions of anchor positions other than the uniform distributions, as well as considering other types of measuring techniques, including Time of Arrival (TOA), Time Difference of Arrival (TDOA), etc. In addition, it is more attractive, but of course extremely difficult, to conduct similar studies for multi-hop sensor localization.

Acknowledgment

B. Huang, C. Yu and B.D.O. Anderson are supported by the ARC (Australian Research Council) under DP-110100538 and NICTA. Tao Li is supported by the National Natural Science Foundation of China under grant 61004029. C. Yu is an ARC Queen Elizabeth II Fellow and



(a) Simulation Mean

(b) Simulation Standard Deviation

(c) Analytical Mean

(d) Analytical Standard Deviation

(e) Mean

(f) Standard deviation

Fig. 3. The means and the standard deviations of $Tr(C_{RSS})$ from the simulations and the formulas, and the corresponding relative errors with $R_0 = 1\text{m}, \alpha = 2.3$ and $\sigma_{dB} = 3.92$.

is also supported by Overseas Expert Program of Shandong Province. B. Huang is also supported by the Key Laboratory of Computer Networks of Shandong Province. This material is based on research sponsored by the Air Force Research Laboratory, under agreement number FA2386-10-1-4102.

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