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On the Relay Pursuit of a Maneuvering Target by a Group of Pursuers

Efstathios Bakolas Panagiotis Tsiotras

***Abstract*— This paper addresses the problem of pursuit of a maneuvering target by a group of pursuers distributed in the plane. This group pursuit problem is solved in a distributed way by employing a relay pursuit strategy, that is, a group pursuit scheme, such that at each instant of time, only one pursuer is assigned the task of capturing the maneuvering target. During the course of the relay pursuit, the pursuer-target assignment changes dynamically with time in accordance with the (time-varying) proximity relations between the pursuers and the target. Simulation results are presented to highlight the theoretical developments.**

I. Introduction

We present a pursuit strategy for the capture of a maneuvering target by a group of pursuers distributed in the plane. Typically, problems of group pursuit of a moving target (or an evader) are dealt with by employing cooperative or non-cooperative pursuit strategies, which are based on local or global information [1]–[8]. One common theme in all these approaches is that more than one pursuers are, at every instant of time, actively participating in the process of capturing the target. However, in many applications involving groups of agents, a more "frugal" assignment of tasks within the group may constitute a more prudent strategy. For example, in the problem of pursuit of a moving target by a group of agents guarding a certain area, the guards may be required to remain close to their initial positions to account for possible deceptive strategies, decoy targets, etc.

In this paper, we propose a *relay pursuit* scheme to address the group pursuit problem. In particular, given a team of pursuers, which are distributed in the plane, we wish to find a scheme such that, at every instant of time, only one pursuer is assigned the task of capturing the moving target, whereas the rest of the pursuers remain stationary. During the course of the pursuit, the scheme dynamically selects the appropriate pursuer in the group in order to minimize the overall capture time. In our problem formulation, we do not constraint the moving target to follow a prescribed trajectory, as it is usually assumed in the literature [9]. Instead, the target can maneuver by applying an "evading" strategy aiming at delaying or, if possible, avoiding capture. In contrast to the pursuer-target assignment schemes presented in [10]–[12], which are fixed with time, in this work, we introduce a dynamic pursuer-target assignment scheme that derives from the (time-varying) proximity relations between the maneuvering target and the group of pursuers. The proximity relations are induced by a generalized proximity metric, namely, the minimum intercept time, and they are encoded in the solution of a dynamic Voronoi-like partitioning problem.

The rest of the paper is organized as follows. Section II presents some key results from our previous work that are used in Section III, where the dynamic pursuer-target assignment problem is formulated and subsequently addressed. Simulation results are presented in Section IV. Finally, Section V concludes the paper with a summary of remarks.

II. Problem Setup

A. Formulation of the Optimal Pursuit Problem

Consider a team of n pursuers located at time $t=0$ at n distinct points in the plane, denoted by $\mathcal{P} := \{\bar{\mathsf{x}}_{\mathcal{P}}^i \in \mathbb{R}^2, i \in \mathcal{I}\}$, where $\mathcal{I} := \{1,\ldots,n\}$. It is assumed that the kinematics of the i^{th} pursuer, where $i \in \mathcal{I}$ are described by

$$\dot{\mathsf{x}}_{\mathcal{P}}^i = u_{\mathcal{P}}^i, \qquad \mathsf{x}_{\mathcal{P}}^i(0) = \bar{\mathsf{x}}_{\mathcal{P}}^i, \tag{1}$$

where $\mathsf{x}_{\mathcal{P}}^i := (x_{\mathcal{P}}^i, y_{\mathcal{P}}^i) \in \mathbb{R}^2$ and $\bar{\mathsf{x}}_{\mathcal{P}}^i := (\bar{x}_{\mathcal{P}}^i, \bar{y}_{\mathcal{P}}^i) \in \mathbb{R}^2$ denote the position vectors of the i^{th} pursuer at time t and time $t=0$, respectively, and $u_{\mathcal{P}}^i$ is the control input of the i^{th} pursuer. We assume that $u_{\mathcal{P}}^i \in \mathcal{U}_{\mathcal{P}}$, where $\mathcal{U}_{\mathcal{P}}$ consists of all piecewise continuous functions taking values in the set $U_{\mathcal{P}} := \{\mathsf{z} \in \mathbb{R}^2 : |\mathsf{z}| \leq \bar{u}_{\mathcal{P}}\}$, where $\bar{u}_{\mathcal{P}}$ is the maximum allowable speed of the pursuers. The goal of each pursuer, located initially at a point in $\mathcal{P}$, is to capture a moving target detected in its vicinity. It is assumed that the kinematics of such a moving target are described by

$$\dot{\mathsf{x}}_{\mathcal{T}} = u_{\mathcal{T}}, \qquad \mathsf{x}_{\mathcal{T}}(0) = \bar{\mathsf{x}}_{\mathcal{T}}, \tag{2}$$

where $\mathsf{x}_{\mathcal{T}} := (x_{\mathcal{T}}, y_{\mathcal{T}}) \in \mathbb{R}^2$ and $\bar{\mathsf{x}}_{\mathcal{T}} := (\bar{x}_{\mathcal{T}}, \bar{y}_{\mathcal{T}}) \in \mathbb{R}^2$ denote the target's position vectors at time t and time $t=0$, respectively, and $u_{\mathcal{T}}$ is the control input of the target. It is assumed that the target can employ a feedback evading strategy, which depends on the relative position of the target from the i^{th} pursuer, that is, $u_{\mathcal{T}} = u_{\mathcal{T}}(\mathsf{x}_{\mathcal{T}} - \mathsf{x}_{\mathcal{P}}^i)$. The objective of the i^{th} pursuer is to determine an admissible pursuit strategy that minimizes the time $T_{\mathsf{f}} = T_{\mathsf{f}}(\bar{\mathsf{x}}_{\mathcal{T}} - \bar{\mathsf{x}}_{\mathcal{P}}^i)$ such that $|\mathsf{x}_{\mathcal{T}}(t; u_{\mathcal{T}}, \bar{\mathsf{x}}_{\mathcal{T}}) - \mathsf{x}_{\mathcal{P}}^i(t; u_{\mathcal{P}}^i, \bar{\mathsf{x}}_{\mathcal{P}}^i)| > \epsilon_c$ for all $t < T_{\mathsf{f}}$ (*time of first capture*), for a sufficiently small $\epsilon_c > 0$, where ϵ_c is the *capturability radius* of the pursuit problem. Henceforth, we shall refer to the problem of characterizing the strategy $u_{\mathcal{P}}^i$ that minimizes the time of capture T_{f} as the *optimal pursuit problem*.

E. Bakolas is a Ph.D. candidate at the School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150, USA, Email: ebakolas@gatech.edu

P. Tsiotras is a Professor at the School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150, USA, Email:tsiotras@gatech.edu

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Let $\mathsf{y}^i := \mathsf{x}_{\mathcal{T}} - \mathsf{x}_{\mathcal{P}}^i$. Equation (1) can then be written in the following compact form

$$\dot{\mathsf{y}}^i = \mathsf{u}^i + u_{\mathcal{T}}(\mathsf{y}^i), \qquad \mathsf{y}^i(0) = \bar{\mathsf{y}}^i := \bar{\mathsf{x}}_{\mathcal{T}} - \bar{\mathsf{x}}_{\mathcal{P}}^i, \quad (3)$$

where $\mathsf{u}^i := -u_{\mathcal{P}}^i$. Thus, the optimal pursuit problem can be interpreted as a problem of steering a single integrator from $\bar{\mathsf{y}}^i$ to a ball of radius ϵ_c centered at the origin, in the presence of a spatially-varying drift $u_{\mathcal{T}}(\mathsf{y}^i)$, which is not precisely known, in minimum-time. Thus the optimal pursuit problem can be reduced to a special case of Zermelo's navigation problem (ZNP for short), which can be solved when $u_{\mathcal{T}}$ is perfectly known a priori to the i^{th} pursuer [13]. Here we employ, however, a different approach, which does not require a priori knowledge of $u_{\mathcal{T}}$ and is based on the following assumption.

Assumption 1: There exists a Lipschitz continuous function $f : [\epsilon_c, \infty) \mapsto \mathbb{R}$ such that the evading strategy $u_{\mathcal{T}}$ of the target satisfies the following condition

$$\langle u_{\mathcal{T}}, \mathsf{x}_{\mathcal{T}} - \mathsf{x}_{\mathcal{P}}^i \rangle = f(|\mathsf{x}_{\mathcal{T}} - \mathsf{x}_{\mathcal{P}}^i|). \quad (4)$$

The interpretation of Assumption 1 is as follows: The projection of the velocity vector of the maneuvering target on the relative position vector between the target and the i^{th} pursuer depends only on the relative distance between the two. In addition, it is assumed that

$$f(\mathsf{z}) \leq \bar{f}(\mathsf{z}), \quad \text{for all} \quad \mathsf{z} \geq \epsilon_c, \quad (5)$$

where $\bar{f} : [\epsilon_c, \infty) \mapsto \mathbb{R}$ is a continuous function that is known to all of the pursuers, whereas both the exact $u_{\mathcal{T}}$ and f may be unknown to them. It can be shown [12] that, under Assumption 1, the unique solution of the optimal pursuit problem is given in closed form by

$$u_{\mathcal{P}}^i = \bar{u}_{\mathcal{P}} \frac{\mathsf{x}_{\mathcal{T}} - \mathsf{x}_{\mathcal{P}}^i}{|\mathsf{x}_{\mathcal{T}} - \mathsf{x}_{\mathcal{P}}^i|} = \bar{u}_{\mathcal{P}} \frac{\mathsf{y}^i}{|\mathsf{y}^i|}. \quad (6)$$

Therefore the optimal pursuit strategy of the i^{th} pursuer is a "pure" pursuit strategy [14], where the pursuer moves with the maximum allowable speed, and its velocity vector is always pointing towards the current position of the target.

B. The Winning Sets of the Pursuers and the Optimal Pursuit Dynamic Voronoi Diagram

The feasibility of the optimal pursuit problem for a given $\bar{\mathsf{y}}^i \in \mathbb{R}^2$ is characterized by the winning set $\mathcal{W}_f(\bar{\mathsf{x}}_{\mathcal{P}}^i)$ of the i^{th} pursuer, that is, the set of the initial positions of the target from which it can be captured by the i^{th} pursuer in finite time. In other words, $\mathcal{W}_f(\bar{\mathsf{x}}_{\mathcal{P}}^i) := \{\mathsf{x} \in \mathbb{R}^2 : T_{\mathsf{f}}(\mathsf{x} - \bar{\mathsf{x}}_{\mathcal{P}}^i) < \infty\}$. In [12] it was shown that the winning set of the i^{th} pursuer is given by

$$\mathcal{W}_f(\bar{\mathsf{x}}_{\mathcal{P}}^i) := \{\mathsf{x} : \ |\bar{\mathsf{x}}_{\mathcal{P}}^i - \mathsf{x}| < \bar{\eta}_f\} \cup \{\mathsf{x} : |\bar{\mathsf{x}}_{\mathcal{P}}^i - \mathsf{x}| \leq \epsilon_c\}, \quad (7)$$

where $\bar{\eta}_f := \inf\{\mathsf{z} \in [\epsilon_c, \infty) : f(\mathsf{z}) \geq \bar{u}_{\mathcal{P}}\mathsf{z}\}$. Note, that since the i^{th} pursuer has only knowledge of an upper bound of f, it can only compute an approximation $\mathcal{W}_{\bar{f}}(\bar{\mathsf{x}}_{\mathcal{P}}^i)$ of its actual winning set. It can easily be shown that

$$\mathcal{W}_{\bar{f}}(\bar{\mathsf{x}}_{\mathcal{P}}^i) := \{\mathsf{x} : \ |\bar{\mathsf{x}}_{\mathcal{P}}^i - \mathsf{x}| < \bar{\eta}_{\bar{f}}\} \cup \{\mathsf{x} : |\bar{\mathsf{x}}_{\mathcal{P}}^i - \mathsf{x}| \leq \epsilon_c\}, \quad (8)$$

where $\bar{\eta}_{\bar{f}} := \inf\{\mathsf{z} \in [\epsilon_c, \infty) : \bar{f}(\mathsf{z}) \geq \bar{u}_{\mathcal{P}}\mathsf{z}\}$, provides a conservative approximation of the winning set of the i^{th} pursuer (that is, $\mathcal{W}_{\bar{f}}(\bar{\mathsf{x}}_{\mathcal{P}}^i) \subseteq \mathcal{W}_f(\bar{\mathsf{x}}_{\mathcal{P}}^i)$).

The minimum time of capture is given by [12]

$$T_{\mathsf{f}}(\bar{\mathsf{y}}^i) := \begin{cases} 0, & \text{if } |\bar{\mathsf{y}}^i| \leq \epsilon_c, \\ \displaystyle\int_{\epsilon_c}^{|\bar{\mathsf{y}}^i|} \frac{\mu \, \mathrm{d}\mu}{\bar{u}_{\mathcal{P}}\mu - f(\mu)}, & \text{if } \epsilon_c < |\bar{\mathsf{y}}^i| < \bar{\eta}_f, \\ \infty, & \text{otherwise}. \end{cases} \quad (9)$$

Assuming that $\bar{\eta}_f > \epsilon_c$, the minimum time of capture, along with the winning sets, induces a Voronoi-like partitioning decomposition of the space, called the Optimal Pursuit-Dynamic Voronoi Diagram (OP-DVD). The OP-DVD is given by [12]

$$\mathcal{V} := \{\mathcal{V}^i, \ i \in \mathcal{I}\}, \quad \mathcal{V}^i = V^i \cap \mathcal{W}_f(\bar{\mathsf{x}}_{\mathcal{P}}^i), \qquad i \in \mathcal{I}, \quad (10)$$

where $V := \{V^i, i \in \mathcal{I}\}$ is the standard Voronoi partition generated by the set $\mathcal{P}$. Since only a conservative approximation of the winning sets is known to the pursuers, the approximate OP-DVD is given by

$$\widetilde{\mathcal{V}} := \{\widetilde{\mathcal{V}}^i, \ i \in \mathcal{I}\}, \quad \widetilde{\mathcal{V}}^i = V^i \cap \mathcal{W}_{\bar{f}}(\bar{\mathsf{x}}_{\mathcal{P}}^i), \qquad i \in \mathcal{I}. \quad (11)$$

III. THE DYNAMIC PURSUER-TARGET ASSIGNMENT PROBLEM AND RELAY-PURSUIT

A. Problem Formulation

Next, we formulate the dynamic pursuer-target assignment problem. To this end, assume that $\bar{\mathsf{x}}_{\mathcal{T}} \in \mathcal{W}$. Without loss of generality[1], let $\bar{\mathsf{x}}_{\mathcal{T}} \in \operatorname{int} \mathcal{V}^i$ for some $i \in \mathcal{I}$. Let $\mathcal{S}$ be the family of right continuous, piecewise constant signals $\sigma : [0, \infty) \mapsto \mathcal{I}$, such that $\sigma(t) = i$ implies that the i^{th} pursuer, at time $t \geq 0$, is the (only) active pursuer; subsequently, we write $\mathsf{x}_{\mathcal{P}}^i \overset{t}{\rightsquigarrow} \mathsf{x}_{\mathcal{T}}$ to denote this fact. The dynamics of the pursuit problem can then be described by the following switched system [15]

$$\dot{\mathsf{y}}^{\sigma(t)} = -\bar{u}_{\mathcal{P}} \frac{\mathsf{y}^{\sigma(t)}}{|\mathsf{y}^{\sigma(t)}|} + u_{\mathcal{T}}(\mathsf{y}^{\sigma(t)}), \quad \mathsf{y}^{\sigma(0)}(0) = \bar{\mathsf{y}}^{\sigma(0)}, \quad (12)$$

$$\dot{\mathsf{y}}^j = 0, \qquad \mathsf{y}^j(0) = \bar{\mathsf{y}}^j, \quad j \neq \sigma(t), \quad (13)$$

where $\sigma(0) = \operatorname{argmin}_{i \in \mathcal{I}} T_{\mathsf{f}}(\bar{\mathsf{y}}^i)$. If, in addition, $0 < \tau_1 < \cdots < \tau_k < \cdots < \infty$ are the switching times of the signal σ, then $\mathsf{y}^{i_k}(\tau_k) = \mathsf{y}^{i_k}(\tau_k^-)$ where $i_k := \sigma(\tau_k) = \sigma(\tau_k^+)$.

Let $\varphi(t; t_0, \mathsf{y}_0, \sigma)$ be the solution of (12) for $t \geq t_0 \geq 0$ and $\mathsf{y}_0 = \varphi(t_0; 0, \bar{\mathsf{y}}^{\sigma(0)}, \sigma)$ for a given $\sigma \in \Sigma$. Given $\sigma \in \mathcal{S}$, we define the minimum capture time as follows

$$\mathrm{T}(t_0, \mathsf{y}_0; \sigma) := \inf\{t \geq t_0 : |\varphi(t; t_0, \mathsf{y}_0, \sigma)| \leq \epsilon_c\}. \quad (14)$$

Henceforth, we will restrict the family of acceptable switching signals to a subset Σ of $\mathcal{S}$, which includes all those signals in $\mathcal{S}$ that satisfy the following switching condition.

[1] If $\bar{\mathsf{x}}_{\mathcal{T}} \in \bigcap_{i \in \mathcal{J}} \mathcal{V}^i$, where $\mathcal{J} \subseteq \mathcal{I}$, we may assign as the initial pursuer any one of the elements of $\mathcal{J}$.

Switching Condition *Let $\sigma \in \mathcal{S}$ and let $\tau > 0$ be a switching time, such that $i = \sigma(\tau^-)$ and $j = \sigma(\tau^+) = \sigma(\tau)$, where $j \neq i$. Then $\sigma \in \Sigma$ if the following conditions hold:*

i) $\mathsf{x}_{\mathcal{T}}(\tau) \in \operatorname{int} \mathcal{V}^j$.

ii) $\mathrm{T}(\tau, \mathsf{y}^j(\tau); \sigma) < \mathrm{T}(\tau, \mathsf{y}^i(\tau); \tilde{\sigma})$, *where*

$$\tilde{\sigma}(t) = \begin{cases} \sigma(t), & t \in [0, \tau), \\ i, & t \geq \tau. \end{cases}$$

The previous condition can be interpreted as follows: For any $\sigma \in \Sigma$, the assignment $\mathsf{x}_{\mathcal{P}}^i \overset{t}{\rightsquigarrow} \mathsf{x}_{\mathcal{T}}$, for $t \geq 0$, is updated only if during the course of the pursuit, the target reaches a position from which, say, the j^{th} pursuer, where $j \neq i$, can capture the target faster than the i^{th} pursuer.

Next, we formulate the dynamic pursuer-moving target assignment problem.

Problem 1: Let $\mathcal{V} = \{\mathcal{V}^i,\ i \in \mathcal{I}\}$ denote the OP-DVD generated by the set $\mathcal{P}$ and assume that $\bar{\mathsf{x}}_{\mathcal{T}} \in \operatorname{int} \mathcal{V}^i$ for some $i \in \mathcal{I}$. Determine a switching signal $\sigma_\star \in \Sigma$ (if one exists) such that $\mathrm{T}(0, \bar{\mathsf{y}}^i, \sigma_\star) < T_{\mathsf{f}}(\bar{\mathsf{y}}^i) = \mathrm{T}(0, \bar{\mathsf{y}}^i; i)$.

B. Analysis of the Pursuer-Target Assignment Problem

Before proceeding to a detailed discussion on the characterization of a solution of Problem 1, we need to introduce a few geometric concepts. In particular, let $\chi_t^{i,j} \subseteq \mathbb{R}^2$ be the moving line in the plane, where, for $t \geq 0$,

$$\chi_t^{i,j} := \{\mathsf{x} : |\mathsf{x} - \mathsf{x}_{\mathcal{P}}^i(t)| = |\mathsf{x} - \mathsf{x}_{\mathcal{P}}^j(t)|\}.$$

The line $\chi_t^{i,j}$ divides, for all $t \geq 0$, the plane into two open half-planes, namely,

$$H_t^i(\mathsf{x}_{\mathcal{P}}^i(t), \mathsf{x}_{\mathcal{P}}^j(t)) := \{\mathsf{x} : |\mathsf{x} - \mathsf{x}_{\mathcal{P}}^i(t)| < |\mathsf{x} - \mathsf{x}_{\mathcal{P}}^j(t)|\},$$
$$H_t^j(\mathsf{x}_{\mathcal{P}}^i(t), \mathsf{x}_{\mathcal{P}}^j(t)) := \{\mathsf{x} : |\mathsf{x} - \mathsf{x}_{\mathcal{P}}^i(t)| > |\mathsf{x} - \mathsf{x}_{\mathcal{P}}^j(t)|\}.$$

The following proposition provides a necessary and sufficient condition for the existence of a solution to Problem 1.

Proposition 1: Let $\mathcal{V} = \{\mathcal{V}^i,\ i \in \mathcal{I}\}$ denote the OP-DVD generated by the set $\mathcal{P}$, and assume that $\bar{\mathsf{x}}_{\mathcal{T}} \in \operatorname{int} \mathcal{V}^i$ for some $i \in \mathcal{I}$. Then, for all $\sigma \in \Sigma$, $\mathrm{T}(0, \bar{\mathsf{y}}^i; \sigma) \geq T_{\mathsf{f}}(\bar{\mathsf{y}}^i)$ if and only if $\mathsf{x}_{\mathcal{T}}(t) \notin H_t^j(\mathsf{x}_{\mathcal{P}}^i(t), \mathsf{x}_{\mathcal{P}}^j(t)) \cap \operatorname{int} \mathcal{V}^j$, for all $j \neq i$ and all $t \geq 0$.

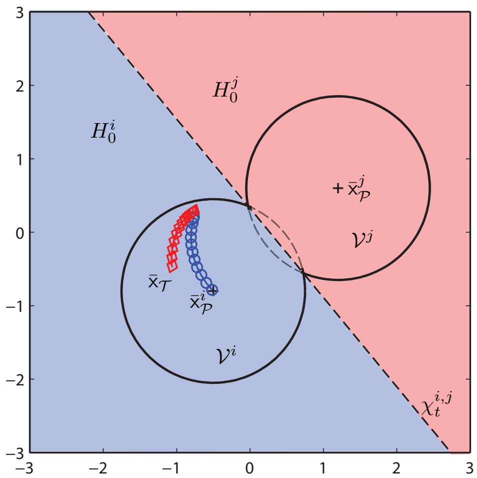
Proof: First we show sufficiency. Let us assume, on the contrary, that there exists a switching signal $\sigma_\star \in \Sigma$ such that $\mathrm{T}(0, \bar{\mathsf{y}}^i; \sigma_\star) < T_{\mathsf{f}}(\bar{\mathsf{y}}^i)$. Clearly, $\sigma_\star \not\equiv i$. If $t_1 > 0$ is the first switching time of the signal $\sigma_\star$, then, in light of the Switching Condition, there exists $j \neq i$, such that $\mathsf{x}_{\mathcal{T}}(t_1) \in \operatorname{int} \mathcal{V}^j$ and $\mathrm{T}(t_1, \mathsf{y}^j(t_1); \tilde{\sigma}) < \mathrm{T}(t_1, \mathsf{y}^i(t_1); i)$, where $\tilde{\sigma}(t) = \sigma_\star(t) = i$ for $t \in [0, t_1)$ and $\tilde{\sigma}(t) = j$ for $t \geq t_1$. Using a similar argument as in the proof of the converse part of Theorem 1 of [12], it follows that $|\mathsf{x}_{\mathcal{T}}(t_1) - \mathsf{x}_{\mathcal{P}}^j(t_1)| < |\mathsf{x}_{\mathcal{T}}(t_1) - \mathsf{x}_{\mathcal{P}}^i(t_1)|$. Hence, $\mathsf{x}_{\mathcal{T}}(t_1) \in H_{t_1}^j(\mathsf{x}_{\mathcal{P}}^i(t_1), \mathsf{x}_{\mathcal{P}}^j(t_1))$, leading to a contradiction.

Conversely, given that $\mathrm{T}(0, \bar{\mathsf{y}}^i; \sigma) \geq T_{\mathsf{f}}(\bar{\mathsf{y}}^i)$, for all $\sigma \in \Sigma$, we wish to show that $\mathsf{x}_{\mathcal{T}}(t) \notin H_t^j(\mathsf{x}_{\mathcal{P}}^i(t), \mathsf{x}_{\mathcal{P}}^j(t)) \cap \operatorname{int} \mathcal{V}^j$, for all $j \neq i$ and $t \geq 0$. Let assume, on the contrary, that there exists $j \neq i$ and $0 < t_1 < T_{\mathsf{f}}(\bar{\mathsf{y}}^i)$ such that $\mathsf{x}_{\mathcal{T}}(t_1) \in H_{t_1}^j(\mathsf{x}_{\mathcal{P}}^i(t_1), \mathsf{x}_{\mathcal{P}}^j(t_1)) \cap \operatorname{int} \mathcal{V}^j$ and let the signal $\sigma_\star \in \Sigma$ be defined such that $\sigma_\star(t) = i$ for $t \in [0, t_1)$ and $\sigma_\star(t) = j$ for $t \geq t_1$. Since $\mathsf{x}_{\mathcal{T}}(t_1) \in H_{t_1}^j(\mathsf{x}_{\mathcal{P}}^i(t_1), \mathsf{x}_{\mathcal{P}}^j(t_1))$, it follows that $|\mathsf{x}_{\mathcal{T}}(t_1) - \mathsf{x}_{\mathcal{P}}^j(t_1)| < |\mathsf{x}_{\mathcal{T}}(t_1) - \mathsf{x}_{\mathcal{P}}^i(t_1)|$. Note that necessarily $|\mathsf{x}_{\mathcal{T}}(t_1) - \mathsf{x}_{\mathcal{P}}^j(t_1)| > \epsilon_c$, otherwise capture would occur at $t_1 < T_{\mathsf{f}}(\bar{\mathsf{y}}^i)$, contradicting the assumption that $\mathrm{T}(0, \bar{\mathsf{y}}^i; \sigma) \geq T_{\mathsf{f}}(\bar{\mathsf{y}}^i)$ for all $\sigma \in \Sigma$. Furthermore, by the definition of the OP-DVD, $\mathsf{x}_{\mathcal{T}}(t_1) \in \operatorname{int} \mathcal{V}^j$ implies that $|\mathsf{x}_{\mathcal{T}}(t_1) - \mathsf{x}_{\mathcal{P}}^j(t_1)| < \bar{\eta}_f$. Note that if $\epsilon_c < |\mathsf{x}_{\mathcal{T}}(t_1) - \mathsf{x}_{\mathcal{P}}^j(t_1)| < \bar{\eta}_f$ and $\epsilon_c < |\mathsf{x}_{\mathcal{T}}(t_1) - \mathsf{x}_{\mathcal{P}}^i(t_1)| < \bar{\eta}_f$, then it follows via Proposition 4 of [12] that $\mathrm{T}(t_1, \mathsf{y}^j(t_1); \sigma_\star) < \mathrm{T}(t_1, \mathsf{y}^i(t_1); i)$. Similarly, if $|\mathsf{x}_{\mathcal{T}}(t_1) - \mathsf{x}_{\mathcal{P}}^i(t_1)| > \bar{\eta}_f$, then it follows from (9) that $\mathrm{T}(t_1, \mathsf{y}^i(t_1); i) = \infty$. Since $\mathsf{x}_{\mathcal{T}}(t_1) \in \operatorname{int} \mathcal{V}^j$, it follows that $\mathrm{T}(t_1, \mathsf{y}^j(t_1); \sigma_\star) < \infty$. Therefore, in both cases $|\mathsf{x}_{\mathcal{T}}(t_1) - \mathsf{x}_{\mathcal{P}}^j(t_1)| < |\mathsf{x}_{\mathcal{T}}(t_1) - \mathsf{x}_{\mathcal{P}}^i(t_1)|$ implies that $\mathrm{T}(t_1, \mathsf{y}^j(t_1); \sigma_\star) < \mathrm{T}(t_1, \mathsf{y}^i(t_1); i)$ for $j \neq i$, where $\mathsf{x}_{\mathcal{T}}(t_1) \in H_{t_1}^j(\mathsf{x}_{\mathcal{P}}^i(t_1), \mathsf{x}_{\mathcal{P}}^j(t_1)) \cap \operatorname{int} \mathcal{V}^j$. Therefore, the signal $\sigma_\star \in \Sigma$ satisfies $\mathrm{T}(0, \bar{\mathsf{y}}^i; \sigma_\star) = t_1 + \mathrm{T}(t_1, \mathsf{y}^j(t_1); \sigma_\star) < t_1 + \mathrm{T}(t_1, \mathsf{y}^i(t_1); i) = \mathrm{T}(0, \bar{\mathsf{y}}^i; i) = T_{\mathsf{f}}(\bar{\mathsf{y}}^i)$. Hence there exists $\sigma_\star \in \Sigma$ such that $\mathrm{T}(0, \bar{\mathsf{y}}^i; \sigma_\star) < T_{\mathsf{f}}(\bar{\mathsf{y}}^i)$, leading to a contradiction. ■ ■

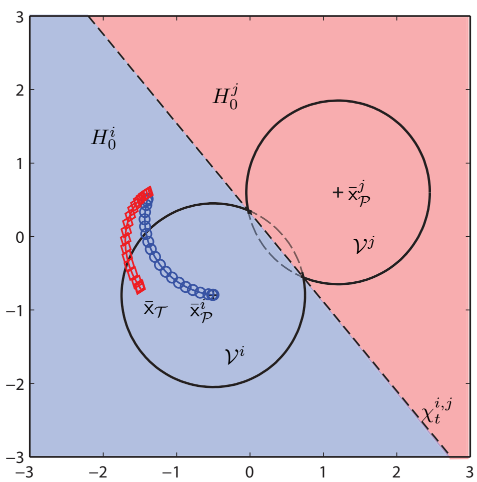
Figures 1-2 illustrate some of the cases that may appear during the pursuit of a target in the special case when $\mathcal{P} = \{\bar{\mathsf{x}}_{\mathcal{P}}^i, \bar{\mathsf{x}}_{\mathcal{P}}^j\}$ and $\bar{\mathsf{x}}_{\mathcal{T}} \in \operatorname{int} \mathcal{V}^i$. In particular, Fig. 1(a) illustrates the scenario where the i^{th} pursuer captures the target at some point in $\mathcal{V}^i$, whereas Fig. 1(b) illustrates the case when capture occurs at some point in $(\mathcal{V}^i \cup \mathcal{V}^j)^c$. Note that in both cases shown in Fig. 1, the initial pursuer-target assignment does not change since the requirements of the Switching Condition are not met. Figure 2 illustrates the case when during the course of the pursuit, the target enters $\mathcal{V}^j$, and subsequently reaches a position within this cell from which it can be captured by the j^{th} pursuer faster than the i^{th} pursuer.

C. Implementation and Analysis of the Relay Pursuit Strategy

Next, we present a simple algorithm that will allow us to solve Problem 1 by dynamically updating the pursuer-target assignment. In particular, we propose the following scheme. First, we construct the OP-DVD generated by the set $\mathcal{P}$, and determine the cell $\mathcal{V}^i$ of the OP-DVD such that $\bar{\mathsf{x}}_{\mathcal{T}} \in \operatorname{int} \mathcal{V}^i$, and let $\mathsf{x}_{\mathcal{P}}^i \overset{t}{\rightsquigarrow} \mathsf{x}_{\mathcal{T}}$ for $t \in [0, T_{\mathsf{f}}(\bar{\mathsf{y}}^i)]$. If, during the course of the pursuit, the target never enters $\operatorname{int} \mathcal{V}^j$, for all $j \neq i$, then it follows that $\mathrm{T}(0, \bar{\mathsf{y}}^i; \sigma) \geq T_{\mathsf{f}}(\bar{\mathsf{y}}^i)$ for all $\sigma \in \Sigma$. Hence, the pursuer target assignment is not updated. If there exists $t_1 > 0$ and $j \neq i$ such that $\mathsf{x}_{\mathcal{T}}(t_1) \in \operatorname{int} \mathcal{V}^j \cap H_{t_1}^j(\mathsf{x}_{\mathcal{P}}^i(t_1), \mathsf{x}_{\mathcal{P}}^j(t_1))$, where $\mathsf{x}_{\mathcal{P}}^j(t_1) = \bar{\mathsf{x}}_{\mathcal{P}}^j$, then the signal σ with $\sigma(t) = i$ for $t \in [0, t_1)$ and $\sigma(t) = j$ for $t \geq t_1$ satisfies $\mathrm{T}(t_1, \mathsf{y}^j(t_1); \sigma) < \mathrm{T}(t_1, \mathsf{y}^i(t_1); i)$. Therefore, by taking $\mathsf{x}_{\mathcal{P}}^j \overset{t}{\rightsquigarrow} \mathsf{x}_{\mathcal{T}}$, for $t \geq t_1$, it follows that capture can be achieved after $t_1 + \mathrm{T}(t_1, \mathsf{y}^j(t_1); \sigma) < t_1 + \mathrm{T}(t_1, \mathsf{y}^i(t_1); i) = T_{\mathsf{f}}(\bar{\mathsf{y}}^i)$ units of time.



(a) Capture occurs in $\mathcal{V}^i$.



(b) Capture occurs in $(\mathcal{V}^i \cup \mathcal{V}^j)^c$.

Fig. 1. If $x_{\mathcal{P}}^i \overset{t}{\rightsquigarrow} x_{\mathcal{T}}$ and $x_{\mathcal{T}}(t) \notin \mathcal{V}^j$ for all $t \geq 0$, then $\mathrm{T}(0, \bar{y}^i; \sigma) \geq T_f(\bar{y}^i)$ for all $\sigma \in \Sigma$.

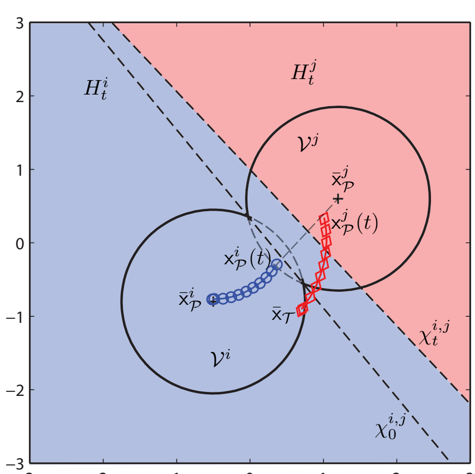


Fig. 2. If $x_{\mathcal{P}}^i \overset{0}{\rightsquigarrow} x_{\mathcal{T}}$ and there exists $t > 0$ such that $x_{\mathcal{T}}(t) \in \operatorname{int} \mathcal{V}^j \cap H_t^j(x_{\mathcal{P}}^i(t), x_{\mathcal{P}}^j(t))$, where $x_{\mathcal{P}}^j(t) = \bar{x}_{\mathcal{P}}^j$, then the j^{th} pursuer will capture the target faster than $\mathrm{T}(t, y^i(t); i)$. Thus $x_{\mathcal{P}}^j \overset{t}{\rightsquigarrow} x_{\mathcal{T}}$.

The previous procedure is repeated every time the target enters a different cell of the OP-DVD during the course of its pursuit. Note that if the pursuer-target assignment is updated at some time t_1, one needs to construct the OP-DVD generated by the set of the pursuers' positions at time t_1. In particular, one needs to compute the OP-DVD generated by the point-set $\mathcal{P}_{t_1} := \big(\mathcal{P} \cup \{x_{\mathcal{P}}^i(t_1)\}\big) \setminus \{\bar{x}_{\mathcal{P}}^i\}$ at time t_1. In this way, the previously described pursuer-target assignment scheme can be applied mutatis mutandis until capture occurs.

The previous scheme may be difficult to be implemented in practice due to the indeterminacy of the pursuer-target assignment scheme when the target lies on the switching line $\chi_t^{i,j}$ at some time $t \geq 0$. This is a well known problem in the theory of switched systems [15], which can be addressed by simply redefining the sets $\chi_t^{i,j}$, H_t^i, H_t^j as follows $\chi_{t,\varepsilon}^{i,j} := \{x : \big||x - x_{\mathcal{P}}^i(t)| - |x - x_{\mathcal{P}}^j(t)|\big| \leq \varepsilon\}$, and

$$H_{t,\varepsilon}^i(x_{\mathcal{P}}^i(t), x_{\mathcal{P}}^j(t)) := \{x : |x - x_{\mathcal{P}}^i(t)| < |x - x_{\mathcal{P}}^j(t)| - \varepsilon\},$$
$$H_{t,\varepsilon}^j(x_{\mathcal{P}}^i(t), x_{\mathcal{P}}^j(t)) := \{x : |x - x_{\mathcal{P}}^i(t)| > |x - x_{\mathcal{P}}^j(t)| + \varepsilon\},$$

where $\varepsilon > 0$ is a *hysteresis* constant. Note that after the target is assigned to, say, the i^{th} pursuer at time $t = 0$, based on the proximity relations encoded in the OP-DVD generated by $\mathcal{P}$, then the pursuer-target assignment cannot be updated as long as the target remains inside the set $H_{t,\varepsilon}^i(x_{\mathcal{P}}^i(t), x_{\mathcal{P}}^j(t)) \cup \chi_{t,\varepsilon}^{i,j}$, for $t > 0$ and for all $j \neq i$. In other words, if $x_{\mathcal{P}}^i \overset{t_0}{\rightsquigarrow} x_{\mathcal{T}}$ for some $t_0 \geq 0$, then the signal σ is allowed to switch at time $t_1 > t_0$ from $i = \sigma(t_0)$ to some $j \neq i$ with $j = \sigma(t_1)$ only if $\mathrm{T}(t_1, y^j(t_1); \sigma)$ is "sufficiently" smaller than $\mathrm{T}(t_1, y^i(t_1); \tilde{\sigma})$, where the signal $\tilde{\sigma}$ is defined such that $\tilde{\sigma}(t) = \sigma(t)$ for $t \in [0, t_1)$ and $\tilde{\sigma}(t) = i$, for $t \geq t_1$. The threshold difference between $\mathrm{T}(t_1, y^j(t_1); \sigma)$ and $\mathrm{T}(t_1, y^i(t_1); \tilde{\sigma})$ depends on the hysteresis constant ε.

Next, we determine a lower bound on the decrease of the capture time of the target that can be achieved by employing the previous dynamic pursuer-target assignment scheme when compared to a static pursuit scheme. In addition, we determine an upper bound on the number of switches of the signal $\sigma_\star \in \Sigma$ that solves Problem 1.

Proposition 2: Let $\mathcal{V} = \{\mathcal{V}^i, i \in \mathcal{I}\}$ denote the OP-DVD generated by the set $\mathcal{P}$, and assume that $\bar{x}_{\mathcal{T}} \in \operatorname{int} \mathcal{V}^i$ for some $i \in \mathcal{I}$. In addition, let $\sigma_\star \in \Sigma$ be a solution of Problem 1 and let $N(\sigma_\star)$ denote the number of switches of $\sigma_\star$. If $\bar{\eta}_f > \epsilon_c$, then

$$\mathrm{T}(0, \bar{y}^i; \sigma_\star) < T_f(\bar{y}^i) - N(\sigma_\star)\bar{\phi}\varepsilon, \tag{15}$$

where $\bar{\phi} := \inf_{[\epsilon_c, \bar{\eta}_f)} z/(\bar{u}_{\mathcal{P}} z - f(z))$. In particular,

$$N(\sigma_\star) < \frac{T_f(\bar{y}^i)}{\varepsilon \bar{\phi}}. \tag{16}$$

Proof: Let τ_k be the k^{th} switching time of $\sigma_\star$, such that $\sigma_\star(\tau_k^-) = \ell_k$ and $\sigma_\star(\tau_k^+) = \sigma_\star(\tau_k) = \ell_{k+1}$, where $\ell_k, \ell_{k+1} \in \mathcal{I}$. Furthermore, let σ^k be the switching signal defined such that $\sigma^k(t) = \sigma_\star(t)$ for $t \in [0, t_k)$ and $\sigma^k(t) = \ell_k$ for $t \geq t_k$. Note that $i \equiv \ell_1$ and $\sigma^1 \equiv i$. By hypothesis, $x_{\mathcal{T}}(\tau_k) \in H_{\tau_k,\varepsilon}^{\ell_{k+1}}(x_{\mathcal{P}}^{\ell_k}(\tau_k), x_{\mathcal{P}}^{\ell_{k+1}}(\tau_k)) \cap \operatorname{int} \mathcal{V}^{\ell_{k+1}}$

which implies that $\epsilon_c < |\mathsf{y}^{\ell_{k+1}}(\tau_k)| + \varepsilon < |\mathsf{y}^{\ell_k}(\tau_k)| < \bar{\eta}_f$, where $\mathsf{y}^{\ell_{k+1}}(\tau_k) := \mathsf{x}_{\mathcal{T}}(\tau_k) - \mathsf{x}_{\mathcal{P}}^{\ell_{k+1}}(\tau_k)$ and $\mathsf{y}^{\ell_k}(\tau_k) := \mathsf{x}_{\mathcal{T}}(\tau_k) - \mathsf{x}_{\mathcal{P}}^{\ell_k}(\tau_k)$. Furthermore,

$$\mathrm{T}(\tau_k, \mathsf{y}^{\ell_k}(\tau_k); \sigma^k) - \mathrm{T}(\tau_k, \mathsf{y}^{\ell_{k+1}}(\tau_k); \sigma^{k+1}) = \int_{|\mathsf{y}^{\ell_{k+1}}(\tau_k)|}^{|\mathsf{y}^{\ell_k}(\tau_k)|} \phi(\mathsf{z})\,\mathrm{d}\mathsf{z}, \qquad (17)$$

where $\phi(\mathsf{z}) := \mathsf{z}/(\bar{u}_{\mathcal{P}}\mathsf{z} - f(\mathsf{z}))$. By virtue of the mean value theorem for Riemann integrals, there exists $\epsilon_c < |\mathsf{y}^{\ell_{k+1}}(\tau_k)| \leq \zeta \leq |\mathsf{y}^{\ell_k}(\tau_k)| < \bar{\eta}_f$, such that

$$\mathrm{T}(\tau_k, \mathsf{y}^{\ell_k}(\tau_k); \sigma^k) - \mathrm{T}(\tau_k, \mathsf{y}^{\ell_{k+1}}(\tau_k); \sigma^{k+1}) = \phi(\zeta)(|\mathsf{y}^{\ell_k}(\tau_k)| - |\mathsf{y}^{\ell_{k+1}}(\tau_k)|) > \phi(\zeta)\varepsilon. \qquad (18)$$

Note that the function ϕ is continuous and strictly positive for all $\mathsf{z} \in [\epsilon_c, \bar{\eta}_f)$. Furthermore, $\lim_{\mathsf{z}\to\bar{\eta}_f} \mathsf{z}/(\bar{u}_{\mathcal{P}}\mathsf{z} - f(\mathsf{z})) = \infty$. Therefore $\bar{\phi} := \inf_{[\epsilon_c, \bar{\eta}_f)} \mathsf{z}/(\bar{u}_{\mathcal{P}}\mathsf{z} - f(\mathsf{z})) > 0$. Then (18) gives $\mathrm{T}(\tau_k, \mathsf{y}^{\ell_k}(\tau_k); \sigma^k) - \mathrm{T}(\tau_k, \mathsf{y}^{\ell_{k+1}}(\tau_k); \sigma^{k+1}) > \bar{\phi}\varepsilon$, which, furthermore, implies that

$$\begin{aligned} T_{\mathsf{f}}(\bar{\mathsf{y}}^i) &= \tau_1 + \mathrm{T}(\tau_1, \mathsf{y}^{\ell_1}(\tau_1); \sigma^1) \\ &> \tau_1 + \mathrm{T}(\tau_1, \mathsf{y}^{\ell_2}(\tau_1); \sigma^2) + \bar{\phi}\varepsilon \\ &= \tau_1 + (\tau_2 - \tau_1) + \mathrm{T}(\tau_2, \mathsf{y}^{\ell_2}(\tau_2); \sigma^2) + \bar{\phi}\varepsilon \\ &> \tau_2 + \mathrm{T}(\tau_2, \mathsf{y}^{\ell_3}(\tau_2); \sigma^3) + 2\bar{\phi}\varepsilon \\ &\;\;\vdots \\ &> \tau_k + \mathrm{T}(\tau_k, \mathsf{y}^{\ell_{k+1}}(\tau_k); \sigma^{k+1}) + k\bar{\phi}\varepsilon. \end{aligned} \qquad (19)$$

Therefore $T_{\mathsf{f}}(\bar{\mathsf{y}}^i) > k\bar{\phi}\varepsilon$ for all $k \geq 1$, which implies that the maximum number of switches, N is bounded. Furthermore, the previous inequality yields

$$\begin{aligned} T_{\mathsf{f}}(\bar{\mathsf{y}}^i) &> \tau_N + \mathrm{T}(\tau_N, \mathsf{y}^{\ell_{N+1}}(\tau_N); \sigma_\star) + N\bar{\phi}\,\varepsilon \\ &= \mathrm{T}(0, \bar{\mathsf{y}}^i; \sigma_\star) + N\bar{\phi}\,\varepsilon. \end{aligned}$$

Thus (15) follows readily. Finally, (16) follows immediately from the fact that $\mathrm{T}(0, \bar{\mathsf{y}}^i; \sigma_\star) > 0$. ■

IV. Simulation Results

In this section, we present simulation results to illustrate the previous developments. We consider a scenario where the maneuvering target is faster than the i^{th} pursuer, but the winning set of the i^{th} pursuer is non-empty as a result of the information pattern employed in Section II. In particular, it is assumed that the target's evading strategy is given by

$$u_{\mathcal{T}}(\mathsf{y}^i) = \begin{cases} \alpha\mathsf{y}^i + \rho(\mathsf{y}^i)\mathsf{S}\mathsf{y}^i, & \text{for } \epsilon_c \leq |\mathsf{y}^i| \leq \dfrac{M}{\alpha}, \\ M\dfrac{\mathsf{y}^i}{|\mathsf{y}^i|}, & \text{for } |\mathsf{y}^i| > \dfrac{M}{\alpha}, \end{cases} \qquad (20)$$

where M and α are some positive constants with $M > \max\{\bar{u}_{\mathcal{P}}, \alpha\}$, S is a nonzero skew symmetric matrix in $\mathbb{R}^{2\times 2}$, and $\rho(\mathsf{y}^i) := \sqrt{M^2 - \alpha^2|\mathsf{y}^i|^2}/|\mathsf{S}\mathsf{y}^i|$. Note that $f(\mathsf{y}^i) := \langle u_{\mathcal{T}}, \mathsf{y}^i \rangle$ satisfies Assumption 1.

The intuition behind the evading strategy (20) is as follows: Let $\mathsf{e}_1(\mathsf{y}^i) := \mathsf{y}^i/|\mathsf{y}^i|$ be the unit vector along the line connecting the target and the i^{th} pursuer ("line-of-sight" direction), and let $\mathsf{e}_2(\mathsf{y}^i)$ be the unit vector orthogonal to $\mathsf{e}_1(\mathsf{y}^i)$ ("tangential" direction). Note that the target has a constant speed $M > u_{\mathcal{P}}$.

Assume for this example that the set $\mathcal{P}$ consists of ten locations, and let

$$\bar{f}(\mathsf{y}^i) := \begin{cases} \bar{\alpha}|\mathsf{y}^i|^2, & \text{for } \epsilon_c \leq |\mathsf{y}^i| \leq \dfrac{M}{\bar{\alpha}}, \\ M|\mathsf{y}^i| & \text{for } |\mathsf{y}^i| > \dfrac{M}{\bar{\alpha}}, \end{cases} \qquad (21)$$

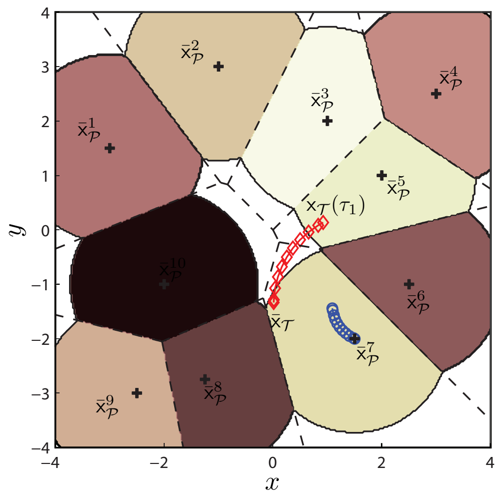
where $\bar{\alpha}$ is a positive scalar with $\alpha \leq \bar{\alpha} < M$. In this case, the capturability condition reduces to $\eta^i(0) < \bar{u}_{\mathcal{P}}/\alpha$, which implies that $\bar{\eta}_f = \bar{u}_{\mathcal{P}}/\alpha < M/\alpha$ and $\bar{\eta}_{\bar{f}} = \bar{u}_{\mathcal{P}}/\bar{\alpha} < M/\bar{\alpha}$. Furthermore, it is easy to show that, in light of (9), the minimum-time of the optimal pursuit problem, for $\epsilon_c < |\bar{\mathsf{y}}^i| < \bar{\eta}_f$, is given by

$$T_{\mathsf{f}}(\bar{\mathsf{y}}^i) = -\frac{1}{\alpha}\ln\left(\frac{\bar{u}_{\mathcal{P}} - \alpha|\bar{\mathsf{y}}^i|}{\bar{u}_{\mathcal{P}} - \alpha\epsilon_c}\right). \qquad (22)$$

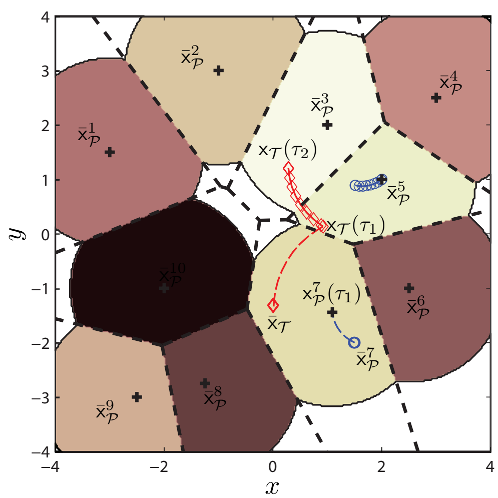
Next, we present simulation results of the relay-pursuit scheme introduced in this paper. In particular, Fig. 3 illustrates the trajectories of the active pursuers and the moving target during the course of the relay pursuit for the following data: $\mathsf{S} = \left[\begin{smallmatrix} 0 & 1.5 \\ -1.5 & 0 \end{smallmatrix}\right]$, $\varepsilon = 0.2$, $\alpha = \bar{\alpha} = 0.7$ and $M = 3$. It is assumed that $\bar{\mathsf{x}}_{\mathcal{T}} \in \mathcal{W}_f(\bar{\mathsf{x}}_{\mathcal{P}}^7)$. Specifically, Fig. 3(a) illustrates the trajectories of the target and the 7th pursuer, which is assigned to the target at $t = 0$, until $t = \tau_1$, when $\mathsf{x}_{\mathcal{T}}(\tau_1) \in \operatorname{int}\mathcal{V}^5 \cap H^5_{\tau_1,\varepsilon}(\mathsf{x}_{\mathcal{P}}^7(\tau_1), \bar{\mathsf{x}}_{\mathcal{P}}^5)$ and the target is assigned to the 5th pursuer. Figure 3(b) illustrates the trajectories of the target and the 5th pursuer, for $\tau_1 \leq t < \tau_2$, where τ_2 is the second switching time, when the target is assigned to the 3rd pursuer. Note that $\mathsf{x}_{\mathcal{T}}(\tau_1)$ resides in the interior of the cell of the OP-DVD generated by the locations of all the pursuers at time $t = \tau_1$, that is, the set $\mathcal{P}_{\tau_1} := \big(\mathcal{P} \cup \{\mathsf{x}_{\mathcal{P}}^7(\tau_1)\}\big) \setminus \{\bar{\mathsf{x}}_{\mathcal{P}}^7\}$, that is associated with the 5th pursuer. Figure 3(c) illustrates the trajectories of the target and the 3rd pursuer for $t \geq \tau_2$. Again, we observe that at time $t = \tau_2$ the target resides inside the cell of the OP-DVD generated by the locations of the pursuers at time $t = \tau_2$, that is, the set $\mathcal{P}_{\tau_2} := \big(\mathcal{P} \cup \{\mathsf{x}_{\mathcal{P}}^7(\tau_1)\} \cup \{\mathsf{x}_{\mathcal{P}}^5(\tau_2)\}\big) \setminus (\{\bar{\mathsf{x}}_{\mathcal{P}}^7\} \cup \{\bar{\mathsf{x}}_{\mathcal{P}}^5\})$, that is associated with the 3rd pursuer. Moreover, we observe that, although at some time instant $\tau_3 > \tau_2$ the target enters the cell associated with the 2nd pursuer at time $t = \tau_2$, the 3rd pursuer remains closer to the target than the 2nd pursuer for all $t \geq \tau_3$. Thus the pursuer-target assignment does not change for $t \geq \tau_2$, and thus the 3rd pursuer will eventually capture the target.

V. Conclusion

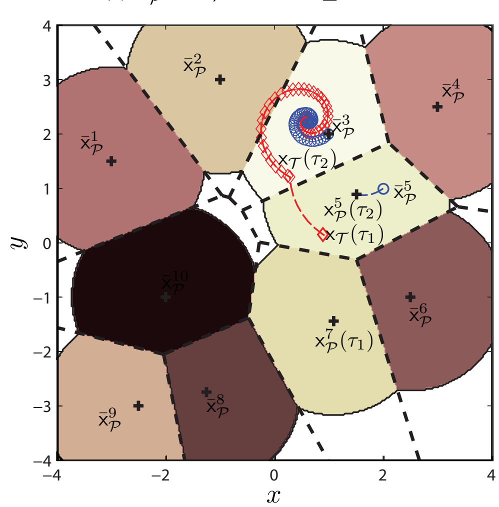
We have proposed a relay pursuit scheme for the capture of a maneuvering target by a group of pursuers distributed in the plane. It is assumed that during the course of the pursuit, only one pursuer can go after the target, whereas the rest of the pursuers remain stationary. The problem of assigning a pursuer from the group of pursuers to the maneuvering target



(a) $x_{\mathcal{P}}^7 \overset{t}{\rightsquigarrow} x_{\mathcal{T}}$ for all $0 \leq t < \tau_1$.



(b) $x_{\mathcal{P}}^5 \overset{t}{\rightsquigarrow} x_{\mathcal{T}}$ for all $\tau_1 \leq t < \tau_2$.



(c) $x_{\mathcal{P}}^3 \overset{t}{\rightsquigarrow} x_{\mathcal{T}}$ for all $t \geq \tau_2$.

Fig. 3. Trajectories of the active pursuers and the moving target during the course of the relay pursuit.

is associated with the solution of a Voronoi-like partitioning problem that deals with the characterization of the sets of initial conditions of the moving target from which a particular pursuer can intercept the target faster than any other pursuer from the same group. Based on this Voronoi-like partition, we have subsequently presented a scheme that dynamically assigns the task of pursuing the maneuvering target to the appropriate pursuers in the group in order to minimize the overall capture time.

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