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Fast-rate Adaptive Output Feedback Control with Adaptive Output Estimator for Non-uniformly Sampled Multirate Systems

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Abstract— **This paper deals with an adaptive output feedback control design problem for non-uniformly sampled multirate systems with a fast uniform input updating rate and slow non-uniform output sampling rates. A design scheme of a fast-rate adaptive output feedback control at the fast sampling period of the input updating rate will be proposed for non-uniformly sampled systems using an adaptive output estimator. The proposed adaptive output feedback control is designed by using only measured outputs of slow rate samplings and fast rate updating input. We also analyze the stability of the obtained fast rate output feedback control system briefly based on ASPR properties of the controlled system.**

I. INTRODUCTION

Because of hardware limitations on sensoring and actuating in several chemical and mechanical systems, multirate systems are well recognized in industries as a system with different output sampling periods and input updating rate. In particular, there are many systems in which faster output sampling periods cannot be taken, even though the input actuating period can be taken at relatively high speeds [1], [2]. Generally, in such systems, feedback controllers are designed according to a single rate based on the measured output at slow sampling rate. In these cases, the control performance within the slow sampling period has not always been ensured. However, if the input can be updated at a faster rate, one can expect an improvement in the control performance. With this in mind, multirate control approaches with a lifting technique in which the control input can be up dated at fast rate have widely researched [3]. By using the lifting technique, one can design a controller with the fast-rate updating rate, however, since the lifted system is represented as a multi-variable system with different input and output numbers, the designed controller were complex. Furthermore, the ripple phenomenon sometimes deteriorates the control performance in the multirate control systems.

On the other hand, several kinds of output estimators for multi-rate systems and fast rate model identification methods have been proposed [4], [5], [6], [7]. If the input can be updated at a faster rate by utilizing the estimated output, one can expect an improvement in the control performance within the slow sampling periods. Unfortunately, however, the majority of them were based on a full order model of the considered system. Since it might be difficult to determine the exact order of the considered systems in practice and for higher order systems, the designed estimator might become complex. With this in mind, recently a novel adaptive output estimator that realizes output estimate at fast rates has been proposed [8] for uniformly sampled dual-rate systems and this method has also been extended for uncertain non-uniformly sampled systems so as to realize a fast-rate output feedback control in which the output signals are non-uniformly sampled with a slow sampling period [9].

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In this paper, we propose a design scheme of a fast-rate adaptive output feedback control for uncertain non-uniformly sampled multirate systems with a fast uniform input updating rate and slow non-uniform output sampling rates. Since one can meet several types of non-uniformly sampled systems, the controller design for non-uniformly sampled systems have been attracted a much attention [10], [11]. The results in [10] and [11] were obtained for a sampled data system with non-uniform sampling period but with uniform frame period. Unlike the previous methods, the proposed method can deal with more general non-uniform sampled systems without uniform frame periods. In the proposed method, we adopt the adaptive output estimator proposed in [9] and, using estimated output at fast sampling period, for uncertain non-uniformly sampled systems we realize a fast-rate adaptive output feedback control based on the almost strictly positive real (ASPR) property [12], [13], [14]. The stability of the designed fast-rate adaptive output feedback control system with the adaptively estimated fast-rate outputs will be briefly analyzed based on ASPR-ness of the controlled system.

II. PROBLEM STATEMENT

A. Non-uniformly Sampled System Representation

Consider a non-uniformly sampled system with the input updated at a time instant kT uniformly and the output sampled at a time instant $\bar{n}_k T$ non-uniformly. Where, denoting the k-th output sampling interval by $n_k T$ ($n_k \geq 1$), $\bar{n}_k$ is defined by

$$\bar{n}_k = \sum_{i=0}^{k-1} n_i \ , \ \bar{n}_0 = 0$$

That is, consider a system for which the input is updated with a fast uniform updating period T and the output is slowly sampled with non-uniform sampling interval $n_k T$.

Denoting $y(k) = y(kT), x(k) = x(kT)$, represent the virtual fast-rate system with sampling period of T of the considered non-uniformly sampled system as

$$\begin{aligned} \boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + \boldsymbol{b}u(k) \\ y(k) &= \boldsymbol{c}^T\boldsymbol{x}(k) \end{aligned} \tag{1}$$

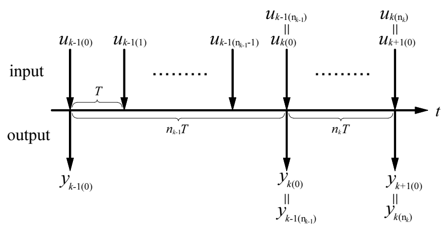


Fig. 1. Non-uniform multirate system

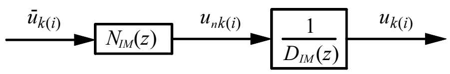


Fig. 2. Internal model filter

Now, defining $y_k = y(\bar{n}_k T)$, denote fast-rate outputs at sampling period of T within y_k to y_{k+1} by

$$y_{k(i)} = y\left((\bar{n}_k + i)T\right) \quad (i = 0, 1, \cdots, n_k)\ , \tag{2}$$

where

$$\begin{aligned} y_{k(0)} &= y_k \\ y_{k(n_k)} &= y_{k+1} = y_{k+1(0)}. \end{aligned} \tag{3}$$

Note that the output $y_{k(i)}$ for $i = 1, 2, \cdots, n_k - 1$ can not be measured practically. Further, denoting the state at a time instant $(\bar{n}_k + i)T$ by $\boldsymbol{x}_{k(i)}$, we have the following virtual fast-rate system representation of the considered non-uniformly sampled system (See Fig. 1):

$$\begin{aligned} \boldsymbol{x}_{k(i+1)} &= A\boldsymbol{x}_{k(i)} + \boldsymbol{b}u_{k(i)} \\ y_{k(i)} &= \boldsymbol{c}^T \boldsymbol{x}_{k(i)} \end{aligned} \tag{4}$$

B. Problem Statement

Suppose that the considered virtual fast-rate system (1) satisfies the following assumptions:

Assumption 1: The virtual fast-rate system has a relative degree of 1.

Assumption 2: The virtual fast-rate system is minimum-phase.

Further the following assumption is imposed for the sampling interval of non-uniformly sampled output y_k.

Assumption 3: There exists an upper bound of the sampling interval as $1 \le n_k \le n_{max}$ and n_{max} is known.

Next, consider a reference signal $y_m(k)$ which satisfies the following relation.

$$\begin{aligned} & R(z)[y_m(k)] = 0, R(z) = z^\rho + d_1 z^{\rho-1} + \cdots + d_\rho \\ & |y_m(k)| \le \bar{r} \end{aligned}$$

where "z" is the forward-shift operator.

The objective here is to design an adaptive control system such that the output $y(k)$ track the reference signal $y_m(k)$ at fast rate.

To this end, firstly introduce an internal model filter $G_{IM}(z) = \frac{N_{IM}(z)}{D_{IM}(z)}$ as shown in Fig. 2, where, $D_{IM}(z) = R(z)$ and $N_{IM}(z)$ is any stable polynomial of order ρ.

The virtual fast-rate error system including the internal model filter can be represented by [14]

$$\begin{aligned} \bar{\boldsymbol{x}}(k+1) &= \bar{A}\bar{\boldsymbol{x}}(k) + \bar{\boldsymbol{b}}\bar{u}(k) \\ e(k) &= \bar{y}(k) = \bar{\boldsymbol{c}}^T \bar{\boldsymbol{x}}(k) \end{aligned} \tag{5}$$

where $\bar{\boldsymbol{x}}(k)$ is the state vector of the system with the internal model filter and $e(k) = y(k) - y_m(k)$ is the output tracking error.

The objective is then to design a fast-rate adaptive output feedback controller which stabilizes the virtual fast-rate error system (5).

III. Adaptive Output Estimator Design

Firstly, an adaptive output estimator which estimates unmeasured outputs $y_{k(i)}$, $i = 1, \cdots, n_k - 1$ will be considered.

From Assumption 1, there exists an appropriate nonsingular transformation $[y(k), \boldsymbol{\eta}(k)^T]^T = \Phi\boldsymbol{x}(k)$ such that the virtual fast rate system (1) can be transformed into the following canonical form [15]:

$$\begin{aligned} y(k+1) &= a_y^* y(k) + b_y^* u(k) + \boldsymbol{c}_\eta^T \boldsymbol{\eta}(k) \\ \boldsymbol{\eta}(k+1) &= A_\eta \boldsymbol{\eta}(k) + \boldsymbol{b}_\eta y(k) \end{aligned} \tag{6}$$

The zero dynamics of the system (6) is stable from Assumption 2, i.e. A_η is a stable matrix.

Using the expression defined in (2), the system's output in (6) can be represented by

$$\begin{aligned} y_{k(0)} &= a_y^* y_{k-1(n_{k-1}-1)} + b_y^* u_{k-1(n_{k-1}-1)} \\ &\quad + \boldsymbol{c}_\eta^T \boldsymbol{\eta}_{k-1(n_{k-1}-1)} \end{aligned} \tag{7}$$

$$y_{k-1(i)} = a_y^* y_{k-1(i-1)} + b_y^* u_{k-1(i-1)} + \boldsymbol{c}_\eta^T \boldsymbol{\eta}_{k-1(i-1)} \tag{8}$$

Furthermore, from (7) and (8), the sampled output $y_{k(0)}$ can be expressed by using measured outputs an d inputs as

$$\begin{aligned} y_{k(0)} &= b_y^* u_{k-1(n_{k-1}-1)} + b_y^* \sum_{i=2}^{n_{k-1}} a_{yi}^* u_{k-1(n_{k-1}-i)} \\ &\quad + a_{yn_{k-1}}^* y_{k-1(0)} + \boldsymbol{c}_{\eta n_{k-1}}^T \boldsymbol{\eta}_{k-1(0)} \end{aligned} \tag{9}$$

with

$$\begin{aligned} a_{yj}^* &= a_{y(j-1)}^* a_y^* + \boldsymbol{c}_{\eta(j-1)}^T \boldsymbol{b}_\eta\ ,\ a_{y0}^* = 1 \\ \boldsymbol{c}_{\eta j}^T &= a_{y(j-1)}^* \boldsymbol{c}_\eta^T + \boldsymbol{c}_{\eta(j-1)}^T A_\eta\ ,\ \boldsymbol{c}_{\eta 0}^T = \boldsymbol{0}^T \\ & (j = 1, \cdots, n_{k-1}) \end{aligned}$$

It follows that $a_{y1}^* = a_y^*$.

Taking the expressin in (8) and (9) in to account, the output estimator is designed as follows [9]:

$$\begin{aligned} \hat{y}_{k(0)} &= \hat{b}_{yk} u_{k-1(n_{k-1}-1)} \\ &\quad + \hat{b}_{yk-1} \sum_{i=2}^{n_{k-1}} \hat{a}_{ik} u_{k-1(n_{k-1}-i)} + \hat{a}_{n_{k-1}k} y_{k-1(0)} \\ \hat{y}_{k(1)} &= \hat{a}_{1k} y_{k(0)} + \hat{b}_{yk} u_{k(0)} \\ \hat{y}_{k(i)} &= \hat{a}_{1k} \hat{y}_{k(i-1)} + \hat{b}_{yk} u_{k(i-1)} \quad , \quad (i = 2, \cdots, n_k - 1) \end{aligned} \tag{10}$$

where $\hat{a}_{ik}$ and $\hat{b}_{yk}$ are estimated values of a_{yi}^* and b_y^* respectively.

$\hat{b}_{yk}$ and $\hat{a}_{ik}$ are estimated by the following parameter adjusting law with a period of n_kT.

$$\hat{b}_{yk} = \bar{\sigma}\hat{b}_{yk-1} - \bar{\sigma}\gamma_b u_{k-1(n_{k-1}-1)}\epsilon_{k(0)} + p_{bk} \tag{11}$$

$$\hat{a}_{ik} = \bar{\sigma}\hat{a}_{ik-1} - \bar{\sigma}\gamma_{ai}\hat{b}_{yk-1}u_{k-1(n_{k-1}-i)}\epsilon_{k(0)} + p_{aik} \quad (i=1,\cdots,n_{k-1}-1) \tag{12}$$

$$\hat{a}_{n_{k-1}k} = \bar{\sigma}\hat{a}_{n_{k-1}k-1} - \bar{\sigma}\gamma_{an_{k-1}}y_{k-1(0)}\epsilon_{k(0)} + p_{an_{k-1}k} \tag{13}$$

$$\hat{a}_{jk} = \hat{a}_{jk-1},\ (j = n_{k-1}+1,\cdots,n_{max}) \tag{14}$$

$$\gamma_{ai},\gamma_b > 0\ ,\ \bar{\sigma} = \frac{1}{1+\sigma}\ ,\ 0<\sigma$$

where p_{bk}, p_{aik} are parameter projections which are given by

$$\begin{aligned} p_{bk} &= \begin{cases} 0 & (\underline{b}_y \le \hat{b}_{yk} \le \bar{b}_y) \\ \bar{\sigma}\gamma_b u_{k-1(n_{k-1}-1)}\epsilon_{k(0)} & \text{otherwise} \end{cases} \\ p_{aik} &= \begin{cases} 0 & (i \ne 1 \texttt{ or } \underline{a}_y \le \hat{a}_{1k} \le \bar{a}_y) \\ \bar{\sigma}\gamma_{a1}\hat{b}_{yk-1}u_{k-1(n_{k-1}-2)}\epsilon_{k(0)} & \text{otherwise} \end{cases} \\ p_{an_{k-1}k} &= \begin{cases} 0 & (n_{k-1} \ne 1 \texttt{ or } \underline{a}_y \le \hat{a}_{1k} \le \bar{a}_y) \\ \bar{\sigma}\gamma_{a1}y_{k-1(0)}\epsilon_{k(0)} & \text{otherwise} \end{cases} \end{aligned} \tag{15}$$

with $\underline{a}_y$, $\bar{a}_y$, $\underline{b}_y$ and $\bar{b}_y$ such that

$$\underline{a}_y \le a_y^* \le \bar{a}_y\ ,\ \underline{b}_y \le b_y^* \le \bar{b}_y$$

and $\epsilon_{k(0)} = \hat{y}_{k(0)} - y_{k(0)}$ is an estimate error which can be generated using the available signals as follows:

$$\begin{aligned} \epsilon_{k(0)} = &\Big\{\bar{\sigma}\hat{b}_{yk-1}u_{k-1(n_{k-1}-1)} + \hat{b}_{yk-1}\sum_{i=2}^{n_{k-1}}\bar{\sigma}\hat{a}_{ik-1}u_{k-1(n_{k-1}-i)} + \bar{\sigma}\hat{a}_{n_{k-1}k-1}y_{k-1(0)} - y_{k(0)}\Big\} \\ &/\Big\{1 + \bar{\sigma}\gamma_b u^2_{k-1(n_{k-1}-1)} + \hat{b}_{yk-1}\sum_{i=2}^{n_{k-1}}\bar{\sigma}\gamma_{ai}\hat{b}_{yk-1}u^2_{k-1(n_{k-1}-i)} + \bar{\sigma}\gamma_{an_{k-1}}y^2_{k-1(0)}\Big\} \end{aligned}$$

We have the following lemma concerning the boundedness of all signals in the obtained adaptive estimator [9].

Lemma 1: Consider the following positive definite function:

$$V_{ak} = \sum_{i=1}^{3} V_{ik} \tag{16}$$

$$V_{1k} = \rho_1\epsilon^2_{k(0)},\ V_{2k} = \rho_2\boldsymbol{\eta}^T_{k(0)}P_\eta\boldsymbol{\eta}_{k(0)},\ V_{3k} = \sum_{i=1}^{4}V_{3ik}$$

$$V_{31k} = \bar{\sigma}\gamma_b^{-1}\Delta b^2_{yk},\ V_{32k} = \bar{\sigma}\sum_{i=1}^{n_{k-1}-1}\gamma_{ai}^{-1}\Delta a^2_{ik}$$

$$V_{33k} = \bar{\sigma}\gamma^{-1}_{an_{k-1}}\Delta a^2_{n_{k-1}k},\ V_{34k} = \bar{\sigma}\sum_{i=n_{k-1}+1}^{n_{max}}\Delta a^2_{ik}$$

$$\rho_i > 0, (i=1,2),\ P_\eta = P_\eta^T > 0$$

where $\Delta a_{ik} = \hat{a}_{ik} - a^*_{yi}$ and $\Delta b_{yk} = \hat{b}_{yk} - b^*_y$. Then the difference $\Delta V_{ak} = V_{ak} - V_{ak-1}$ can be evaluated by

$$\begin{aligned} &\Delta V_{ak} \\ &\le -\alpha_\epsilon|\epsilon_k|^2 - \rho_1|\epsilon_{k-1}|^2 \\ &\quad -\alpha_\eta\|\boldsymbol{\eta}_{k-1(0)}\|^2 - (\rho_2\alpha_{21} - \delta_2)\sum_{i=1}^{n_{k-1}-1}\|\boldsymbol{\eta}_{k-1(i)}\|^2 \\ &\quad +\alpha_y|y_{k-1(0)}|^2 + \rho_2\left(\alpha_{23} + \frac{\rho_2\alpha_{22}^2}{\delta_2}\right)\sum_{i=1}^{n_{k-1}-1}|y_{k-1(i)}|^2 \\ &\quad -\bar{\sigma}\sigma^2\gamma_b^{-1}b_y^{*2} - \bar{\sigma}\sigma^2\gamma^{-1}_{an_{k-1}}a^{*2}_{yn_{k-1}} \\ &\quad -\sum_{i=1}^{n_{k-1}-1}(1-\delta_{ai})\,\bar{\sigma}\gamma_{ai}\hat{b}^2_{yk-1}u^2_{k-1(n_{k-1}-i-1)}\epsilon_k^2 \\ &\quad -\alpha_{ab}\left\{\gamma_b^{-1}\Delta b^2_{yk} + \sum_{i=1}^{n_{k-1}}\gamma_{ai}^{-1}\Delta a^2_{ik}\right\} \\ &\quad +\sum_{i=1}^{n_{k-1}-1}\bar{\sigma}\left(\frac{1}{\delta^2_{ai}} - \sigma^2\right)\gamma_{ai}^{-1}a^{*2}_{yi} \\ &\quad +\frac{1}{\delta_3}\left\{\frac{b_y^{*2}}{\gamma_b} + \sum_{i=1}^{n_{k-1}}\frac{a^{*2}_{yi}}{\gamma_{ai}}\right\} + \tilde{a}_y^2\tilde{b}_y^2\sum_{i=0}^{n_{k-1}-2}\frac{1}{\delta_{ui}}|u_{k-1(i)}|^2 \\ &\quad +\frac{\tilde{b}_y}{\delta_{uy\epsilon}}|u_{k-1(n_{k-1}-1)}|^2 + \frac{\tilde{a}_y}{\delta_{uy\epsilon}}|u_{k-1(n_{k-1}-2)}|^2 \end{aligned} \tag{17}$$

for any positive constants δ_2, δ_3, δ_ϵ, $\delta_{uy\epsilon}$ δ_{ai} and δ_{ui}, where

$$\begin{aligned} \alpha_\epsilon &= 2 - \rho_1 - \delta_\epsilon - \sum_{i=1}^{n_{k-1}-2}\delta_{ui} - 3\delta_{uy\epsilon} \\ \alpha_\eta &= \rho_2\alpha_{21} - \delta_2 - \frac{1}{\delta_\epsilon}\|\boldsymbol{c}_{\eta n_{k-1}}\|^2 \\ \alpha_{ab} &= \left\{\bar{\sigma}^{-1} - \bar{\sigma} - \delta_3\right\} \\ \alpha_y &= \rho_2\left(\alpha_{23} + \frac{\rho_2\alpha_{22}^2}{\delta_2}\right) + \frac{\tilde{a}_y}{\delta_{uy\epsilon}} \end{aligned}$$

and

$$\begin{aligned} \alpha_{21} &= \lambda_{\min}[Q_\eta],\ \alpha_{22} = \|A_\eta\|\|P_\eta\|\|\boldsymbol{b}_\eta\|, \\ \alpha_{23} &= \|P_\eta\|\|\boldsymbol{b}_\eta\|^2,\ \delta_2 > 0 \end{aligned}$$

Thus, considering sufficient small ρ_1 and sufficient large ρ_2, we can confirm that there exist a small positive constants δ_2, δ_3, δ_ϵ, $\delta_{uy\epsilon}$, δ_{ai} and δ_{ui} such that $\alpha_\epsilon > 0$, $\alpha_\eta > 0$, $\alpha_{ab} > 0$, $1 - \delta_{ai} > 0$ and $\rho_2\alpha_{21} - \delta_2 > 0$, and then all the signals in the adaptive output estimator are uniformly bounded with bounded inputs and outputs.

IV. Fast-Rate Adaptive Output Feedback Control System

A. Adaptive Controller Design

Let's impose the following assumption:

Assumption 4: There exists, for the virtual fast-rate error system (5) with the internal model filter, a known stable parallel feedforward compensator (PFC):

$$\begin{aligned} \boldsymbol{x}_f(k+1) &= A_f\boldsymbol{x}_f(k) + \boldsymbol{b}_f\bar{u}(k) \\ y_f(k) &= \boldsymbol{c}_f^T\boldsymbol{x}_f(k) + d_f\bar{u}(k) \end{aligned} \tag{18}$$

Such that the resulting augmented system

$$\begin{aligned} \boldsymbol{x}_a(k+1) &= A_a\boldsymbol{x}_a(k) + \boldsymbol{b}_a\bar{u}(k) \\ e_a(k) &= \boldsymbol{c}_a^T\boldsymbol{x}_a(k) + d_f\bar{u}(k) \end{aligned} \tag{19}$$

$$\boldsymbol{x}_a(k) = \begin{bmatrix} \bar{\boldsymbol{x}}(k) \\ \boldsymbol{x}_f(k) \end{bmatrix}, A_a = \begin{bmatrix} \bar{A} & 0 \\ 0 & A_f \end{bmatrix},$$

$$\boldsymbol{b}_a = \begin{bmatrix} \bar{\boldsymbol{b}} \\ \boldsymbol{b}_f \end{bmatrix}, \boldsymbol{c}_a^T = \begin{bmatrix} \bar{\boldsymbol{c}}^T & \boldsymbol{c}_f^T \end{bmatrix}$$

is ASPR.

The augmented system with a PFC can be expressed as follows using the form given in (4).

$$\begin{aligned} \boldsymbol{x}_{ak(i+1)} &= A_a\boldsymbol{x}_{ak(i)} + \boldsymbol{b}_a\bar{u}_{k(i)} \\ e_{ak(i)} &= \boldsymbol{c}_a^T\boldsymbol{x}_{ak(i)} + d_f\bar{u}_{k(i)} \\ &= e_{k(i)} + c_f^T\boldsymbol{x}_{fk(i)} + d_f\bar{u}_{k(i)} \\ e_{k(i)} &= y_{k(i)} - y_{mk(i)} \end{aligned} \tag{20}$$

For this ASPR system, if we can design the controller by an output feedback with ideal feedback gain k^* as

$$\bar{u}^*_{k(i)} = -k^* e_{ak(i)} \tag{21}$$

the resulting control system should be SPR.

The input given in (21) cannot be realized due to unmeasurable output error signals $e_{ak(i)}$ and causality problem from the feedthrough input term. Therefore, in practice, the controller is designed using available signals as follows:

$$\begin{aligned} \bar{u}_{k(i)} &= -\tilde{k}^*\hat{\hat{e}}_{ak(i)}\ ,\ \tilde{k}^* = k^*(1+d_f k^*)^{-1} \\ \hat{\hat{e}}_{ak(i)} &= \hat{e}_{k(i)} + \boldsymbol{c}_f^T\boldsymbol{x}_{fk(i)} \end{aligned} \tag{22}$$

where

$$\begin{aligned} \hat{e}_{ak(i)} &= \hat{y}^*_{k(i)} + y_{fk(i)} - y_{mk(i)} \\ \hat{y}^*_{k(i)} &= \begin{cases} y_{k(i)} & \\ \hat{y}_{k(i)} & (i = 1, 2, \cdots, n_k - 1) \end{cases} \end{aligned}$$

If the ideal gain k^* is known, using the controller given in (22), one can obtain a stable control system [9]. Unfortunately, it might be hard to obtain an optimal and/or ideal feedback gain for uncertain systems. Here, we consider adaptively adjusting the feedback gain in (22).

The fast-rate adaptive output feedback controller is designed as follows:

$$\bar{u}_{k(i)} = -\tilde{\theta}_{pk(i)}\hat{\hat{e}}_{ak(i)} \tag{23}$$

$$\tilde{\theta}_{pk(i)} = \bar{\sigma}_p\tilde{\theta}_{pk(i-1)} + \bar{\sigma}_p\gamma\hat{\hat{e}}\hat{e}_{ak(i)} + p_{k(i)} \tag{24}$$

where $p_{k(i)}$ is a parameter projection given by

$$p_{k(i)} = \begin{cases} 0 & \text{if} \quad \tilde{\theta}_{p\min} \le f_{pk(i)} < \frac{1}{d_f} \\ -\bar{\sigma}_p\gamma\hat{\hat{e}}_{ak(i)}\hat{e}_{ak(i)} & \text{otherwise} \end{cases}$$

$$f_{pk(i)} = \bar{\sigma}_p\tilde{\theta}_{pk(i-1)} + \bar{\sigma}_p\gamma\hat{\hat{e}}_{ak(i)}\hat{e}_{ak(i)}$$

with $\tilde{\theta}_{p0(0)} = \tilde{\theta}_{p\min}$, and $\hat{e}_{ak(i)}$ is obtained via available signals as follows:

$$\hat{e}_{ak(i)} = \frac{\left\{1 - d_f\bar{\sigma}_p\tilde{\theta}_{pk(i-1)}\right\}\hat{\hat{e}}_{ak(i)}}{1 + d_f\bar{\sigma}_p\gamma\hat{\hat{e}}^2_{ak(i)}} \tag{25}$$

The designed control system is illustrated in Fig. 3, where S is a sampler at sampling periods of n_kT.

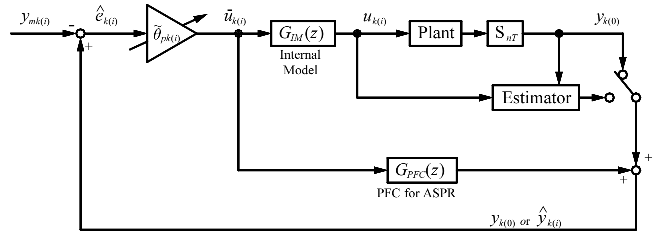


Fig. 3. Non-uniform multirate control system with fast-rate output feedback

B. Stability Analysis of the Control System

The designed control input can be expressed by

$$\begin{aligned} \bar{u}_{k(i)} &= -\tilde{\theta}_{pk(i)}\hat{\hat{e}}_{ak(i)} \\ &= -\tilde{k}^*\tilde{e}_{ak(i)} + \psi_{k(i)} \end{aligned}$$

where

$$\begin{aligned} \tilde{e}_{ak(i)} &= e_{k(i)} + \boldsymbol{c}_f^T\boldsymbol{x}_{fk(i)} \\ \psi_{k(i)} &= -\Delta\tilde{\theta}_{pk(i)}\hat{\hat{e}}_{ak(i)} - \tilde{k}^*\epsilon_{k(i)} \\ \epsilon_{k(i)} &= \hat{e}_{ak(i)} - e_{ak(i)} \end{aligned}$$

Thus, using the fact that $e_{ak(i)}$ can be expressed by

$$\begin{aligned} e_{ak(i)} &= \tilde{e}_{ak(i)} + d_f\bar{u}_{k(i)} \\ &= \left(1 - d_f\tilde{k}^*\right)\tilde{e}_{ak(i)} + d_f\psi_{k(i)} \end{aligned} \tag{26}$$

the obtained closed-loop fast-rate system can be represented as

$$\begin{aligned} \boldsymbol{x}_{ak(i+1)} &= A_{c\epsilon}\boldsymbol{x}_{ak(i)} + \boldsymbol{b}_{c\epsilon}\psi_{k(i)} \\ e_{ak(i)} &= \boldsymbol{c}_{c\epsilon}^T\boldsymbol{x}_{ak(i)} + d_{c\epsilon}\psi_{k(i)} \end{aligned} \tag{27}$$

$$\begin{aligned} A_{c\epsilon} &= A_a - \boldsymbol{b}_a\tilde{k}^*\boldsymbol{c}_a^T \quad , \quad \boldsymbol{b}_{c\epsilon} = \boldsymbol{b}_a \\ \boldsymbol{c}_{c\epsilon} &= \{1 + d_f k^*\}^{-1}\boldsymbol{c}_a \quad , \quad d_{c\epsilon} = d_f \end{aligned}$$

Under Assumption 4, this closed-loop system is SPR and then there exist positive symmetric matrices $P = P^T > 0$, $Q = Q^T > 0$, a vector $\boldsymbol{l}$ and a constant w such tat the following KYP-Lemma is satisfied

$$\begin{aligned} A_{c\epsilon}^T P A_{c\epsilon} - P &= -\boldsymbol{l}\boldsymbol{l}^T - Q \\ A_{c\epsilon}^T P\boldsymbol{b}_{c\epsilon} &= \boldsymbol{c}_{c\epsilon}^T - w\boldsymbol{l} \\ \boldsymbol{b}_{c\epsilon}^T P\boldsymbol{b}_{c\epsilon} &= 2d_{c\epsilon} - w^2 \end{aligned} \tag{28}$$

Let's consider the following positive definite function:

$$V_k = V_{ak} + V_{bk} \tag{29}$$

where,V_{ak} has been given in (16) and V_{bk} is given by

$$V_{bk} = V_{4k} + \frac{\rho_4}{(1+\sigma_p)\,\gamma}V_{5k} \tag{30}$$

$$V_{4k} = \rho_4\boldsymbol{x}_{ak(0)}^T P\boldsymbol{x}_{ak(0)} \quad , \quad \rho_4 > 0 \tag{31}$$

$$V_{5k} = \Delta\tilde{\theta}^2_{pk-1(n_k-1)} \tag{32}$$

The difference $\Delta V_{4k} = V_{4k} - V_{4k-1}$ can be represented by

$$\begin{aligned} \Delta V_{4k} &= \rho_4\boldsymbol{x}_{ak(0)}^T P\boldsymbol{x}_{ak(0)} - \rho_4\boldsymbol{x}_{ak-1(0)}^T P\boldsymbol{x}_{ak-1(0)} \\ &= \sum_{i=0}^{n_{k-1}-1}\Delta V_{4ik} \end{aligned} \tag{33}$$

where,

$$\Delta V_{4ik} = \rho_4 \boldsymbol{x}_{ak-1(i+1)}^T P \boldsymbol{x}_{ak-1(i+1)} - \rho_4 \boldsymbol{x}_{ak-1(i)}^T P \boldsymbol{x}_{ak-1(i)}$$

From KYP-Lemma, ΔV_{4ik} can be evaluated as

$$\begin{aligned} &\Delta V_{4ik} \\ &\quad \le -\rho_4 \lambda_{min}[Q] \|\boldsymbol{x}_{ak-1(i)}\|^2 - 2\rho_4 \hat{e}_{ak-1(i)} \Delta\tilde{\theta}_{pk(i)} \hat{\bar{e}}_{ak(i)} \\ &\qquad +2\rho_4 \left\{ \epsilon_{k-1(i)} \Delta\tilde{\theta}_{pk-1(i)} \hat{\bar{e}}_{ak(i)} - e_{ak-1(i)} \tilde{k}^* \epsilon_{k-1(i)} \right\} \end{aligned} \tag{34}$$

Since we have from (20),(23) that

$$|\epsilon_{k-1(i)}| \le \frac{\alpha_{xi}\|\boldsymbol{x}_{ak-1(0)}\| + \alpha_{ri}\bar{r}}{1 + 2d_f \theta_{p\min}} \tag{35}$$

$$|u_{k-1(i)}| \le \alpha_{uxi}\|\boldsymbol{x}_{ak-1(0)}\| + \alpha_{uri}\bar{r} \tag{36}$$

$$|e_{ak-1(i)}| \le \frac{\bar{\bar{\alpha}}_{xi}\|\boldsymbol{x}_{ak-1(0)}\| + \bar{\bar{\alpha}}_{ri}\bar{r}}{1 + d_f \theta_{p\min}} \tag{37}$$

$$|\hat{\bar{e}}_{ak-1(i)}| \le \hat{\bar{\alpha}}_{xi}\|\boldsymbol{x}_{ak-1(0)}\| + \hat{\bar{\alpha}}_{ri}\bar{r} \tag{38}$$

with appropriate positive constants α_{xi}, α_{ri}, α_{uxi}, α_{uri}, $\bar{\bar{\alpha}}_{xi}$, $\bar{\bar{\alpha}}_{ri}$, $\hat{\bar{\alpha}}_{xi}$, $\hat{\bar{\alpha}}_{ri}$ which are bound for any magnitude of k^*, it follows from (35), (37) that

$$\begin{aligned} &\epsilon_{k-1(i)} \Delta\tilde{\theta}_{pk-1(i)} \hat{\bar{e}}_{ak-1(i)} - e_{ak-1(i)} \tilde{k}^* \epsilon_{k-1(i)} \\ &\le (\delta_{b1i} + \alpha_{b1i}) \|\boldsymbol{x}_{ak-1(0)}\|^2 + \left(\frac{\alpha_{b2i}^2}{4\delta_{b1i}} + \alpha_{b3i} \right) \bar{r}^2 \end{aligned} \tag{39}$$

for $\delta_{b1i} > 0$, where $\alpha_{b1i}, \alpha_{b2i}, \alpha_{b3i}$ are also bounded constants for any k^*.

Consequently, ΔV_{4k} can be evaluated by

$$\begin{aligned} \Delta V_{4k} \le& -\rho_4 \lambda_{min}[Q] \sum_{i=0}^{n_{k-1}-1} \|\boldsymbol{x}_{ak-1(i)}\|^2 \\ &+2\rho_4 \sum_{i=1}^{n_{k-1}-1} \left\{ (\delta_{b1i} + \alpha_{b1i}) \|\boldsymbol{x}_{ak-1(0)}\|^2 \right. \\ &\qquad \left. + \left(\frac{\alpha_{b2i}^2}{4\delta_{b1i}} + \alpha_{b3i} \right) \bar{r}^2 \right\} \\ &-2\rho_4 \sum_{i=0}^{n_{k-1}-1} \hat{\bar{e}}_{ak-1(i)} \Delta\tilde{\theta}_{pk-1(i)} \hat{e}_{ak-1(i)} \end{aligned} \tag{40}$$

Next, the difference $\Delta V_{5k} = V_{5k} - V_{5k-1}$ can be represented as

$$\Delta V_{5k} = \sum_{i=0}^{n_{k-1}-1} \Delta V_{5ik} \tag{41}$$

$$\Delta V_{5ik} = \Delta\tilde{\theta}_{pk-1(i)}^2 - \Delta\tilde{\theta}_{pk-1(i-1)}^2$$

and then it can be evaluated from (24) that

$$\begin{aligned} \Delta V_{5ik} \le& -\left\{ (1+\sigma_p)^2 - 1 - \delta_\theta \right\} \Delta\tilde{\theta}_{pk-1(i)}^2 \\ &+ \frac{\sigma_p^2 (1+\sigma_p)^2}{\delta_\theta} \tilde{k}^{*2} \\ &+2(1+\sigma_p)\gamma \hat{\bar{e}}_{ak-1(i)} \Delta\tilde{\theta}_{pk-1(i)} \hat{e}_{ak-1(i)} \end{aligned} \tag{42}$$

Finally, using the fact from (4),(36) that

$$\begin{aligned} |y_{k-1(i)}|^2 &\le \|\bar{\boldsymbol{c}}\|^2 \|\boldsymbol{x}_{ak-1(i)}\|^2 + 2\|\bar{\boldsymbol{c}}\| \|\boldsymbol{x}_{ak-1(i)}\| \bar{r} + \bar{r}^2 \\ |u_{k-1(i)}|^2 &\le \alpha_{uxi}^2 \|\boldsymbol{x}_{ak-1(0)}\|^2 \\ &\qquad +2\alpha_{uxi}\alpha_{uri}\bar{r}\|\boldsymbol{x}_{ak-1(0)}\| + \alpha_{uri}^2 \bar{r}^2 \end{aligned}$$

and using the result in Lemma 1, $\Delta V_k = \Delta V_{ak} + \Delta V_{bk}$ can be evaluated by

$$\begin{aligned} \Delta V_k \le& -\alpha_\epsilon |\epsilon_{k(0)}|^2 - \rho_1 |\epsilon_{k-1(0)}|^2 - \alpha_\eta \|\boldsymbol{\eta}_{k-1(0)}\|^2 \\ &- (\rho_2 \alpha_{21} - \delta_2) \sum_{i=1}^{n_{k-1}-1} \|\boldsymbol{\eta}_{k-1(i)}\|^2 \\ &-\alpha_{ab} \left\{ \gamma_b^{-1} \Delta b_{yk}^2 + \sum_{i=1}^{n_{k-1}} \gamma_{ai}^{-1} \Delta a_{ik}^2 \right\} \\ &-\alpha_{xa0} \|\boldsymbol{x}_{ak-1(0)}\|^2 - \alpha_{xai} \sum_{i=1}^{n_{k-1}-1} \|\boldsymbol{x}_{ak-1(i)}\|^2 \\ &-\alpha_p \sum_{i=0}^{n_{k-1}-1} \Delta\tilde{\theta}_{pk-1(i)}^2 + R \end{aligned} \tag{43}$$

where

$$\begin{aligned} \alpha_p =& \frac{\rho_4 \left\{ (1+\sigma_p)^2 - 1 - \delta_\theta \right\}}{(1+\sigma_p)\gamma} \\ \alpha_r =& \alpha_y + \rho_2 \left(\alpha_{23} + \frac{\rho_2 \alpha_{22}^2}{\delta_2} \right) (n_{k-1} - 1) \tilde{a}_y^2 \tilde{b}_y^2 \\ &\times \sum_{i=0}^{n_{k-1}-2} \frac{1}{\delta_{ui}} \alpha_{uri}^2 + \frac{\tilde{b}_y}{\delta_{uy\epsilon}} \alpha_{urn_{k-1}-1}^2 \\ &+ \frac{\tilde{a}_y}{\delta_{uy\epsilon}} \alpha_{urn_{k-1}-2}^2 + 2\rho_4 \sum_{i=1}^{n_{k-1}-1} \left(\frac{\alpha_{b2i}^2}{4\delta_{b1i}} + \alpha_{b3i} \right) \\ &+ \frac{1}{\delta_{xa0}} \alpha_{\bar{r}x0}^2 + \frac{1}{\delta_{xai}} \rho_2^2 \left(\alpha_{23} + \frac{\rho_2 \alpha_{22}^2}{\delta_2} \right)^2 \|\bar{\boldsymbol{c}}\|^2 \\ \alpha_{xa0} =& \rho_4 \lambda_{min}[Q] - \alpha_y \|\bar{\boldsymbol{c}}\|^2 - 2\rho_4 \sum_{i=1}^{n_{k-1}-1} (\delta_{b1i} + \alpha_{b1i}) \\ &- \tilde{a}_y^2 \tilde{b}_y^2 \sum_{i=0}^{n_{k-1}-2} \frac{1}{\delta_{ui}} \alpha_{uxi}^2 \\ &- \frac{\tilde{b}_y}{\delta_{uy\epsilon}} \alpha_{uxn_{k-1}-1}^2 - \frac{\tilde{a}_y}{\delta_{uy\epsilon}} \alpha_{uxn_{k-1}-2}^2 - \delta_{xa0} \\ \alpha_{xai} =& \rho_4 \lambda_{min}[Q] - \rho_2 \left(\alpha_{23} + \frac{\rho_2 \alpha_{22}^2}{\delta_2} \right) \|\bar{\boldsymbol{c}}\|^2 - \sum_{i=1}^{n_{k-1}-1} \delta_{xai} \\ R =& \alpha_r \bar{r}^2 + \sum_{i=1}^{n_{k-1}-1} \bar{\sigma} \left(\frac{1}{\delta_{ai}^2} - \sigma^2 \right) \gamma_{ai}^{-1} a_{yi}^{*2} \\ &+ \frac{1}{\delta_3} \left\{ \frac{b_y^{*2}}{\gamma_b} + \sum_{i=1}^{n_{k-1}} \frac{a_{yi}^{*2}}{\gamma_{ai}} \right\} + n\rho_4 \frac{\sigma_p^2 (1+\sigma_p)}{\delta_\theta \gamma} \tilde{k}^{*2} \\ &\delta_0 > 0 \ , \ \delta_i > 0 \end{aligned}$$

Considering a ρ_4 such that $\rho_4 \gg \rho_2 > 0$, one can confirm that there exists a sufficient large k^* for sufficient small δ_{b1i}, δ_1 such that $\alpha_{xa0}, \alpha_{xai} > 0$. Thus, ΔV_k can be evaluated for a sufficient large k^* by

$$\Delta V_k \leq -\alpha V_k + R \tag{44}$$

$$\alpha = \min\left[\frac{\alpha_\epsilon}{\rho_1}, \frac{\alpha_\eta}{\rho_2\lambda_{\max}[P_\eta]}, \frac{\bar{\sigma}^{-1}-\bar{\sigma}-\delta_3}{\max\left\{\gamma_b^{-1}, \gamma_{ai}^{-1}\right\}}, \frac{\alpha_{xa0}}{\rho_4\lambda_{\max}[P]}, \frac{\alpha_{xai}}{\rho_4\lambda_{\max}[P]}, \alpha_p\right]$$

Then we can conclude that all the signals in the control system are uniformly bounded.

Theorem 1: Under Assumptions 1 to 4, designing a control system with the adaptive output estimator (10) and the adaptive output feedback (23) with the PFC (18), all the signals in the resulting control system are uniformly bounded.

V. Validation through Numerical Simulation

The effectiveness of the proposed method is confirmed through numerical simulations. In this simulation, we assume that the output $y(t)$ is sampled randomly at a maximum period of $10T = 100$ [sec] but the input signal $u(t)$ can be updated through a zero-order hold at a fast period $T = 10$ [sec].

In the simulation, as an unknown controlled system, we consider the following system which is expressed by a fast-rate model as in (1):

$$\begin{aligned} \boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + \boldsymbol{b}u(k) \\ y(k) &= \boldsymbol{c}^T\boldsymbol{x}(k) \end{aligned} \tag{45}$$

Where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 3.28\times10^{-5} & 7.74\times10^{3} & 0.416 & -1.83 & 2.41 \end{bmatrix}$$

$$\boldsymbol{b} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$\boldsymbol{c} = \begin{bmatrix} -6.95\times10^{-7} & -1.88\times10^{-4} & -0.014 & 0.025 & 0.032 \end{bmatrix}^T$$

The transfer function representation of the system can be given by

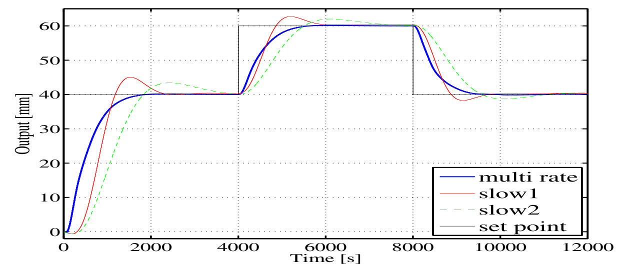
$$G_p(z) = \frac{0.0323z^4 + 0.0249z^3 - 0.0137z^2}{z^5 - 2.41z^4 + 1.83z^3 - 0.416z^2} \quad \frac{-1.87\times10^{-4}z + 6.95\times10^{-7}}{+7.37\times10^{-3}z - 3.28\times10^{-5}} \tag{46}$$

This system is non-minimum phase and it is supposed that this system is unknown but the approximated fast rate model $G_p^*(z)$ is known. In this simulation, we set $G_p^*(z)$ as follows:

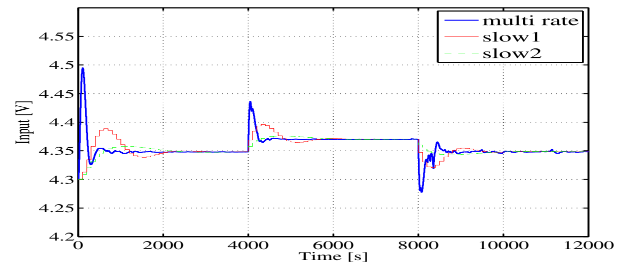
$$G_p^*(z) = \frac{0.0258z^4 + 0.020z^3 - 0.011z^2}{z^5 - 2.41z^4 + 1.83z^3 - 0.416z^2} \quad \frac{-1.503\times10^{-4}z + 5.558\times10^{-7}}{+7.37\times10^{-3}z - 3.28\times10^{-5}} \tag{47}$$

Using this approximated model, a PFC which renders the augmented fast rate system ASPR was designed as follows according to the model based PFC design scheme [14].

$$G_{PFC1}(z) = G_{ASPR}(z) - G_p^*(z) \tag{48}$$



(a) Outputs



(b) Inputs

Fig. 4. Simulation results; the proposed control method and slow-rate controls

with an ideal ASPR model of

$$G_{ASPR}(z) = \frac{20z - 15.3}{z - 0.967} \tag{49}$$

Furthermore, since the model is not minimum-phase, we introduced additional PFC for output estimation. The PFC was designed as follows in order to render a minimum-phase augmented system with a relative degree of 1.

$$G_{PFC2}(z) = G_{est}(z) - G_p^*(z) \tag{50}$$

$$G_{est}(z) = \frac{1}{z - 0.1}$$

In the simulations, the design parameters in the output estimator and the adaptive controller are set as:

$$\gamma_b = 500,\ \gamma_{ai} = 1(i = 1, \cdots, 10),\ \sigma = 10^{-5}$$

$$\gamma = 10^{-6},\quad \sigma_p = 10^{-5}$$

and we set in the parameter projection that

$$0 \leq a_y \leq 500, \quad -500 \leq b_y \leq 500.$$

The internal model was given by

$$\bar{G}(z) = \frac{z}{z-1}$$

Fig. 4 show the simulation results. The output reaches the given set points quickly and a good control performance is shown compared with slow-rate control.

VI. Conclusion

In this paper, we proposed a design scheme of an adaptive output feedback control for uncertain non-uniformly sampled systems in which the output signals are non-uniformly sampled with a slow sampling period. An adaptive fast-rate output estimator was applied to design fast-rate adaptive output feedback for a system which satisfies the almost strictly positive real (ASPR) property.

REFERENCES

[1] D. Li, S. L. Shah, and T. Chen, "Identification of fast-rate models from multirate data," *International Journal of Control*, vol. 74, no. 7, pp. 680–689, 2001.

[2] D.Li, S.L.Shah, T.Chen, and K.Z.Qi, "Application of dual-rate modeling to capital octane quality inferential control," *IEEE Transactions on Control System Technology*, vol. 11, no. 1, pp. 43–51, 2003.

[3] T. Chen and B. A. Francis, *Optimal sampled-data control systems*. London: Springer, 1995.

[4] W.Lu and D.G.Fisher, "Output estimation with multi-rate sampling," *International Journal of Control*, vol. 48, no. 1, pp. 149–160, 1988.

[5] W.Lu and D.G.Fisher, "Least-squares output estimation with multirate sampling," *IEEE Transactions on Automatic Control*, vol. AC-34, no. 6, pp. 669–672, 1989.

[6] F.Ding and T.Chen, "Combined parameter and output estimation of dual-rate systems using an auxiliary model," *Automatica*, vol. 40, pp. 1739–1748, 2004.

[7] J.Wang, T.Chen, and B.Huamg, "Multirate sampled-data systems computing fast-rate models," *Journal of Process Control*, vol. 14, pp. 79–88, 2004.

[8] I. Mizumoto, S. Ohdaira, N. Watanabe, H. Tanaka, H. Harada, Y. Fujimoto, H. Kinoshita, and Z. Iwai, "Output feedback control of multirate sampled systems with an adaptive output estimator and its application to a liquid level process control," *Journal of System Design and Dynamics*, vol. 4, no. 2, pp. 314–330, 2010a.

[9] I. Mizumoto, Y. Fujimoto, and Z. Iwai, "Fast-rate output feedback control with adaptive output estimator for non-uniformly sampled multirate systems," *Proc. of IFAC World Congress 2011*, 2011 (to be published).

[10] J. Sheng, T. Chen, and S. L. Shah, "Generalized predictive control for non-uniformly sampled systems," *Journal of Process Control*, vol. 12, pp. 875–885, 2002.

[11] I. Mizumoto, S. Ohdaira, M. Kumon, and Z. Iwai, "Model output following control of non-uniformly sampled systems," *Trans. of Society of Instrument and Control Engineers (in Japanese)*, vol. 42, no. 12, pp. 1295–1304, 2006.

[12] I. Barkana, "Parallel feedforward and simplified adaptive control," *International Journal of Adaptive Control and Signal Processing*, vol. 1, no. 2, pp. 95–109, 1987.

[13] H. Kaufman, I. Barkana, and K. Sobel, *Direct Adaptive Control Algorithms*. Springer, 2nd ed., 1997.

[14] I. Mizumoto, D. Ikeda, T. Hirahata, and Z. Iwai, "Design of discrete time adaptive pid control systems with parallel feedforward compensator," *Control Engineering Practice*, vol. 18, no. 2, pp. 168–176, 2010b.

[15] A. Ishidori, *Nonlinear Control Systems*. Springer, 3rd ed., 1985.