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A Model Predictive Control Approach to Attitude Stabilization and Trajectory Tracking Control of a 3D Universal Joint Space Robot with an Initial Angular Momentum

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***Abstract*— In this paper, we deal with two types of control problems for a 3D space robot of two rigid bodies with an initial angular momentum. In our previous work, we studied a near-optimal control approach to the state transition control problem for the 3D space robots with initial angular momenta. However, for the approach, we need huge computation amount and other control purposes cannot be treated. In order to overcome these disadvantages, in this paper we apply model predictive control to the attitude stabilization control problem and the trajectory tracking control problem for a 3D universal joint space robot with an initial angular momentum. As a results of simulations, we can see that both attitude stabilization control and trajectory tracking control for the 3D space robot are accomplished by the model predictive control approach. Moreover, it can be also confirmed that this approach considerably reduces computation amount and achieves real-time control of the 3D space robot.**

I. INTRODUCTION

It is well known that for a space robot in 3-dimensional outer space, its conversation law of total angular momentum plays a role of nonholonomic constraints [1], and hence the robot's attitude can be changed by transforming its shape. A lot of researches on such a space robot have been done in the fields of analytic mechanics, control theory and robotics [1], [2], [4], [5], [6]. In most researches on control of space robots, it is assumed that space robots do not have initial angular momentum. In realistic situations, for example, when a mother ship gives a space robot out, space robots often have initial angular momenta. Hence we have focused on 3D space robots with initial angular momentum and derived a control strategy based on the near-optimal control method [9], [10]. However, since the model of a space robot with initial angular momentum is quite complicated and the proposed control law is feedforward-type, a huge quantities of calculation amount (from a few hours to a few days) is needed. In addition, we consider only the state transition control problem in which we make the state of the space robot transfer to a desired one at a desired time.

The purpose of this study is to overcome these disadvantages mentioned above by using model predictive control that consists of feedback-type control laws. The contents of this paper is as follows. In section II, we first show the model of the 3D universal joint space robot with an initial angular momentum (the universal joint model with an initial angular momentum) and explain some characteristics of it. Next, Section III considers the attitude stabilization control problem for the universal joint model with an initial angular momentum from the viewpoint of model predictive control and the C/GMRES method, We then treat the trajectory tracking control for the universal joint model with an initial angular momentum in Section IV. Simulations are illustrated in order to show the availability of our approach.

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II. 3D UNIVERSAL JOINT SPACE ROBOT WITH INITIAL ANGULAR MOMENTUM

This section derives a space robot model, which is our controlled object and dealt with through this paper. First, we give the problem setting of the space robot. We consider the space robot that consists of two rigid bodies in the 3-dimensional space as shown in Fig. 1. Two rigid bodies (Rigid Body 1 and 2) are connected by a joint via two links (Link 1 and 2), respectively. We denote coordinates of the inertial space, Rigid Body 1 and 2 by C_0, C_1 and C_2, respectively. In addition, we assume that the origins of C_1 and C_2 correspond to the centroids of Rigid Body 1 and 2, respectively.

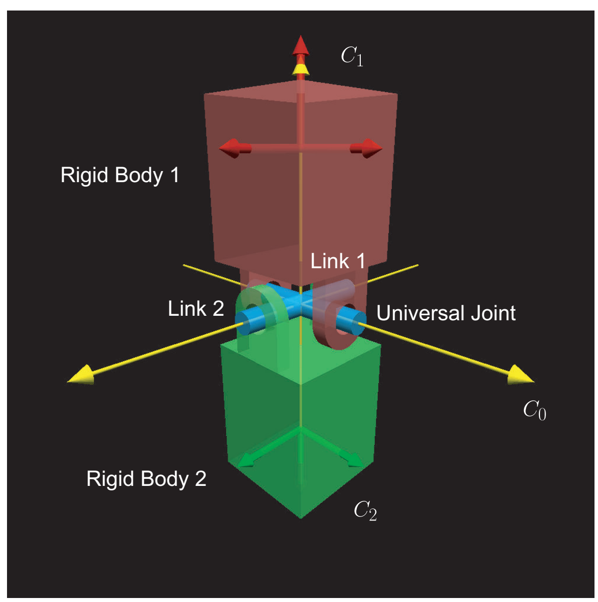


Fig. 1 : 3D Universal Joint Space Robot

Let $A_i \in SO(3)$ be the attitude of Rigid Body i $(i = 1, 2)$ with respective to the inertial space C_0, and $w_i \in \mathbf{R}^3$ be the angular velocity of Rigid Body i. Note that $\hat{w}_i = A_i^{\mathsf{T}} \dot{A}_i$ holds. We use the following notations; m_i: the mass of Rigid Body i ($\epsilon = m_1 m_2/(m_1 + m_2)$), l_i: the lenght of Link i,

$$A_1(\alpha) = \frac{1}{1+||\alpha||^2}\begin{bmatrix} 1+\alpha_1^2-\alpha_2^2-\alpha_3^2 & 2(\alpha_1\alpha_2-\alpha_3) & 2(\alpha_1\alpha_3+\alpha_2) \\ 2(\alpha_1\alpha_2+\alpha_3) & 1-\alpha_1^2+\alpha_2^2-\alpha_3^2 & 2(\alpha_2\alpha_3-\alpha_1) \\ 2(\alpha_1\alpha_3-\alpha_2) & 2(\alpha_2\alpha_3+\alpha_1) & 1-\alpha_1^2-\alpha_2^2+\alpha_3^2 \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\alpha \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ U_1^{-1}I_u^{-1}A_1^\mathsf{T}P_0 \end{bmatrix}}_{f(q)} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -U_1^{-1}I_u^{-1}(AJ_2+J_{12})b_1 & -U_1^{-1}I_u^{-1}(AJ_2+J_{12})b_2 \end{bmatrix}}_{g(q)} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (11)$$

$s_i = [0\,0\,-l_i]^\mathsf{T} \in \mathbf{R}^3$: the vector showing the position of the joint w.r.t. C_0, $I_i \in \mathbf{R}^3$: The inertia tensor of Rigid Body i ($J_i = I_i + \epsilon\hat{s}_i^\mathsf{T}\hat{s}_i$, $J_{12} = \epsilon\hat{s}_1^\mathsf{T}A_1^\mathsf{T}A_2\hat{s}_2$), where $\hat{}$ is the operator that transforms a 3-dimensional vector $v = [v_1\, v_2\, v_3]^\mathsf{T} \in \mathbf{R}^3$ into a 3×3 skew-symmetric matrix:

$$\hat{v} = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}. \quad (1)$$

It is also noted that $A := A_1^\mathsf{T}A_2$ represents the shape of the space robot and

$$w_2 = A^\mathsf{T}w_1 + w \quad (2)$$

holds for the angular velocity of the joint $w \in \mathbf{R}^3$, $\hat{w} = A^\mathsf{T}\dot{A}$. In this paper, we use the universal joint depicted in Fig. 2 as a joint connecting the two rigid bodies. Note that the universal joint can twist and the degree of freedom is 2. Let $\theta_1 \in \mathbf{R}$ and $\theta_2 \in \mathbf{R}$ be angles of Link 1 and 2, respectively, and we use the notation: $\theta = [\,\theta_1\ \ \theta_2\,]^\mathsf{T} \in \mathbf{R}^2$. By considering coordinates of the space robot, we can show the following:

$$A = \begin{bmatrix} \sin\theta_1\sin\theta_2 & \cos\theta_1 & -\sin\theta_1\cos\theta_2 \\ \cos\theta_2 & 0 & \sin\theta_2 \\ \cos\theta_1\sin\theta_2 & -\sin\theta_1 & -\cos\theta_1\cos\theta_2 \end{bmatrix}. \quad (3)$$

In this paper, we consider the case where the universal model has an initial angular momentum, so we denote it by $P_0 \in \mathbf{R}^3$. The conservation law of total angular momentum of the space robot is given by

$$(A_1J_1 + A_2J_{12}^\mathsf{T})w_1 + (A_2J_2 + A_1J_{12})w_2 = P_0, \quad (4)$$

and we can easily confirm that (4) is represented as $A(q) + B(q)\dot{q} = 0$ using the generalized coordinate q, and thus this is an affine constraint [8]. From the result in [8], it can be checked that (4) is completely nonholonomic. Assume that angular velocities of the universal joint can be controlled, that is, $u_1 := \dot\theta_1$, $u_2 := \dot\theta_2$. Then, we have

$$w = \underbrace{\begin{bmatrix} \cos\theta_2 \\ 0 \\ \sin\theta_2 \end{bmatrix}}_{b_1} u_1 + \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{b_2} u_2. \quad (5)$$

Substituting (2) and (5) into (4), we obtain

$$A_1I_uw_1 + A_1(AJ_2 + J_{12})(b_1u_1 + b_2u_2) = P_0, \quad (6)$$

where we define the new notation:

$$I_u := J_1 + AJ_2A^\mathsf{T} + AJ_{12}^\mathsf{T} + J_{12}A^\mathsf{T}. \quad (7)$$

To represent the attitudes of Rigid Body 1, we use the Cayley-Rodrigues parameter, and hence the attitude of Rigid Body 1 A_1 is expressed as (8) using the parameter $\alpha = [\,\alpha_1\ \ \alpha_2\ \ \alpha_3\,]^\mathsf{T} \in \mathbf{R}^3$. The relationship between w_1 and α is expressed by

$$w_1 = U_1(\alpha)\dot\alpha,\ \ U_1(\alpha) = \frac{2(I-\hat\alpha)}{1+\alpha^\mathsf{T}\alpha}. \quad (9)$$

Substiting (9) into (6) and solving for $\dot\alpha$, we have

$$\begin{aligned} \dot\alpha = {} & U_1^{-1}I_u^{-1}A_1^\mathsf{T}P_0 \\ & - U_1^{-1}I_u^{-1}(AJ_2 + J_{12})(b_1u_1 + b_2u_2). \end{aligned} \quad (10)$$

So, we obtain *the universal joint model with an initial angular momentum* (11) with the variables $q := [\theta^\mathsf{T}\ \alpha^\mathsf{T}]^\mathsf{T} \in \mathbf{R}^5$, $u := [\,u_1\ \ u_2\,]^\mathsf{T} \in \mathbf{R}^2$, which is represented as a 5-state and 2-input nonlinear affine control system.

Finally, we show some characteristics of the universal joint model with an initial angular momentum (11) from the viewpoint of nonlinear control theory. When an initial angular momentum exists, that is, $P_0 \neq 0$, the drift term of (11) satisfies $f(q) \neq 0$, $\forall q$. This fact states that (11) does not have any equilibrium point, that is, the space robot cannot stop and keeps moving. For nonlinear control systems, it is quite important to check properties such as local accessibility and local controllability. We show the next proposition on local accessibility and local controllability of (11) [9].

Proposition 1 : The universal joint model with an an initial angular momentum (11) is locally strongly accessible at any point $q = [\,\theta^\mathsf{T}\ \alpha^\mathsf{T}\,]^\mathsf{T} \in \mathbf{R}^5$. Moreover, if the control input u is sufficiently large, (11) is small-time locally controllable at any point q. ∎

Since the universal joint model with an initial angular momentum (11) does not have any equilibrium point, we cannot consider a normal stabilization problem for (11). However, Proposition 1 guarantees possibilities of other control purposes except normal stabilization. For the universal joint model with an initial angular momentum (11), we can consider the next three types of control purposes; (i) the state transition control problem: we transfer the space robot to a desired state at a desired time, (ii) the attitude stabilization control problem: we stabilize only the attitude of the space robot with ignoring the shape of it, (iii) the trajectory tracking control problem: we make the space robot track a given trajectory. The problem (i) has already dealt with in our previous work [9], [10], so, in this paper we will mainly tackle the problems (ii) and (iii).

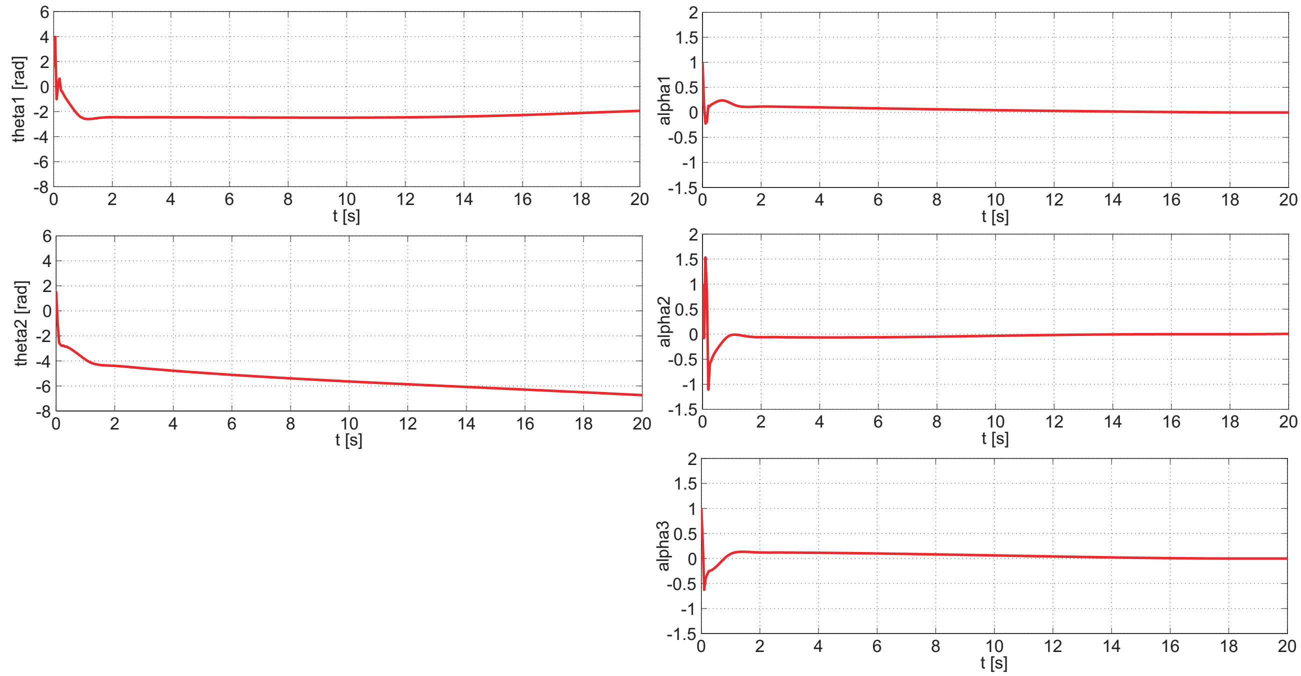


Fig. 2 Time Series of θ and α in Attitude Stabilization Control

III. ATTITUDE STABILIZATION CONTROL

In this section, we consider the attitude stabilization control problem for the universal joint model with an initial angular momentum (11). The aim of this control problem is that we stabilize only the attitude of the space robot α with ignoring th shape θ, and this control problem contains, for example, the situation where we move the space robot to the direction of a given point of the earth in order to send and receive information. The attitude stabilization control problem is formulated as follows.

Problem 1 [Attitude Stabilization Control Problem] : For the universal joint model with initial angular momentum (11), find control inputs such that the attitude of Rigid Body 1 α is stabilized to a desired value α_d. ∎

In this paper, we take the model predictive control approach in order to solve Problem 1. Especially, we use *the C/GMRES method* [7], which is a real-time optimization algorithm. In a simulation, we use the parameters of the universal joint model: $l_1 = l_2 = 1$, $m_1 = m_2 = 1$, $I_1 = I_2 = \mathrm{diag}\{\,1/2,\ 1/2,\ 1\,\}$, initial angular momentum: $P_0 = [0.1\ 0.1\ -0.1]^\mathsf{T}$, the initial state: $q_0 = [\pi/2\ \pi/2\ 1\ 1\ 1]^\mathsf{T}$, the desired attitude: $\alpha_d = [0\ 0\ 0]^\mathsf{T}$. For the C/GMRES method, we use the next cost function:

$$\begin{aligned} J =& \frac{1}{2}\int_t^{t+T(t)} (\alpha(\tau)-\alpha_d)^\mathsf{T} Q(\alpha(\tau)-\alpha_d)d\tau \\ &+ \frac{1}{2}\int_t^{t+T(t)} u(\tau)^\mathsf{T} R u(\tau) d\tau \\ &+ \frac{1}{2}(\alpha(t+T)-\alpha_d)^\mathsf{T} S(\alpha(t+T)-\alpha_d) \end{aligned} \tag{12}$$

with the weight matrices: $Q = \mathrm{diag}\{4.0,\ 1.5,\ 5.0\}$, $R = \mathrm{diag}\{0.01,\ 0.01\}$, $S = \mathrm{diag}\{0.8,\ 0.2,\ 0.4\}$ and the evaluation interval $T(t) = T(1-e^{-at})$, $T = 6.5$, $a = 0.05$. Moreover, we also use the parameters of controller: the division number of the evaluation interval: $N = 50$, the stabilization parameter of the continuation method: $\zeta = 20$, the number of iterations of the GMRES method: $k_{max} = 3$, the sampling time: $\Delta t = 0.05\,[\mathrm{s}]$, the simulation time: $20\,[\mathrm{s}]$.

The simulation results are shown in Fig. 2 and 3. Fig. 2 illustrates the time series of θ and α of the space robot, and Fig. 3 depicts a snapshot of the space robot. From these results, it can be confirmed that the attitude of Rigid Body 1 α is stabilized to the desired value $\alpha_d = 0$. The computation time of this simulation is $1.45\,[\mathrm{s}]$, and hence we can see that the computation time is drastically reduced in comparison with the case of the near-optimal control method [9], [10]. We also confirm that the control purposes can be achieved for other problem settings (the parameters of the space robot, initial and desired states, and an initial angular momentum) with tuning the weight matrices in (12).

IV. TRAJECTORY TRACKING CONTROL

In this section, we next deal with another control problem, the trajectory tracking control problem, for the universal joint model with an initial angular momentum (11). The aim of this control problem is that we make the state of space robot track to a desired trajectory data, and this control problem contains, for example, the situation where we track a solar panel installed into the space robot to the direction of the sun. We define the trajectory tracking control problem as follows.

Problem 2 [Trajectory Tracking Control Problem] : For the universal joint model with an initial angular momentum (11), find control inputs such that the state q tracks to a desired trajectory $q_d(t)$. ∎

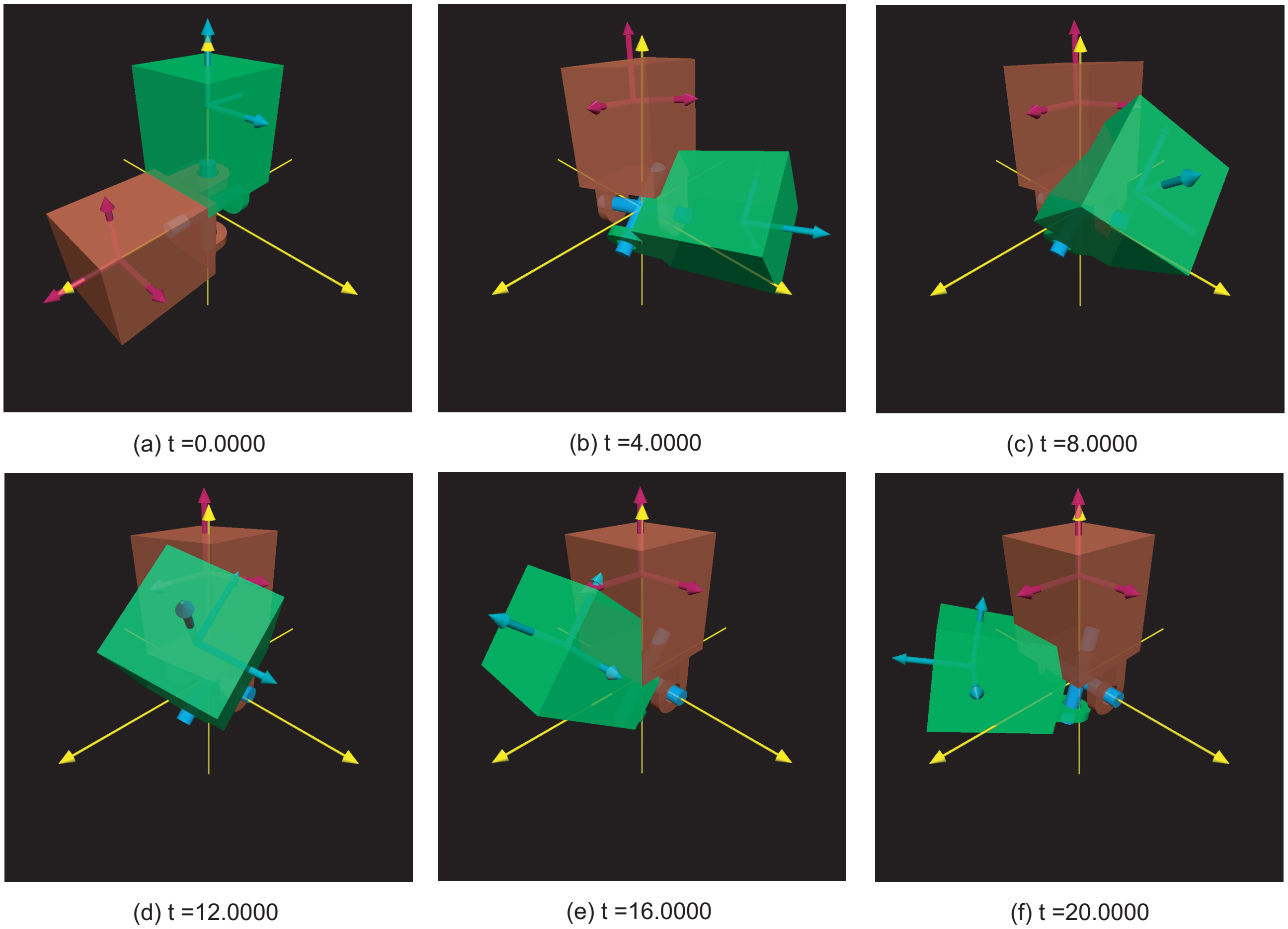


Fig. 3 Snapshot in Attitude Stabilization Control

To solve Problem 2, we also utilize the C/GMRES method [7]. In a simulation, we use the parameters of the universal joint model: $l_1 = l_2 = 1$, $m_1 = m_2 = 1$, $I_1 = I_2 = \mathrm{diag}\{1/2, 1/2, 1\}$, initial angular momentum: $P_0 = [0.1\ -0.1\ 0.2]^\mathsf{T}$, the initial state: $q_0 = [-\pi/2\ -\pi/2\ -1\ -1\ -1]^\mathsf{T}$. The desired trajectory $q(t)$ is generated from (11) in advance (see Fig. 2). For the C/GMRES method, we use the next cost function:

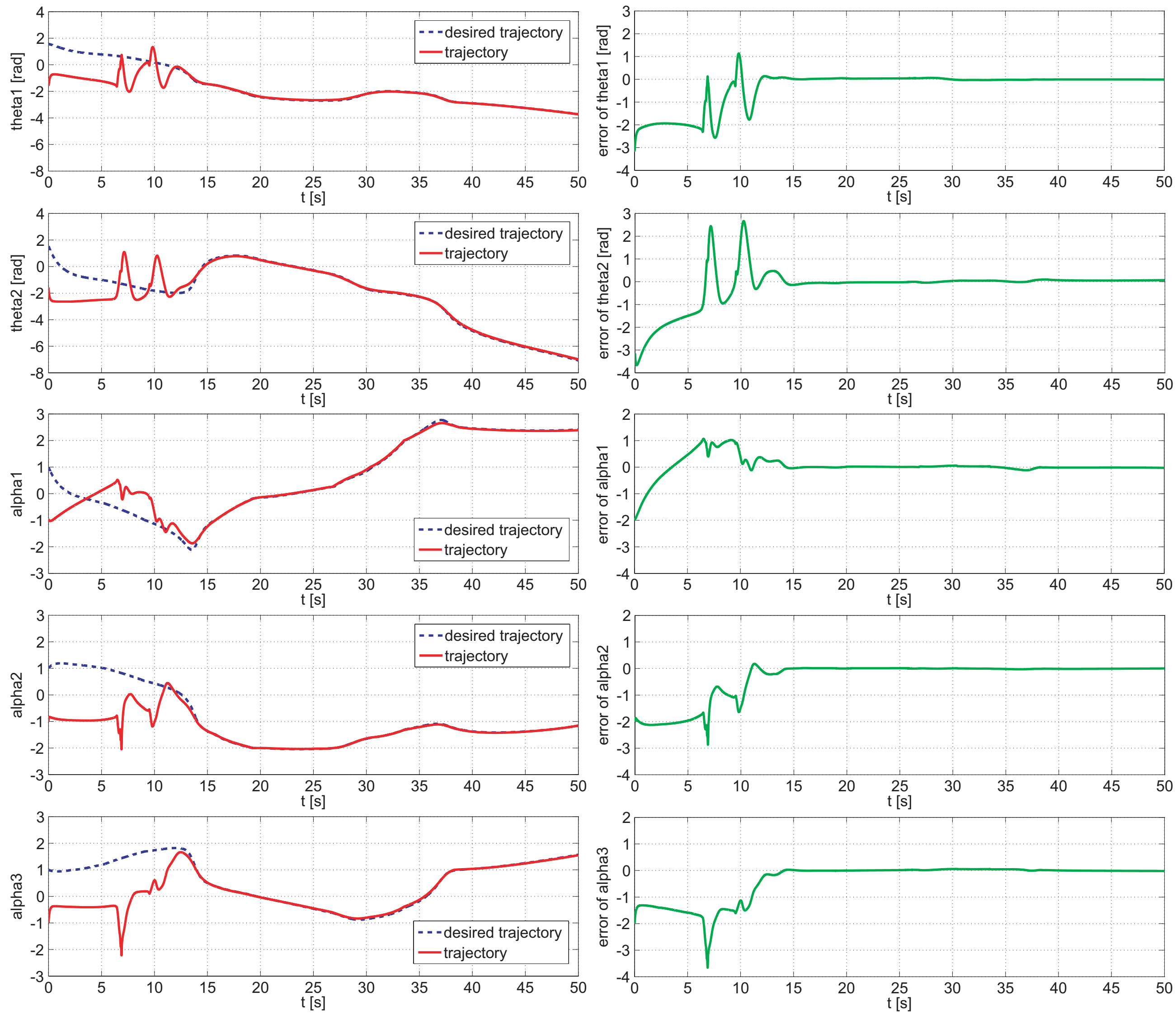
$$\begin{aligned} J =& \frac{1}{2}\int_t^{t+T(t)} (q(\tau) - q_d(\tau))^\mathsf{T} Q (q(\tau) - q_d(\tau)) d\tau \\ &+ \frac{1}{2}\int_t^{t+T(t)} u(\tau)^\mathsf{T} R u(\tau) d\tau \\ &+ \frac{1}{2}(q(t+T) - q_d(t))^\mathsf{T} S (q(t+T) - q_d(t)), \end{aligned} \tag{13}$$

where $Q = \mathrm{diag}\{0.015, 0.015, 0.04, 0.04, 0.04\}$, $R = \mathrm{diag}\{0.01, 0.01\}$, $S = \mathrm{diag}\{0.1, 0.1, 0.2, 0.2, 0.2\}$ and the evaluation interval $T(t) = T(1 - e^{-at})$, $T = 6.5$, $a = 0.05$. Moreover, we also use the parameters of controller: the division number of the evaluation interval: $N = 50$, the stabilization parameter of the continuation method: $\zeta = 200$, the number of iterations of the GMRES method: $k_{max} = 3$, the sampling time: $\Delta t = 0.005\,[\mathrm{s}]$, the simulation time: $50\,[\mathrm{s}]$.

Fig. 4 and 5 illustrate the simulation results. In the left figure in Fig. 4, both the time series of θ, α of the space robot and the desired trajectory are shown. The right figure in Fig. 4 depicts the time series of the error defined by $e(t) := q(t) - q_d(t)$. In addition, a snapshot the space robot for both desired trajectory and the result is illustrated in Fig. 5. From these results, we can confirm that the state of the space robot tracks to the desired trajectory $q_d(t)$ and then the error $e(t)$ converges to 0. The computation time of this simulation is $16.67\,[\mathrm{s}]$, and hence it turns out that the computation time of the simulation is also considerably reduced in comparison with the case of the near-optimal control method [9], [10]. It is also confirmed that the control purposes can be achieved for other problem settings (the parameters of the space robot, initial state, desired trajectory, and an initial angular momentum) with tuning the weight matrices in (12).

V. CONCLUSION

In this paper, we have considered two types of control problems for the 3D universal joint space robot with an initial angular momentum, which have not dealt with so far, from the standpoint of the model predictive control approach. the simulation results have shown that computation amount has



(a) Time Series of State q (b) Time Series of Error e

Fig. 4 Simulation Results in Trajectory Tracking Control

been drastically reduced in comparison with our previous results, and hence real-time control has been achieved.

Our future work are as follows: limit-cycle-like control of the 3D space robots with initial angular momenta, and modeling and control the space robots with initial angular momenta by the quaternion representation.

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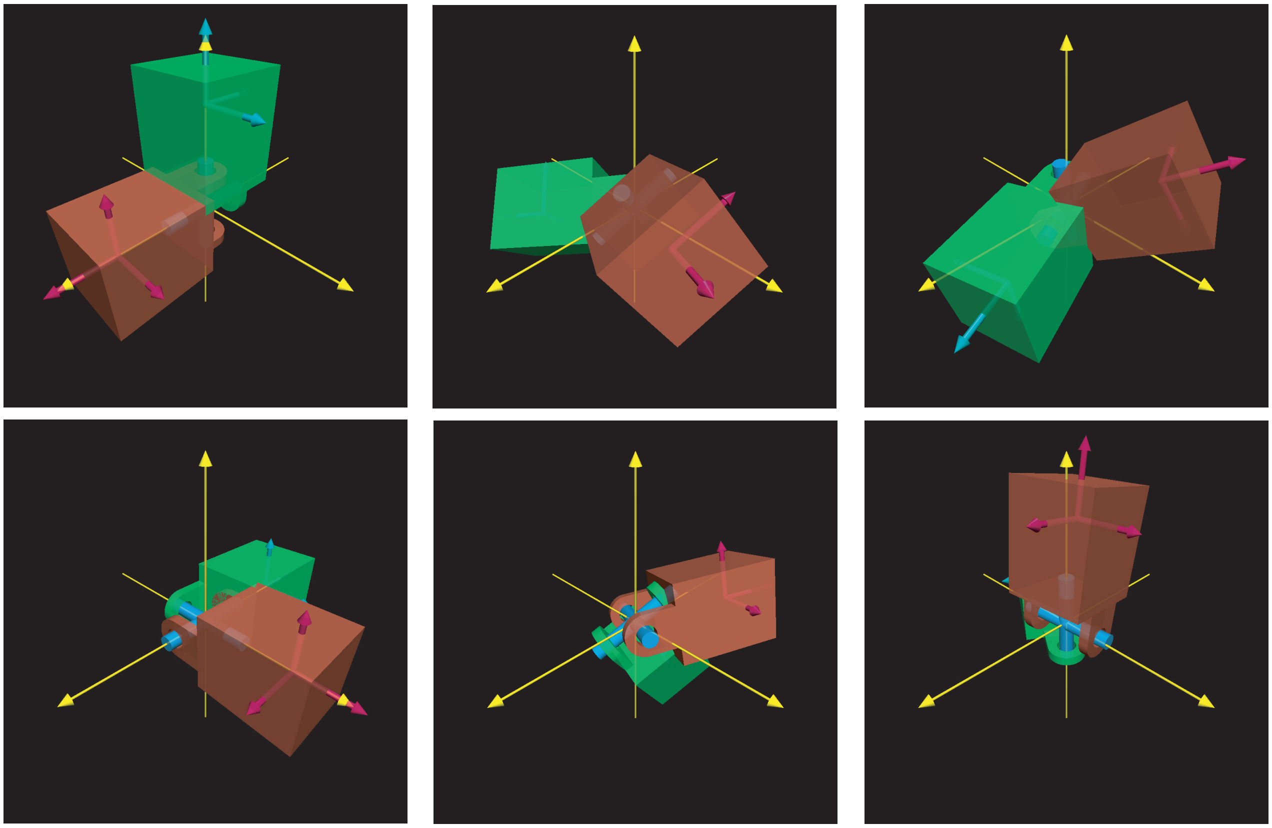
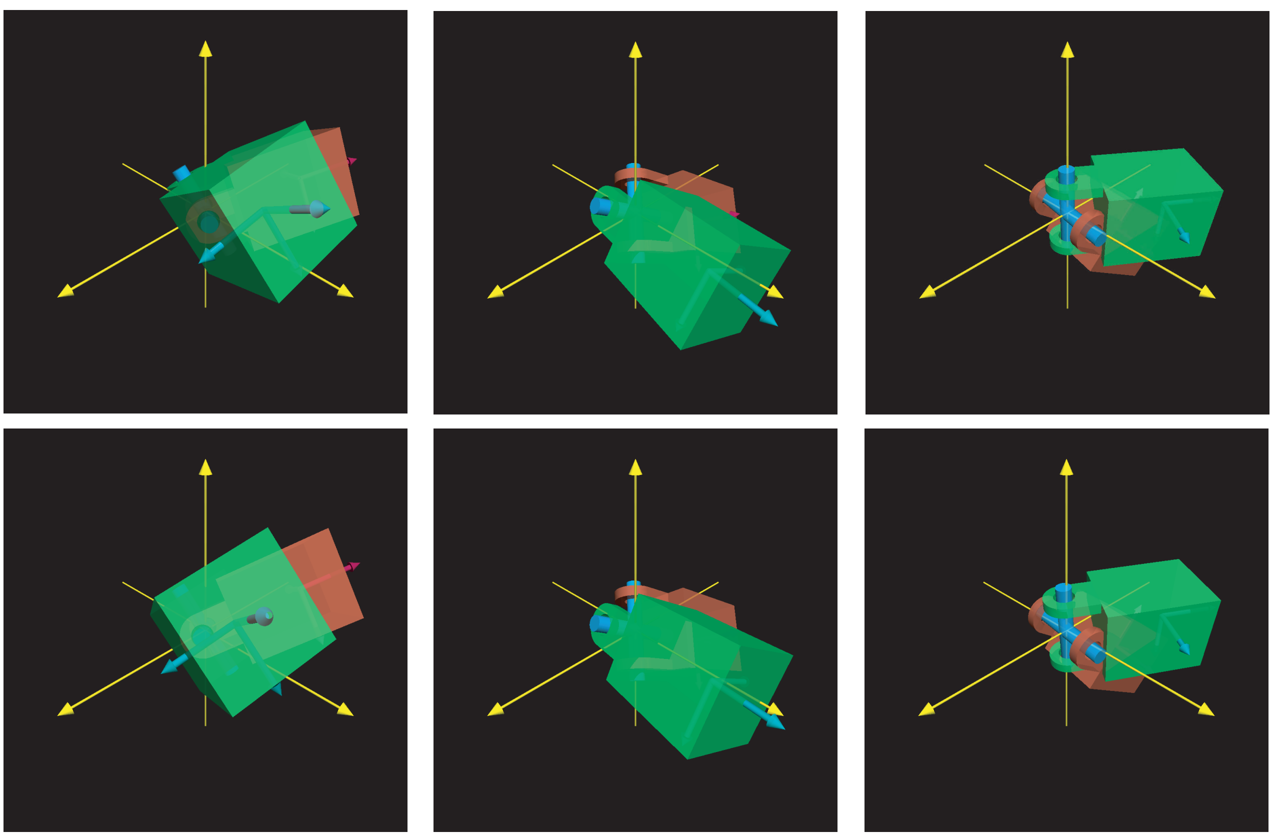


(a) t =0.0000 (b) t =4.0000 (c) t =8.0000



(d) t =12.0000 (e) t =16.0000 (f) t =20.0000

Fig. 5 Snapshot in Trajectory Tracking Control (Upper: Desired Trajectory, Lower: Result)