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Inverse Optimal Trajectory Tracking for Discrete Time Nonlinear Positive Systems

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Abstract— **In this paper, discrete time inverse optimal trajectory tracking for a class of non-linear positive systems is proposed. The scheme is developed for MIMO (multi-input, multi-output) affine systems. This approach is adapted for glycemic control of type 1 diabetes mellitus (T1DM) patients. The control law calculates the insulin delivery rate in order to prevent hyperglycemia levels. A neural model is obtained from an on-line neural identifier, which uses a recurrent neural network, trained with the extended Kalman filter (EKF); this neural model has an affine form, which permits the applicability of inverse optimal control scheme. The proposed algorithm is tuned to follow a desired trajectory; this trajectory reproduces the glucose absorption of a healthy person. Simulation results illustrate the aplicability of the control law in biological processes.**

I. Introduction

Many physical systems involve variables which are always positive. These class of systems are called positive systems [1]. The state variables of positive systems are confined within a "cone" located in the positive orthant rather than in the whole space R^n. This feature makes the analysis and synthesis of positive systems a challenging and interesting task [2] [3]. Considering this class of systems, the goal of this paper is design a feedback controller which stabilizes a non-linear positive system along a desired trajectory. This paper reports an important extension to positive systems which is inspired by [4]. The scheme control is focused in the application to Type 1 Diabetes Mellitus (T1DM) Patients. Recently, many authors have worked intensively to develop an appropriate T1DM controller [5] [6] [7]. Progress has been important, as there are insulin pumps and glucose sensors very sophisticated which improve the quality of life patients.

For the case of T1DM, there are different models representing glucose-insulin dynamics [8] [9]. However, synthesizing a control law for these models is complicated due to the complexity associated with the measurements and the uncertainty of related parameters [10]. Hence, neural identification is an excellent option to determine mathematical models for glucose dynamics. The neural identification used in this paper comes from the compartmental model proposed by Sorensen [8]. The present paper reports the modeling and the control for glucose-insulin dynamics, which allows representing a virtual patient, for prediction purposes and for performance evaluation of the proposed controller.

This paper is organized as follows: First the inverse optimal control strategy is developed for positive systems; then the neural model in an affine form is obtained from the compartmental model developed by Sorensen; and finally, simulation results are described and important conclusions are stated.

II. Inverse Optimal Trajectory Tracking for Positive Systems

Let consider a nonlinear affine system with an output (to achieve passivity) given as

$$x_{k+1} = f\left(x_k\right) + g(x_k)\, u_k \tag{1}$$

$$y_k = h(x_k) + J(x_k)\, u_k \tag{2}$$

where (1) is a positive system if $x \in \Re^n_+$ which is the state of the system at time $k \in \mathcal{N}$, $u, y \in \Re^m_+$ $f : \Re^n_+ \to \Re^n_+$, $g : \Re^n_+ \to \Re^{n\times m}_+$, $h : \Re^n_+ \to \Re^m_+$, and $J : \Re^n_+ \to \Re^{m\times m}_+$ are smooth and bounded mappings. For all the last definitions $\bullet_+$ indicates non-negative elements including zero. We assume $f(0) = 0$ and $h(0) = 0$. $\mathcal{N}$ denotes the set of nonnegative integers. It is worth to note that, the output which renders the system passive is not in general the variable we wish to control.

The problem considered in this paper is to find a feedback control law which stabilizes system (1) along a desired trajectory, and to establish that this controller is inverse optimal with respect to a cost functional given as

$$C = \sum_{k=0}^{\infty} L(x_k, u_k) \tag{3}$$

where $L(\bullet)$ is a non-negative function. Similar to the continuous-time case, the discrete-time Hamiltonian becomes [11]

$$H(x_k, u_k) = L(x_k, u_k) + V(x_{,k+1}) - V(x_k) \tag{4}$$

where $H(x_k, u_k) = 0$ for $x \in \Re^n_+$ and the optimal control law $u \in \Re^m_+$; $V : \Re^n_+ \to \Re_+$ is a nonnegative definite function such that $V(0) = 0$ and $V(x_k) > 0$ (positive definite function), $\forall x_k \neq 0$.

Due to the fact that the inverse optimal control is based on a Lyapunov function, we establish the following definitions:

Definition 1 (DTCLF [12]) Let V_1 be a radially unbounded, positive definite function, with $V_1\left(x_k\right) > 0$,

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$\forall x_k \neq 0$ and $V_1(0) = 0$. If for any $x_k \in \Re^n_+$, there exist real values u_k such that

$$\Delta V_1(x_k, u_k) < 0$$

where $\Delta V_1(x_k, u_k)$ is defined as

$$V_1\left(f(x_k) + g(x_k)\, u_k\right) - V_1(x_k)$$

Then $V_1(\cdot)$ is said to be a "discrete-time control Lyapunov function" (DTCLF) for system (1).

Definition 2 (Passivity [13]) The system (1)-(2) is said to be passive if there exists a non-negative function V, called storage function, such that for all u_k

$$V(x_{k+1}) - V(x_k) \leq y_k^T\, u_k \tag{5}$$

where $(\cdot)^T$ denotes transpose. This storage function can be selected as a DTCLF if it is a positive definite function.

Definition 3 [14]. System (1)-(2) is locally zero-state observable (respectively locally zero-state detectable) if there exists a neighborhood $\mathcal{Z}$ of $x_k = 0$ such that $x_0 = x_k \in \mathcal{Z}$.

$$y_k|_{u(k)=0} = h(\Phi(k, x_k, 0)) = 0 \qquad \forall k \Rightarrow x_k = 0$$

$$(resp. \lim_{k\to\infty} \Phi(k, x_k, 0) = 0)$$

where $\phi(k, x_k, 0) = f^k(x_k)$ is the trajectory of the unforced dynamics $x_{k+1} = f(x_k)$ from $x_0 = x_k$; $\mathcal{Z}$ is in general a neighborhood of the origin in $\Re^n_+$. If $\mathcal{Z} = \Re^n_+$, the system is zero-state observable (respectively zero-state detectable).

It is important to note that the output, with respect to which the system is rendered passive, will not be the variable which we wish to control. The passive output will only be a preliminary step for control synthesis; additionally, we have to define the signals which ensure the output variables, which we want to control, behaves as desired.

To achieve tracking trajectory, first we define a DTCLF as

$$V(x_k, x_{\delta,k}) = \frac{1}{2}(x_{\delta,k} - x_k)^T\, {K_1}^T\, PK_1\, (x_{\delta,k} - x_k) \tag{6}$$

where $x_{\delta,k} \in R^n_+$ is the desired trajectory and K_1 is an additional gain matrix to modify the convergence rate of the tracking error.

Theorem 1. Assume an affine discrete-time positive nonlinear system (1), and define an output as

$$y_k = h(x_k, x_{\delta,k+1}) + J(x_k)\, u_k \tag{7}$$

which is zero-state detectable with a candidate DTCLF defined by (6), and satisfies the modified passivity condition

$$V(x_{k+1}, x_{\delta,k+1}) - V(x_k, x_{\delta,k}) \leq y_k^T\, u_k \tag{8}$$

If there exists a $\overline{P} = \overline{P}^T > 0$ such that

$$\begin{gathered} f^T\, \overline{P}\, f + x^T_{\delta,k+1}\, \overline{P}\, x_{\delta,k+1} - f^T\, \overline{P}\, x_{\delta,k+1} \\ -x^T_{\delta,k+1}\, \overline{P}\, f - (x_k - x_{\delta,k})^T\, \overline{P}\, (x_k - x_{\delta,k}) \\ \leq 0 \end{gathered}$$

where $\overline{P} = K_1^T P K_1$ is a positive definite matrix. Then, system (1) with output (7), is globally asymptotically stabilized, along the desired trajectory $(x_{\delta,k})$, by the output feedback

$$u_k = -\left(I_m + J(x_k)\right)^{-1} h(x_k, x_{\delta,k+1}) \tag{9}$$

with

$$h(\bullet) = \begin{cases} g^T(x_k)\, \overline{P}\, (x_{\delta,k+1} - f(x_k)) \text{ for } f(x_k) \succeq x_{\delta,k+1} \\ g^T(x_k)\, \overline{P}\, (f(x_k) - x_{\delta,k+1}) \text{ for } f(x_k) \prec x_{\delta,k+1} \end{cases} \tag{10}$$

and

$$J(x_k) = \frac{1}{2}\, g^T(x_k)\, \overline{P}\, g(x_k) \tag{11}$$

Moreover, with (6) as a DTCLF, this control law is inverse optimal in the sense that minimizes the cost functional given as [4]

$$C = \sum_{k=0}^{\infty} L\left(x_k, x_{\delta,k}, u_k\right) \tag{12}$$

Proof: **Case 1** ($h(x_k, x_{\delta,k+1}) = g^T(x_k)\, \overline{P}\, (x_{\delta,k+1} - f(x_k))$). Let (6) be a candidate DTCLF. System (1) with output (7), must be rendered passive, such that the inequality (8) is fulfilled. Then, from (8), and considering one step ahead for $x_{\delta,k}$, we have

$$\begin{aligned} & \frac{(x_{\delta,k+1} - x_{k+1})^T\, K_1^T\, P\, K_1\, (x_{\delta,k+1} - x_{k+1})}{2} \\ & -\frac{(x_{\delta,k} - x_k)^T\, K_1^T\, P\, K_1\, (x_{\delta,k} - x_k)}{2} \\ \leq\ & h^T(x_k, x_{\delta,k}) u_k + u_k^T\, J^T(x_k) u_k. \end{aligned} \tag{13}$$

Defining $\overline{P} = K_1^T\, P\, K_1$ and substituting (1) in (13), we have

$$\begin{aligned} & \frac{(x_{\delta,k+1} - f - g\, u_k)^T\, \overline{P}\, (x_{\delta,k+1} - f - g\, u_k)}{2} \\ & -\frac{(x_{\delta,k} - x_k)^T\, \overline{P}\, (x_{\delta,k} - x_k)}{2} \\ \leq\ & h^T\, u_k + u_k^T\, J^T\, u_k \end{aligned} \tag{14}$$

Hence, (14) becomes

$$\begin{aligned} & f^T\, \overline{P}\, f + x^T_{\delta,k+1}\, \overline{P}\, x_{\delta,k+1} - f^T\, \overline{P}\, x_{\delta,k+1} \\ & -x^T_{\delta,k+1}\, \overline{P}\ f - (x_{\delta,k} - x_k)^T\, \overline{P}\, (x_{\delta,k} - x_k) \\ & +(2f^T\, \overline{P}\, g - 2x^T_{\delta,k+1}\, \overline{P}\, g) u_k + u_k^T g^T\, \overline{P}\, g\, u_k \\ \leq\ & 2\, h^T u_k + 2\, u_k^T J^T u_k. \end{aligned} \tag{15}$$

From (15), passivity is achieved if:

1) from the first term of (15), we can find $\overline{P} > 0$ such that

$$\begin{gathered} f^T\, \overline{P}\, f + x^T_{\delta,k+1}\, \overline{P}\, x_{\delta,k+1} - f^T\, \overline{P}\, x_{\delta,k+1} \\ -x^T_{\delta,k+1}\, \overline{P}\, f - (x_k - x_{\delta,k})^T\, \overline{P}\, (x_k - x_{\delta,k}) \leq 0, \end{gathered} \tag{16}$$

2) with $(2f^T \overline{P} g - 2x^T_{\delta,k+1}\, \overline{P}\, g) u_k = 2\, h^T\, u_k$, thus

$$h(x_k, x_{\delta,k+1}) = g^T(x_k)\, \overline{P}\, (f(x_k) - x_{\delta,k+1}), \tag{17}$$

3) and $u^T g^T \overline{P} g u_k = 2 u_k^T J^T u_k$, thus

$$J(x_k) = \frac{1}{2} g^T(x_k) \overline{P} g(x_k). \tag{18}$$

If system (1) with output (7) fulfill the zero-state detectability property and if, from 1), 2), and 3) we deduce that, if there exist a $\overline{P}$, such that is satisfied (16), then the system (1) with output (2) is passive.

To guarantee asymptotic trajectory tracking, we choose $u_k = -y_k$ and then $V(x_{k+1}, x_{\delta,k+1}) - V(x_k, x_{\delta,k}) \leq -y_k^T y_k \leq 0$, which satisfies the Lyapunov forward difference of V.

In order to establish the inverse optimality, (4) is minimize w.r.t. u_k, with:

$$L(x_k, u_k) = l(x_k) + u_k^T u_k$$

$$= l(x_k) - y_k^T u_k$$

where $l(x_k) = -\left(f^T(x_k) P f(x_k) - x_k^T P x_k\right) \geq 0$; thus, we have

$$0 = \min_{u_k} \{L(x_k, u_k) + V(x_{k+1}) - V(x_k)\}$$

$$= \min_{u_k} \left\{l(x_k) - y_k^T u_k + V(x_{k+1}) - V(x_k)\right\}$$

$$= -h^T - u_k^T \left(J + J^T\right) + \left(f^T - y_k^T g^T\right) Pg$$

$$= -h^T - u_k^T \left(J + J^T\right) + f^T Pg - y_k^T g^T Pg$$

Considering $h^T = f^T Pg$ and $J = J^T$, it is obtained

$$0 = -u_k^T J - h^T J - u_k^T J^T J$$

$$u_k^T \left(J + J^T J\right) = -h^T J$$

$$\left(J + J^2\right) u_k = -Jh$$

and solving for u_k, the proposed inverse optimal control law is given as

$$u_k = -\left(I_m + J(x_k)\right)^{-1} h(x_k, x_{\delta,k+1}) \tag{19}$$

with $h(x_k, x_{\delta,k+1}) = g^T(x_k) \overline{P} \left(x_{\delta,k+1} - f(x_k)\right)$

Now, solving (4) for $L(x_k, u_k)$ and summing over $[0, N]$, where $N \in \mathcal{N}$ yields

$$\sum_{k=0}^{N} L(x_k, u_k) = -V(x_N) + V(x_0) + \sum_{k=0}^{N} H(x_k, u_k)$$

Letting $N \to \infty$ and nothing that $V(x_N) \longrightarrow 0$ for all x_0, and $H(x_k, u_k) = 0$ for the inverse optimal control u_k, then $C(x_0, u_k) = V(x_0)$, which is called the optimal value function. Finally, if $V(x_k)$ is a radially unbounded function, i.e., $V(x_k) \longrightarrow \infty$ as $\|x_k\| \longrightarrow \infty$, then the solution $x_k = 0$, $k \in \mathcal{N}$, of the closed-loop system (1) is globally asymptotically stable.

Case 2 ($h(x_k, x_{\delta,k+1}) = g^T(x_k) \overline{P} (f(x_k) - x_{\delta,k+1})$). It can be derived as explained in [4]. Then the proposed inverse optimal control law is given as

$$u_k = -\left(I_m + J(x_k)\right)^{-1} h(x_k, x_{\delta,k+1}) \tag{20}$$

with $h(x_k, x_{\delta,k+1}) = g^T(x_k) \overline{P} \left(f(x_k) - x_{\delta,k+1}\right)$

Finally combining (19) and (20) the control law is given as

$$u_k = abs\left[-\left(I_m + J(x_k)\right)^{-1} h(x_k, x_{\delta,k+1})\right] \tag{21}$$

which ensures that $h(\bullet)$ satisfies (10); the absolute value in the control law is used to simplify the calculations for the implementations. ■

III. Neural Glucose metabolism model

A. On-line Neural Identification

In this paper for the neural model identification the RMLP (Recurrent Multi-Layer Perceptron) is chosen, then the neural model structure definition reduces to dealing with the following issues: selecting the inputs to the network and 2) selecting the internal architecture of the network.

The structure selected in this paper is NNARX [15] (acronym for Neural Network AutoRegressive eXternal input); the output vector for the artificial neural network is defined as the regression vector of an AutorRegressive eXternal input linear model structure (ARX) [16].

It is common to consider a general nonlinear system; however, for many control applications is preferred to express the model in an affine form, which can be represented by the following equations

$$y_{k+1} = f(y_k, y_{k-1}, ..., y_{k-q+1}) + g u_k \tag{22}$$

where q is the dimension of the state space and g is the input matrix. In other words, a nonlinear mapping f exists, for which the present value of the output y_{k+1} is uniquely defined in terms of its past values $y_k, ..., y_{k-q+1}$ and the present values of the input u_k.

Considering that it is possible to define:

$$\phi_k = \left[y_k ... y_{(k-q+1)}\right]^T$$

which is similar to the regression vector of a ARX linear model structure [15], then the nonlinear mapping f can be approximated by a neural network defined as

$$y_{k+1} = \varphi(\phi_k, w^*) + w'^* u_k + \varepsilon$$

where w^* is an ideal weight vector, w'^* is an ideal weight vector for inputs and ε is the modeling error; such neural network can be implemented on predictor form as

$$\hat{y}_{k+1} = \varphi(\phi_k, w) + w' u_k \tag{23}$$

where w is the vector containing the adjustable parameters in the neural network and w' is a fixed weight vector for inputs, which is used to ensure controllability of the neural model [4]. It is worth to note that for identification, adequate inputs signals should be used [17].

B. EKF Training Algorithm

For KF-based neural network training, the network weights become the states to be estimated. Due to the fact that the neural network mapping is nonlinear, an EKF-type is required [18]. The training goal is to find the optimal weight values which minimize the prediction error. The modified Extended Kalman Filter (EKF) algorithm is defined by:

$$w_{k+1} = w_k + K_k \left[y_k - \hat{y}_k \right] \tag{24}$$

$$K_k = P_k H_k^T M_k$$

$$P_{k+1} = P_k - K_k H_k P_k + Q_k$$

with

$$M_k = \left[R_k + H_k P_k H_k^T \right]^{-1}$$

$$e_k = y_k - \hat{y}_k$$

where $e(k) \in \Re$ is the respective approximation error, is the prediction error associated covariance matrix at step k, $w \in \Re^L$ is the weight (state) vector, L is the respective number of neural network weights, y is the system output, $\hat{y}$ is the neural network output, $K \in \Re^L$ is the Kalman gain vector, $Q \in \Re^{L\times L}$ is the state noise associated covariance matrix, $R \in \Re$ is the measurement noise associated covariance; $H \in \Re$ is a vector, in which each entry (H_{IJ}) is the derivative of one of the neural network output, $\left(\hat{y}\right)$, with respect to one neural network weight, (w_j) defined as follows

$$H_{ijk} = \left[\frac{\partial \hat{y}_k}{\partial w_{jk}} \right]^T_{w_{i,k} = w_{i,k+1}}$$

where $i = 1, ..., m$; $j = 1, ..., L$. Usually P and Q are initialized as diagonal matrices, with entries $P(0)$ and $Q(0)$, respectively. It is important to remark that H_k, K_k and P_k for the EKF are bounded; for a detailed explanation of this fact see [19]. The measurement and process noises are typically characterized as zero-mean, white noises with covariance given by $\delta_{k,,j} R_k$ and $\delta_{k,,j} Q_k$, respectively with $\delta_{k,j}$ a Kronecker delta function (zero for $k \neq 1$ and l for $k = l$) [20]. In order to simplify the notation in this paper the covariance will be represented by their respective associated matrices, R_k and Q_k for the noises and P_k for the prediction error.

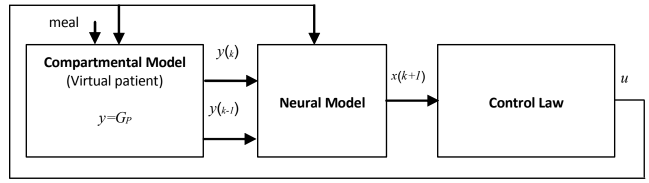

Fig. 1. Closed loop diagram for the control law.

C. Tracking objective

Proposition 1. Given a desired output trajectory $x_d = x_\delta$, a dynamic system with state x, and a neural network identifier with state $\widehat{x}$, the following inequality holds [16]:

$$\|x_d - x\| \leq \|\widehat{x} - x\| + \|x_d - \widehat{x}\| \tag{25}$$

where $x_d - x$ is the system state tracking error, and $\widehat{x} - x$ is the state estimation error and $x_d - \widehat{x}$ is the tracking error for the neural network.

It is possible to establish *Proposition 1* due to the separation principle for discrete-time nonlinear systems [21]. Based on (25), it is possible to divide the tracking objective into two parts:

1. Minimization of $\widehat{x} - x$, which can be achieved by the proposed on-line EKF-learning algorithm (24) for the neural identifier (23).
2. Minimization of $x_d - \widehat{x}$. This minimization is obtained by the control law (9).

IV. Simulation results

In this section inverse optimal trajectory tracking is adapted for glycemic control of type 1 diabetes mellitus (T1DM) patients. Fig. 1 is a block diagram which portrays how the compartmental model proposed by Sorensen [8] is connected to the on-line neural identifier, and how the neural model is used to determining the control law. The compartmental model takes as system input the total glucose absorbed by the patient gut with every meal and the insulin in the plasma after bolus and basal subcutaneous dose as initial conditions; then the on-line neural identifier captures the dynamics of the compartmental model. The neural model is used to calculate the inverse optimal law control and to obtain the insulin dose to be supplied to the compartmental model and to the neural model. Simulations are implemented using Matlab, which is a trade mark of the MathWorks, Inc.

A. On-line Identification

The on-line identification comes from the compartmental model proposed by Sorensen [8]. The compartmental model represents the insulin-glucose dynamics (for more details of the compartmental model see [8]). In order to do the identification, the system output of the compartmental model is the glucose in the periphery interstitial fluid space, which is represented by $y = G_p$; the system input of the compartmental model are the total glucose absorbed by the patient gut with every meal which is represented by Γ_{meal} and the time evolution for insulin in plasma after bolus and basal subcutaneous dose provided to the patient by the Paradigm® Real-time Insulin pump which is represented by $i(t)$ then, $u = [\Gamma_{meal}, i(t)]$. Time evolution of insulin in plasma is obtained using the model proposed by Berger *et al* [22]. The simulation of the compartmental model is done with real data taken from a T1DM patient. To obtain the values for Γ_{meal}, the patient has to register the quantity of carbohydrates every time he takes a meal; then, with the quantity of carbohydrates and using the model proposed by Lehmann *et al.* [23] the appearance of glucose via glucose absorption from the gut is calculated.

It is important to note that the compartmental model [8] is used as a virtual patient in order to update the patient glucose level.

The RMLP used in this paper contains sigmoid units only in the hidden layer (5 neurons); the output layer is a linear one (one neuron). The sigmoid function $S(\bullet)$ is defined as

$$S(\varsigma) = \frac{1}{1 + \exp(-\beta\varsigma)}, \beta > 0$$

where ς is any real value variable. For the RMLP model the input is the insulin, the output is the peripheral glucose and the glucose absorbed by the patient gut with every meal is taken as an unknown disturbance.

The neural model (23) can be represented in state space as follows:

$$x_{1,k+1} = f_1(x_k) \tag{26}$$

$$x_{2,k+1} = f_2(x_k) + g(x_k)u(x_k) \tag{27}$$

$$\hat{y}_k = x_{2,k}$$

$$y_k = h(x_k, x_{\delta,k+1}) + J(x_k)u_k$$

$$f_1(x_k) = x_{2,k}$$

$$f_2(x_k) = \sum_{i=0}^{5} w_{1i}^{(2)} v_i \text{ with } v_0 = +1$$

$$v_i = \left[S\left(\sum_{j=0}^{2} w_{ij}^{(1)} x_j \right) \right] \text{ with } x_0 = +1$$

$$g(x_k) = \begin{bmatrix} 0 \\ w' \end{bmatrix}$$

where $h(x_k, x_{\delta,k+1})$ is equal to (10) and $J(x_k)$ is equal to (11), $x_{2,k+1}$ is the glucose level, u_k is the insulin dose.

The initial values for the covariance matrices (R, Q, P) are $R_0 = Q_0 = P_0 = 10000$. The lagged recurrent inputs to the RMLP are equal to 2. The training is performed on-line, using an EKF-learning algorithm in a series-parallel configuration; the delayed outputs are taken from the Sorensen model, which is fed with experimental data. The specified target prediction error is 10^{-4}. The results are presented in Fig. 2, for the neural network identifier. The sample time is 5 minutes because the Paradigm® Real-time Continuous Glucose Monitoring System takes a sample every 5 minutes. The total of samples is 1400 equivalent to almost 5 days of monitoring.

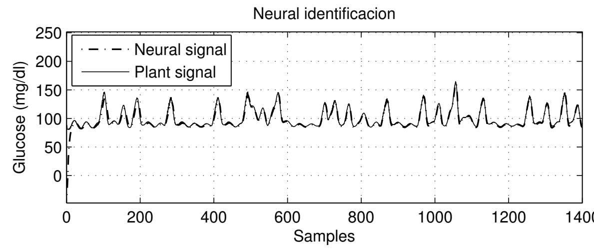

Fig. 2. Neural network identification.

B. Control Law Implementation

P and K_1 are select as follows:

$$P = \begin{bmatrix} 14 & 11.5 \\ 11.5 & 14 \end{bmatrix} \text{ and } K_1 = \begin{bmatrix} 46 & 0 \\ 0 & 6.5 \end{bmatrix}$$

and $\overline{P} = K_1^T P K_1$

The desired trajectory $(x_{\delta,k})$ is obtained using the model proposed by Lehmann *et Al.* [23] in order to take into account the postprandial effect. We select the trajectory to be tracked as the glucose level of a healthy person in order to improve the T1DM patient well-being. The control law is (21), which ensures that $h(\bullet)$ satisfies (10). The tracking performance and the tracking error are displayed in Fig. 3; it can be seen that the two errors have the same sign so the vector comparison of (10) is valid for this application. Fig. 4 presents the difference between the insulin supplied to the patient with the insulin pump (open loop) and the insulin which is calculated by the proposed control law. The mean of the insulin supplied by the pump is 25.47mU/min. and the mean of the insulin calculated by the proposed control law is 26.81mU/min. Fig. 4 also displays the difference between the glucose in the plasma taken from the patient with the Continuous Glucose Monitoring System by MiniMed Inc (open loop) and the glucose in the plasma with the proposed control law. It can be noticed that the glucose reaches values above 150mg/dl, due to the carbohydrates ingested by the patient, which act like a disturbance; therefore the control law corrects this situation quickly through the insulin infusion. These are preview results; we are tuning the algorithm for clinical trial.

V. Conclusions

A controller for positive systems is proposed. This class of systems has high relevance as illustrates its applicability to T1DM patients. The proposed scheme stabilizes the system along a desired trajectory. The controller is applied to an affine positive model representing the glucose-insulin dynamics. Simulation results show how the control law is

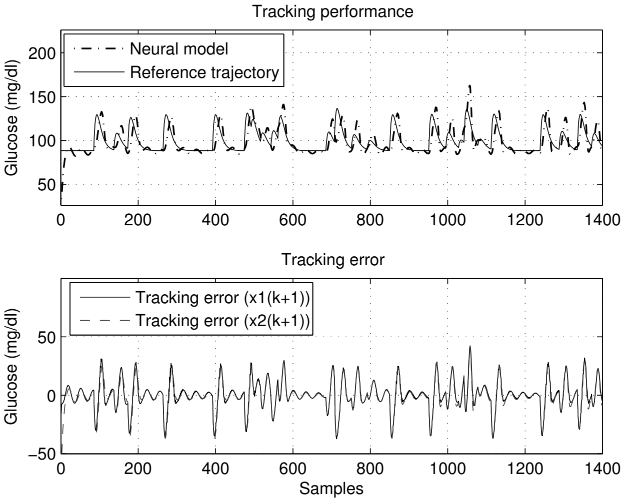

Fig. 3. Tracking performance of glucose in plasma from a patient with T1DM.

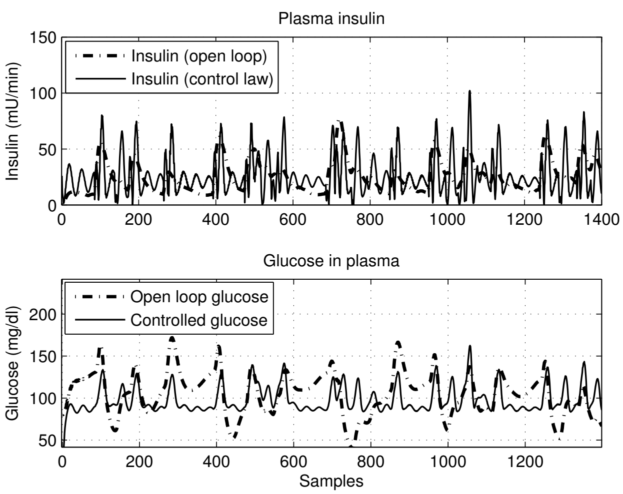

Fig. 4. Open loop vs. controller time responses.

able to stabilize the blood glucose levels along a desired trajectory. Indeed, this scheme improves the regulation of the blood glucose level in T1DM patients, increasing slightly the insulin quantity. This technique is an important result since most of the biological systems are positive. It is also illustrated that the proposed RMLP, used in our experiments, captures very well the complexity associated with blood glucose level for T1DM patients.

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