

Model Predictive Control of BSM1 benchmark of wastewater treatment process: a tuning procedure

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Abstract: The BSM1 (Benchmark Simulation Model N° 1) has become the standard simulation tool for performance assessment of control techniques applied to wastewater treatment plants (WWTP). Control of WWTP is not trivial because of large disturbances in the influent, nonlinearities, delays, and interaction between variables. In this work a multivariable Model Predictive Controller (MPC) is implemented and optimally tuned. The paper presents results for different tuning situations without the use of long time consuming simulations and considering uncertainty by means of multiple linearized models. Control of the dissolved oxygen in the last aerated reactor and nitrate level in the last anoxic compartment has been tested in simulations using the MPC control strategy. The obtained results show the MPC benefits when it is properly tuned.

Keywords: Model predictive control, BSM1 benchmark, wastewater treatment, tuning, mixed sensitivity problem, robust control.

1. INTRODUCTION

Environmental issues, particularly those related to water resources, are receiving increasing consideration by the present society. One example is the European legislation about the quality of effluents from urban or industrial discharges, which have become very strict, making the wastewater treatment process crucial to keep water free of pollutants.

The most widespread biological treatment process is the activated sludge process (ASP), where microorganisms transform organic contaminants into biological sludge, removed later by settling. For nitrogen removal, the process layout consists of both aerated and non-aerated (anoxic) reactors. In the aerated reactors the bacteria oxidize ammonium to nitrate by the nitrification process. In the anoxic reactors bacteria change nitrate into nitrogen gas, and this transformation is denoted as the denitrification process. The effluent requirements can be obtained with the application of proper control techniques to the ASP, but this is usually difficult because of the large variety of mechanisms involved, nonlinearities due to the complex biological processes, and large disturbances in the influent flow.

In order to test the different control techniques developed by the research community, the BSM1 simulation model has

become a standard in the area (Coop, 2002), and it has also been used in this paper, together with the sets of disturbances and data provided.

Model Predictive Control (MPC) is widely used in the process industry with successful results and seems to be a good candidate for the challenging BSM1 control tasks. In particular, WWTP are inherently multivariable and MPC is specially designed for that kind of control. The popularity of MPC is also due to the natural way of incorporating constraints, and its relative simplicity (Maciejowski, 2002).

MPC controllers have been tuned traditionally by a trial and error procedure, determining input and output weights in the objective function, prediction and control horizons, and constraints. The tuning task can be particularly difficult if the system is multivariable, since the whole set represents a formidable array of possible tuning combinations and also because many of these parameters have overlapping effects on the closed-loop performance and robustness. In these cases the advantages of using automatic MPC tuning methods is clear. In the literature there are many works dealing with MPC tuning, for a review see (Garriga, 2010), but due to their complexity and the difficulty to perform analytical studies, their application to such a complex process as the BSM1 is still a challenge.

The MPC tuning has been frequently tackled by solving different optimization problems, for example (Ali, 1993)

(Francisco, 2006, 2007), but to speed up tuning procedures avoiding dynamical simulations, the authors propose in (Francisco, 2010b) a tuning procedure by solving a constrained H_∞ mixed sensitivity problem using a linearized model of the process (closed loop transfer functions). Other indexes related to the range of manipulated and controlled variables expressed using the l_1 norm were considered to keep the process away from physical and operational bounds. Moreover, multiple linear models were considered to ensure robust performance in the face of uncertainty.

The main objective of this work is to apply the mentioned tuning MPC method (Francisco, 2010b) to a multivariable controller applied to the full BSM1, considering also the linearized full ASM1 as internal prediction model, extending the results of (Francisco, 2010a) where the tuning procedure was applied to a very simple ASP model with a SISO MPC controller. In addition to that, in the WWTP disturbances are clearly asymmetrical with respect to the steady state value and manipulated variables are constrained asymmetrically due to zero saturation or other technological reasons. The l_1 norm in the MPC tuning procedure does not take into account that fact, so another objective of this paper is the inclusion of asymmetrical bounds for those variables.

The paper is organized as follows. First, the BSM1 is explained, followed by the MPC formulation and closed loop transfer functions definition. Then, the method for MPC automatic tuning is posed, detailing the controllability indices considered. Finally, some control results applied to the BSM1 are presented for different tuning requirements, to end up with conclusions and references.

2. BSM1 MODEL AND CONTROL PROBLEM DESCRIPTION

The purpose of WWTP is to process sewage and return clean water to the river. ASP is a very important part of the cleaning procedure in those plants, and its proper control is the aim of this work. The BSM1 has been used as a standard ASP model for performance assessment of control strategies.

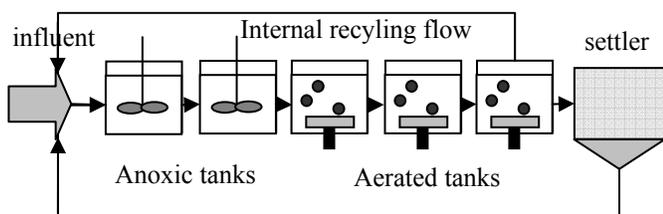


Fig. 1: The Benchmark Simulation Model n°1 (BSM1)

The Benchmark Simulation Model n°1 (BSM1) consists of five biological reactors connected in series. The first two are anoxic and the last three are aerated reactors. The reactors are modelled according to the Activated Sludge Model n° 1 (ASM1) developed by IWAQ (International Association on Water Quality) (Henze, 1987).

As presented in Fig.1 the nitrogen removal is achieved using a first denitrification step performed in the anoxic tanks, placed before the aerated tanks where the nitrification step is carried out. The first two anoxic tanks are assumed perfectly mixed and have a volume of 1000 m³ each. The rest of the three aerated tanks have a volume of 1333 m³ each. Eight different processes are modelled, involving thirteen state variables. An internal recycle from the last tank to the first one is used to supply the denitrification step with nitrate. For the secondary settler the one-dimensional ten-layer system implementing the double exponential settling velocity model has been used.

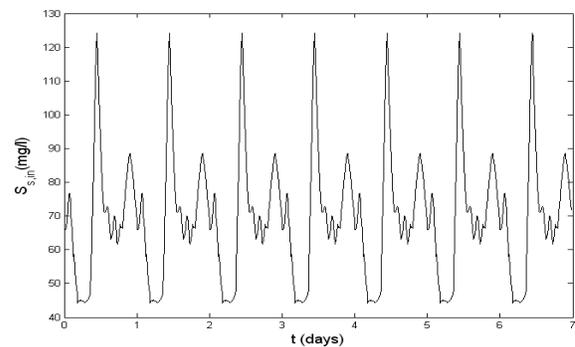


Fig. 2: $S_{s,in}$ disturbances at the influent

The control of this process aims to keep nitrates and nitrites at the second anoxic tank ($S_{NO,dnit}$) limited, and dissolved oxygen concentration in the last aerated tank ($S_{O,nit}$) controlled, despite the large variations of the influent flow rate (Q_{in}) and ready biodegradable substrate concentration in the incoming water ($S_{s,in}$), which are the input disturbances (see Fig. 2). The selected manipulated variables are the internal recycling flow (Q_a) and the air flow rate introduced in the last aerated reactor (by changing the oxygen transfer coefficient K_{La5}) (Coop, 2002; Shen, 2008; Cristea, 2008). In order to investigate the control performance for disturbance rejection a set of specially designed dry weather conditions provided with the BSM1 platform have been considered.

3. MPC FORMULATION AND TUNING

4.1 Basic MPC formulation

The MPC controller calculates on line the future control moves $\Delta \mathbf{u}$ by solving a constrained optimization problem subject to constraints on inputs, predicted outputs and inputs increments. The objective function for zero reference is the following:

$$V(k) = \|\hat{\mathbf{x}}(k+H_c)\|_p^2 + \sum_{i=0}^{H_c-1} \left(\|\hat{\mathbf{x}}(k+i)\|_Q^2 + \|\Delta \mathbf{u}(k+i)\|_R^2 \right) \quad (1)$$

where k denotes the current sampling point, $\hat{\mathbf{x}}(k+i)$ is the predicted state at time $k+i$, depending of measurements up to time k , $\Delta \mathbf{u}$ are the changes in the manipulated variables, H_c is the control horizon, R and Q are positive definite constant matrices representing the weights of the change of control variables and the weights of the set-point tracking errors respectively. The MPC is implemented with a terminal penalty term obtained from the solution of a Riccati equation, guaranteeing closed loop stability of the system (Scokaert, 1998). This formulation is equivalent to an infinite prediction horizon, and therefore only H_c appears. See (Maciejowski, 2002) for a more detailed description of the control algorithm.

The MPC prediction model used is a linear discrete state space model of the plant obtained by linearizing the first-principles nonlinear model of the process:

$$\begin{cases} \mathbf{x}(k+1) = A\mathbf{x}(k) + B\mathbf{u}(k) + B_d\mathbf{d}(k) \\ \mathbf{y}(k) = C\mathbf{x}(k) \end{cases} \quad (2)$$

where $\mathbf{x}(k)$ is the state vector, $\mathbf{u}(k)$ is the input vector and $\mathbf{d}(k)$ the disturbance vector. Matrices A , B and B_d are of adequate dimensions. For the BSM1 control, in order to simplify the formulation, the internal prediction state space model considers only one anoxic tank, one aerated tank, and a secondary settler modelled with one layer. Then, the vector \mathbf{x} includes 26 states for the reactors and 5 for the settler (only for particulate components).

4.2 Closed loop transfer functions

In this point the SISO closed loop transfer functions are obtained, but for MIMO systems, the matrices can be also calculated easily in the same way. The linear MPC control law obtained by solving problem (1) without constraints provides the following closed loop system output (Fig. 3):

$$y(s) = \frac{G_0 K_1}{1 + G_0 K_1} r(s) + \frac{1}{1 + G_0 K_1} \tilde{d}(s) \quad (3)$$

where $\tilde{d}(s)$ are the filtered disturbances $\tilde{d}(s) = (G_0 K_2 + G_{d0})d(s)$, G_0 and G_{d0} are the nominal transfer functions, K_i are the transfer functions between the control signal and the different inputs (error and disturbances) which in turn depend on the control system tuning parameters (Q , R , H_c). Note that the proposed MPC structure is a combined feedforward-feedback system.

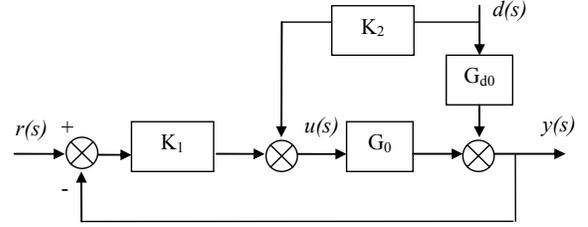


Fig. 3: Nominal closed loop system

Equation (3) can be expressed substituting the sensitivity function and complementary sensitivity functions:

$$y(s) = T_0(s)r(s) + S_0(s)R_{d0}(s)d(s) \quad (4)$$

where

$$S_0 = \frac{1}{1 + G_0 K_1}; \quad T_0 = \frac{G_0 K_1}{1 + G_0 K_1} \quad (5)$$

$$R_{d0}(s) = G_{d0}(s) - K_2 G_0(s) \quad (6)$$

In order to state the automatic tuning problem for the nominal case, the sensitivity function $S_0(s)R_{d0}(s)$ between the load disturbances and the outputs will be considered, as well as the control sensitivity transfer function $M_0(s)$ between the load disturbances and the control signals when the reference is set to zero.

$$M_0(s) = \frac{u(s)}{d(s)} = \frac{K_2 - K_1 G_{d0}}{1 + G_0 K_1} \quad (7)$$

4.3 MPC automatic tuning

The MPC applied to the BSM1 has been tuned using a procedure developed by (Francisco, 2010b), which is based on the H_∞ and l_1 norms of some closed loop transfer functions of the system as controllability indices. The H_c is fixed in advance here for simplicity but the methodology is extensive for that parameter. The MPC is tuned to achieve good disturbance rejection and the proposed method provides the optimum weight R by solving a constrained multiobjective optimization problem.

The calculation of the closed loop transfer functions is only possible in the unconstrained case, and when the plant works with no active constraints as is the typical situation. For that reason, the constraints (13) or (14) are used to keep the plant within the feasibility region.

The MPC tuning is stated as a mixed sensitivity optimization problem that takes into account both disturbance rejection and control effort objectives in the same function. The problem definition is:

$$\min_c \|N_0\|_\infty = \min_c \left(\max_\omega |N_0(j\omega)| \right) \quad (8)$$

where
$$N_0 = \left(\frac{Wp \cdot S_0 \cdot R_{d0}}{Wesf \cdot s \cdot M_0} \right) \quad (9)$$

$c = (H_c, R)$ are the tuning parameters, and the dependence on s of the transfer functions N_0 , S_0 and M_0 has been omitted for brevity.

$Wp(s)$ and $Wesf(s)$ are suitable weights to achieve closed loop performance specifications and to reduce the control efforts respectively. Note that control efforts rather than magnitudes of control are included in the objective function by considering the derivative of the transfer function M_0 . The SQP method has been used to solve the optimization problem (8) with the constraints presented below.

In order to ensure disturbance rejection (considering normalized disturbances) the following equation needs to be satisfied in the disturbances frequency range:

$$|S_0(j\omega)| \cdot |d(\omega)| < 1 \quad \omega \in (\omega_1, \omega_2) \quad (10)$$

where $S_0(j\omega)$ is the frequency response of the sensitivity function, $d(\omega)$ is the disturbance spectra, and (ω_1, ω_2) is the disturbances frequency range. By choosing a proper weight $Wp(s)$, disturbance rejection can be assured including the following constraint in the optimization (8)

$$\|Wp \cdot S_0 \cdot R_{d0}\|_\infty < 1 \quad (11)$$

For robust performance, the condition (11) must be satisfied for every plant inside the uncertainty region, which in this work is defined by linear local models. A simple way to verify robustness is the strategy followed in this paper, which consists of imposing (11) for the vertices of the uncertainty region and assuming convexity.

Regarding to bounds on the manipulated variables, a new constraint is included using the following index (Francisco,

2010a) Suppose that $z(k)$ is a signal bounded in an asymmetrical domain:

$$-z_{\min} \leq z(k) \leq z_{\max} \quad \text{where } z_{\min} \geq 0 ; z_{\max} \geq 0 ; z_{\min} \neq z_{\max}$$

Due to that, it is possible to use a simple norm that takes into account this asymmetry in the limits. The functional proposed (Naib, 2003) gives information about that, and for a generic signal $z(k)$ is defined as follows:

$$\|z(k)\|_a = \left[\max \left\{ 0, \max_k (z(k)) \right\} \max \left\{ 0, -\min_k (z(k)) \right\} \right]^t \quad (12)$$

Then, the constraint imposed in the automatic tuning optimization problem is:

$$\|z(k)\|_a \leq Z \quad \text{where } Z^t = [z_{\max}, z_{\min}] \quad (13)$$

This index replaces the classical l_1 norm that only takes into account symmetric limits.

In order to evaluate the satisfaction of those kinds of constraints, the following result has been considered (Naib, 2003). Let us consider $u(k)$ and $y(k)$ as inputs and outputs of a generic discrete linear system described by a transfer function $G(z^{-1})$. A necessary and sufficient condition for the output of the system $G(z^{-1})$ be constrained in the asymmetrical domain $\|y(k)\|_a \leq Y$, $Y^t = [y_{\max}, y_{\min}]$, for any input in another asymmetrical domain $\|u(k)\|_a \leq U$, $U^t = [u_{\max}, u_{\min}]$, is the following equation:

$$\Psi U \leq Y, \quad \text{where } \Psi = \begin{bmatrix} \sum_{i=1}^{\infty} \Phi_i^+ & \sum_{i=1}^{\infty} \Phi_i^- \\ \sum_{i=1}^{\infty} \Phi_i^- & \sum_{i=1}^{\infty} \Phi_i^+ \end{bmatrix} \quad (14)$$

$\Phi_i^+ = \max \{ \Phi_i, 0 \}$; $\Phi_i^- = \max \{ -\Phi_i, 0 \}$; and $\{ \Phi_i, i = 1, \dots, \infty \}$ are the impulse response terms of the transfer function $G(z^{-1})$ $y(k) = \sum_{i=1}^k \Phi_i u(k-i)$

4. RESULTS

4.1 Problem setup

In this point some results of BSM1 control with automatically tuned MPC are presented. The MPC implemented is multivariable in order to control nitrates and nitrites in the second anoxic tank ($S_{NO,dnii}$) and the dissolved

oxygen in the last aerated tank ($S_{O,mit}$). The sampling period selected is $T_s=0.0104$ days. The influent dry weather disturbances are assumed to be measured, and their nominal values are: $S_{Sin}=69.5$ mg/l and $Q_{in}=18446$ m³/d. For simplicity, only variations in S_{Sin} and Q_{in} are considered. For simulations, 7 days have been considered because a steady state point has been calculated in advance.

The influent disturbances variations are asymmetric. For example, for influent flow, $Q_{imin}=9997.4$ m³/d and $Q_{imax}=32197$ m³/d, whose normalized value (deviations from the steady state) are $\bar{d}_{max}=1$; $\bar{d}_{min}=0.61529$. Then, the Q_{in} domain is determined by $D'=[\bar{d}_{max}, \bar{d}_{min}]=[1, 0.61529]$. The same applies for disturbance S_{Sin} . On the other hand the zero saturation of input Q_a puts a stricter bound $\bar{Q}_{a,min}$ while $\bar{Q}_{a,max}$ can be much larger depending on the pump characteristics. Then, the functional (12) is used, and the constraint (14) is included in the tuning procedure. The same reasoning is valid for K_La_5 , which is also asymmetrically constrained.

Weights $Wp(s)$ have been chosen considering condition (11) with dry weather disturbance spectra. Their values for both outputs are the same because the MPC internal model is fully normalized. For each disturbance there is one weight:

$$Wp_{S_{Sin}} = 8 \cdot \frac{s+20}{s+0.2}; \quad Wp_{Q_{in}} = 8 \cdot \frac{s+8}{s+1.6}$$

MPC constraints include the following limits on inputs:

$$0 < \Delta Q_a < 100000 \text{ m}^3 / d, \quad 0 < Q_a < 92230 \text{ m}^3 / d \\ 0 < \Delta K_La_5 < 100000 \text{ d}^{-1}, \quad 0 < K_La_5 < 240 \text{ d}^{-1}$$

The local linear models to define the uncertainty region have been calculated around steady state points obtained varying 1% the values of Q_a , K_La_5 , S_{Sin} , Q_{in} , because those changes imply typical variations in the working point of the process.

4.2 Simulation results

The first results are a comparison between two tuning cases with different weights $Wesf$ (Case 1 and Case 2) imposing a penalty on control moves, keeping Wp constant. Their value is the same for both manipulated inputs due to the normalization:

$$\text{Case 1: } Wesf_{S_{Sin}} = 5 \cdot \frac{0.33s+4}{s+4 \cdot 10^{-4}}; \quad Wesf_{Q_{in}} = 3 \cdot \frac{0.33s+4}{s+4 \cdot 10^{-4}}$$

$$\text{Case 2: } Wesf_{S_{Sin}} = 8 \cdot \frac{0.33s+4}{s+4 \cdot 10^{-4}}; \quad Wesf_{Q_{in}} = 10 \cdot \frac{0.33s+4}{s+4 \cdot 10^{-4}}$$

In Table 1 some numerical results are shown, where concentrations are in mg/l, EQ is in Kg/d and PE is in KWh/d. In both cases, the asymmetrical constraints over the manipulated variables are $U'=[10, 8.5]$ for Q_a and $U'=[5, 4.4]$ for K_La_5 , in normalized values. Note that those constraints impose bounds for the worst combination of disturbances, and for that reason the values are much higher than the obtained when simulating with a dry weather benchmark disturbances). MPC weight R is a matrix with two nonzero elements: $R(1,1)$ representing the weight for the input Q_a and $R(2,2)$ representing the weight for K_La_5 .

The optimal MPC obtained in Case 2 rejects better disturbances than the controller in Case 1 (by observing the peak values in the table) for both $S_{O,mit}$ and $S_{NO,dnit}$ concentrations because control efforts are less penalized.

Table 1: Results for BSM1 control with tuned MPC

	Case 1	Case 2	Case 3
$R(1,1)$	0.0200	0.3547	28.42
$R(2,2)$	0.0206	0.0932	36.61
H_c	10	10	10
$\max(S_{NO,dnit})$	5.6071	5.6659	5.0014
$\max(S_{O,mit})$	0.9344	0.9859	1.2447
$\ N_o\ _\infty$	1.9763	6.4459	107.42
$\ \bar{Q}_a\ _\infty$	[9.96 8.49]	[9.93 8.45]	[8 6.46]
$\ \bar{K}_La_5\ _\infty$	[4.77 4.40]	[4.75 4.37]	[3.07 2.68]
$\ Wp \cdot S_o \cdot R_{d0}\ _\infty$	0.1518	0.9692	107.24
$\ Wp \cdot S \cdot R_d\ _\infty$	0.1118	0.9947	107.82
PE	485.6543	434.1247	414.9294
EQ	5385.86	5405.76	5385.85

The last column in Table 1 (Case 3) is a tuning result imposing more strict asymmetrical bounds for manipulated variables, $U'=[8, 7]$ for Q_a and $U'=[5, 3.8]$ for K_La_5 . In this case the constraint for disturbance rejection (11) is not included. In Fig. 4 and 5 a comparison of Cases 1 and 3 is presented, showing that disturbance rejection is significantly better in Case 1, because the constraint for performance (11) is included in the tuning procedure and the bounds for control variables are more relaxed. The tradeoff between costs and performance is also shown by looking at the PE values. The MPC performance in the worse point of the region defined by a local model has been also presented by

means of $\|W_p \cdot S \cdot R_d\|_\infty$ value, in order to prove that the MPC tuned satisfies robust performance condition.

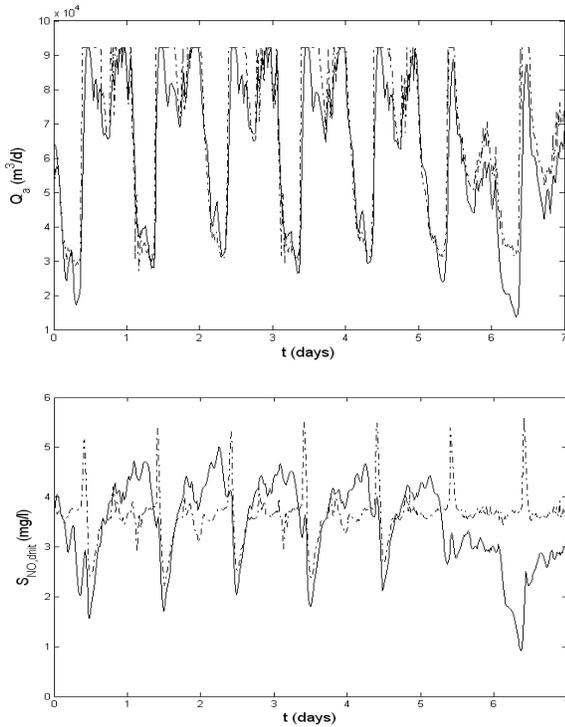


Fig. 4: $S_{NO,dnit}$ and internal recycling flow Q_a in the 2nd tank for Case 1 (dashed dotted) and Case 3 (solid)

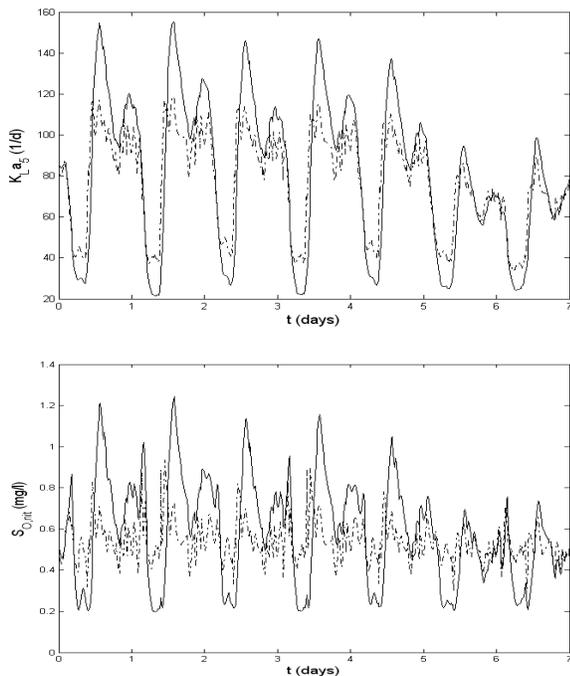


Fig. 5: $S_{O,dnit}$ and oxygen transfer coefficient ($K_L a_5$) in the 5th tank for Case 1 (dashed dotted) and Case 3 (solid)

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